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Angaben zur Veröffentlichung / Publication details:

Vollhardt, Dieter, Y. R. Lin-Liu, and Kazumi Maki. 1981. “Magnetic resonances of helical textures in 3He-A.” *Journal of Low Temperature Physics* 43 (1-2): 189–96. https://doi.org/10.1007/bf00118630.

Magnetic Resonances of Helical Textures in ^{3}He-A*

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The NMR frequencies of the helical texture in ^{3}He-A in the presence of a magnetic field parallel to the superflow are reexamined in the entire stability region. The effects of the anisotropic contribution to the spin susceptibility are explicitly included and are shown to leave previous results qualitatively unchanged. On the other hand, we find that the anisotropic contribution gives rise to quantitative corrections of maximally 20% at $T/T_c = 0.95$*, while its contribution in the immediate vicinity of* T_c *is negligibly small.*

We have recently studied the stability regime of helical textures in ^{3}He-A in the presence of a magnetic field parallel to the superflow.[1–3] In particular, assuming that the stability of the helical texture is determined by the spectrum of the longitudinal fluctuations, we have determined the phase diagram of the helical texture in the Ginzburg–Landau regime[3] (this paper is referred to as I hereafter). Since the helical texture breaks the chiral symmetry, the appearance and the existence of the helical texture can be most readily detected by the dramatic splittings in the nuclear magnetic resonance frequencies.[1–3]

We calculated all of these frequencies earlier.[3] However, in the previous calculations we neglected contributions from the anisotropy term of the spin susceptibility. Therefore, although the predicted frequencies are qualitatively correct, they are not quantitatively exact. For example, at $T/T_c = 0.99$, the corrections due the anisotropic term are still negligible (i.e., less than a few percent); on the other hand, at $T/T_c = 0.95$, where the

*Work supported in part by the National Science Foundation under grant number DMR 76-21032.

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resonance frequencies are calculated, these corrections are quite appreciable, although they are always less than 20%.

The purpose of the present paper is two-fold. First, we shall present a spin Lagrangian, which includes the anisotropic term in the spin susceptibility.[4] Second, we shall present our results for the resonance frequencies in terms of this new Lagrangian.

The kinetic energy associated with the spin rotation in ^{3}He-A is given by

$$T = \tfrac{1}{2}(\boldsymbol{\omega} - \boldsymbol{\omega}_0)\boldsymbol{\chi}(\boldsymbol{\omega} - \boldsymbol{\omega}_0) \tag{1}$$

$$= \tfrac{1}{2}\chi_N\{(\alpha_t - \omega_0)^2 + \beta_t^2 + \gamma_t^2 + 2(\alpha_t - \omega_0)\gamma_t \cos\beta - a[\gamma_t + (\alpha_t - \omega_0)\cos\beta]^2\} \tag{2}$$

so here

$$\begin{aligned} \omega_1 &= (\cos\alpha \sin\beta)\gamma_t - (\sin\alpha)\beta_t \\ \omega_2 &= (\sin\alpha \sin\beta)\gamma_t + (\cos\alpha)\beta_t \\ \omega_3 &= \alpha_t + (\cos\beta)\gamma_t \\ \boldsymbol{\omega}_0 &= \omega_0\hat{z} \end{aligned} \tag{3}$$

and α, β, γ are Euler angles, ω_0 is the Larmor frequency, and we assumed that a magnetic field is applied along the z axis.

Here we have introduced an anisotropic spin susceptibility tensor[5] $\boldsymbol{\chi}$ with components

$$\chi_{ij} = \chi_N(\delta_{ij} - a\hat{d}_i\hat{d}_j) \tag{4}$$

which is appropriate for ^{3}He-A, while in our earlier analysis, we neglected the anisotropy term a. In fact in the vicinity of the melting pressure and the transition temperature, a is approximated by[6]

$$a = 4.8(1 - T/T_c) \tag{5}$$

The anistropy term vanishes linearly as T approaches T_c. Therefore, the anisotropy term is completely negligible in the immediate vicinity of the transition temperature T_c.

In terms of Euler angles, the d vector is given by

$$\hat{d} = (\sin\beta \cos\alpha, \sin\beta \sin\alpha, \cos\beta) \tag{6}$$

where in the equilibrium configuration of the helical texture α and β are given by[3]

$$\beta = \theta \quad \text{and} \quad \alpha = kz \tag{7}$$

and θ and k are determined by minimizing the Gibbs free energy in the presence of superflow.

It is very important to note that in the helical texture $\hat{d}$ does not depend on γ. This implies that γ is a cyclic variable.

The present system has a conserved angular momentum

$$P = \gamma_t + (\alpha_t - \omega_0) \cos \beta \tag{8}$$

In the equilibrium configuration (i.e., $\alpha_t = \gamma_t = 0$) we have

$$P = -\omega_0 \cos \theta \tag{9}$$

Then we can eliminate γ_t from T defined in (2). The new kinetic energy T^* is given by

$$\begin{aligned} T^* &\equiv T - \gamma_t(\partial T/\partial \gamma_t) \\ &= \tfrac{1}{2}\chi_N[(\alpha_t - \omega_0)^2 \sin^2 \beta + \omega_0^2 + \beta_t^2 - (1-a)P^2 \\ &\quad + 2(1-a)P(\alpha_t - \omega_0) \cos \beta] \end{aligned} \tag{10}$$

Now small fluctuations of $\hat{d}$ around the equilibrium configuration are parameterized by

$$\beta = \theta + g \qquad \text{and} \qquad \alpha = kz + f \tag{11}$$

where g and f are small parameters.

The equations of motion of g and f are obtained from the effective Lagrangian density

$$L = T^* - \tfrac{1}{2}\chi_N \Omega_A^2 G \tag{12}$$

where G is the normalized Gibbs free energy[2] in the presence of superflow and Ω_A if the Leggett frequency,

$$\begin{aligned} G = &-\frac{p^2}{1+s} + \frac{1}{2}(3-2s)\chi_z^2 + (1+s)\beta_z^2 + 2\frac{(1-s)^{1/2}}{1+s} p\psi_z \\ &+ s\left(\frac{2}{1+s} - \frac{1}{2}\right)\psi_z^2 + (\sin^2 \beta)(1+s)\alpha_z^2 + 1 \\ &- [\cos \chi \cos \beta + \sin \chi \sin \beta \cos(\psi - \alpha)]^2 \end{aligned} \tag{13}$$

Here $s = \sin^2 \chi$, and χ and ψ are the angles describing the spatial orientation of the $\hat{l}$ vector:

$$\hat{l} = (\sin \chi \cos \psi, \sin \chi \sin \psi, \cos \chi) \tag{14}$$

In (13) we have dropped the term $h^2 \cos^2 \beta$ for the magnetic anisotropy energy which was previously present in the free energy functional,[2] because it is already contained in T^* in the present formulation.

From (12) we obtain

$$f_{tt}\sin^2\theta-\omega_0(1+a)(\sin\theta\cos\theta)g_t=-\tfrac{1}{2}\Omega_A^2\,\partial G/\partial f \tag{15}$$

$$\begin{aligned}g_{tt}-\omega_0^2 a\sin\theta\cos\theta+\omega_0^2(\sin^2\theta-a\cos^2\theta)g&\\ +\omega_0(1+a)(\sin\theta\cos\theta)f_t&=-\tfrac{1}{2}\Omega_A^2\,\partial G/\partial g\end{aligned} \tag{16}$$

where we have linearized the left-hand sides of (15) and (16) in f and g.

Substituting the expression for G into (15) and (16), we obtain a system of coupled linear equations for f and g. In particular, the resonance frequencies are now determined by the dispersion relation

$$\begin{aligned}[\omega^2-L_{11}\Omega_A^2][\omega^2-\omega_0^2(\sin^2\theta-a\cos^2\theta)-L_{22}\Omega_A^2]&\\ -[\omega\omega_0(1+a)-L_{12}\Omega_A^2]^2\cos^2\theta=0&\end{aligned} \tag{17}$$

where

$$\begin{aligned}L_{11}&=\frac{\sin\chi\cos(\chi-\theta)}{\sin\theta}+(1+s)q^2\\ L_{22}&=\frac{\sin 2\chi}{\sin 2\theta}+h^2\cos 2\theta+(1+s)q^2\\ L_{12}&=2kq(1+s)\end{aligned} \tag{18}$$

and $h=H/H_0$, $H_0=27.44$ Oe, and q is the wave vector of the spin wave. The expressions for L_{11}, etc. have been determined previously,[2] but we have corrected the sign of the $h^2\cos 2\theta$ term in L_{22}. Note, however, that the change thus introduced is exactly of the same order of smallness as the anisotropy term of the spin susceptibility (because $\Omega_A^2 h^2=a\omega_0^2$), which had been neglected anyway. Equation (17) reduces to the earlier equation[2] in the limit $a\to 0$. Here we have used $H_0=27.44$ Oe, which is consistent with a finite anisotropy term given in (5), rather than $H_0=20$ Oe used before for zero anistropy. The corresponding "dipole velocity" v_{s0} is then given by 1.37 mm/sec.

Very recently Bromley[7] derived an equation for the resonance frequencies of the helical texture very similar to (17), starting from the Leggett equation. A direct comparison between actual numerical results of the two works is difficult, however, because of the different physical situations considered; e.g., Bromley has not minimized the free energy with respect to the helix pitch k. While in a torus this can be justified by arguing that k will be restricted by the quantization condition imposed on the texture by the size of the torus, one cannot expect it to be correct in the bulk situation (which we considered[2]) because then one does not obtain the equilibrium configuration, i.e., lowest energy configuration. One of the consequences is that, as we showed,[2] the true equilibrium configuration in the bulk does not allow for large-angle helices as obtained by Bromley.[7] Although Bromley's criticism[7] of our earlier calculations (where we neglected the anisotropic contribution to the susceptibility) is valid in principle, it should be pointed out that at the temperatures at which he presents his numerical results (i.e., $T/T_c = 0.99$ and 0.998) the anisotropic contribution is negligibly small. In this particular temperature region our previous results are sufficiently accurate (the error is less than a few percent).

This can be seen for example, by investigating the spin wave dispersion relation (17) in the vicinity of the phase boundary between the uniform and the helical texture. In this region the inclination angles χ and θ are small and (17) can be solved exactly. One obtains

$$\omega = [\tfrac{1}{4}\omega_0^2(1+a)^2 + \Omega_A^2(\lambda_l + q^2 \mp 2kq)]^{1/2} \pm \tfrac{1}{2}\omega_0(1+a) \tag{19}$$

where[2] $\lambda_l = 1 + k^2 - h^2$ is the ratio of χ and θ near the phase boundary. Making use of the fact[3] that the longitudinal rf field picks up the spin wave with $q = 0$, while the transverse rf field couples to the spin mode with $q = \pm k$, one obtains two longitudinal and four transverse resonance frequencies (two

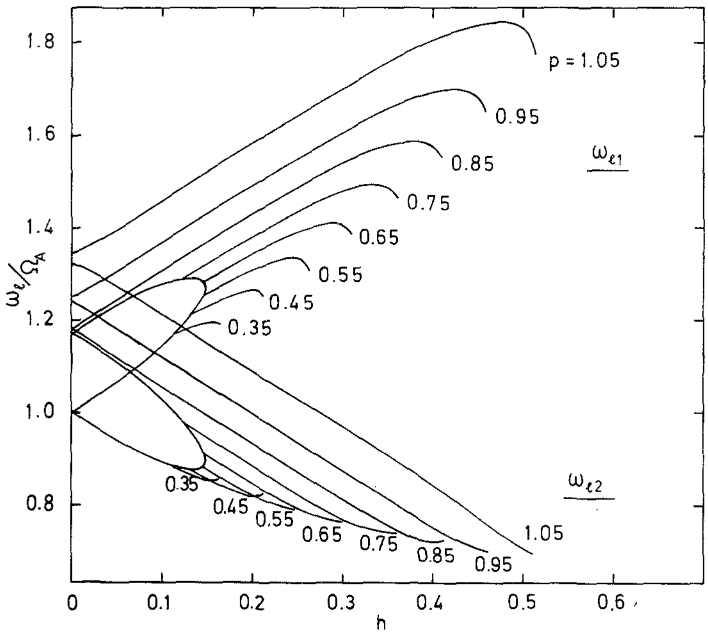

Fig. 1. The longitudinal resonance frequencies ω_{l1} and ω_{l2} of the helical texture for fixed superflow p as functions of the reduced magnetic field h at $T = 0.95T_c$.

different modes for each of the two different polarizations, i.e., helicities). For the longitudinal frequencies (19) yields

$$\omega_{l1} = [\lambda_l \Omega_A^2 + \tfrac{1}{4}\omega_0^2(1+a)^2]^{1/2} \pm \tfrac{1}{2}\omega_0(1+a) \tag{20}$$

and for the transverse modes

$$\begin{aligned}
\omega_{t1}^{+} &= [\lambda_t \Omega_A^2 + \tfrac{1}{4}\omega_0^2(1+a)^{1/2} + \tfrac{1}{2}\omega_0(1+a) \\
\omega_{t2}^{+} &= [\Omega_A^2 + \tfrac{1}{4}\omega_0^2(1-a)^2]^{1/2} - \tfrac{1}{2}\omega_0(1+a) \\
\omega_{t1}^{-} &= [\Omega_A^2 + \tfrac{1}{4}\omega_0^2(1-a)^2]^{1/2} + \tfrac{1}{2}\omega_0(1+a) \\
\omega_{t2}^{-} &= [\lambda_t \Omega_A^2 + \tfrac{1}{4}\omega_0^2(1+a)^2]^{1/2} - \tfrac{1}{2}\omega_0(1+a)
\end{aligned} \tag{21}$$

where $\lambda_t = \lambda_l + 3k^2$. If we compare these results with the previous ones in I we can see that the anisotropic contribution to the spin susceptibility only appears in the terms $\omega_0(1 \pm a)$, which clearly shows that its contribution is unimportant in the immediate vicinity of T_c.

Equation (17) allows us to calculate the six resonance frequencies for the whole stability region of the helical texture in the p–h phase diagram.[3] We present the numerical results for the resonance frequencies at $T = 0.95T_c$ in Figs. 1 and 2 as functions of the reduced magnetic field h ($\equiv H/H_0$) for fixed p [$\equiv -s(1-s)^{1/2}k + (1+s)v_s/v_{s0}$], the reduced mass superflow. In Fig. 1 the two branches of the longitudinal resonance frequency are shown, while in Fig. 2 the two branches of the transverse resonance frequencies with different polarization (two modes each) are depicted.

In particular, in Fig. 2b, where ω_t^- is shown, the frequency axis has been split up so that the two branches of ω_{t1}^- and ω_{t2}^- can be viewed separately. Also, fewer curves have been drawn, to avoid confusion. This is necessary because ω_{t1}^- initially decreases while ω_{t2}^- increases, so it appears as if both modes intersected. This, however, is not the case: the branches repel each other (but come very close to each other), as can be seen, for example, for $p = 1.05$ in the insert in Fig. 2b, where an enlarged section of the hybridization region is shown, using the same ω scale. Repulsion rather than crossing takes place because both modes are coupled and have the same symmetry.

Comparing the corresponding figures in I (i.e., Figs. 3 and 4), we note that the anisotropic term has an appreciable effect only in the higher field region ($h \leqslant 0.5$). In general, the anisotropy term pushes the higher frequency branches even higher while the lower frequency ones are pushed further down.

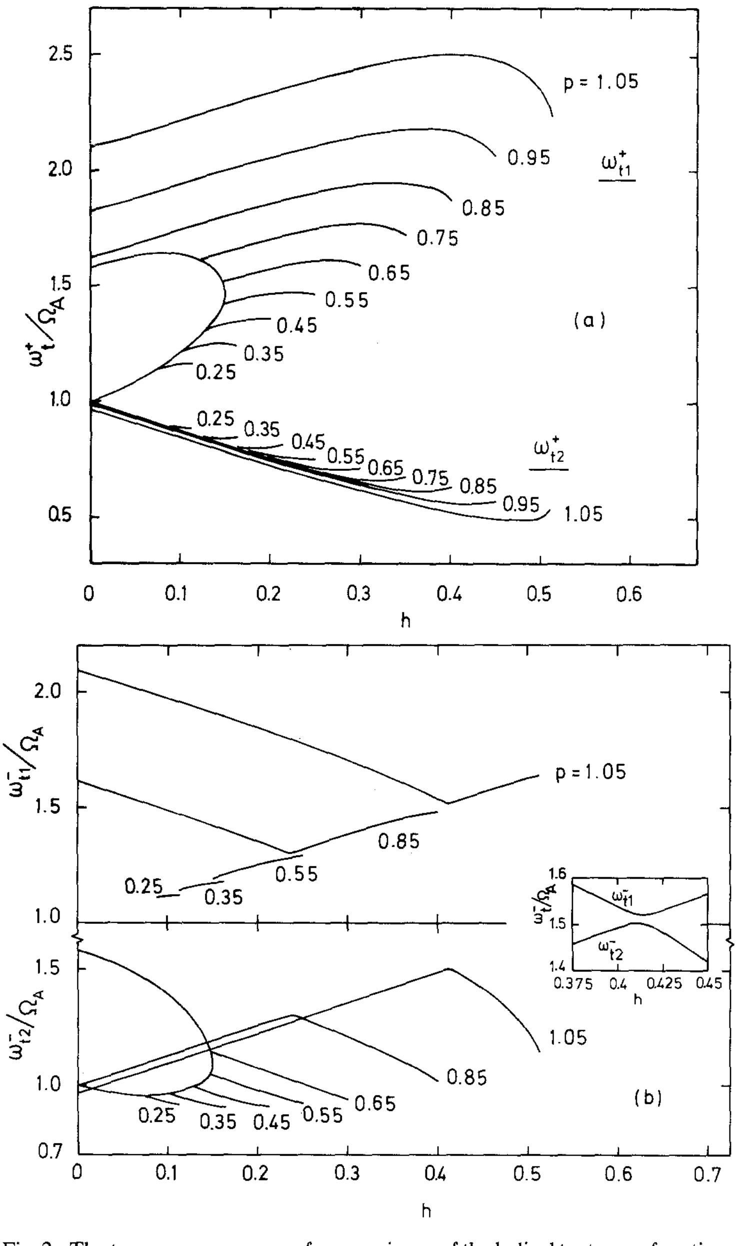

Fig. 2. The transverse resonance frequencies ω_t of the helical texture as functions of the reduced magnetic field h for fixed superflow p at $T = 0.95T_c$: (a) the two branches of ω_t^+, coupling to M^+; (b) the two branches of ω_t^-, coupling to M^-. For reasons of clarity different ω axes have been used in the case of ω_t^- to separate the branches (see text).

ACKNOWLEDGMENTS

One of us (KM) would like to thank I. Fomin for a discussion of possible effects of the anisotropy term on the spin dynamics in ^{3}He-A. One of us (DV) acknowledges discussions with D. Bromley on his paper.

REFERENCES

1. Y. R. Lin-Liu, K. Maki, and D. Vollhardt, *J. Phys. (Paris) Lett.* **39**, L-381 (1978).
2. Y. R. Lin-Liu, D. Vollhardt, and K. Maki, *Phys. Rev.* **20**, 159 (1979).
3. D. Vollhardt, Y. R. Lin-Liu, and K. Maki, *J. Low Temp. Phys.* **37**, 627 (1979).
4. K. Maki, *Phys. Rev. B* **22**, 3493 (1980).
5. A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
6. D. N. Paulson, M. Krusius, and J. C. Wheatley, *Phys. Rev. Lett.* **36**, 1322 (1976).
7. D. J. Bromley, *Phys. Rev. B* **21**, 2754 (1980).