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# FLOW-INDUCED SOLITON LATTICE IN SUPERFLUID $^3$He-A ☆

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It is shown that in the presence of superflow, $\mathbf{v}_s$, parallel to a strong magnetic field $H$ ($|H| \gg 20$ Oe), the uniform texture becomes unstable against the formation of a one dimensional soliton lattice for $\mathbf{v}_s \gg \mathbf{v}_{c2}$ ($\approx 1$ mm/s). The period of the resulting soliton lattice as well as the related NMR frequencies are determined as functions of $\mathbf{v}_s$.

We have shown recently [1] that in a parallel geometry ($H \parallel \mathbf{v}_s$) and in a strong magnetic field $H$ ($|H| \gg 20$ Oe), the uniform texture becomes unstable against formation of domain walls (i.e. $\hat{l}$-solitons) when $v_s$, the superfluid velocity, exceeds $v_{c2}$ ($\equiv 0.893$ $(\hbar\xi_\perp^{-1}/2m)$), where $\xi_\perp$ ($\approx 10\ \mu$) is the dipole coherence distance and $m$ is the mass of the $^3$He atom.

When the superfluid velocity is further increased, a regular soliton lattice is formed with lower Gibbs energy than the uniform texture. The purpose of this letter is to report on the structure of the soliton lattice and associated NMR signals.

As in the earlier work [1], we assume that the $\hat{l}$ and $\hat{d}$ configurations depend only on $z$, the coordinate parallel to the superflow. Here $\hat{l}$ indicates the symmetry axis of the quasi-particle energy gap, while $\hat{d}$ is the spin component of the $^3$He-A order parameter. Then the Gibbs free energy density in the presence of superflow $\mathbf{v}_s$ is given in the Ginzburg–Landau regime (i.e., $T \gtrsim T_c$) as [1]:

$$\mathcal{G} = \frac{1}{2}A\left\{-\frac{2q^2}{1+s} + \frac{s(3-s)\gamma_z^2}{1+s} - \frac{4(1-s)^{1/2}q\gamma_z}{1+s} + (3-2s)\chi_z^2 + 2(1+s)\phi_z^2 + 4\xi_\perp^{-2}[1 - s\cos^2(\gamma-\phi)]\right\}, \tag{1}$$

where

$$A = \frac{3}{5}\left(\frac{N}{8m^*}\right)\frac{7\zeta(3)}{(2\pi T_c)^2}\Delta_0^2, \quad q = \frac{2m}{\hbar}v_s, \quad s = \sin^2\chi, \tag{2}$$

and the suffix $z$ on $\chi$, $\gamma$ and $\phi$ implies the derivative with respect to $z$.

Here we have parameterized $\hat{l}$ and $\hat{d}$ as

$$\hat{l} = \sin\chi\,(\cos\gamma\hat{x} + \sin\gamma\hat{y}) + \cos\chi\hat{z}\,,$$
$$\hat{d} = \cos\phi\hat{x} + \sin\phi\hat{y}\,. \tag{3}$$

(Note that in the equilibrium configuration $\hat{d}$ is perpendicular to $H$.)

In the following we shall limit our consideration to a pure $\hat{l}$ texture with localized twist at the center of the soliton, as this type of domain wall has the smallest energy and this energy becomes negative when $v_s > v_{c2}$. For these types of textures the Gibbs free energy per unit length is given by

$$g = \frac{A}{2L_0}\left\{\int_0^{L_0} dz\left[(1+2\cos^2\chi)\chi_z^2 - q^2\frac{\cos^2\chi}{1+\sin^2\chi} + 4\xi_\perp^{-2}\cos^2\chi\right] - 4\pi q\right\}, \tag{4}$$

where we have put $\phi = 0$ and $\gamma_z = \pi\delta(z - \frac{1}{2}L_0)$. Here we have assumed that $\chi$ is a periodic function of $z$ with period $L_0$. In the definition of $g$ we have substracted the Gibbs energy corresponding to the uniform texture ($g_0 = \frac{1}{2}Aq^2$). Therefore $g < 0$ means that the uniform texture is thermodynamically unstable against forma-

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tion of a soliton lattice. The Euler–Lagrange equation for $\chi$ is easily integrated as

$$\chi_z^2 = 4\xi_\perp^{-2}(1+2\cos^2\chi)^{-1} \times \left[k^2 + \cos^2\chi\left(1 - \frac{1}{4}\frac{(q\xi_\perp)^2}{1+\sin^2\chi}\right)\right], \tag{5}$$

where $k$ is an integral constant. Then $g$ is rewritten as

$$g = 2A\xi^{-1}\left\{\frac{1}{L_0}\int_0^\pi d\chi\,(1+2\cos^2\chi)^{1/2}(k^2+F)^{1/2} - \pi(q\xi_\perp)\,L_0^{-1} - \xi_\perp^{-1}k^2\right\}, \tag{6}$$

and

$$L_0 = \tfrac{1}{2}\xi_\perp \int_0^\pi d\chi\,(1+2\cos^2\chi)^{1/2}(k^2+F)^{-1/2}, \tag{7}$$

where

$$F = \cos^2\chi\,[1 - \tfrac{1}{4}(q\xi_\perp)^2(1+\sin^2\chi)^{-1}]. \tag{8}$$

Minimizing $g$ with respect to $k^2$, we find

$$g = -2A\xi_\perp^{-2}k^2,$$

and

$$\int_0^\pi d\chi\,(1+2\cos^2\chi)^{1/2}(k^2+F)^{1/2} = \pi(q\xi_\perp). \tag{9}$$

Eq. (9) is solved numerically. For $q\xi_\perp \leqslant 10$ the result is very well approximated by

$$k^2 = 0.6363\,[(q\xi_\perp)^2 - (0.885)^2]. \tag{10}$$

As expected $k$ starts to increase from 0 at $q\xi_\perp = q_{c2}\xi_\perp$ ($\equiv 0.885$), implying a second-order transition into the soliton lattice. The free energy $g$ and the soliton density $N$ ($\equiv L_0^{-1}$) are shown in fig. 1 as functions of $q\xi_\perp$. The soliton density increases rapidly *from 0* when $q$ becomes larger than $q_{c2}$ and reaches the order of $\xi_\perp^{-1}$ ($\approx 10^3$ cm$^{-1}$) where $q\xi_\perp \approx 3$. Then $N$ increases almost linearly with $q\xi_\perp$ ($N = 0.329\,(q\xi_\perp)$).

The appearance of the soliton lattice may be most easily detected by the nuclear magnetic resonance. The small $\hat{d}$ oscillation around the equilibrium configuration is parameterized as

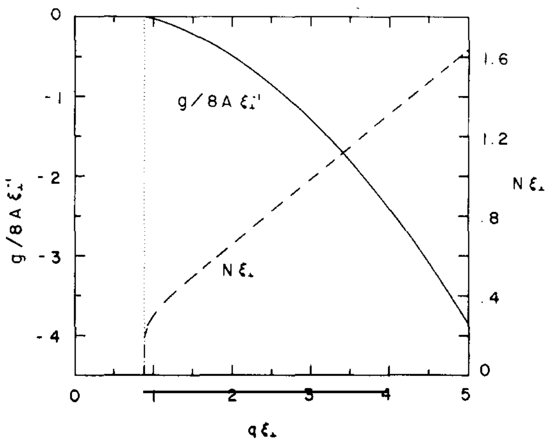


Fig. 1. The normalized Gibbs free energy $g$ of the soliton lattice and the soliton density $N$ are shown as functions of the superflow, $q\xi_\perp$ ($\equiv v_s/v_0$) with $v_0 \equiv (\hbar/2m)\,\xi_\perp^{-1} \approx 0.8$ mm/s.

$$\hat{d} = (\cos f\hat{x} + \sin f\hat{y})\cos g + \sin g\hat{z}. \tag{11}$$

Then from the fluctuation free energy, which is quadratic in both $f$ and $g$, we can construct the eigenequations [2]

$$\lambda_f f = -\frac{1}{2}\xi_\perp^2\frac{\partial}{\partial z}[(1+\sin^2\chi)f_z] + \sin^2\chi f,$$
$$\lambda_g g = -\frac{1}{2}\xi_\perp^2\frac{\partial}{\partial z}[(1+\sin^2\chi)g_z] + (1-2\cos^2\chi)g, \tag{12}$$

where $\chi$ describes the equilibrium $\hat{l}$ texture. The longitudinal and transversal resonance frequencies are then

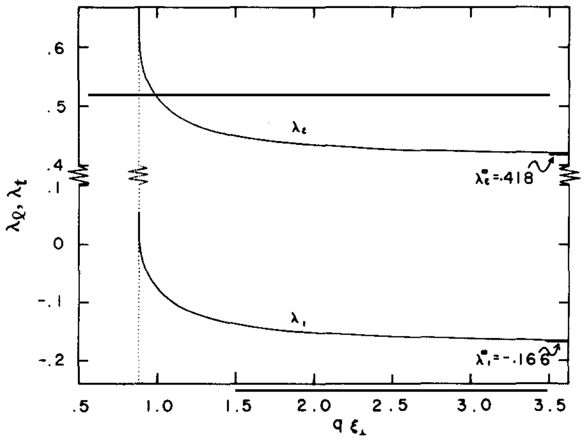


Fig. 2. Eigenvalues $\lambda_f$ and $\lambda_g$, which appear in the expressions of NMR frequencies, are shown as functions of $q\xi_\perp$.

expressed in terms of eigenvalues $\lambda_f$ and $\lambda_g$ as:

$$\omega_\ell = (\lambda_f)^{1/2}\Omega_A\,, \quad \omega_t = (\omega_0^2 + \lambda_g \Omega_A^2)^{1/2}\,, \tag{13}$$

where $\Omega_A$ is the Leggett frequency [3] in $^3$He-A and $\omega_0$ is the Larmor frequency.

The lowest eigenvalues for $\lambda_f$ and $\lambda_g$ are obtained numerically and plotted in fig. 2 as functions of $q\xi_\perp$. In the uniform texture we have $\lambda_f = \lambda_g = 1$. At $q = q_{c2}$, both $\lambda_f$ and $\lambda_g$ drop to smaller values indicating the appearance of the soliton lattice. Then both $\lambda_f$ and $\lambda_g$ decrease monotonically to their limiting values ($\lambda_f^\infty$ = 0.418 and $\lambda_g^\infty = -0.166$) as $q$ is increased.

When $q\xi_\perp \geqslant 2$ the resonance should be exhausted by the lowest eigenmodes [4]. However, in the intermediate region ($0.885 < q\xi_\perp < 2$) it is expected that a series of resonances appear both for the longitudinal and the transversal resonance. The details will be published elsewhere.

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