# Quantum Statistical Enhancement of the Collective Performance of Multiple Bosonic Engines

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(Received 21 April 2019; revised manuscript received 9 April 2020; accepted 1 May 2020; published 27 May 2020)

We consider an ensemble of indistinguishable quantum machines and show that quantum statistical effects can give rise to a genuine quantum enhancement of the collective thermodynamic performance. When multiple indistinguishable bosonic work resources are coupled to an external system, the internal energy change of the external system exhibits an enhancement arising from permutation symmetry in the ensemble, which is absent when the latter consists of distinguishable work resources.

DOI: 10.1103/PhysRevLett.124.210603

*Introduction.*—Technological advances have allowed us to miniaturize thermal machines to the nanoscale and beyond, where quantum effects can play an important role [1]. A paradigmatic instance of a thermal machine is a quantum heat engine (QHE). First conceived in the late 1950s, a QHE is a quantum system that serves as the working fuel of a thermodynamic cycle [2–7]. More recently, a synergy between technology and progress on the foundations of quantum thermodynamics [8–13] has led to a surge of activity on the study of quantum machines [14–22], consolidating it as an active area of research [23]. A prominent challenge in this context is the identification of situations where quantum effects govern and lead to an enhanced performance with no classical counterpart [19,24–26]. One strategy to identify such situations is to consider thermal machines composed of multiple components [17,24,27–42] described by collective quantum states with mutual coherence. In this setting, the enhancement requires either interaction among the components or the performance of collective unitary operations on the constituents. Furthermore, the natural process of extracting work from a QHE by outcoupling it to drive another quantum system [43,44] or even the process of storing work in a quantum system [32,39] can also lead to the manifestation of genuine quantum effects. In this Letter, we identify a third route, in which quantum statistics leads to a genuine quantum enhancement of the performance as a result of the statistical indistinguishability of the constituent work resources. Specifically, we consider multiple work resources, each composed of a single QHE with an individual piston [20,45], coupled to a single external system and show that the internal energy change of the external system displays quantum enhancement when the QHEs are indistinguishable. We note that such a setting is fundamentally different from a single QHE with a working fluid consisting of multiple particles [24,30,34,37,41,46–48].

*Setup.*—Consider a collective work resource $R$ made of $N$ heat engines $E_1, \ldots, E_N$ interacting with two heat baths ($B_1$ and $B_2$) and an external quantum system $S$ on which the work is performed. The coupling between $R$ and $S$ is solely established via the heat engines; see Fig. 1. The global Hamiltonian of the whole system is the sum of that of the work resources, the external system, and the coupling $C$ between them:

$$H(t) = H_R(t) + H_C(t) + H_S, \tag{1}$$

where the external system is assumed to be time independent. If the work resources are QHEs, $H_R(t)$ collectively represents the Hamiltonian for $N$ engines and the two common baths. For simplicity, we consider the following form of the coupling Hamiltonian $H_C$:

$$H_C(t) = g_C(t) V_R \otimes V_S, \tag{2}$$

where $g_C(t)$ is a time-dependent coupling constant, and $V_R$ and $V_S$ are operators of the work resource and of the external system, respectively. In the analytical treatment below, we assume a sufficiently weak coupling between the work resources and the external system justifying a perturbative treatment [49].

In this Letter, we characterize the work performed by the work resources on the system $S$ by its internal energy change $\Delta U$. A more in-depth discussion as to what extent $\Delta U$ represents work will be provided in the conclusions. $\Delta U$ is evaluated by energy measurements on the external system $S$ at the beginning and the end of the cycle at $t = 0$ and $T$, respectively [44]. For simplicity, we turn off the coupling $g_C(t)$ at $t = 0$ and $T$, and, thus, $[H_S, H(t)] = 0$ at these moments in time. Consequently, measurements of the system energy $H_S$ at these two times do not affect the state of the work resources $R$. The external system is initially prepared in its ground state $|0\rangle_S$, and the initial state $\rho_0$ of the total system is $\rho_0 = \rho_0^R \otimes |0\rangle_{SS}\langle 0|$ with $\rho_0^R$ being the initial state of the work resources.

*Average energy delivered by outcoupled work resources.*—We consider the average internal energy change of the system $S$, $\langle \Delta U\rangle_N$, caused by the coupling to the $N$ work resources. In the rotating frame with respect to $H_0(t) \equiv H_R(t) + H_S$, the propagator in the interaction picture is $U^{(I)}(t,0) = \mathcal{T}\exp[-i\int_0^t H_C^{(I)}(t')dt']$, where $\mathcal{T}$ is the time-ordering operator and $H_C^{(I)}(t) \equiv U_0^\dagger(t,0)H_C(t)U_0(t,0)$ with $U_0(t,0)\equiv\mathcal{T}\exp[-i\int_0^t H_0(t')dt']$. Regarding the coupling Hamiltonian $H_C$ in Eq. (2), $H_C^{(I)}(t)$ reads $H_C^{(I)}(t) = g_C(t)V_R^{(I)}(t) \otimes V_S^{(I)}(t)$ with $V_R^{(I)}(t) \equiv U_R^\dagger(t,0)V_R U_R(t,0)$, $V_S^{(I)}(t) \equiv e^{iH_S t}V_S e^{-iH_S t}$, and $U_R(t,0) \equiv \mathcal{T}\exp[-i\int_0^t H_R(t')dt']$.

Setting the energy of the ground state $|0\rangle_S$ of the system to be zero without loss of generality, the average internal energy change is given by $\langle \Delta U\rangle_N = \sum_{i\neq 0}\varepsilon_i^S p_i$, where the probability $p_i$ for measuring the $i$th eigenvalue $\varepsilon_i^S$ of $H_S$ as an outcome of the energy measurement at $t = T$ reads

$$p_i = \mathrm{Tr}_R[{}_S\langle i|U^{(I)}(T,0)\rho_0 U^{(I)\dagger}(T,0)|i\rangle_S]. \tag{3}$$

Here, $\mathrm{Tr}_R[\cdots]$ is the trace over the Hilbert space of the work resources and $|i\rangle_S$ is the $i$th eigenvector of $H_S$. To gain an analytical insight, we first resort to the weak coupling regime where $\int_0^T g_C(t)dt \ll 1$. In this limit, expanding the propagator to leading order as $U^{(I)}(T,0) \approx I - i\int_0^T dt g_C(t)V_R^{(I)}(t) \otimes V_S^{(I)}(t)$ in Eq. (3), the excitation probability of the system reduces to

$$p_i \simeq \int_0^T dt \int_0^T dt' g_C(t) g_C(t')\, {}_S\langle i|V_S^{(I)}(t)|0\rangle_{SS}\langle 0|V_S^{(I)}(t')|i\rangle_S \times \langle V_R^{(I)}(t')V_R^{(I)}(t)\rangle_{\rho_0^R}, \tag{4}$$

with $\langle\cdots\rangle_{\rho_0^R} \equiv \mathrm{Tr}_R[\cdots\rho_0^R]$.

*Quantum statistical enhancement.*—To demonstrate the genuinely quantum mechanical advantage of indistinguishable bosons in comparison to distinguishable particles as the work resources, we consider $N$ QHEs, each performing an Otto cycle with the two lowest internal energy levels of a bosonic atom prepared in its center of mass (COM) ground state as a working fluid, i.e., the temperature $\beta_{\mathrm{COM}}^{-1}$ of the COM degrees of freedom is set to be zero. As sketched in Fig. 1, the work resources $R$ contain these engines, together with the hot and cold heat baths.

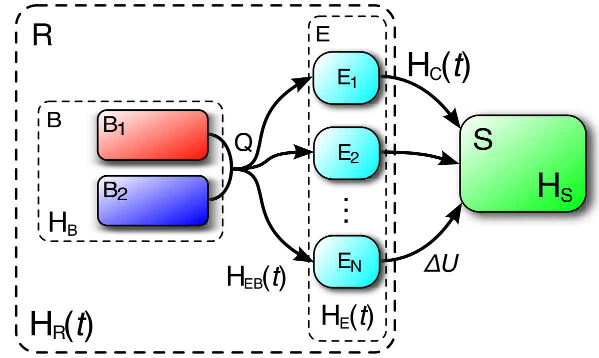


FIG. 1. Schematic picture of the setup. Multiple work resources collectively denoted by $R$ deliver energy $\Delta U$ to an external system $S$ through the coupling Hamiltonian $H_C$. If the work resources are $N$ quantum heat engines $E_1, \ldots, E_N$, all the engines and the heat baths collectively denoted by $E$ and $B$, respectively, are included in $R$.

The four strokes of the Otto cycle are performed as follows: (0) *Initial state.*—In the absence of the coupling to the external system $S$, $g_C(0) = 0$, all two-level atoms are prepared in thermal equilibrium with the common cold bath at inverse temperature $\beta_c$. Thus, the initial reduced density matrix $\rho_0^E \equiv \mathrm{Tr}_B\rho_0^R$ of the engine part is $\rho_0^E = Z_{\beta_c}^{-1}\exp[-\beta_c H_E(0)]$ with $Z_{\beta_c} \equiv \mathrm{Tr}_E\exp[-\beta_c H_E(0)]$, where $\mathrm{Tr}_E[\cdots]$ and $\mathrm{Tr}_B[\cdots]$ are the trace over the Hilbert space of the engines and that of the baths, respectively. The baths are assumed to be time independent and in the canonical state of $H_B$ throughout the cycle. (1) *Isentropic compression.*—From $0 < t < T/2$, all the engines are decoupled from the baths, $H_{EB} = 0$, and the level distance of all the two-level atoms is slowly increased in the same manner. (2) *Hot isochore.*—At $t = T/2$, setting $g_C = 0$, all the two-level atoms are brought into weak contact with a common hot bath and thermalized at inverse temperature $\beta_h$. At the end of this process, the state of the engine is given by $\rho_{T/2}^E = \mathrm{Tr}_B\rho_{T/2}^R = Z_{\beta_h}^{-1}\exp[-\beta_h H_E(T/2)]$ with $Z_{\beta_h} \equiv \mathrm{Tr}_E\exp[-\beta_h H_E(T/2)]$. (3) *Isentropic expansion.*—From $T/2 < t < T$, all engines are decoupled from the baths, $H_{EB} = 0$, and the energy separation of each two-level atoms is decreased slowly in the same way. (4) *Cold isochore.*—At $t = T$, setting $g_C = 0$, and all the two-level atoms are brought into contact with the common cold bath again and quickly return to the initial state.

First, we focus on the case of indistinguishable atoms. We choose $V_R = 2S_x$ in the coupling Hamiltonian (2):

$$H_C(t) = g_C(t) 2S_x \otimes V_S, \tag{5}$$

with $S_x \equiv (a^\dagger b + b^\dagger a)/2$, where $a^\dagger$ and $a$ are creation and annihilation operators of the ground-state atoms in the lowest COM level, and $b^\dagger$ and $b$ are those of the excited-state atoms, respectively. While we keep the external system general in this discussion, we note that if the external system is a harmonic oscillator (HO) and $V_S = c^\dagger + c$ with $c$ ($c^\dagger$) denoting the annihilation (creation) operator of the HO, Eq. (5) reduces to the standard dipole coupling between an ensemble of atoms and a single-mode HO. In order to compute $\langle \Delta U \rangle_N$ using the probability in Eq. (4), we now choose the following engine Hamiltonian:

$$H_E(t) = 2\Omega(t) S_z + 2\Delta S_x, \tag{6}$$

with $S_z \equiv (a^\dagger a - b^\dagger b)/2$. In the Otto cycle, both the compression and expansion strokes are done without coupling to the heat baths, and, hence, they are described by unitary dynamics governed by the engine Hamiltonian (6). For quasistatic changes of $\Omega(t)$, the propagator can be written as

$$U_R(t, t_0) \approx \sum_{m=-N/2}^{N/2} |m, \theta_t\rangle_{EE} \langle m, \theta_{t_0}| e^{-im\phi(t,t_0)}, \tag{7}$$

with $\phi(t, t_0) = \int_{t_0}^{t} dt' 2E_{t'}$. Here, we denote by $|m, \theta_t\rangle_E$ the eigenstate of the instantaneous engine Hamiltonian $H_E(t)$ with eigenvalue $2E_t m$, where $E_t \equiv \sqrt{\Omega(t)^2 + \Delta^2}$. Furthermore, $\theta_t$ is defined by $\tan\theta_t = -\Omega(t)/\Delta$. The initial time is $t_0 = 0$ for the isentropic compression and $t_0 = T/2$ for the isentropic expansion strokes. With this adiabatic propagator, we obtain the autocorrelation function of the operator $V_R$ (see [50] for details), when $t_0 \le \{t, t'\} \le t_0 + T/2$, in Eq. (4) as [54]

$$\langle V_R^{(I)}(t') V_R^{(I)}(t) \rangle_{\rho_{t_0}^R} = 4\cos\theta_t \cos\theta_{t'} \langle m^2 \rangle_{t_0} + \sin\theta_t \sin\theta_{t'} \sum_{\sigma=\pm} e^{-i\sigma\phi(t',t)} \left[\frac{N}{2}\left(\frac{N}{2}+1\right) - F_\sigma(N, \beta_{t_0} E_{t_0})\right], \tag{8}$$

with $\beta_0 \equiv \beta_c$, $\beta_{T/2} \equiv \beta_h$, and $F_\pm(N, \beta_{t_0} E_{t_0}) \equiv \langle m^2 \rangle_{t_0} \pm \langle m \rangle_{t_0}$, where the expectation values are defined with respect to the thermal state of $H_E(t_0)$ at $\beta_{t_0}$. On the other hand, if $t$ and $t'$ are separated by the thermalization process at $t = T/2$, for instance, $t < T/2$ and $t' > T/2$, the autocorrelation function takes the factorized form $\langle V_R^{(I)}(t') V_R^{(I)}(t) \rangle_{\rho_0^R} = \langle V_R^{(I)}(t') \rangle_{\rho_{T/2}^R} \langle V_R^{(I)}(t) \rangle_{\rho_0^R}$ with

$$\langle V_R^{(I)}(t) \rangle_{\rho_{t_0}^R} = 2\cos\theta_t \langle m \rangle_{t_0}. \tag{9}$$

Next, we examine the case in which all the atoms are distinguishable. In this case, the coupling Hamiltonian (5) reduces to $H_C(t) = g_C(t) 2 \sum_{j=1}^N (\sigma_{j,x}/2) \otimes V_S$ and the engine Hamiltonian (6) to $H_E(t) = 2\Omega(t) \sum_{j=1}^N (\sigma_{j,z}/2) + 2\Delta \sum_{j=1}^N (\sigma_{j,x}/2)$, where $\sigma_{j,x}$ and $\sigma_{j,z}$ are the Pauli matrices of the $j$th atom. Assuming quasistatic changes of $\Omega(t)$ like in the indistinguishable case, we find the following autocorrelation function:

$$\langle V_R^{(I)}(t') V_R^{(I)}(t) \rangle_{\rho_{t_0}^R} = \cos\theta_t \cos\theta_{t'} [N + N(N-1)\tanh^2(\beta_{t_0} E_{t_0})] + \frac{N}{2} \frac{\sin\theta_t \sin\theta_{t'}}{\cosh(\beta_{t_0} E_{t_0})} \sum_{\sigma=\pm} e^{\sigma[i\phi(t',t) - \beta_{t_0} E_{t_0}]}, \tag{10}$$

and the average

$$\langle V_R^{(I)}(t) \rangle_{\rho_{t_0}^R} = -N\cos\theta_t \tanh(\beta_{t_0} E_{t_0}), \tag{11}$$

allowing the calculation of the probability in Eq. (4) for the distinguishable case. We emphasize that in the distinguishable case, while taking the trace over the engine states, all the possible $2^N$ configurations of the atomic pseudospins have to be considered, while in the indistinguishable case the trace is taken over only $N + 1$ symmetrized eigenstates of $S_z$. Thus, the collective nature of the latter set of states gives rise to an enhanced coupling with the external system and results in the enhancement of $\Delta U$ that we demonstrate next.

To clearly evidence that Bose statistics leads to a quantum advantage, we specialize the coupling protocol to the impulse form $g_C(t) = g\delta(t - t_1)$, with $0 < t_1 < T/2$. In this case, the expressions for the probability (4) are simplified greatly, and the average internal energy change equals

$$\langle \Delta U \rangle_N \simeq g^2 \langle [V_R^{(I)}(t_1)]^2 \rangle_{\rho_0^R} \sum_{i \neq 0} \varepsilon_i^S |{}_S\langle i| V_S^{(I)}(t_1) |0\rangle_S|^2. \tag{12}$$

Thus, for the impulse form of the coupling, $\langle \Delta U \rangle_N$ for the indistinguishable and distinguishable cases differ by the value of the variance $\langle [V_R^{(I)}(t_1)]^2 \rangle_{\rho_0^R}$ that can be evaluated from Eqs. (8) and (10). Remarkably, we find that, in the indistinguishable case, this variance is larger than or equal to

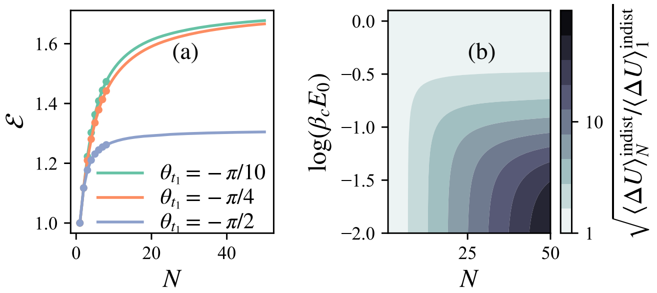


FIG. 2. Quantum-enhanced performance of multiple identical engines in the perturbative impulse coupling. (a) Ratio $\mathcal{E} \equiv \langle \Delta U\rangle_N^{\rm indist}/\langle \Delta U\rangle_N^{\rm dist}$ for an outcoupling with an impulse of strength $g = 0.01$ to an external system. The atomic energy gap is driven with a linear speed of $v = 0.1\Omega(0)^2$ over a total protocol time of $T = 20/\Omega(0)$. The impulse kick occurs at $t_1 = 0.35T/2$, and $\theta_{t_1}$ is tuned by changing $\Delta = \{4.2\Omega(0), 1.4\Omega(0), 0\}$ (different curves). The bath temperatures are given by $\beta_c E_0 = 2$ and $\beta_h E_{T/2} = 1/4$. Solid lines are from analytical expressions derived in the perturbative limit, and dots are from a numerical calculation with the external system given by a HO of frequency $\omega = 2\pi \times 0.05/T$. (b) $\langle\Delta U\rangle_N^{\rm indist}$ produced by indistinguishable particles when $\Delta = 0$ and $\beta_h E_{T/2} = 1/4$ as a function of $N$ and $\beta_c E_0$ for impulse-type coupling and other parameters as in (a). Regions with linear scaling in $N$ of the contour gradients correspond to quadratic scaling of $\langle\Delta U\rangle_N$ with $N$.

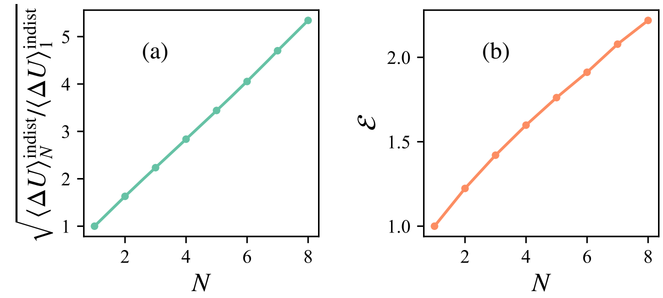


FIG. 3. Quantum performance of multiple identical engines with continuous nonperturbative coupling. (a) $\langle\Delta U\rangle_N^{\rm indist}$ done by $N$ indistinguishable engines outcoupled via a continuous nonperturbative coupling. A quadratic scaling is shown for small $N$. (b) Scaling of the enhancement $\mathcal{E}$. Engine parameters are $v = 0.1\Omega(0)^2$, $T = 20/\Omega(0)$, $\Delta = 0$, and $\Omega(0) = 1$. The coupling $g_C(t)$, to the external HO system of frequency $\omega = 2\pi \times 0.05/T$, is chosen with $g = 0.5$, $\delta_t = 0.9$, $\alpha = 2142/T$, $t_{\rm on} = (1-\delta_t)T/4$, and $t_{\rm off} = t_{\rm on} + \delta_t T/2$.

that of the distinguishable case for any values of the parameters $N \geq 1$, $\beta_c E_0$, and $\theta_{t_1}$ (see [50]). This fact guarantees that $\langle\Delta U\rangle_N^{\rm indist}$ due to $N$ indistinguishable bosonic engines is always larger than $\langle\Delta U\rangle_N^{\rm dist}$ arising from the same number of distinguishable engines. The resulting enhancement can be quantified by the ratio $\mathcal{E} = \langle\Delta U\rangle_N^{\rm indist}/\langle\Delta U\rangle_N^{\rm dist}$.

In Fig. 2(a), we compare our analytical result for the enhancement $\mathcal{E}$, for different values of $\theta_{t_1}$ and $N$, with numerical simulations for a specific choice of the system as a HO with frequency $\omega$, i.e., $H_S = \omega c^\dagger c$ and $V_S = c^\dagger + c$ in the coupling Hamiltonian (5). For the engine Hamiltonian $H_E(t)$, we consider linear sweeps of $\Omega(t)$ as $\Omega(t) = \Omega(0) + vt$ and $\Omega(t) = \Omega(0) + v(T-t)$ for the isentropic compression and expansion strokes, respectively. Hereafter, let us focus on the situation with $\Delta = 0$ (i.e., $\theta_t = -\pi/2$), where the difference between the indistinguishable and distinguishable cases is most prominent [55]. When $\Delta = 0$, using Eq. (10) we see that $\langle\Delta U\rangle_N \propto N$ for the distinguishable case, while in the indistinguishable case using Eq. (8) the dependence on $N, \beta_c E_0$ is more involved (see [50]). In general, although $\langle[V_R^{(I)}(t_1)]^2\rangle_{\rho_0^R}$ contains both terms proportional to $N^2$ and $N$, we find that $\langle\Delta U\rangle_N$ shows $N^2$ scaling for moderate values of $N$ with $N\beta_c\Omega(0) \lesssim 1$ [56]. We see this behavior in Fig. 2(b), where we plot $\sqrt{\langle\Delta U\rangle_N/\langle\Delta U\rangle_1}$ to bring out the quadratic scaling. We also note that, for sufficiently large $N$, the $N^2$ scaling of $\langle[V_R^{(I)}(t_1)]^2\rangle_{\rho_0^R}$ for the indistinguishable case turns into a linear scaling with an enhanced slope of $\coth\beta_c\Omega(0) > 1$, while it is unity in the distinguishable case.

Considering general coupling protocols $g_C(t)$, when $\Delta = 0$, we find that the autocorrelation $\langle V_R^{(I)}(t')V_R^{(I)}(t)\rangle_{\rho_0^R}$ is factorized and, hence, vanishes when $t$ and $t'$ are separated by a thermalization step with the hot bath. This allows us to simplify (4) and write the probability of excitation of the driven system in the indistinguishable case as

$$p_i^{\rm indist} \simeq \sum_{\substack{t_0=0,T/2\\ \sigma=\pm}} |c_i^\sigma(t_0)|^2 \left[\frac{N(N+2)}{4} - F_\sigma(N, \beta_{t_0}E_{t_0})\right] \qquad (13)$$

and in the distinguishable case as

$$p_i^{\rm dist} \simeq \sum_{\substack{t_0=0,T/2\\ \sigma=\pm}} |c_i^\sigma(t_0)|^2 \left[\frac{N}{2}(1 + \sigma \tanh(\beta_{t_0}E_{t_0}))\right], \qquad (14)$$

where the positive, coupling-protocol-dependent terms are determined by the amplitudes $c_i^\pm(t_0) = \int_{t_0}^{t_0+T/2} dt g_C(t) \times {}_S\langle i|V_S^{(I)}(t)|0\rangle_S e^{\pm i\phi(t,t_0)}$. Comparing the terms in brackets in Eqs. (13) and (14), we have that $p_i^{\rm indist} \geq p_i^{\rm dist}$. Thus, for $\Delta = 0$, our central result of the enhancement of internal energy change for indistinguishable bosonic engines still holds for arbitrary coupling protocols and external system Hamiltonians.

In order to widen the scope of our results, we also consider engine strokes with $\Delta \neq 0$ as described in more detail in Ref. [50]. There, we find that the enhancement persists for small values of $N$ independently of the form of the coupling and the external system Hamiltonian. The fact that the enhancement is guaranteed for small $N$ most likely will be of particular relevance to experiments in the near future that will presumably have access to small $N$. Furthermore, we demonstrate for a given choice of the Hamiltonian how to identify generic parameter regimes and coupling protocols that lead to enhancement. We also extend our results to nonperturbative continuous coupling $g_C(t)$ [see [50] for the exact functional form of $g_C(t)$] using numerical simulations for a HO external system. From Fig. 3, we see that, for small values of $N$ and moderate values of the coupling strength, there is enhancement and $\langle \Delta U \rangle_N^{\mathrm{indist}}$ scales as $N^2$. The fact that we are able to find enhancement for a generic set of parameters as in Fig. 3 without fine-tuning suggests the general applicability of our result. Finally, we note that the performance in the case of engines made of $N$ identical noninteracting fermionic two-level atoms, considered in Ref. [50], is generally diminished for even $N$ and converges to that by a single engine for odd $N$ in the limit of small $\beta_{\mathrm{COM}}^{-1}$ due to the Pauli blocking effect.

In conclusion, we have demonstrated that the statistical indistinguishability of work resources can be exploited to gain a genuine quantum enhancement in quantum thermodynamics. While we identify this enhancement in terms of the internal energy change of an external system coupled to the engines, the question arises as to how much of this change is attributable to the actual action of the engines and how much results from the time dependence of the part of the Hamiltonian describing the interaction between the system and the engines. Our preliminary analysis indicates that an accordingly corrected work contribution of engines also displays enhancement in the parameter regimes considered here [50]. The predicted enhancement of the energy output from multiple indistinguishable heat engines to a generic external system is readily testable with current or near-future experimental realizations of quantum heat engines, e.g., in nitrogen-vacancy centers [17], trapped ions [59], and ultracold gases. While we have considered bosonic and fermionic statistics [50], exotic fractional statistics [41,60] may lead to further interesting results.

We thank Yanming Che, Iñigo L. Egusquiza, and Kosuke Ito for helpful discussions and comments. G. W. is supported by the Zhejiang Provincial Natural Science Foundation Key Project (Grant No. LZ19A050001), by National Natural Science Foundation of China (Grants No. 11975199 and No. 11674283), by the Fundamental Research Funds for the Central Universities (2017QNA3005 and 2018QNA3004), and by the Zhejiang University 100 Plan. B. P. V. is supported by the Research Initiation Grant, Excellence-in-Research Fellowship of IIT Gandhinagar, and a Department of Science & Technology-Science and Engineering Research Board (India) Start-up Research Grant No. SRG/2019/001585. M.-J. H. is supported by the startup fund by Duke Kunshan University and the European Research Council Synergy grant BioQ. G. W. and B. P. V. led the project and contributed equally.

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