# Field-dependent specific heat and magnetization for the $S = \frac{1}{2}$ antiferromagnetic chain Yb<sub>4</sub>As<sub>3</sub>: simulation and experiments

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#### Abstract

The  $S = \frac{1}{2}$  antiferromagnetic Heisenberg model with the transverse staggered field and uniform magnetic field perpendicular to the staggered field is applied to the semimetallic compound Yb<sub>4</sub>As<sub>3</sub>. The field-dependent specific heat for infinite and finite chains as well as the magnetization for infinite chains are calculated by the numerical quantum transfer-matrix method. Specific heat data for polydomain samples of Yb<sub>4</sub>As<sub>3</sub> and (Yb<sub>0.99</sub>Lu<sub>0.01</sub>)<sub>4</sub>As<sub>3</sub> at B = 12T are presented and compared with numerical results obtained for microscopic parameters taken from theoretical predictions. Magnetization data for single domain and polydomain samples of Yb<sub>4</sub>As<sub>3</sub> are also compared with simulation results.

#### 1. Introduction

The high-temperature phase of Yb<sub>4</sub>As<sub>3</sub> is cubic with lattice constant a = 8.788 Å and has anti-Th<sub>3</sub>P<sub>4</sub> crystal structure [1]. Above  $T_{co} \approx 295$  K Yb<sub>4</sub>As<sub>3</sub> is a homogeneous intermediate valence metal (IV) with a valence ratio of Yb<sup>2+</sup>/Yb<sup>3+</sup> = 3 : 1, where the Yb ions reside statistically on four equivalent families of chains along the space diagonals of a cube [1]. At  $T_{co} \approx 295$  K the IV state exhibits such a charge-ordering instability that far

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below  $T_{co}$  one of the four Yb ions becomes trivalent and forms a one-dimensional spin  $S = \frac{1}{2}$  chain along the (111) direction. The remaining Yb ions take nonmagnetic divalent states. The crystal structure is trigonal with the angle 90.8° [1]. The Yb<sup>3+</sup> ion has one hole in the 4f closed shell. The  $J = \frac{7}{2}$  ground multiplet splits into four doublets under the crystal field. Thus, the lowtemperature dynamics is described by an effective  $S = \frac{1}{2}$ spin chain. The neutron scattering experiments on Yb<sub>4</sub>As<sub>3</sub> confirmed that the excitation spectrum is well described by the one-dimensional  $S = \frac{1}{2}$  isotropic Heisenberg model [2] in the absence of magnetic field. The interchain interactions are small and ferromagnetic, leading to a low-T spin-glass freezing [1]. On the other hand, the field-dependent data confirm the existence of a gap related to the Dzyaloshinskii-Moriya (DM)

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interaction [3] which can be mapped onto the anisotropic Heisenberg model with both uniform field  $B^x$  and staggered field  $B^y_s$  [4].

Below we also focus on  $(Yb_{0.99}Lu_{0.01})_4As_3$ . The nonmagnetic Lu-atoms are randomly distributed over all different Yb sites—both on the magnetic chain and on the nonmagnetic chains. The partial substitution of Yb with nonmagnetic Lu-atoms effectively dilutes the magnetic  $S = \frac{1}{2}$  chain by the introduction of nonmagnetic static defects [5] and should affect the fielddependent specific heat behaviour.

#### 2. Model and simulation technique

To characterize the finite-temperature properties of the Yb<sub>4</sub>As<sub>3</sub>, we consider the  $S = \frac{1}{2}$  Heisenberg model with the DM interaction [4,6]. The DM interaction is eliminated by rotating the spins in the x - y plane by the angle  $\theta$  [4]. Then the model is mapped onto

$$\mathcal{H} = -J \sum_{i=1}^{N} \mathbf{S}_{i} \mathbf{S}_{i+1} - g_{\perp} \mu_{\mathrm{B}} B^{x} \sum_{i=1}^{N} S_{i}^{x} - g_{\perp} \mu_{\mathrm{B}} B_{\mathrm{s}}^{y} \sum_{i=1}^{N} (-1)^{i} S_{i}^{y}, \qquad (1)$$

where  $B^x = B \cos(\theta)$ ,  $B_s^y = B \sin(\theta)$  and *B* is the uniform external magnetic field perpendicular to the one-dimensional spin-chain. Eq. (1) describes the effective isotropic Heisenberg model with both the uniform field  $B^x$  and the transverse staggered field  $B_s^y$ . We apply the quantum transfer-matrix (QTM) simulation technique. Following the scheme for infinite chains described in Ref. [6], we have calculated the partition function from the largest eigenvalue of the global transfer-matrix.

For finite chains we have applied the vectors  $|a\rangle$  and  $\langle b|$  described in Ref. [7] and we have used the following equations to calculate the *m*th classical approximant to the partition function of Eq. (1).

$$Z_m = \langle b | (W_1 W_2)^{(N-1)/2} | a \rangle \quad \text{for} \quad \text{odd } N,$$
(2)

$$Z_m = \langle b | (W_1 W_2)^{N/2 - 1} | a \rangle \quad \text{for even } N.$$
(3)

We have tested our simulations for the finite chains using the results obtained from the exact diagonalization technique for the limit B = 0 [8]. In Fig. 1 we present the quantum transfer-matrix results for the finite chains (open symbols) as a function of  $1/m^2$ . The exact diagonalization data (full symbols) for N = 8 and 15 stand for the expected limit of our approximants when  $m \to \infty$ . For the macroscopic chains, the convergence is similar but deteriorates down to 5% for  $B \neq 0$  in (1) and  $k_BT/J \approx 0.15$ .



Fig. 1. Variation of the zero-field specific heat per spin at  $k_BT/J = 0.05$  against  $1/m^2$  for the finite-chain QTM results (m = 5, ..., 12). Full symbols represent the exact diagonalization data.

## 3. Experimental results and numerical data

The experiments were carried out on high-quality single crystals of  $(Yb_{1-x}Lu_x)_4As_3$  prepared as described in Refs. [5,9]. For the specific heat measurements a microcalorimeter from Oxford Instruments was used. The transfer-matrix simulations of the specific heat and magnetization were performed using the parameters:  $g_{\parallel} = 2.9, g_{\perp} = 1.3$  [10],  $J/k_{\rm B} = -26$  K [2] and  $\tan(\theta) =$ 0.19 [4].

The specific heat results obtained in the magnetic fields from extrapolations of the largest eigenvalues of the transfer matrices ( $4 \le m \le 12$ , N infinite), are presented in Fig. 2. The uncertainties are smaller than the size of the symbols at higher temperatures and reach the size of the symbols near T = 4 K. The open symbols represent our experimental results for a polydomain sample with the magnetic field applied along the cubic (111) direction [3] and the remaining symbols are numerical results. For the experimental data the phonon contribution  $C_{\rm ph} = 2.05 \times 10^{-3} T^3 \,\mathrm{J/(K^4 \, mol)}$  has been subtracted [3]. The computer calculations were performed for a polydomain sample in which 25% of the domains were oriented with the spin chains parallel to the applied field B and about 75 of the domains were aligned so that the effective field component  $B \sin(70^\circ)$ was perpendicular to the spin chains. With increasing magnetic field the maximum in specific heat divided by temperature increases, shifts to the right and the C/Tcurves intersect at about 9K, which is consistent with the experimental findings.



Fig. 2. Comparison of the QTM results and the measured specific heat for  $Yb_4As_3$  after phonon subtraction.



Fig. 3. Comparison of the QTM results (square symbols) and polydomian specific heat for  $Yb_4As_3$  and  $Yb_{0.99}Lu_{0.01})_4As_3$  (Circle symbols).

Similar behaviour we have recovered for the 1% diluted system. In Fig. 3, using Eqs. (2) and (3) we have compared specific heat experimental results for a polydomain  $(Yb_{0.99}Lu_{0.01})_4As_3$  sample in the applied field B = 12 T and numerical data for the finite N = 99 chain. We expect that the reduction of the specific heat will be stronger when the distribution of chain segments is taken into account in the simulations. Our theoretical specific-heat data systematically overestimate the experimental results. This has not been understood so far.



Fig. 4. The field-dependent magnetization for Yb<sub>4</sub>As<sub>3</sub>: experimental data (single domain and polydomain sample) and numerical data.

We have also analysed the magnetization data. To achieve the convergence of our extrapolations up to B = 25 T, we performed simulations of the field-dependent magnetization at T = 3 K. In Fig. 4 we present these results for the field perpendicular and parallel to the chains (full and open circles, respectively), where vanVleck-type ( $\chi_{\perp} = 0.0324$  and  $\chi_{\parallel} = 0.0205$ ) contributions [10] are added. We also plot the experimental data obtained at 0.6 K for a polydomain sample [3] by continues line. Single-domain magnetization results [10] for T = 1.5 K and 1.7 K for the field perpendicular and parallel to the chains, respectively, are represented by squares. Our estimates in the low-field limit are consistent with experiment and the DMRG calculations [10].

## 4. Conclusions

We have presented experimental data of field-dependent specific heat and magnetization measurements. The former refer both to the pure and diluted system. We have successfully compared them with quantum transfer-matrix calculations performed for the Heisenberg model with the DM interactions, revealing at least a qualitative agreement with the experiments.

#### Acknowledgements

We thank Dr. B. Schmidt and Dr. P. Thalmeier for helpful discussions, the Committee for Scientific Research—for a partial support via the grant 4 T11F 014 24 and PSNC—for an access to the supercomputing platforms.

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