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Superconductivity versus quantum criticality: what can we learn from heavy fermions?

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Abstract
Two quantum critical point (QCP) scenarios are being discussed for different classes of antiferromagnetic (AF) heavy-fermion (HF) systems. In the itinerant one, where AF order is of the spin-density wave (SDW) type, the heavy 'composite' charge carriers keep their integrity at the QCP. The second one implies a breakdown of the Kondo effect and a disintegration of the composite fermions at the AF QCP. We discuss two isostructural compounds as exemplary materials for these two different scenarios: $CeCu_2Si_2$ exhibits a three-dimensional (3D) SDW QCP and superconductivity, presumably mediated by SDW fluctuations, as strongly suggested by recent inelastic neutron scattering experiments. In $YbRh_2Si_2$, the AF QCP is found to coincide with a Kondo-destroying one. However, in the latter compound these two QCPs can be detached by varying the average unit-cell volume, e.g. through the application of chemical pressure, as realized by partial substitution of either Ir or Co for Rh. A comparison of $CeCu_2Si_2$ and $YbRh_2Si_2$ indicates that the apparent differences in quantum critical behaviour go along with disparate behaviour concerning the (non-) existence of superconductivity (SC). No sign of SC could be detected in $YbRh_2Si_2$ down to mK temperatures. A potential correlation between the specific nature of the QCP and the occurrence of SC, however, requires detailed studies on further quantum critical HF superconductors, e.g. on β-$YbAlB_4$, UBe_{13}, $CeCoIn_5$ and $CeRhIn_5$.

1. Quantum criticality in heavy fermions

Quantum critical points (QCPs) are of extensive current interest in condensed-matter physics as they can give rise to exotic finite-temperature properties. In addition, they promote the formation of novel phases, notably unconventional superconductivity. Heavy-fermion (HF) compounds have emerged as prototypical materials to study quantum criticality. A QCP occurs at the critical value of a non-thermal control parameter, e.g. pressure p or magnetic field B, at which two ground states compete only at temperature $T = 0$. In HF compounds, these are usually a Landau–Fermi liquid (LFL) phase and an antiferromagnetically ordered one [1–3].

Theoretically, QCPs in HF metals have mostly been treated by applying the quantum version of the theory of order parameter fluctuations [4–7]. In this 'conventional' approach it is assumed that the heavy 'composite' charge carriers keep their integrity at the QCP, i.e. they exist on either side of it. This implies the antiferromagnetic (AF) order to be of itinerant, spin-density-wave (SDW) type. The observation, through inelastic neutron scattering (INS), that the AF correlations in the quantum critical material $CeCu_{5.9}Au_{0.1}$ are of a local character [8] has prompted theoretical descriptions of the QCP which include the destruction of the Kondo effect, resp. the disintegration of the composite fermions [9–11].

Within the SDW scenario, superconductivity (SC) mediated by AF spin fluctuations (paramagnons) had been predicted for HF compounds already in 1986 to occur in the vicinity of an SDW instability [12]. In fact, the formation of SC appears to provide an efficient way to dispose of the huge amount of entropy that is accumulated at a QCP [13]. More recently, the possibility of unconventional SC in the LFL phase near the Kondo-destroying (KD) QCP has been proposed theoretically [14]. The latter may be considered a Mott transition selective to the f-component of the composite quasiparticles.

In this paper, the isostructural compounds $CeCu_2Si_2$ and $YbRh_2Si_2$ are discussed as prototypical materials for the two QCP scenarios described above. The following section is devoted to $CeCu_2Si_2$ at ambient pressure. According to early resistivity and specific-heat studies [15, 16] as well as to more recent neutron-diffraction [17] data, this compound exhibits a three-dimensional (3D) SDW QCP, masked by SC. INS results will be presented which are consistent with the concept of SC being mediated by SDW fluctuations [18], in agreement with the theoretical prediction [12]. In section 3, results of the magnetic Grüneisen ratio of $YbRh_2Si_2$ [19] are presented in the context of previous resistivity, specific heat [20, 21], magnetostriction and magnetization [22] as well as Hall effect [23] results which, in fact, reveal a KD QCP coinciding with a field-induced AF one [24]. In addition, recent investigations will be briefly described, in which the temperature–magnetic-field phase diagram has been expanded by including the average unit-cell volume as an additional parameter. To this end, positive and negative chemical pressure was applied to $YbRh_2Si_2$ by substituting either small amounts of Co or Ir for Rh, respectively [25]. In the resulting 'global' phase diagram, one finds a detachment of the KD from the AF QCP which, in particular, implies a novel 'spin-liquid'-type phase to form in the volume-expanded (Ir-doped) material. A section devoted to the interplay of quantum criticality and SC in other quantum critical HF metals is added before concluding the paper.

2. Spin excitations and superconductivity in quantum critical $CeCu_2Si_2$

When SC below $T_c \approx 0.6$ K in $CeCu_2Si_2$ was discovered in 1979 [26], all superconductors known at that time were classical (phonon-mediated) BCS superconductors. The latter typically exhibit a high sensitivity against magnetic dopants: usually, they lose SC when doped with small amounts ($\leqslant$1 at.%) of magnetic impurities [27]. In contrast, 100 at.% of magnetic Ce^{3+} ions were found to be a prerequisite to generate SC in $CeCu_2Si_2$: a small concentration of *non-magnetic* impurities is sufficient to suppress SC completely [28], and the non-f reference compound $LaCu_2Si_2$ is *not* a superconductor [27]. Since the heavy, i.e. slow, charge carriers can hardly escape their own 'polarization cloud', the BCS-type electron–phonon pairing mechanism had to be discarded from the outset. A few years after the discovery, magnetically mediated pairing in HF SC was proposed [29].

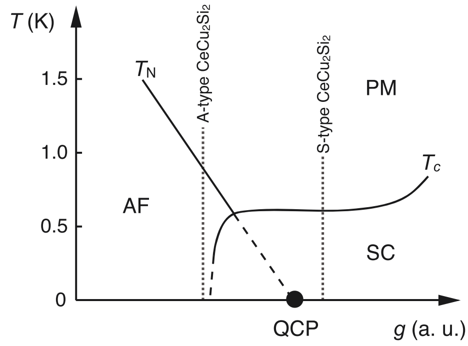

Figure 1. Schematic T–g phase diagram of $CeCu_2Si_2$ in the vicinity of the quantum critical point (QCP) where the antiferromagnetic (AF) phase vanishes as a function of the effective coupling parameter g. Superconductivity (SC) is observed around the QCP and extends far into the paramagnetic (PM) regime. Composition as well as hydrostatic pressure can be used to change the coupling parameter g in order to tune the system to the QCP. The positions of the A-type and the S-type single crystals in the phase diagram are marked.

As mentioned before, for HF metals AF paramagnons were thought rather early to act as superconducting glue near an SDW instability [12]. The pressure-induced superconductor $CePd_2Si_2$ is commonly regarded as a good example for paramagnon-mediated SC [30]. However, the very high critical pressure ($p_c \approx 2.8$ GPa) necessary to suppress AF order in $CePd_2Si_2$ did not yet allow one to explore the nature of its ordered state and spin fluctuation spectrum near the QCP. Compared to $CePd_2Si_2$, $CeCu_2Si_2$ has the advantage of its AF QCP being accessible already at ambient pressure. As explained elsewhere [31], homogeneous $CeCu_2Si_2$ samples can be prepared both from the antiferromagnetically ordered ('A') phase (by preparing samples with a slight deficit of Cu) and from the paramagnetic superconducting ('S') phase (with a tiny Cu excess). Samples very close to the exactly stoichiometric composition exhibit a so-called 'A/S'-type ground state in which AF order and SC compete for stability without microscopic coexistence [31]. Compared to $CePd_2Si_2$, where a rather narrow superconducting dome is centred around the QCP, $CeCu_2Si_2$ exhibits a very wide superconducting existence range (cf figure 1), at least up to pressures of the order of 10 GPa [32]. This is due to a second superconducting dome centred at around $p \approx 5$ GPa, which overlaps with the one at low pressure [33]. Since the high-pressure dome is intersected by a first-order valence transition line terminating in a low-lying (10–20 K) critical end point [34], critical valence fluctuations were proposed to be relevant for the superconducting pairing mechanism in highly pressurized $CeCu_2Si_2$ [33, 35]. Ambient-pressure measurements of the Cu–NQR of S-type polycrystals revealed a spin-lattice relaxation rate $1/T_1 \sim T^3$ and the absence of a Hebel–Slichter peak at T_c, consistent with d-wave SC, as well as a gap ratio $2\Delta_0/k_BT_c \approx 5$ both at $p = 0$ [36, 37] and at $p = 4.2$ GPa [37]. From recent specific-heat results performed on an A/S-type single crystal, different pairing symmetries, i.e. $d_{x^2-y^2}$ and d_{xy}, were suggested at low and high pressure, respectively [38]. For A-type $CeCu_2Si_2$, the nature of the AF order could be determined by neutron diffraction to be of an incommensurate SDW type, with a low-temperature ordered moment of only $\mu_S \approx 0.1\ \mu_B$ and an incommensurate ordering wavevector due to Fermi-surface nesting, $\tau \approx (0.215\ 0.215\ 0.53)$ [17]. Recent INS studies on a high-quality S-type single crystal [18] have revealed, inside the ambient-pressure superconducting phase, some 50–60 Å wide regions with *static* SDW correlations, which occur exactly at the same nesting wavevector τ and exhibit the same temperature dependence as observed before [17] for the long-range SDW order in an A-type single crystal.

Pronounced *dynamical* SDW correlations, as inferred from the INS results on this same S-type crystal, are presented in figure 2. The magnetic response inside the superconducting state at the nesting wavevector Q_{AF} exhibits a strong inelastic signal [18, 39]. The missing spectral weight at very low energies is an indication for a gap in the spin excitation spectrum within the superconducting state. Furthermore, this missing spectral weight is transferred to energies just above the gap value, resulting in an inelastic line. In the normal state of $CeCu_2Si_2$ only a broad quasielastic magnetic response is observed. Such a broad response is expected for overdamped paramagnons which are then gapped in the superconducting state. In contrast, the HF superconductors UPd_2Al_3 [40] and $CeCoIn_5$ [41] display resolution-limited inelastic peaks, called spin resonances, below T_c. In $CeCu_2Si_2$, the low-T value of the spin gap $\hbar\omega_{gap} \approx 0.2$ meV (i.e. $\hbar\omega_{gap}/k_B T_c \approx 3.9$), as extracted from fits to the data, is within 20% accuracy identical to the amplitude of the spin gap determined by Cu–NQR as mentioned before. The spin gap completely disappears at T_c, giving way to the quasielastic magnetic response in the normal state. The inelastic peak at $\hbar\omega_{gap} \approx 0.2$ meV at $T \ll T_c$ does not seem to be a singular point in (Q, ω) space, but is most likely part of an overdamped propagating mode. This is inferred from recent measurements of the Q dependence of the magnetic response at different energy transfers, to be published elsewhere [42]. Our results strongly suggest that these overdamped AF paramagnons may be the driving force in the superconducting pairing mechanism. They are highly consistent with the proposal of SC mediated by strong SDW fluctuations as predicted for HF superconductors [12].

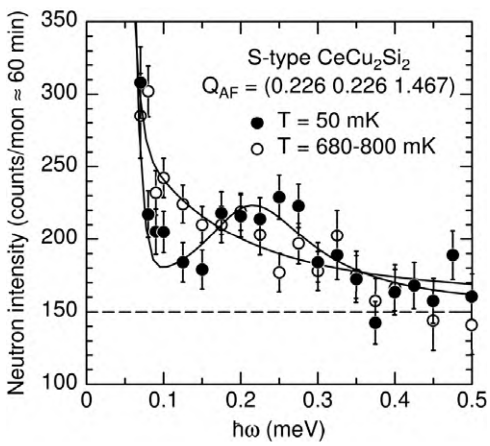

Figure 2. Energy scans (neutron intensity $S = S_{ela} + S_{qe/ine,mag}$ versus energy transfer $\hbar\omega$) in S-type $CeCu_2Si_2$ at $Q = Q_{AF} = (0.226\ 0.226\ 1.467)$ in the superconducting state at $T = 0.05$ K, $B = 0$. For comparison the magnetic response at Q_{AF} in the normal state at $T = (0.68–0.8)$ K $> T_c = 0.6$ K, $B = 0$ is also plotted. The solid lines represent fits to the data comprised of the incoherent and coherent elastic contributions, and the inelastic or quasielastic magnetic signal at Q_{AF}, convoluted with the resolution. The magnetic signal is modelled by a Lorentzian line. The dashed line represents the constant background as determined at negative energy transfers (reproduced with permission from [18]. Copyright 2007, Elsevier).

3. Unconventional quantum criticality and global phase diagram of $YbRh_2Si_2$

$YbRh_2Si_2$ is a clean, stoichiometric HF metal exhibiting weak AF order below $T_N \approx 70$ mK [20], which can be continuously suppressed by a small magnetic field and finally vanishes at $B_c \approx 60$ mT, applied within the easy magnetic *ab* plane $(B \perp c)$ [24]. For $B > B_c$ a heavy LFL phase develops at low temperatures, $T < T_{LFL}$. Within the LFL phase, the largely enhanced, T-independent values of the Sommerfeld coefficient $\gamma = C_{el}(T)/T$ and of the coefficient A in the T dependence of the resistivity, $\Delta\rho = \rho - \rho_0 = AT^2$, were found to diverge upon approaching the QCP, $B \to B_c$ [24]. For slightly Ge-doped $YbRh_2Si_2$, for example, $\gamma_0 \sim (B - B_c)^{-1/3}$ for $B - B_c < 0.3$ T [21]. Pronounced non-Fermi-liquid (NFL) effects were observed in the quantum critical regime: $\Delta\rho(T)$ is proportional to T at sufficiently low temperatures, i.e. below 10 K for a 5 at% Ge-doped single crystal [21]. The Sommerfeld coefficient diverges logarithmically below $T \approx 10$ K, but follows $\gamma \sim 1/T^{1/3}$ below $T \approx 0.3$ K. In contrast, $\Delta\rho(T) \sim T$ continues to hold down to the lowest temperatures [21]. NMR [43] and susceptibility [44] experiments highlight dominating ferromagnetic critical fluctuations over extended parts of the T–B phase diagram. Only very close to the AF phase boundary, $T_N(B)$, do the AF quantum critical fluctuations dominate [43, 44, 21]. Strong ferromagnetic (FM) correlations can also be inferred from a Sommerfeld–Wilson ratio being as large as 30 within the LFL phase slightly above the critical field [44]. It is important to stress, however, that neither the T dependence of the spin-lattice relaxation rate nor that of the bulk susceptibility can be explained by the itinerant theory for (2D or 3D) ferromagnetic fluctuations [22].

Below $T = 0.6$ K, the dimensionless *thermal* Grüneisen ratio $\Gamma_{therm} = (V_{mol}/\kappa_T)\cdot(\beta_{cr}/C_{cr})$ (V_{mol}: molar volume, κ_T: isothermal compressibility), where β_{cr} and C_{cr} denote the volume thermal expansion and electronic specific heat after subtraction of normal (LFL) contributions, has been investigated at $B = 0$ and down to $T = 80$ mK [45]. In this T range $\Gamma_{therm} \sim 1/T^{0.7}$ was found for slightly Ge-doped $YbRh_2Si_2$, which cannot be explained within the SDW scenario; rather, it appears to be consistent with a KD QCP [45]. While $\Gamma_{therm}(T)$ has to diverge for any pressure-sensitive QCP [45], a divergence of the *magnetic* Grüneisen ratio $\Gamma_{mag}(T) = -M'(T)/C(T)$ was predicted [46] for any magnetic-field-induced QCP. Here, $M'(T)$ is the temperature derivative of the magnetization. In figure 3(a), $-M'(T)/T$ of pure $YbRh_2Si_2$ for different fields is compared with the temperature dependence of the coefficient of the volume

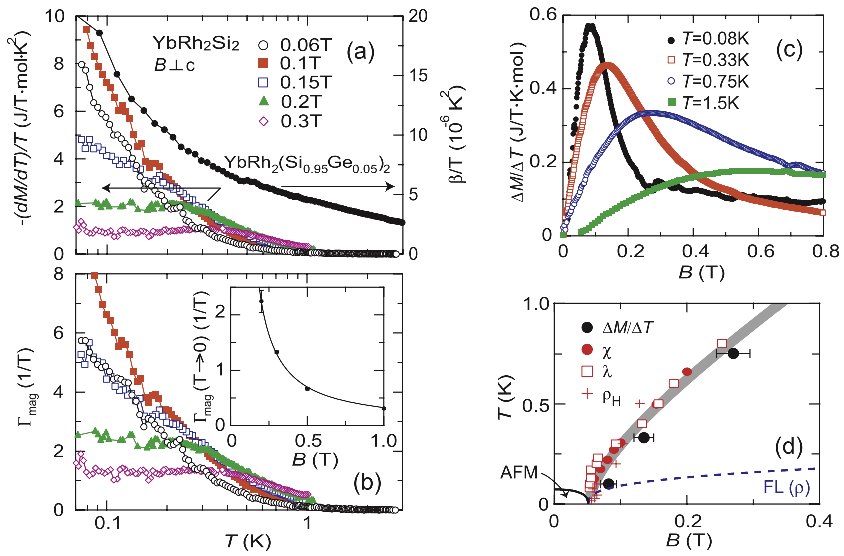

Figure 3. (a) Temperature derivative of the magnetization as $-(\mathrm{d}M/\mathrm{d}T)/T$ (left axis) and (b) magnetic Grüneisen ratio Γ_{mag} as a function of temperature (on logarithmic scales) for $YbRh_2Si_2$ at different magnetic fields applied perpendicular to the c axis. Volume thermal expansion of $YbRh_2(Si_{0.95}Ge_{0.05})_2$ as $-\beta/T$ at zero field is shown for comparison in (a) (right axis). Inset in (b): field dependence of saturated Γ_{mag}, as extrapolated to $T = 0$. The solid line indicates $-G(B - B_c^{\mathrm{fit}})^{-1}$ with $G = -0.3$ and $B_c^{\mathrm{fit}} = 0.065$ T. (c) Change in magnetization divided by temperature increment, $-\Delta M/\Delta T$, versus magnetic field for $YbRh_2Si_2$ at $T = 0.08$ K (black), 0.33 K (red), 0.75 K (blue) and 1.5 K (green). (d) Temperature–field phase diagram including peak positions in $-\Delta M/\Delta T$ and $T^*(B)$ line (grey line) from transport and thermodynamic properties (red symbols). The solid black and the dashed blue lines represent the AF phase boundary and the NFL–LFL crossover, respectively (reproduced with permission from [19]. Copyright 2009 by the American Physical Society).

thermal expansion, β/T, for $YbRh_2(Si_{0.95}Ge_{0.05})_2$ [19]. While $-M'(T)/T$ diverges more rapidly than $\beta(T)/T$ at elevated temperature, they both become comparable below $T \sim 0.2$ K. At the critical field $B_c = 0.06$ T, both $-M'(T)/T$ and the Sommerfeld coefficient $C(T)/T$ diverge upon cooling, but the divergence in $-M'(T)/T$ is much stronger, resulting in a divergence of $\Gamma_{\mathrm{mag}}(T)$ (frame (b)). A crossover at about 0.3 K is found within the quantum critical regime at $B = 0.06$ T, separating a strong divergence $\Gamma_{\mathrm{mag}} \sim 1/T^2$ at elevated temperatures from a weaker one at lower T ($\sim 1/T^{0.7}$) [19]. At the same temperature, a crossover in the magnetic susceptibility arises due to the interplay of AF and FM fluctuations [44]. Remarkably, the critical exponent below 0.3 K is nearly identical to that observed in the thermal Grüneisen ratio $\Gamma_{\mathrm{therm}}(T)$ [45]. According to Zhu *et al* [46], $\Gamma_{\mathrm{mag}}(T, B_c) \sim 1/T^{1/\nu z}$, where ν and z are the correlation length exponent and the dynamical exponent, respectively.

In the LFL regime and for constant $B > B_c$, $-M'(T)/T$, $C(T)/T$ and $\Gamma_{\mathrm{mag}}(T)$ saturate at sufficiently low T at 'plateau' values which diverge with decreasing field: $\Gamma_{\mathrm{mag}}(0) \sim (B - B_c)^{-1}$, cf inset of figure 3(b). This agrees well with the prediction of scaling analysis: $\Gamma_{\mathrm{mag}}(0, B) = -G(B - B_c)^{-1}$. Here, $G = \nu(d - z)$ (d: dimensionality of the critical fluctuations) has to agree with the exponent in the field divergence of the Sommerfeld coefficient, $\gamma_0 \sim (B - B_c)^G$ [46, 47]. The latter was determined from low-T specific-heat experiments in slightly Ge-doped $YbRh_2Si_2$ to yield $G = -0.33$ [21]. From the fit of the 'plateau' values $\Gamma_{\mathrm{mag}}(T = 0, B)$ in the LFL phase, $B_c = 0.065$ T and $G = -0.30 \pm 0.01$, close to the critical exponent in γ_0 versus $(B - B_c)$, were obtained. This proves the thermodynamic consistency of the data. We note that the experimental value of G cannot be understood within the SDW model ($\nu = 1/2, z = 2$), neither for $d = 2$ nor $d = 3$, while $G = -1/3$ was predicted within a critical Fermi-surface model which may be applied to a KD QCP [48]. As shown in figures 3(c) and (d), the maxima in $-\Delta M(T)/\Delta T$ versus B (at T = const) which correspond to inflection points in the isothermal field dependence of the entropy agree well with the KD crossover line $T^*(B)$ as determined from Hall effect [23] and thermodynamic [22] measurements.

As mentioned in the introductory section, the crossover line $T^*(B)$ merges with the AF phase boundary, $T_N(B)$, at the field-induced QCP ($T = 0$, $B = B_c$). Since both the RKKY and the Kondo interaction should be sensitive to the average unit-cell volume, studying the evolution of $T_N(B)$ and $T^*(B)$ under variations of the average unit-cell volume appears to be straightforward. Therefore, positive or negative chemical pressure has been applied to $YbRh_2Si_2$ by substituting Co or Ir for Rh, respectively [25]. The results of combined ac susceptibility and magnetoresistance measurements on such single crystals are summarized in figure 4: (i) the AF state is stabilized/weakened by volume compression/expansion, as expected. Unexpectedly, however, the KD crossover line is almost independent of chemical pressure. (ii) Under volume compression (Co-doping) the AF QCP occurs at a field substantially higher than B^* at which $T^* \to 0$. In this situation, the SDW theory is expected to be applicable to the AF QCP [9, 3], which is confirmed by the field dependence of the Néel temperature for $Yb(Rh_{0.93}Co_{0.07})_2Si_2$, $T_N \sim (B_N - B)^{\varepsilon}$, with $\varepsilon = 0.65$ [25]. In fact, $\varepsilon = 2/3$ was calculated for an SDW QCP with 3D critical fluctuations [2].

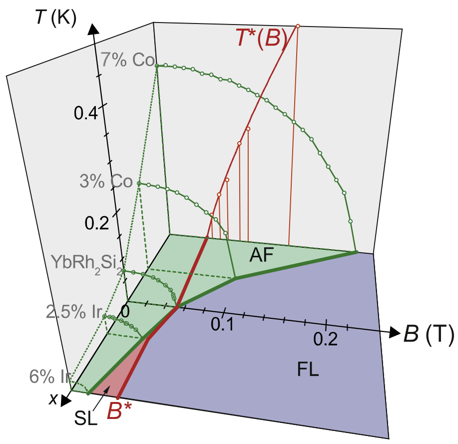

Figure 4. Evolution of the T–B phase diagram of $YbRh_2Si_2$ under positive and negative chemical pressure, determined via ac susceptibility and electrical resistivity measurements. The zero-temperature plane displays B versus average unit-cell volume, which increases from Co- to Ir-doped $YbRh_2Si_2$. Red/green lines indicate lines of Kondo-destroying/antiferromagnetic quantum critical points. Blue/red areas mark Landau–Fermi liquid and 'spin-liquid'-type phases, respectively (reproduced with permission from [25]. Copyright 2009 by Macmillan Publishers Limited).

Interestingly, the NFL–LFL crossover line, $T_{\mathrm{LFL}}(B)$ is linked to the magnetic phase transition line in this case. (iii) Under volume expansion (Ir-doping), B_{N} (corresponding to T_{N}) is substantially smaller than B^* (corresponding to T^*) and the NFL–LFL crossover appears to be linked to the $T^*(B)$ line. For $Yb(Rh_{0.94}Ir_{0.06})_2Si_2$, AF order and the LFL ground state are not connected by a single QCP but are separated by an extended, possibly 'spin-liquid'-type phase, in which the 4f moments are neither Kondo screened nor magnetically ordered. This poses a formidable challenge to those theories which deal with the destruction of the Kondo effect near an AF QCP in Kondo-lattice systems.

In studying $Yb(Rh_{0.94}Ir_{0.06})_2Si_2$, no SC was observed, at least for $T \geqslant 20$ mK [49], which may be ascribed to doping-induced disorder in this single crystal. Therefore, in view of the proposal of unconventional SC in the LFL phase near the KD QCP [14], the low-T state of pure $YbRh_2Si_2$ both below and above $B_{\mathrm{c}} = 60$ mT ($B \perp c$) has to be carefully scrutinized for SC in the future. Preliminary magnetization measurements did not reveal any sign of SC above 15 mK in extremely clean $YbRh_2Si_2$ single crystals at $B < 25$ mT ($B \perp c$) [50]. In the following section we will thus survey other candidates for HF SC near a potential KD QCP.

4. Interplay of quantum criticality and heavy-fermion superconductivity

UBe_{13}, a cubic HF superconductor [51], is especially fascinating since (i) SC develops out of a highly unusual normal state characterized by a large and strongly T-dependent resistivity [51] and (ii) upon substituting a small amount of Th for U in $U_{1-x}Th_xBe_{13}$, a non-monotonic evolution of $T_{\mathrm{c}}(x)$ and a second (as yet not fully identified) phase transition at T_{c2} (below the superconducting T_{c1}) was discovered within a critical concentration range $0.019 < x < 0.045$ [52]. A pronounced hump in $\rho(T)$ at about 2 K is shifted to higher temperatures by a magnetic field. This gives way to the observation of a universal temperature dependence in a plot of $\rho(T)/\rho(1\ \mathrm{K})$ versus T between the respective $T_{\mathrm{c}}(B)$ and about 1.2 K in a surprisingly wide range of magnetic fields, $4\ \mathrm{T} \lesssim B \lesssim 10\ \mathrm{T}$ [53]. Above $T \approx 0.7$ K, $\rho(T)$ is *proportional* to T, but becomes superlinear at lower T: $(\rho - \rho_0) \sim T^{1.5}$. The normal-state specific-heat coefficient $\gamma(T) = C(T)/T$ diverges logarithmically below $T \approx 2$ K. At $B = 12$ T ($> B_{\mathrm{c2}}(0)$), this divergence becomes degraded below $T \approx 0.3$ K, in accordance with $\gamma(T) = \gamma_0 - \beta T^{0.5}$ [53]. The NFL phenomena inferred from the temperature dependence of the resistivity and the specific heat are compatible with a field-induced 3D SDW QCP. The latter has tentatively been assigned to the smooth disappearance of clearly resolved anomalies in both specific heat and thermal expansion of UBe_{13} at $B_{\mathrm{cr}} \approx 4.5$ T [54]. In zero magnetic field, these anomalies show up at $T_{\mathrm{L}} \approx 0.7$ K inside the superconducting state. In $U_{1-x}Th_xBe_{13}$ with subcritical Th concentration they were found to be the 'precursor' of the lower phase transition which occurs at T_{c2} within the critical concentration range [54]. As the phase transition at T_{c2} was ascribed to the formation of an SDW wave [55], motivated by a huge ultrasound attenuation peak, a field-induced (broadened) SDW QCP at $B_{\mathrm{cr}} \approx 4.5$ T may indeed be the source for the pronounced NFL phenomena observed in an extended range of magnetic fields in the normal state of undoped UBe_{13} [53].

$CeRhIn_5$ [56] and $CeCoIn_5$ [57] are tetragonal variants of cubic $CeIn_3$—a HF superconductor ($T_{\mathrm{c}} \approx 0.2$ K) in a narrow pressure range close to an AF quantum phase transition at $p_{\mathrm{c}} \approx 2.6$ GPa. Both compounds were found to be superconducting ($T_{\mathrm{c}} \approx 2$ K) in wide ranges of pressure, $p \geqslant 1$ GPa [56] and $p \geqslant 0$ [57]. The strong T_{c} enhancement has been attributed to the layered crystal structure, giving rise to strongly anisotropic magnetic fluctuations [58].

Thermal-expansion experiments [59], performed in the normal state of $CeCoIn_5$ at different magnetic fields $B > B_{\mathrm{c2}}(0) \approx 4.5$ T, hint at two-dimensional (2D) SDW fluctuations at elevated temperatures. However, a 2D–3D crossover was concluded from these experiments to occur at lower T ($\approx$0.25 K for $B = 5$ T and $\approx$1.2 K for 10 T) [59]. Consequently, the field-induced QCP of $CeCoIn_5$ at [60, 61], or $\approx$10% below [62], $B_{\mathrm{c2}}(0)$ may well be of the 3D SDW type, too. Unfortunately, observation of AF order of this material is prevented by SC.

Measurements of the de Haas–van Alphen (dHvA) effect performed on $CeRhIn_5$ as a function of pressure and at magnetic fields up to 17 T ($> B_{\mathrm{c2}}$) suggest a pronounced change from a 'smaller' Fermi-surface volume at $p < p_{\mathrm{c}} \approx 2.4$ GPa to a 'larger' one at $p > p_{\mathrm{c}}$ [63]. This is frequently considered evidence for a KD QCP [64], but has also been ascribed to a valence change (abrupt jump/'sharp crossover' in the valence of Ce) [65]. The latter scenario assumes itinerant 4f states above *and* below the critical pressure. Thus, an experimental investigation of the Ce valence below and above p_{c} appears to be of great timely interest. Interestingly, AF order at $p < 2.4$ GPa *above* the upper critical field

$B_{c2}(p)$ appears to be of the *commensurate* variety, but becomes *incommensurate* below $B_{c2}(p)$ [66]. For $T \to 0$, this latter (SDW) order coexists with SC on the low-pressure side of a *quantum critical line* between $p_1 \approx 1.8$ GPa ($B = 0$) and $p_2 \approx 2.4$ GPa ($B = B_{c2}(p)$). On the high-pressure side of this line, exclusively bulk SC was observed [64]. For $p \leqslant p_1$, neutron diffractometry revealed a significant change in the incommensurate magnetic ordering wavevector at $T = T_c(p)$, which indicates the SDW order to be indeed affected by SC [67].

β-$YbAlB_4$ is the first Yb-based compound for which HF SC has been observed [68]. Here, the resistivity measured within the orthorhombic plane at $B = 0$ follows a $T^{1.5}$ dependence in the low-T normal state, i.e. at $T \geqslant T_c = 80$ mK. This is consistent with an *itinerant* (3D SDW) QCP scenario, although very recent low-T ESR investigations suggest a localized 4f state [69]. No AF order was detected at ambient pressure down to $T = 35$ mK. As usual, an LFL state is established in applied magnetic field ($B \geqslant 0.3$ T).

5. Summary and outlook

The NFL phenomena in $CeCu_2Si_2$ can be traced back to a 3D SDW QCP. As previously suggested for the pressure-induced superconductor $CePd_2Si_2$ [30], SC in $CeCu_2Si_2$ is very likely mediated by strong SDW fluctuations, which was concluded from the analysis of recent INS results. For the isostructural compound $YbRh_2Si_2$ no SC could be detected down to mK temperatures. It has yet to be clarified if SC in this case (i) occurs at even lower temperatures, (ii) does not form because of the competition between AF and ferromagnetic correlations [44] or (iii) is inhibited due to the unconventional nature of the QCP, i.e. the coincidence of an AF QCP with a KD one. For the HF superconductor β-$YbAlB_4$, the nature of the putative QCP has yet to be resolved. The Fermi-surface reconstruction [63] as well as pronounced anomalies in various thermodynamic and transport properties [64, 70] at $p_c \approx 2.4$ GPa for the pressure-induced superconductor $CeRhIn_5$ could not be explained unequivocally either to date. For UBe_{13} and $CeCoIn_5$, a 3D SDW QCP may be anticipated at a finite field, below B_{c2}. This seems either to coexist with (UBe_{13}) or to be masked by ($CeCoIn_5$) the superconducting phase. In order to verify the occurrence of SC near a KD QCP as recently predicted [14], it would be highly desirable to find a clean, stoichiometric HF metal showing an LFL to spin-liquid transition only, i.e. one which is well separated from magnetic order.

In conclusion, much more future work is required to unravel the potential correlation between the type of quantum criticality and the occurrence of unconventional SC.

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