Comment on "Interacting-Electron Model Exactly Solvable in Any Dimension"

In a recent Letter [1] Montorsi and Rasetti investigated a special fermionic lattice model

$$H = t\sum_{\langle ij\rangle}\kappa_{ij}A_i^\dagger A_j + U\sum_i N_i D_i - \mu\sum_i (N_i + D_i)\,, \qquad (1)$$

where the first term (kinetic energy H_0) commutes with the second term (Hubbard interaction H_1). Hence the double occupancy of lattice sites is conserved. Here $A_i^\dagger$, A_i, with $N_i = A_i^\dagger A_i$, correspond to spinless fermions and $D_i = D_i^2$ commutes with A_i. The amplitude $\kappa_{ij} = 1 - D_i - D_j + \eta D_i D_j$ describes restricted hopping, where η is arbitrary. According to the authors they obtained the exact free energy density for (1) in *any* dimension d [Eq. (20) of Ref. [1]]; it has the general form

$$f = f_0 + N^{-1}\sum_{\mathbf{k}} F(\epsilon_{\mathbf{k}})\,, \qquad (2)$$

where $\epsilon_{\mathbf{k}}$ is the tight-binding dispersion.

In this Comment we would like to point out a discrepancy between the known properties of the model and the result of Ref. [1]. Using the notation $A_i \equiv c_{i\uparrow}$, etc., $N_i \equiv n_{i\uparrow}$ and $D_i \equiv n_{i\downarrow}$, and choosing $\eta = 1$, we see that for zero double occupancy the Hamiltonian (1) reduces to

$$H = t\sum_{\langle ij\rangle}(1 - n_{i\downarrow})c_{i\uparrow}^\dagger c_{j\uparrow}(1 - n_{j\downarrow}) \qquad (3)$$

which is identical with a simplified Hubbard model ("Falicov-Kimball model") at $U = \infty$, where only $\uparrow$ spins are mobile. This model is known to be nontrivial even for $U = \infty$ if $n < 1$ [2]. By contrast, the dimension dependence of (2) is *trivial*, since the integrand of the $\mathbf{k}$ sum depends on momentum $\mathbf{k}$ only via $\epsilon_{\mathbf{k}}$. Using $N^{-1}\sum_{\mathbf{k}} F(\epsilon_{\mathbf{k}}) = \int dE\,\rho_d(E)F(E)$, where $\rho_d(E)$ is the d-dimensional density of states of the noninteracting system, Eq. (2) is seen to have a *mean-field* form. The exact solution of (3), namely, in the limit of high dimensions $d \to \infty$ [3], has recently been obtained by different methods [4,5]. In particular, in this limit and for $U = \infty$ the free energy is given by [5]

$$f = \frac{1}{\pi}\int_{-\infty}^{\infty} d\omega f_T(\omega)\left[\int_{-\infty}^{\infty} dE\,\rho_\infty(E)\,\mathrm{Im}\ln[\omega + \mu - E - \Sigma(\omega) + i0^+] + \mathrm{Im}\ln[1 + G(\omega)\Sigma(\omega)]\right] - \beta^{-1}\ln[1 + e^{\beta(\mu-\epsilon)}]\,, \qquad (4)$$

where

$$\epsilon = \frac{1}{\pi}\int d\omega f_T(\omega)\tan^{-1}\left[\frac{-\mathrm{Im}[G^{-1}(\omega) + \Sigma(\omega)]}{\mathrm{Re}[G^{-1}(\omega) + \Sigma(\omega)]}\right]$$

and $f_T(\omega)$ is the Fermi function. Here $G(\omega)$ is the diagonal element of the Green function of mobile electrons and $\Sigma(\omega)$ is the self-energy; they are determined by $\delta f/\delta\Sigma = \delta f/\delta G = 0$. To be able to compare (2) and (4) we introduce the limit $d \to \infty$, i.e., $\rho_\infty(E)$, into (2) and put $U = \infty$ [3]. The resulting expression is very different from (4). In view of this discrepancy in such a well-defined limit and the simplistic mean-field d dependence of (2) we conclude that (2) is not exact.

Furthermore, we would like to comment on the "metal-insulator transition" discussed in Ref. [1]. Taking, as in Ref. [1], $\mathbf{k} \in \mathcal{D}$ in (2) ($\mathcal{D}$: *half* a Brillouin zone), half filling, and $\rho = \pm 1$, the ground-state energy in arbitrary dimensions d follows as

$$E_g = -2\int_{U/4}^{2dt} dE\,\rho_d(E)(E - \tfrac{1}{4}U)\,. \qquad (5)$$

Even if (5) were correct, the transition at $U = 8dt$ would *not* be of the Mott-Hubbard-type [6] (e.g., there is no band narrowing in this model because $[H_0, H_1] = 0$), but merely due to the choice of the ensemble with the optimal number of doubly occupied sites. Finally we note that the axes in Fig. 1 of Ref. [1] should read $u/4|t|$ and $U/4|t|$, respectively, to yield the correct Lieb-Wu result for the Hubbard model in $d = 1$. Even then the ground-state energy of the $1-d$ Hubbard model at $U = 0$, $E^0_{\mathrm{Hubb}}/t = -4/\pi$, does not coincide with the value from (5) for $d = 2$ and $U = 0$, $E_g^0/t = -16/\pi^2$.

V. Janiš,[1] F. Gebhard,[2] R. Strack,[1] and D. Vollhardt[1]

[1]Institut für Theoretische Physik C
Technische Hochschule Aachen
D-5100 Aachen, Federal Republic of Germany
[2]Physics Department
Rutgers University
Piscataway, New Jersey 08855

Received 10 July 1991
PACS numbers: 05.30.Fk, 71.10.+x

[1] A. Montorsi and M. Rasetti, Phys. Rev. Lett. **66**, 1383 (1991).
[2] T. Kennedy and E. H. Lieb, Physica (Amsterdam) **138A**, 320 (1986); E. H. Lieb, Physica (Amsterdam) **140A**, 240 (1986); U. Brandt and R. Schmidt, Z. Phys. B **63**, 45 (1986); **67**, 43 (1987).
[3] W. Metzner and D. Vollhardt, Phys. Rev. Lett. **62**, 324 (1989).
[4] U. Brandt and C. Mielsch, Z. Phys. B **82**, 37 (1991).
[5] V. Janiš, Z. Phys. B **83**, 227 (1991).
[6] J. Hubbard, Proc. R. Soc. London A **281**, 401 (1964).