

## Depairing critical currents in superfluid $^3\text{He}$

Dieter Vollhardt, Kazumi Maki

### Angaben zur Veröffentlichung / Publication details:

Vollhardt, Dieter, and Kazumi Maki. 1978. "Depairing critical currents in superfluid  $^3\text{He}$ ." *Journal of Low Temperature Physics* 31 (3-4): 457-74.  
<https://doi.org/10.1007/bf00115096>.

### Nutzungsbedingungen / Terms of use:

licgercopyright

Dieses Dokument wird unter folgenden Bedingungen zur Verfügung gestellt: / This document is made available under the following conditions:

#### Deutsches Urheberrecht

Weitere Informationen finden Sie unter: / For more information see:

<https://www.uni-augsburg.de/de/organisation/bibliothek/publizieren-zitieren-archivieren/publizieren>



# Depairing Critical Currents in Superfluid $^3\text{He}^*$

Dieter Vollhardt and Kazumi Maki

*Department of Physics, University of Southern California, Los Angeles, California*

*Critical currents in both superfluid  $^3\text{He-A}$  and  $^3\text{He-B}$  are calculated within a weak coupling model. The Fermi liquid correction is explicitly included. As a by-product we obtain the nonlinear superfluid densities in  $^3\text{He-A}$  and  $^3\text{He-B}$ , which depend strongly on the current.*

## 1. INTRODUCTION

The superfluidity<sup>1,2</sup> is certainly one of the most remarkable characteristics of the new phases in liquid  $^3\text{He}$  below 3 mK. In the present paper we will study theoretically the current-dependent features of superfluid  $^3\text{He}$  when a (large) uniform current (with flow velocity  $v_s \approx 10^{-3}-10^{-2} v_F$ ) is applied to the system. In particular we will show that superfluidity can be completely destroyed by a large current (i.e., a current larger than the depairing critical current), although to our knowledge experiments in the presence of such large a current have not been done yet. However, we find that even a small current affects significantly the quasiparticle energy gap as well as the superfluid density. Therefore, it is certainly of practical interest to study the effects of a uniform current on the properties of the superfluid phases of  $^3\text{He}$ , since even a small temperature gradient introduces an appreciable superflow in liquid  $^3\text{He}$ . It may be of importance to distinguish from the outset two kinds of critical currents; the "textural" critical currents<sup>3</sup> and the depairing critical currents. The former is defined as the current that induces a drastic modification in textures associated with the condensate. The associated critical velocity is of the order of  $\Omega_A/p_F$  in  $^3\text{He-A}$ , where  $\Omega_A$  is the Leggett dipolar shift in NMR frequencies of  $^3\text{He-A}$  and  $p_F$  is the Fermi momentum. On the other hand, the depairing critical current is of the order of  $\Delta(T)/p_F$ , where  $\Delta(T)$  is the quasiparticle energy gap.

\*Work supported by National Science Foundation under Grant No. DMR76-21032.

Therefore the depairing critical current is roughly by a factor  $10^2$  larger than the textural critical current in  $^3\text{He-A}$ . In such a large current we expect a variety of nonlinear effects as in the case of superconductivity.<sup>4,5</sup> The analogy is particularly useful for  $^3\text{He-B}$ . In spite of the difference between the singlet pairing in the BCS superconductor and the triplet pairing in the Balian–Werthamer state,<sup>6</sup> which characterizes the condensate of  $^3\text{He-B}$ , we have the same expressions for the energy gap and the superfluid density in both cases, except that in superfluid  $^3\text{He}$  the Fermi liquid correction is significant and has to be included in the theoretical analysis.

An advantage of considering such a large current is that we are now most likely dealing with a uniform texture (see, e.g., Ref. 7); in particular in  $^3\text{He-A}$ , we may assume that  $\mathbf{v}_s \parallel \hat{\ell}$ , where  $\hat{\ell}$  indicates the direction of the symmetry axis of the energy gap in  $^3\text{He-A}$ . However, recently Hall and Hook<sup>8</sup> suggested (based on their numerical analysis) that uniform  $\hat{\ell}$  textures are unstable under large superfluid current. We have looked into this possibility. However, contrary to them the uniform texture in the bulk appears to be stable at least in the vicinity of the transition temperature even under large current (see also Bhattacharyya *et al.*<sup>9</sup>). Furthermore, in contrast to the case of a superconductor, a uniform current situation appears quite natural in superfluid  $^3\text{He}$ , especially when the current is relatively large. In the following we will calculate the quasiparticle energy gap  $\Delta(T)$  and the superfluid density in the presence of a uniform current in both  $^3\text{He-A}$  (with  $\mathbf{v}_s \parallel \hat{\ell}$ ) and  $^3\text{He-B}$ .

Following the formalism developed by Maki and Tsuneto,<sup>5</sup> we introduce the effect of a uniform current as a frequency shift in the single-particle Green's function. The effects of the Fermi liquid correction are then included by renormalizing the quasiparticle mass and the superfluid velocity as done by Leggett.<sup>10</sup> Furthermore, we adopt the weak coupling model for simplicity, since the results can be comparable to experiments at least at semiquantitative levels. In order to have some insight into the qualitative features, the weak coupling model is quite adequate, except possibly for the stability question of the axial state.<sup>11</sup> In particular in the zero current limit ( $\mathbf{v}_s = 0$ ) the superfluid densities obtained agree with the previous results by Combescot.<sup>12</sup>

The calculated superfluid densities in  $^3\text{He-A}$  and  $^3\text{He-B}$  are strongly nonlinear in the superfluid velocity  $\mathbf{v}_s$ , mostly due to the large Fermi liquid correction. The results appear to be readily accessible experimentally. For example, it may be seen as the amplitude dependence of the fourth-sound velocity  $c_4$ . Furthermore, although the calculated critical currents vary monotonically as functions of temperature, the calculated (depairing) critical velocities  $v_{sc}$  exhibit nonmonotonic temperature dependence in both  $^3\text{He-A}$  and  $^3\text{He-B}$ , again due to the Fermi liquid correction. (In the absence of the Fermi liquid correction, the critical velocities behave

monotonically though!) This could probably be seen in the ion mobility experiments, since we expect that the ion mobility should change dramatically when the ion velocity exceeds the depairing critical velocity. In the experimental data on negative ion mobilities in  $^3\text{He-A}$  and  $^3\text{He-B}$  published by Ahonen *et al.*<sup>13</sup> there is no indication of such a nonmonotonic behavior. However, we hope that experiments on the ion mobilities at lower temperatures will soon resolve this question.

## 2. FORMULATION

The single-particle Green's function for superfluid  $^3\text{He-A}$  and  $^3\text{He-B}$  in the absence of currents is given in the Nambu representation by

$$G_0(\mathbf{p}, \omega_n) = [i\omega_n - \xi\rho_3 - \sigma_1\Delta(p_1\rho_2 - \hat{p}_2\rho_1)]^{-1} \quad (1)$$

and

$$G_0(\mathbf{p}, \omega_n) = [i\omega_n - \xi\rho_3 - \sigma_2\rho_1\Delta(\hat{\mathbf{p}} \cdot \boldsymbol{\alpha})]^{-1} \quad (2)$$

respectively, where

$$\xi = (1/2m^*)p^2 - \mu, \quad \hat{\mathbf{p}} = \mathbf{p}/p_F, \quad \boldsymbol{\alpha} = (\sigma_1\rho_3, \sigma_2, \sigma_3\rho_3) \quad (3)$$

$\Delta$  is the order parameter (which we will refer simply to the energy gap);  $m^*$  and  $\mu$  are the effective mass of the quasiparticle and the chemical potential; and  $\sigma_i$  and  $\rho_i$  are Pauli matrices operating on the ordinary spin space and the particle-hole space, respectively.

Following Maki and Tsuneto,<sup>5</sup> the effect of a uniform current is introduced into the above Green's function by replacing  $i\omega_n$  by  $i\omega_n + \mathbf{v}_s \cdot \mathbf{p}$ , where  $\mathbf{v}_s$  is the superfluid velocity and  $\mathbf{p}$  is the quasiparticle momentum. This replacement gives the correct Green's function in the absence of the Fermi liquid correction and follows from a gauge-like transformation of the condensate order parameter  $\Delta(\mathbf{r}) \rightarrow e^{i\Phi}\Delta(\mathbf{r})$  with  $\Phi = 2m\mathbf{v}_s \cdot \mathbf{r}$ , where  $m$  is the mass of the  $^3\text{He}$  atom.

In the presence of the interaction between quasiparticles the effect of the current will be reduced by  $(1 + \frac{1}{3}F_1\phi)^{-1}$  due to the polarization of the liquid.<sup>10</sup> The above coefficient describes a mean-field screening of the effect of the velocity  $\mathbf{v}_s$ , where  $\phi$  will be determined later. Therefore in this more general situation the effect of the current is introduced into the Green's function by replacing  $i\omega_n$  by  $i\omega_n + \mathbf{v}_s^* \cdot \mathbf{p}$ , where

$$\mathbf{v}_s^* = (1 + \frac{1}{3}F_1\phi)^{-1}\mathbf{v}_s \quad (4)$$

Then the Green's functions in the presence of a uniform current are given by

$$G(\mathbf{p}, \omega_n) = G_0(\mathbf{p}, \omega_n - i s \cos \theta)$$

with

$$s = (1 + \frac{1}{3}F_1\phi)^{-1}v_s p_F \quad (5)$$

where  $G_0(\mathbf{p}, \omega_n)$  is given by Eqs. (1) and (2) for the axial and the isotropic states, respectively. Here  $\theta$  is the angle between  $\mathbf{v}_s$  and  $\mathbf{p}$ .

Within the weak coupling limit the gap equation is given by

$$\Delta_i(\hat{\mathbf{p}}) = -3g_1 T \sum \int \frac{d^3 p'}{(2\pi)^3} (\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}') \times \frac{1}{4} \text{Tr} [\sigma_2(\rho_1 + i\rho_2)\alpha_i G_0(\mathbf{p}', \omega_n - is \cos \theta')] \quad (6)$$

for both  ${}^3\text{He-A}$  and  ${}^3\text{He-B}$ . The above equation is further simplified to

$$1 = 2\pi\lambda T \sum_{n=0}^{n_0} \int \frac{d\Omega}{4\pi} \left( \frac{3}{2} \sin^2 \theta \right) \times \text{Re} \{ [(\omega_n - is \cos \theta)^2 + \Delta^2 \sin^2 \theta]^{-1/2} \} \quad (7)$$

and

$$1 = 2\pi\lambda T \sum_{n=0}^{n_0} \int \frac{d\Omega}{4\pi} \{ \text{Re} [(\omega_n - is \cos \theta)^2 + \Delta^2]^{-1/2} \} \quad (8)$$

for the axial and the isotropic states, respectively, where in the axial state we assumed that  $\hat{l}$  is parallel to  $\mathbf{v}_s$ . Here  $\lambda = N(0)|g_1|$ ,  $\omega_c = \omega_{n_0} = 2\pi T(n_0 + 1/2)$  is the cutoff frequency as in the BCS case, and  $N(0)$  is the density of states at the Fermi level. In the weak coupling limit the cutoff frequency  $\omega_c$  as well as the coupling constant  $\lambda$  can be eliminated from Eqs. (7) and (8) by subtracting the corresponding equations at  $T = T_c$ , the transition temperature, or those at  $T = 0$ . In this case the energy is scaled by  $T_c$  or  $\Delta_{00}$ , the energy gap at  $T = 0$  and  $s = 0$ .

For example, at  $T = 0$ , Eqs. (7) and (8) are further reduced to

$$-\ln (\Delta/\Delta_{00}) = \frac{1}{2} \ln [1 + (s/\Delta)^2] - \frac{1}{2} s^2 / (\Delta^2 + s^2) \quad (9)$$

and<sup>4</sup>

$$-\ln (\Delta/\Delta_{00}) = \theta(s - \Delta) [\ln \{ (s/\Delta) + [(s/\Delta)^2 - 1]^{1/2} \} - [1 - (\Delta/s)^2]^{1/2}] \quad (10)$$

for the axial and the isotropic states, respectively, where  $\theta(x)$  is the step function

$$\begin{aligned} \theta(x) &= 1 & \text{for } x \geq 0 \\ &= 0 & \text{for } x < 0 \end{aligned} \quad (11)$$

Here  $\Delta_{00} = \frac{1}{2} e^{5/6} \Delta_{\text{BCS}}(0)$  and  $\Delta_{00} = \Delta_{\text{BCS}}(0)$  for the axial and the isotropic states, respectively. [Note:  $\Delta_{\text{BCS}}(0) = (\pi/\gamma)T_c \approx 1.76T_c$ .] In general, Eqs.

(7) and (8) determine the energy gap (or more precisely the order parameter) in the presence of a uniform current.

The superfluid mass current  $\mathbf{J}_s$ , due to the superfluid velocity  $\mathbf{v}_s$  is given by

$$\mathbf{J}_s = mN\mathbf{v}_s - \mathbf{J}_n \quad (12)$$

with

$$\mathbf{J}_n = T \sum_n \int \frac{d^3p}{(2\pi)^3} \mathbf{p} \frac{1}{4} \text{Tr} [G_0(\mathbf{p}, \omega_n - is \cos \theta)] \quad (13)$$

which reduces to

$$|\mathbf{J}_s| = 2\pi TN(0)p_F \sum_{n=0}^{\infty} \int_{-1}^1 \frac{dz}{2} z \text{Re} \frac{i\omega_n + sz}{[(\omega_n - isz)^2 + \Delta^2(1-z^2)]^{1/2}} \quad (14)$$

and

$$|\mathbf{J}_s| = 2\pi TN(0)p_F \sum_{n=0}^{\infty} \int_{-1}^1 \frac{dz}{2} z \text{Re} \frac{i\omega_n + sz}{[(\omega_n - isz)^2 + \Delta^2]^{1/2}} \quad (15)$$

for the axial and the isotropic states, respectively, where  $z = \cos \theta$ . In particular, at  $T = 0$  K, Eqs. (14) and (15) are simplified further to

$$\mathbf{J}_s = \rho_s^0 \mathbf{v}_s^* \quad (16)$$

where

$$\rho_s^0/\rho = \Delta^2/(\Delta^2 + s^2) \quad (17)$$

and

$$\rho_s^0/\rho = 1 - \theta(s - \Delta)[1 - (\Delta/s)^2]^{3/2} \quad (18)$$

for the axial and isotropic states, respectively. Here  $\rho = mN$  is the (mass) density of liquid  $^3\text{He}$ .

In terms of  $\rho_s^0/\rho$ , we can now define  $\phi$  introduced in Eq. (4) as

$$\rho_s^0/\rho \equiv 1 - \phi(s) \quad (19)$$

Therefore Eq. (16) may be rewritten as

$$\mathbf{J}_s = \rho_s \mathbf{v}_s$$

with

$$\rho_s/\rho = (1 - \phi)(1 + \frac{1}{3}F_1\phi)^{-1} \quad (20)$$

which is a natural generalization of a similar expression in the linear

regime.<sup>12</sup> At  $T = 0$  K, Eq. (19) indicates that  $\phi$  is given by

$$\phi = s^2/(\Delta^2 + s^2) \quad (17')$$

and<sup>4</sup>

$$\phi = \theta(s - \Delta)[1 - (\Delta/s)^2]^{3/2} \quad (18')$$

for the axial and the isotropic states, respectively.

### 3. LIMITING CASES

In general Eq. (7) [or Eq. (8)] has to be solved for given  $s$ . Then, making use of  $\Delta$  thus determined, we can extract  $\phi(s)$  from Eq. (14) [or Eq. (15)], which enables us to calculate  $\mathbf{v}_s$  as well as  $\mathbf{J}_s$  (or  $\rho_s/\rho$ ). Besides the zero-temperature case described above, there are a few instances where we have relatively simple expressions, although in general the above program can be executed only numerically. We will discuss briefly these simple cases.

#### 3.1. $T \ll T_c$

First let us consider the case of  $^3\text{He-A}$ . Since the quasiparticle spectrum is gapless in  $^3\text{He-A}$ , we expect that both  $\Delta(T)$  and  $\phi(s, T)$  can be expanded in some powers of  $T$ , the temperature. The gap equation (7) can be transformed as

$$\begin{aligned} 1 &= 2\pi\lambda T \sum_{n=0}^{n_0} \int_{-1}^1 dz \frac{3}{4}(1-z^2) \\ &\quad \times \text{Re} \{[(\omega_n - isz)^2 + \Delta^2(1-z^2)]^{-1/2}\} \\ &= 2\pi\lambda T \sum_{n=0}^{n_0} I_A(\omega_n) \end{aligned} \quad (21)$$

where

$$\begin{aligned} I_A(\omega_n) &= \frac{3}{2} \frac{1}{(\Delta^2 + s^2)^{1/2}} \left\{ \left[ 1 - \frac{1}{2} \frac{\Delta^2}{\Delta^2 + s^2} + \frac{1}{2} \frac{(2s^2 - \Delta^2)\omega_n^2}{(\Delta^2 + s^2)^2} \right] \cot^{-1} \frac{\omega_n}{(\Delta^2 + s^2)^{1/2}} \right. \\ &\quad \left. + \frac{1}{2} \frac{\Delta^2 - 2s^2}{(\Delta^2 + s^2)^{3/2}} \omega_n \right\} \end{aligned} \quad (22)$$

Then, making use of the Euler-Maclaurin formula, we can evaluate the right-hand side of Eq. (21) as

$$1 = \lambda \left\{ \ln \frac{\omega_c}{(\Delta^2 + s^2)^{1/2}} + \frac{5}{6} + \frac{1}{2} \frac{1}{\Delta^2 + s^2} \left[ s^2 - s^2 \eta_+ - \frac{7}{30} (\Delta^2 - s^2) \eta_+^2 + 0 \eta_+^3 \right] \right\}$$

or

$$\ln \frac{(\Delta^2 + s^2)^{1/2}}{\Delta_{00}} = \frac{1}{2} \frac{1}{\Delta^2 + s^2} \left[ s^2 - s^2 \eta_+ - \frac{7}{30} (\Delta^2 - s^2) \eta_+^2 + 0 \eta_+^3 \right]$$

with

$$\eta_{\pm} = (\pi T)^2 / (s^2 \pm \Delta^2) \quad (23)$$

[\$\eta\_-\$ appears in Eqs. (33) and (34)].

Similarly, the expression for \$\phi\$ can be transformed as

$$1 - \phi(s) = \frac{2\pi T}{s} \sum_{n=0}^{\infty} J_A(\omega_n) \quad (24)$$

with

$$J_A(\omega_n) = \frac{3}{2} \frac{s\Delta^2}{(\Delta^2 + s^2)^{2,5/2}} \left[ (\Delta^2 + s^2 + 3\omega_n^2) \times \cot^{-1} \left( \frac{\omega_n}{(\Delta^2 + s^2)^{1/2}} \right) - 3\omega_n (\Delta^2 + s^2)^{1/2} \right] \quad (25)$$

The summation over the Matsubara frequency \$\omega\_n\$ yields

$$\phi(s) = \frac{s^2}{\Delta^2 + s^2} + \frac{\Delta^2}{\Delta^2 + s^2} \left( \eta_+ - \frac{7}{15} \eta_+^2 - \frac{31}{105} \eta_+^3 - 0 \eta_+^4 \right) \quad (26)$$

In  $^3\text{He-B}$ , on the other hand, we have to consider two regions separately at  $T \approx 0$  K. For  $\Delta - s > 0$ , the quasiparticle spectrum has a nonvanishing energy gap and the temperature-dependent corrections are exponentially small; we have for the order parameter and \$\phi\$

$$\Delta \approx \Delta_{00} \left[ 1 - \left( \frac{2\pi T}{\Delta} \right)^{1/2} \frac{T}{s} \sinh \left( \frac{s}{T} \right) e^{-\Delta/T} \right] \quad (27)$$

and

$$\phi = 3 \left( \frac{2\pi T}{\Delta} \right)^{1/2} \frac{\Delta T}{s^2} \left[ \cosh \left( \frac{s}{T} \right) - \frac{T}{s} \sinh \left( \frac{s}{T} \right) \right] e^{-\Delta/T} \quad (28)$$

In the second regime,  $\Delta - s < 0$ , the energy spectrum is gapless. Then we can exploit a similar method used for deriving \$\Delta\$ and \$\phi\$ in  $^3\text{He-A}$ ; Eqs. (8) and (15) reduce to

$$1 = 2\pi\lambda T \sum_{n=0}^{n_0} I_B(\omega_n) \quad (29)$$

and

$$1 - \phi = (2\pi T/s) \sum_{n=0}^{\infty} J_B(\omega_n) \tag{30}$$

with

$$I_B(\omega_n) = \frac{1}{s} \operatorname{Im} \left( \sinh^{-1} \frac{is - \omega_n}{\Delta} \right) \tag{31}$$

and

$$J_B(\omega_n) = -\frac{3}{2} \left( \frac{\Delta}{s} \right)^2 \operatorname{Im} \left\{ \frac{\omega_n + is \left[ \left( \frac{\omega_n - is}{\Delta} \right)^2 + 1 \right]^{1/2}}{\Delta} - \sinh^{-1} \frac{is - \omega_n}{\Delta} \right\} \tag{32}$$

respectively.

Then, making use of the Euler–Maclaurin formula again, we obtain for  $T \ll T_c$  (and for  $s > \Delta$ )

$$-\ln \frac{\Delta_0}{\Delta_{00}} = \cosh^{-1} \left( \frac{s}{\Delta} \right) - \left[ 1 - \left( \frac{\Delta}{s} \right)^2 \right]^{1/2} \left[ 1 - \frac{1}{6} \eta - + \frac{7}{360} \frac{2s^2 + \Delta^2}{s^2 - \Delta^2} \eta^2 + 0\eta^3 \right] \tag{33}$$

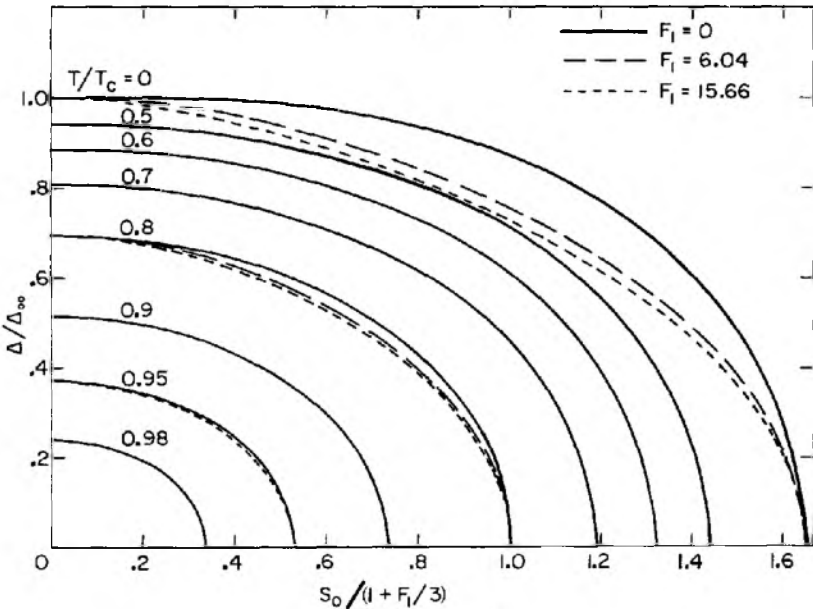


Fig. 1. The order parameter  $\Delta$  of  $^3\text{He-A}$  v.  $s_0/(1 + \frac{1}{3}F_1)$  for several reduced temperatures. The solid curves are for  $F_1 = 0$  (no Fermi liquid correction). The dashed curves are for  $F_1 = 6.04$  and  $F_1 = 15.66$  with  $T/T_c = 0, 0.8,$  and  $0.95$ .

and

$$\phi = \left[ 1 - \left( \frac{\Delta}{s} \right)^2 \right]^{3/2} + \left[ 1 - \left( \frac{\Delta}{s} \right)^2 \right]^{1/2} \eta_- \left[ 1 + \frac{7}{60} \frac{4s^2 - \Delta^2}{s^2 - \Delta^2} \eta_- + 0\eta_-^2 \right] \quad (34)$$

respectively, where  $\eta_-$  has already been defined in Eq. (23).

In both  $^3\text{He-A}$  and  $^3\text{He-B}$ , the temperature-dependent corrections start with  $T^2$ , as it should be in the gapless regime.

### 3.2. $T \approx T_c$

In the vicinity of the transition temperature (i.e., the Ginzburg-Landau regime), we can expand Eqs. (7), (8), (14), and (15) in powers of  $\Delta$ , the order parameter. In  $^3\text{He-A}$  we have then

$$\Delta^2 = \Delta_0^2(T) - \frac{1}{2}s^2 \quad (35)$$

and

$$1 - \phi = \frac{14}{5} \zeta(3) \left( \frac{\Delta}{2\pi T} \right)^2 = \left( 1 - \frac{T}{T_c} \right) - \frac{7}{5} \zeta(3) \left( \frac{s}{2\pi T_c} \right)^2 \quad (36)$$

where

$$\Delta_0(T) = \left[ \frac{10}{7\zeta(3)} \right]^{1/2} \left( 1 - \frac{T}{T_c} \right)^{1/2} (\pi T_c) \quad (37)$$

is the energy gap in the absence of the current.

Similarly, in  $^3\text{He-B}$  we have

$$\Delta^2 = \Delta_0^2(T) - \frac{2}{3}s^2 \quad (38)$$

and

$$1 - \phi = 7\zeta(3) \left( \frac{\Delta}{2\pi T} \right)^2 = 2 \left( 1 - \frac{T}{T_c} \right) - \frac{14}{3} \zeta(3) \left( \frac{s}{2\pi T_c} \right)^2 \quad (39)$$

where

$$\Delta_0(T) = \left[ \frac{8}{7\zeta(3)} \right]^{1/2} \left( 1 - \frac{T}{T_c} \right)^{1/2} (\pi T_c) \quad (40)$$

In a more general situation Eqs. (7), (8), (14), and (15) are evaluated numerically. The results are shown in Figs. 1 and 2 for  $^3\text{He-A}$  and Figs. 4 and 5 for  $^3\text{He-B}$ . In Fig. 1 the energy gap  $\Delta$  (or more precisely the order parameter) is plotted as a function of  $s_0/(1 + \frac{1}{3}F_1)$  for several reduced temperatures  $T/T_c$ , where  $s_0 = v_s p_F / \Delta_{00}$ . The solid curves are calculated for  $F_1 = 0$  (no Fermi liquid correction) while the results for  $F_1 = 6.04$  and

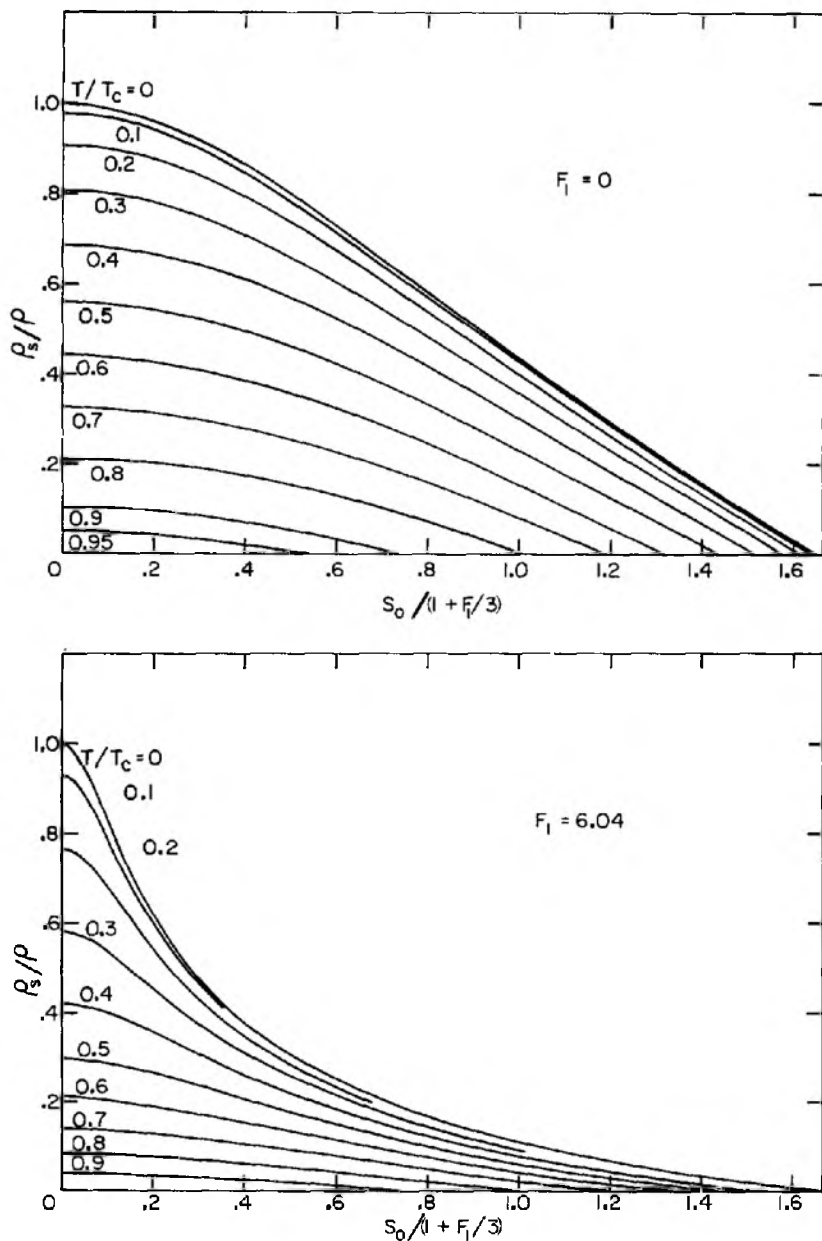


Fig. 2. The superfluid density  $\rho_s$  of  $^3\text{He-A}$  v.  $s_0/(1 + \frac{1}{3}F_1)$  for several reduced temperatures and for  $F_1 = 0$  (no Fermi liquid correction),  $F_1 = 6.04$  (corresponding to zero pressure), and  $F_1 = 15.66$  (corresponding to melting pressure).

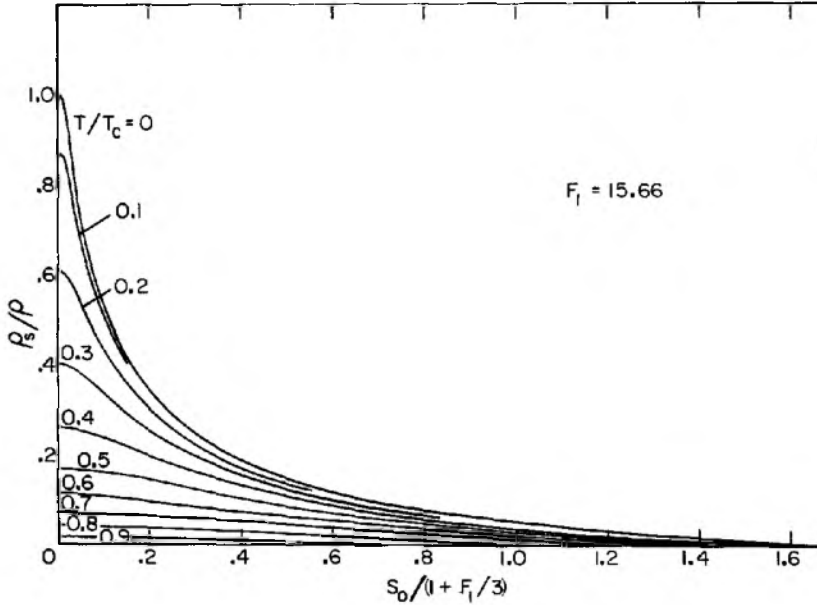


Fig. 2. Continued.

$F_1 = 15.66$  are shown only for  $T/T_c = 0, 0.8$ , and  $0.95$  for comparison. The above  $F_1$  correspond to the case  $P = 0$  and the case at the melting curve, respectively.<sup>14</sup>

If  $\Delta$  is plotted against  $s_0/(1 + \frac{1}{3}F_1)$ , the effect of the Fermi liquid correction is significant only at low temperatures. In the high-temperature regime (i.e.,  $T/T_c > 0.9$ ) the effect of the Fermi liquid correction can be accounted for by simply replacing  $\mathbf{v}_s$  by  $\mathbf{v}_s/(1 + \frac{1}{3}F_1)$  (i.e., the Fermi liquid correction with  $\phi = 1$ ). In Fig. 2 the superfluid density  $\rho_s$  is plotted again as

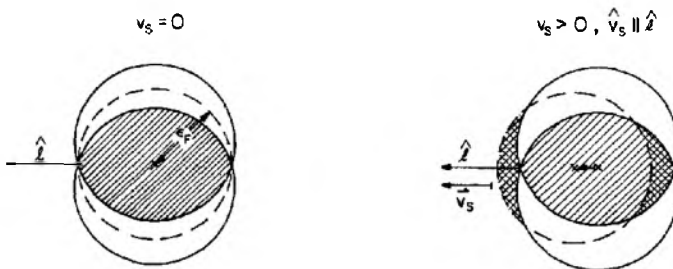


Fig. 3. Anisotropic energy gap of  $^3\text{He-A}$  in the presence of a current for  $v_s = 0$  and  $v_s > 0$ , respectively.

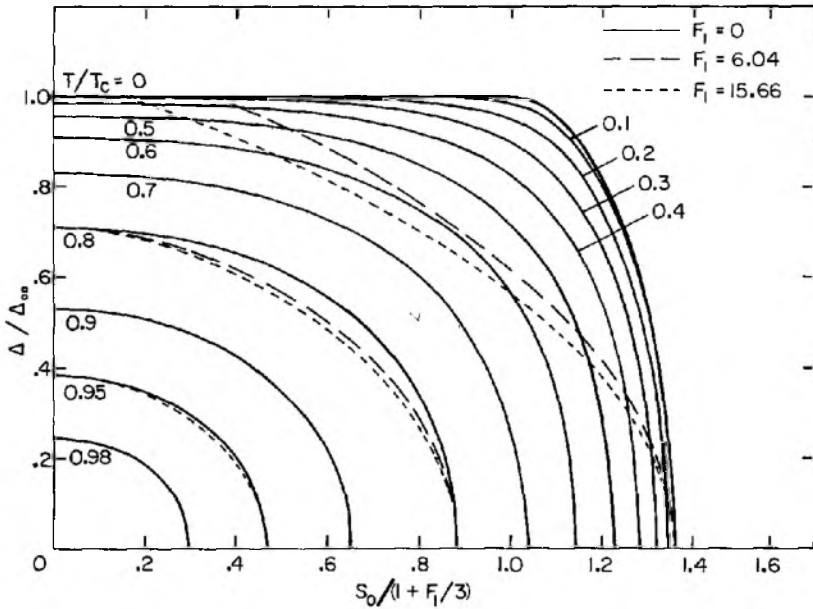


Fig. 4. The order parameter  $\Delta$  of  ${}^3\text{He-B}$  v.  $s_0/(1 + \frac{1}{3}F_1)$ . The solid curves are for  $F_1 = 0$ , while the dashed curves are for  $F_1 = 6.04$  and  $15.66$ .

a function of  $s_0/(1 + \frac{1}{3}F_1)$  for  $F_1 = 0, 6.04,$  and  $15.66$ . We can see immediately that the Fermi liquid correction has a significant effect on  $\rho_s$ . In the presence of the Fermi liquid correction,  $\rho_s$  at lower temperatures decreases rapidly as  $s_0/(1 + \frac{1}{3}F_1)$  increases. Such a strong nonlinearity in  $s_0$  appears readily accessible experimentally. We note further that  $\Delta$  and  $\rho_s$  vary smoothly as  $v_s$  increases. This can be understood by looking at the quasiparticle energy gap of  ${}^3\text{He-A}$  in the presence of a uniform current (see Fig. 3). In  ${}^3\text{He-A}$  the energy gap is anisotropic and the pair-breaking starts continuously from  $v_s = 0$ , implying continuous reductions of  $\Delta$  and  $\rho_s$  even for very small  $v_s$ .

In Fig. 4 the order parameter  $\Delta$  in  ${}^3\text{He-B}$  is shown as a function of  $s_0/(1 + \frac{1}{3}F_1)$  for several reduced temperatures. As in Fig. 1, the solid curves are drawn for the case  $F_1 = 0$ , while results for  $F_1 = 6.04$  and  $15.66$  are shown only for  $T/T_c = 0, 0.8,$  and  $0.95$  for comparison. At low temperatures  $\Delta$  as a function of  $s_0$  has a sharp break where the system becomes gapless. In such regimes the Fermi liquid correction has a rather significant effect on the  $v_s$  dependence of the energy gap. At high temperatures (say  $T/T_c > 0.9$ ) the effect of the Fermi liquid correction can be included by reducing  $v_s$  by a factor  $(1 + \frac{1}{3}F_1)^{-1}$  as in the case of  ${}^3\text{He-A}$ . In Fig. 5, the

superfluid density  $\rho_s$  is shown as a function of  $s_0/(1 + \frac{1}{3}F_1)$  for  $F_1 = 0, 6.04,$  and  $15.66,$  respectively.

As in the case of  $^3\text{He-A},$  the Fermi liquid correction suppresses  $\rho_s$  very rapidly as  $v_s$  increases. Compared with the case of  $^3\text{He-A},$   $\Delta$  and  $\rho_s$  stay constant at  $T = 0$  until  $v_s$  exceeds  $\Delta_{00}/p_F.$  This is easily seen from Fig. 6. At  $T = 0,$  as long as  $v_s < \Delta_{00}/p_F,$  no pair-breaking takes place due to the current. Only when  $v_s$  exceeds this critical value are the quasiparticles excited across the energy gap, drastically reducing both  $\Delta$  and  $\rho_s.$

#### 4. DEPAIRING CRITICAL CURRENT

So far we have considered  $\Delta$  and  $\rho_s$  as functions of  $v_s.$  The superflow is given in terms of  $\rho_s$  as

$$\mathbf{J}_s = \rho_s \mathbf{v}_s \quad (41)$$

Since  $\rho_s$  decreases monotonically as  $v_s$  increases,  $\mathbf{J}_s$  takes the maximal value for  $v_s$  defined by

$$\partial J_s / \partial v_s = 0 \quad (42)$$

where  $J_s = |\mathbf{J}_s|$  and  $v_s = |\mathbf{v}_s|.$  Making use of Eq. (20), we find that Eq. (42) is reduced to

$$d\phi/ds = (1/s)(1 - \phi) \quad (43)$$

which determines the critical velocity  $v_{sc}.$

At  $T = 0 \text{ K},$  Eq. (43) is solved explicitly. For  $^3\text{He-A},$  Eq. (43) together with Eqs. (9) and (17') yields

$$\phi = \sqrt{2} - 1, \quad \text{or} \quad s = (\sqrt{2} - 1)^{1/2} \{ \exp [ \frac{1}{2}(\sqrt{2} - 1) ] \} \Delta_{00} \quad (44)$$

This can be solved for  $v_{sc}$  as

$$v_{sc} = [ 1 + \frac{1}{3}(\sqrt{2} - 1)F_1 ] (\sqrt{2} - 1)^{1/2} \{ \exp [ \frac{1}{2}(\sqrt{2} - 1) ] \} \Delta_{00} / p_F \quad (45)$$

and the critical current

$$J_{sc} = \sqrt{2}(\sqrt{2} - 1)^{3/2} \{ \exp [ \frac{1}{2}(\sqrt{2} - 1) ] \} \Delta_{00} p_F^{-1} \rho \quad (46)$$

It may be of interest to note that the critical current is independent of the Fermi liquid correction, while the critical superflow velocity does depend on  $F_1.$  For  $^3\text{He-B},$  a similar analysis yields

$$\phi = (2^{1/3} - 1)^3, \quad \text{or} \quad s = 2^{-1/3} [ \exp (2^{1/3} - 1) ] \Delta_{00} \quad (47)$$

This gives

$$v_{sc} = [ 1 + \frac{1}{3}(2^{1/3} - 1)^3 F_1 ] 2^{-1/3} [ \exp (2^{1/3} - 1) ] \Delta_{00} / p_F \quad (48)$$

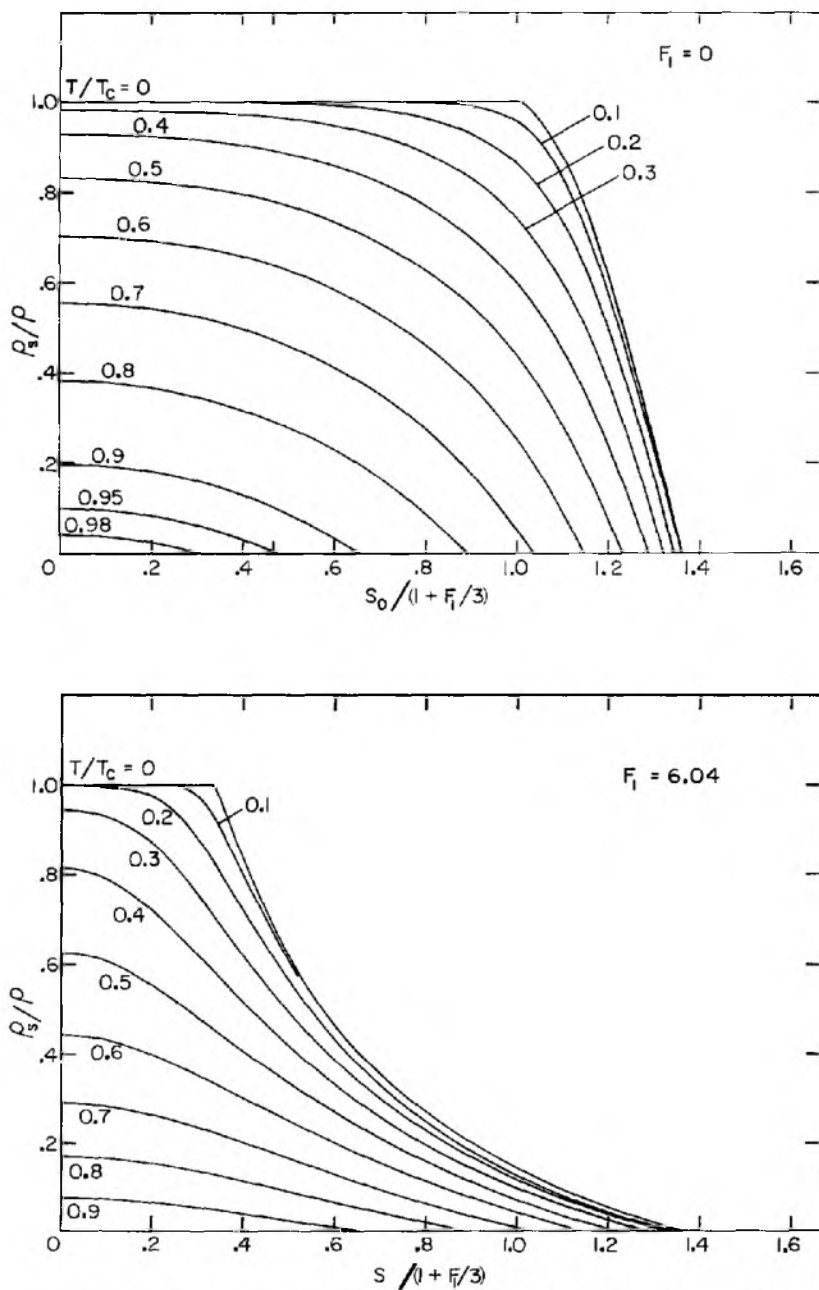


Fig. 5. The superfluid density  $\rho_s$  of  $^3\text{He-B}$  v.  $s_0/(1 + \frac{1}{3}F_1)$  for  $F_1 = 0, 6.04,$  and  $15.66$ .

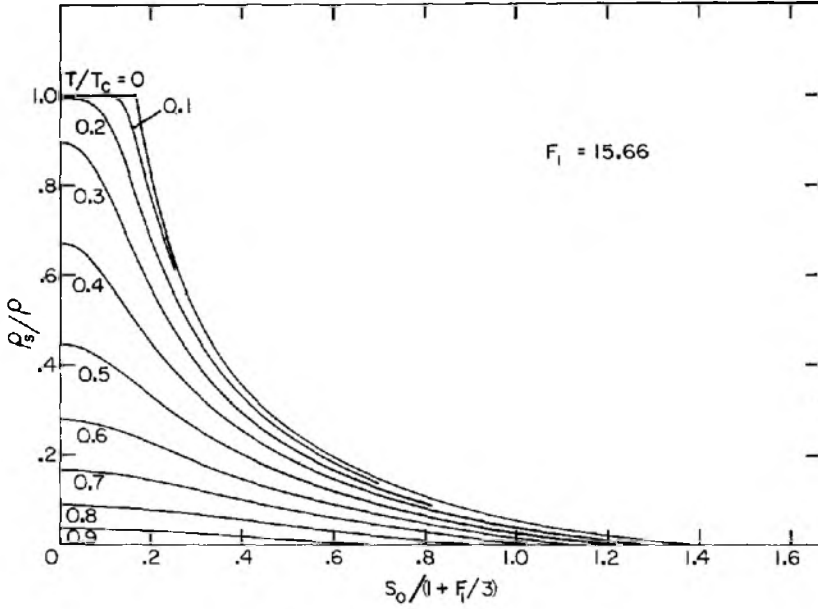


Fig. 5. Continued.

and

$$J_{sc} = [1 - (2^{1/3} - 1)^3] 2^{-1/3} [\exp(2^{1/3} - 1)] \Delta_{00} \rho_F^{-1} \rho \quad (49)$$

Again we find that  $J_{sc}$  is independent of the Fermi liquid correction, while  $v_{sc}$  depends on  $F_1$ .

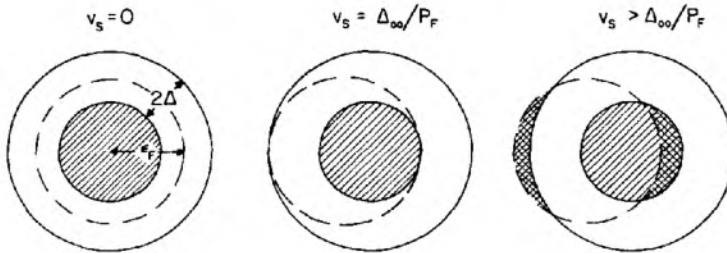


Fig. 6. The energy gap of  $^3\text{He-B}$  in the presence of current for  $v_s = 0$ ,  $v_s = \Delta_{00}/\rho_F$ , and  $v_s > \Delta_{00}/\rho_F$ , respectively.

In the vicinity of the transition temperature we can also solve Eq. (43). For  $^3\text{He-A}$  we have then

$$s = \left(\frac{5}{21\zeta(3)}\right)^{1/2} \left(1 - \frac{T}{T_c}\right)^{1/2} 2\pi T_c \tag{50}$$

$$v_{sc} \approx \left(1 + \frac{1}{3}F_1\right) \left(\frac{5}{21\zeta(3)}\right)^{1/2} \left(1 - \frac{T}{T_c}\right)^{1/2} \frac{2\pi T_c}{\rho_F} \tag{51}$$

and

$$J_{sc} = \frac{2}{3} \left(\frac{5}{21\zeta(3)}\right)^{1/2} \left(1 - \frac{T}{T_c}\right)^{3/2} \frac{2\pi T_c \rho}{\rho_F} \tag{52}$$

Similarly, for  $^3\text{He-B}$  we have

$$s = [7\zeta(3)]^{-1/2} (1 - T/T_c)^{1/2} 2\pi T_c \tag{53}$$

$$v_{sc} \approx (1 + \frac{1}{3}F_1) [7\zeta(3)]^{-1/2} (1 - T/T_c)^{1/2} 2\pi T_c \rho_F^{-1} \tag{54}$$

and

$$J_{sc} = \frac{4}{3} [7\zeta(3)]^{-1/2} (1 - T/T_c)^{3/2} 2\pi T_c \rho_F^{-1} \rho \tag{55}$$

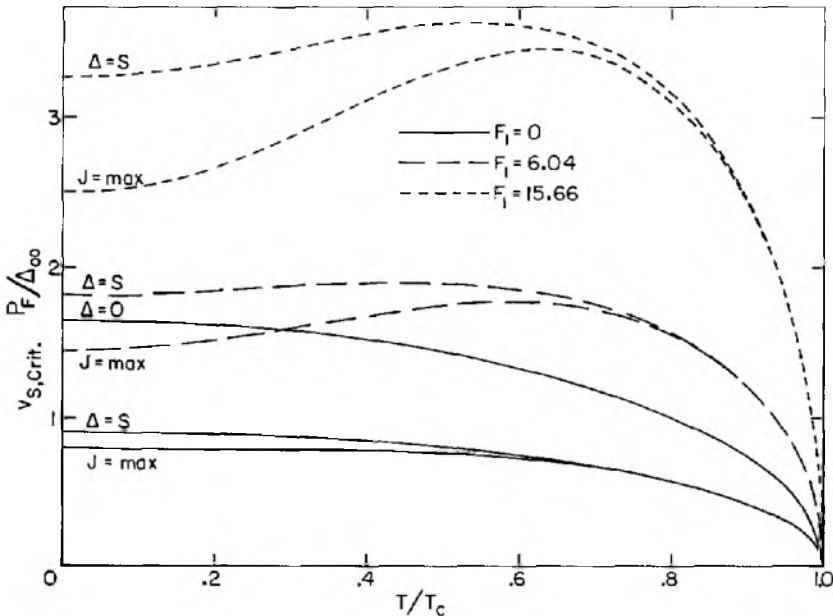


Fig. 7. Critical velocities of  $^3\text{He-A}$  defined by (1)  $J_s = \text{max}$ , (2)  $v_s = \Delta \rho_F^{-1}$ , and (3)  $\Delta = 0$ , as functions of the reduced temperature  $T/T_c$ . The solid curves are for  $F_1 = 0$ , while the dashed curves are for  $F_1 = 6.04$  and  $15.66$ .

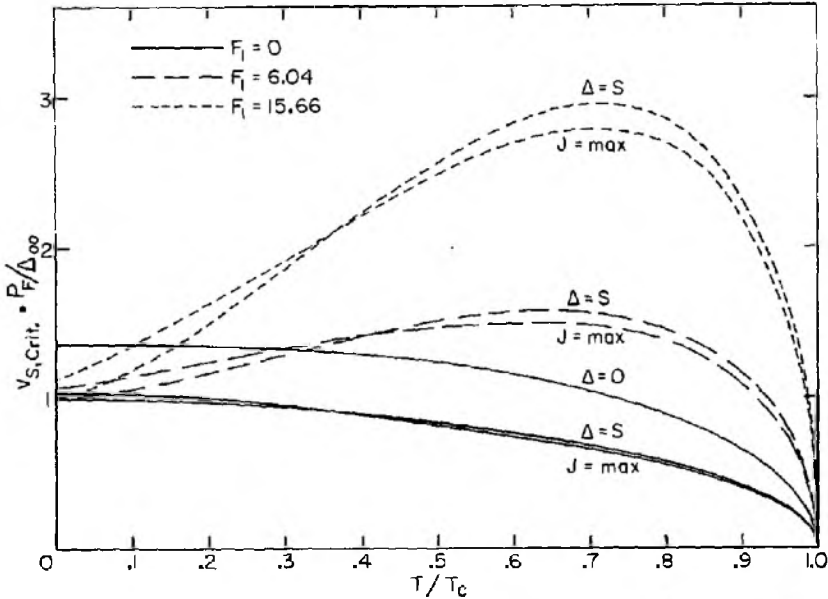


Fig. 8. Critical velocities of  ${}^3\text{He-B}$  defined by (1)  $J_s = \text{max}$ , (2)  $v_s = \Delta p_F^{-1}$ , and (3)  $\Delta = 0$ . The solid curves are for  $F_1 = 0$ , while the dashed curves are for  $F_1 = 6.04$  and  $15.66$ .

At intermediate temperatures the critical velocity is evaluated numerically for  ${}^3\text{He-A}$  and  ${}^3\text{He-B}$ , and is shown in Figs. 7 and 8, respectively. In both figures we have included other critical velocities: the pair-breaking critical velocity, defined as  $v_s$  corresponding to  $\Delta = s$ , and another critical velocity corresponding to  $\Delta = 0$ . The latter critical velocity is shown only for  $F_1 = 0$ , as  $\Delta = 0$  implies  $\phi = 1$  and consequently the corresponding critical velocity for nonvanishing  $F_1$  is obtained by multiplying  $(1 + \frac{1}{3}F_1)$  by the one for  $F_1 = 0$ . The former critical velocity behaves very similarly to the departing critical velocity (for which  $J_s$  has a maximum), although physically this critical velocity (at least in  ${}^3\text{He-B}$ ) corresponds to the velocity where the pair creation of quasiparticles begins to take place. The latter critical velocity may be completely inaccessible, although the velocity gives the instability limit of the normal liquid against formation of the superfluid condensate. In Figs. 7 and 8 we include the critical departing velocities in the presence of the Fermi liquid correction. In the presence of a large Fermi liquid correction  $v_{sc}$  is no longer monotonic but has a broad peak around  $T/T_c = 0.56$  and  $T/T_c = 0.7$  for  ${}^3\text{He-A}$  and  ${}^3\text{He-B}$ , respectively. This unusual behavior of the critical velocity is uniquely due to the Fermi liquid correction, since, as we see from these figures,  $v_{sc}$  is monotonic for  $F_1 = 0$ . We would like to stress here that although the critical velocity

behaves nonmonotonically in the presence of the Fermi liquid correction, the critical current is independent of the Fermi liquid correction (i.e.,  $F_1$ ) and is a monotonic function of the temperature.

As already mentioned in the introduction, we think that this effect could probably be seen in the ion mobility experiment, although the published results<sup>13</sup> on the ion mobility do not indicate this unusual temperature dependence.

## 5. CONCLUDING REMARKS

Making use of the Green's function technique, we have calculated the order parameter  $\Delta$  and the superfluid density  $\rho_s$  in the presence of a large current ( $v_s = 10$  cm/sec) of  $^3\text{He-A}$  and  $^3\text{He-B}$  within the weak coupling model.

We have shown that  $\rho_s$  is strongly nonlinear in  $v_s$  in both  $^3\text{He-A}$  and  $^3\text{He-B}$  due to the large Fermi liquid correction. We have also found that the critical velocity is a nonmonotonic function of temperature, which should have a variety of interesting consequences in the behavior of superfluid  $^3\text{He}$  with a large, uniform current.

## ACKNOWLEDGMENT

We are indebted to Gary Williams for suggesting the present problem and for useful discussions on related subjects. One of us (D.V.) gratefully acknowledges a scholarship granted by the Studienstiftung des Deutschen Volkes, which enabled him to stay at USC.

## REFERENCES

1. H. Kojima, D. N. Paulson, and J. C. Wheatley, *Phys. Rev. Lett.* **32**, 141 (1974); *J. Low Temp. Phys.* **21**, 283 (1975).
2. A. W. Yanof and J. D. Reppy, *Phys. Rev. Lett.* **33**, 631 (1974); J. E. Berthold, R. W. Giannetta, E. N. Smith, and J. D. Reppy, *Phys. Rev. Lett.* **37**, 1138 (1976).
3. P. G. de Gennes and D. Rainer, *Phys. Lett. A* **46**, 429 (1974); A. L. Fetter, *Phys. Rev. B* **15**, 1350 (1977).
4. K. T. Rogers, Superconductivity in Small Systems, thesis, Univ. of Illinois, Urbana (1960), unpublished; J. Bardeen, *Rev. Mod. Phys.* **34**, 667 (1962).
5. K. Maki and T. Tsuneto, *Prog. Theor. Phys. (Kyoto)* **27**, 228 (1960).
6. R. Balian and N. R. Werthamer, *Phys. Rev.* **131**, 1533 (1963).
7. A. J. Leggett, *Rev. Mod. Phys.* **47**, 331 (1975).
8. H. E. Hall and J. R. Hook, *J. Phys. C* **10**, L91 (1977); J. R. Hook and H. E. Hall, preprint.
9. P. Bhattacharyya, T. L. Ho, and N. D. Mermin, *Phys. Rev. Lett.* **39**, 1290 (1977).
10. A. J. Leggett, *Phys. Rev.* **140A**, 1869 (1965).
11. P. W. Anderson and W. F. Brinkman, *Phys. Rev. Lett.* **30**, 1108 (1975).
12. R. Combescot, *J. Low Temp. Phys.* **18**, 537 (1975).
13. A. I. Ahonen, J. Kokko, O. V. Lounasmaa, M. A. Paalanen, R. C. Richardson, W. Schoepe, and Y. Takano, *Phys. Rev. Lett.* **37**, 511 (1976).
14. J. C. Wheatley, *Rev. Mod. Phys.* **47**, 415 (1975).