A model and solution approach for store-wide shelf space allocation☆

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Store space is limited and one of the most costly resources of retailers. Retailers have to apportion available store space among the individual product categories of a store and therefore assign a certain share of shelf space to each category. Assigning more shelf space to one category requires reducing the number of shelves for another category as total space is limited. Reducing available shelf space in turn decreases assortment size and lessens the presentation quantity of products and vice versa. Both affect the demand of products and ultimately the profitability of the entire category such that the profit contribution of a category depends on its shelf size. This interrelation between category sizes and store profits needs to be taken into account for the shelf space assignment to categories and the space allocation for individual products.

We introduce a store-wide shelf space model that optimizes shelf space assignment for categories based on the profit contribution of the corresponding product allocations. We decompose the problem into two hierarchically interlinked subproblems and show that the solution approach suggested works efficiently and provides solutions that are applicable to large problems in retail practice. In a case study with a major European retailer, we show that profits at stores can be improved by 3.2% using our approach. Further, we use simulated data to generalize the findings and derive managerial insights.

1. Introduction

The sales area of retail stores is limited and one of retailers' most cost-intensive resources. Its efficient use is one of the main drivers for a store's success. Retailers decide on space use with respect to different planning horizons and a hierarchical planning process. The planning process is illustrated in Fig. 1 and consists of store layout planning, category space assignment and product allocation. The starting point is the store layout planning that includes defining the role of product divisions (e.g., as traffic driver), their location within the store (e.g., dedicated locations for promotional items), and upper and lower bounds for the division sizes. The store layout planning and definition of divisions (also referred as departments) is mainly driven by the retailer's general strategy and marketing philosophy. It is usually set for multiple years. Multiple product categories (e.g., milk, yoghurt, cheese) form one division (e.g., dairy products), and the sequence of categories within the division is also defined within store layout planning. Secondly, retailers determine the size for each product category within the respective division. When adjusting the category space, planners redefine the number of shelf elements (i.e., the number of racks) of each category (see e.g., [1]). This may lead to replacement or redesign of individual shelf racks. These instore adjustments are usually executed once per year. Lastly, retailers allocate products of a category to given shelves (see e.g., [2,3]). This imposes changes in assortment sizes, the position of products within the shelves and the number of facings of each product. This is usually executed frequently throughout the year.

This paper addresses the assignment of store space to categories that constitutes the central part of this hierarchical planning concept by splitting up the total space into categories, while certain guardrails are set by the retailers store layout planning (e.g., given locations of categories), and the product allocation to shelves is anticipated. In this step, retailers need to assign the space to a category by deciding how many shelf elements they allocate to each category. The assignment of category space implies different effects. If a category size increases, it is more likely that customers will decide to purchase within this category [4]. Furthermore, more category space gives the opportunity to expand the assortment and increase the presentation quantity of the products selected. However, as total store space is limited this also implies that less shelf space is left for the remaining categories. This therefore reduces

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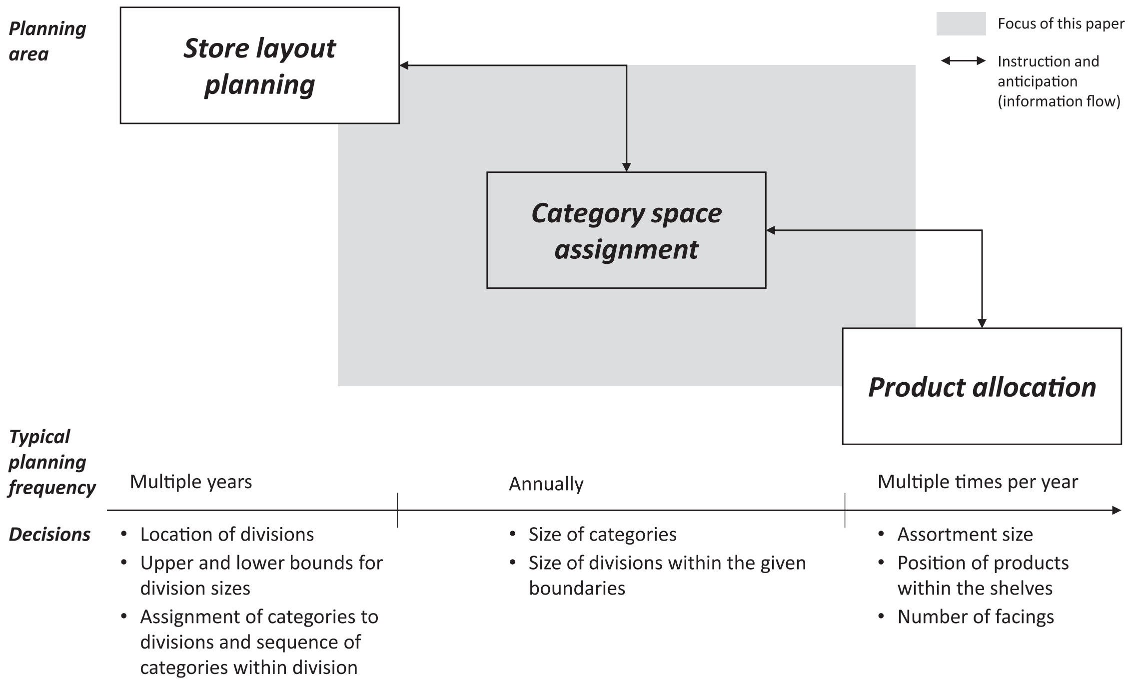

Fig. 1. Hierarchical planning concept for store-wide shelf space planning.

the demand of items in the corresponding categories. The challenge from a modelling and data view is to evaluate the potential contribution of the various sizes of a category. For example, what is the impact on overall profit if one additional shelf element is assigned to one category, while the number of shelf elements is reduced for another? This requires information on the optimal product allocations for each category and all potential shelf sizes of a category. As such, this would require the results of the subordinated product allocation (e.g., detailed plans about shelf layout, assortment sizes, product positioning and shelf quantities) as input to the superordinated assignment of category space. The superordinated assignment of category space on the other hand provides the shelf size limits of each category for the subordinated product allocation. One option for solving this problem is the application of a monolithic product allocation model for the superordinated assignment of category space. This is, however, computationally intractable as we have to deal with problem sizes of 10,000 or more items and the best-known models can usually only handle problem sizes of several hundreds of items [5]. A comprehensive planning framework as introduced above and applied in practice is therefore indispensable.

There are various publications related to the store layout planning (see e.g., [6–8]) and the product allocation problem (see the reviews of [5,9,10]), but little research on the detailed assignment of space to categories. This implies that the store layout models focus on different decisions (e.g., location within store, effect of impulse buying), and that the product allocation literature in general assumes the available shelf space for each category as given. Our work fills this gap in research and presents a comprehensive model formulation to assign space to categories dependent on the category-specific profit contributions. The resulting NP-hard optimization problem is solved via decomposition. In doing this, we calculate the value of each potential size of a category by solving the underlying product allocation problem. These results are then input into the model that defines the sizes of each category. Within this hierarchical approach of dividing the problem into dependent subproblems, we also anticipate the overarching and subsequent decisions and demand effects. In this sense, our approach extends the product allocation literature by optimizing shelf sizes for each category and bridges the gap to the store layout literature.

The remaining paper is structured as follows. Section 2 details the planning problem and relates it to the literature. The mathematical model for the assignment of category space is presented in Section 3 together with the solution approach proposed. We provide numerical tests in Section 4 and show that the approach is able to solve large instances efficiently. This includes a case study with our cooperation partner from grocery retailing. Section 5 summarizes our key findings and identifies areas for future research.

2. Problem description

This section details the underlying problem and its main features in terms of scope and demand impacts. It is based on a collaboration with a major European grocery retailer and a review of related literature. This builds the foundation for identifying the existing gap in research and for specifying the contribution of this work. Despite our focus on grocery retailing, our problem is also of relevance for other retailers such as DIY (do-it-yourself), electronic or department stores, where different categories compete for available shelf space within a store. Despite the fact that some manufacturers play a more important role in managing product allocation of single categories (see e.g., [11] and [12]), the total store-wide shelf space management is usually fully under control of the retailer himself. Retailer-manufacturer collaboration is done within category management, such as minimum space requirements for a category that can be incorporated as constraints in the store planning [13].

2.1. Definition of the store-wide shelf space allocation problem

Overview. Fig. 2 specifies the three different aggregation levels for planning the total store space (divisions, categories and products) and their relationship. Our focus is on the assignment of store space to categories. As such, we elaborate the specific requirements for this aspect, but we also need to detail the interdependencies

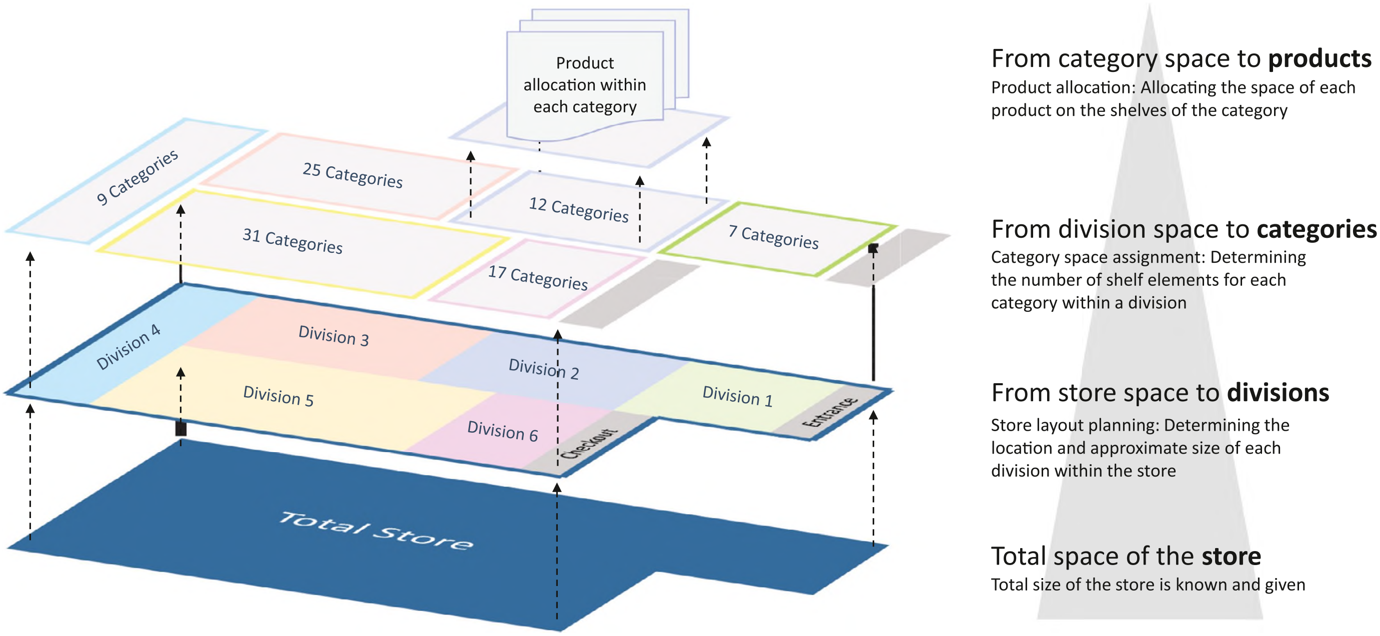

Fig. 2. Different aggregation levels for planning total store space - example.

with the overarching and subsequent aggregation levels. Total store space in our context is measured in running shelf meters. The assignment of store space to divisions and categories is therefore measured in a one-dimensional value, indicating the consumption of store meters.

Based on customer preferences and long-term marketing plans, retailers set up *divisions* for each store type (e.g., hypermarket vs. supermarket). Divisions represent the most aggregated level and divide the total store space into distinct areas. A division summarizes all categories that belong to the same kind of product. In grocery stores, typical examples of divisions are dry food, fresh foods, refrigerated display cases, beverages, print media, household articles, frozen foods, bakery, etc. Within each division, a given number of *categories* are considered, which represent the second aggregation level. A category groups products of the same type (e.g., frozen pizzas). All categories of a division compete for the store space of the respective division. The sizing of categories therefore depends on corresponding division sizes. The final and most granular level comprises *products*, which need to be allocated to shelves of the category. It needs to be decided which products are actually included in the assortment and how much space is allocated to individual products. While product allocation is the most granular level, it affects the ultimate demand and sales for the individual products, and therefore determines the contribution of a category and a division to overall store profit. All three levels are interdependent. The store space allocated to a division influences the shelf space allowance for all related categories. The total shelf space of a category determines the available shelf space for individual products, which in turn determines the profit contribution of the category and division. In the following, we analyze the related scope and demand impact of each aggregation level separately.

From store space to division space. The partitioning of the store space into divisions constitutes the first step. Stores are designed using divisions to group categories of the same kind, define the sequence of categories within the store (i.e., which kinds of product are found at the entrance, which at the checkout), and to ensure a certain display size. The number and size of divisions depends on the value proposition, store type, location and general product variety. Furthermore, the location of the divisions within the store follows long-term marketing plans (e.g., conveying the message of being a retailer with high degree of fresh products by putting the fresh division at the store entrance), insights on shopper paths (e.g., putting a high volume division at the end of the store to increase the walking path), role of divisions (e.g., impulse-buying products closer to checkout), or simply some constructional or design guidelines (e.g., cooled shelves at the walls). At this stage, retailers also select some divisions to serve as traffic drivers (see e.g., [6]). Moreover, the sequence of divisions within a store follows general rules for all stores, i.e., each store with the same arrangement of divisions. Retailers want to create a familiar atmosphere for customers, independent of which of the associated stores is visited.

Updates of the general store layout plan are usually made when major strategy plans are implemented, e.g., adding new divisions or changing the layout of all stores. Store layout planners define at this stage the scope of divisions (i.e., which categories are included in a division) and the dimensions of divisions. To incorporate some flexibility for further plan adjustments during annual category reviews, the division space is here defined within an upper and lower limit. For example, the minimum size of a division may reflect strategic goals of the retailer (e.g., minimum share of fresh products), support to grow strategically important divisions or to enable a basic range of products across the categories involved. A further major decision at this point is defining the location of division within the store. This ultimately also includes the sequencing of categories within a division.

The store layout planning has two demand implications. (1) Increasing size of divisions, categories and products together with a growing visibility increases the demand (see e.g., [14,15]). Total demand ultimately depends on the visibility of individual products to customers. For this reason it is indispensable to understand the detailed impact of space assignment of products. We will hence detail this demand effect below when discussing the allocation of products. (2) The location of a division within a store also impacts visibility and traffic. The actual location of shelves (e.g., near the entrance/check-out) as well as the types of products and categories assigned to nearby shelves impact overall visibility to customers and consequently profits of categories due to corresponding cus-

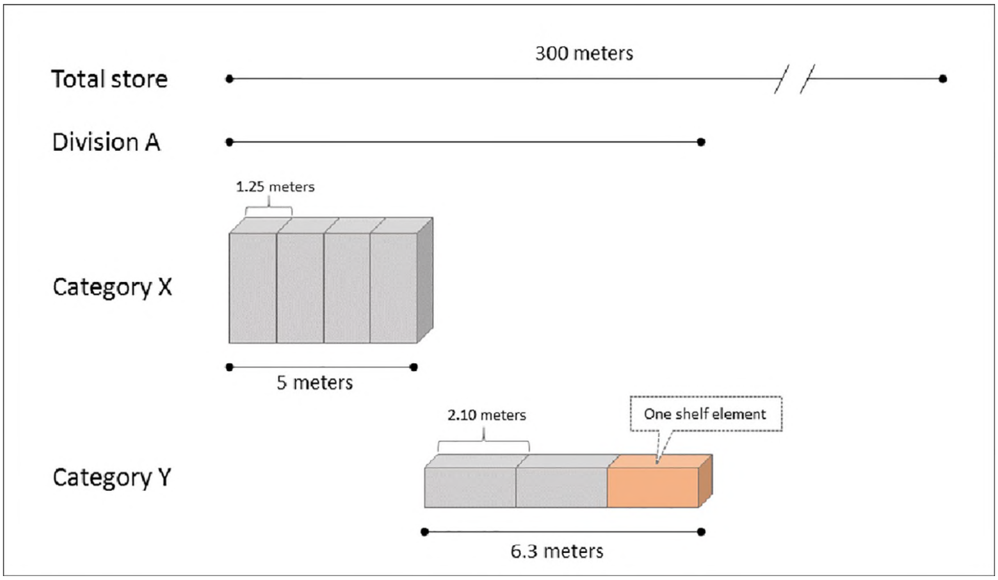

Fig. 3. Example of space assignment to categories with different shelf elements.

tomer traffic [16–19]. As locating divisions is not within the focus for the category sizing, these demand effects need to be implicitly respected when determining the basic demand of products.

From division space to category space. The allocation of division space to categories is the second and subordinate step. Related categories are grouped together in divisions. A category comprises products of the same type (e.g., frozen pizza and frozen vegetables are two categories belonging to the frozen division). Each category is associated with a certain type of shelf rack, indicated in the following as shelf element, depending on the related product characteristics (e.g., dry, chilled, frozen or fresh goods). Individual shelf elements may therefore differ across categories, i.e., category-specific shelf types and sizes are considered. Fig. 3 illustrates different shelf elements for two categories, as examples. While Category X requires standard shelf elements with a width of 1.25 m, Category Y requires special shelf elements with a width of 2.10m. The space consumption of one shelf element therefore differs across categories and it needs to be decided how many category-specific shelf elements are assigned to each category.

The category planning is done at regular intervals and takes place when the performance of categories is reviewed and when categories are added or removed [9]. Retail planners need to decide how many (category-specific) shelf elements are selected for each category, while the existing store layout (i.e., the number of divisions and categories as well as their location within stores) is not affected. To allow some flexibility, category sizes can usually be adjusted within certain boundaries. There are guidelines on the number of shelf elements. Lower limits ensure a certain minimum range of products within each category and reasonable upper bounds are set in line with the maximum assortment. Furthermore, only full shelf elements can be assigned as categories do not share shelf elements. In our example, only multiples of 1.25 or 2.10 m can be taken into account. The demand is impacted by the product allocation that will be detailed below.

From category space to product space. Product allocation comprises decisions on how to place a given set of products (i.e., the assortment of a category) on a limited area of shelf space [5,9,10,20], and constitutes the third step. It is usually updated after major assortment changes (e.g., after regular negotiations with suppliers, delisting of low performing items), and happens more frequently than category planning. Product allocation defines assortment size, shelf quantity of items, and the position on the shelves of each product of a category. Offering broader assortments with more products limits the space available per product and vice versa. This makes it necessary to specify the products to be carried on each shelf and determine the space and quantity to be assigned to selected items. A facing is the first visible unit of an item in the front row of a shelf. In this sense, retailers define the number of units per product in the front row of a shelf that are visible to the customer. The option of lining up products one behind the other (i.e., putting units of an item behind a facing) depends on the shelf and item depth. Common retail practice is to line up as many items as possible [21,22] to fully utilize the available space. The explicit decision is consequently the number of facings for each item, while the units behind a facing are derived from item and shelf sizes. This results in the total shelf inventory of an item. It is common retail practice to assign a product only to one shelf element, not spreading it across multiple elements, to keep facings together. The position of an item on the shelf is therefore described by its vertical and horizontal position on a shelf element. Furthermore, retailers apply minimum inventory limits that ensure a certain service level (e.g., using safety stocks) to minimize out-of-stock situations [23]. Upper limits on the other hand are necessary to limit maximum inventory reach. Similarly, a minimum number of facings can be applied to ensure a certain shelf representation (e.g., for newly listed products with low current demand) or to fulfill supplier targets (e.g., contractual agreements for shelf shares; see e.g., [11]). A maximum number of facings sets an upper bound to limit, for example, the shelf share for certain products.

Customer demand depends on product allocation in four ways: (1) *space allocation to individual items*, (2) *space allocation across items*, (3) *positioning of items on the shelves* and (4) *substitutions* when products are delisted. (1) Item demand depends on the visible quantity on the shelf. The higher the visibility of an item, the higher its demand. The visibility of an item increases with the number of facings assigned to it. This effect is called "space-elasticity" and has been analyzed in various empirical studies (see e.g., [14,15,24,25]). Chandon et al. [26] show that the number of facings is the most important instore factor affecting customer demand. (2) Product allocation may also affect the demand across items. Cross-space elasticity describes the impact on the demand of items when the space assigned to one item is changed. However, [27] show that the impact of this demand source on product allocations and retail profit is limited. This also holds true if elasticities are significantly higher than the empirical values obtained so far. (3) Demand may depend on the position of the products within the shelves as products may be put on different vertical levels. Following [28,29], this means that some levels lie within a specific zone running approximately from eye- to knee-level, where products are more likely to be seen by customers than outside this zone. Further demand impacts may arise from how products are arranged next to each other, how far a product is positioned from the edge of a category (i.e., beginning of an aisle), and the way product facings are arranged, e.g., in rectangular shapes or as a family grouping. Generally these effects are attributed to a lower demand impact (see e.g., [26,30]). Nevertheless, shelf layout may be subject to some layout restrictions that may require keeping certain products together (e.g., brand grouping), but without changing demand [31–33]. (4) The shelf space is limited and hence limits assortment sizes. When a desired product is not listed, customers may decide to replace the product for an alternative. This is called substitution demand (see e.g., [4,34]). Empirical studies indicate substitution rates of 45% to 84% of the initial demand, where the magnitude depends on attributes of the product, situation and customer (e.g., [35,36]).

With respect to the demand affected by sizing categories, the impact of space-elastic demand is unequivocal for our planning problem whereas cross-space elasticity has a negligible impact due to a lower magnitude. Cross-product demand is more relevant for assortment decisions, e.g., when assortment sizes are reduced to fit on the available shelves. The demand arising from product positioning on the shelf elements may impact demand when a detailed product allocation decision is made for different shelf levels. In the context of our aggregated problem of sizing categories, we neglect this positioning effect as we do not consider detailed planograms but the overall space assignment to products in relationship to category sizes.

2.2. Related literature

Two literature streams are related to our setting. The first stream deals with the store related issues, whereas the second is based on the allocating of products to shelves.

Related literature on store planning problems. The contributions in this stream mainly relate to the store layout planning. Campo et al. [4] are the first to deal with the sizing of divisions. In a fundamental empirical paper they determine division sizes and locations by considering specific attraction factors. Please note that their unit of analysis is according to our definition above “division”, but the term used in their paper is “category”. The attraction factors depend on each division’s share of sales, size, and location within the store. In sum, the approach is limited to division-based data and does not consider individual product data in detail. Further, an individual consideration of product-specific shelf elements (e.g., fridges) is missing, and related assortment and substitution effects cannot be considered due to the disregard of product data. Botsali [37] analyze different store layout designs and their impact on impulse buying (i.e., unplanned purchases), revenue, and customer travel distance. Ghoniem et al. [38] present an approach that aims at maximizing the impulse-buying profit by allocating “items” to segments of a “knapsack” (i.e., shelf) with different attractiveness. The authors discuss a single-shelf problem (with up to 70 shelf segments) as well as a multi-shelf problem. Ghoniem et al. [39] present a variable neighborhood search to solve this problem for large problem instances, i.e., up to 210 categories on 42 shelves. Flamand et al. [6] adjust the work of [38] to consider predefined groups of categories. In detail, the authors allocate groups of categories to existing shelves, based on a given store layout to determine the location of each category group (i.e., a specific aisle and shelf) in relation to other categories, define the position of each category within the corresponding group, and decide on the shelf space of single categories. In a subsequent work, Flamand et al. [7] extend their approach using an assortment decision that decides on a category level whether to include a whole category within the store or not. Dorismond [40] also study store layout designs using data-driven models to increase impulse sales. Ozgormus and Smith [8] determine division sizes, but do not further spilt these up into categories. They focus on the sequencing of divisions while considering related layout rules. An adjacency matrix is used where the coherence as well as the contrariness among divisions is defined. Space elasticity and impulse buying rates are considered on a category level. Product level data are not included. The focus of these models is on the location and sequence of divisions and categories with respect to impulse buying effects, while detailed product allocation decisions (and corresponding effects such as substitution or space elasticity) are not included to determine the individual profit contributions of categories. The authors consider predetermined profits for the used category groups, and the actual shelf space assigned to a group does not impact category profits but impulse buying effects. Further, this stream of literature does not consider category-specific shelf elements and therefore different shelf types but assume that all category groups can be assigned to the existing shelf structure of a store. Given these differences, these papers can be seen as contributions to the store layout problem as it determines divisions and category locations and sequences (that are assumed as given in our work) within stores, while we analyze a detailed shelf space allocation to single categories with individual shelf elements and profits.

Another, but different approach for store planning is provided by Irion et al. [1]. They provide two models where solutions of a product allocation model adopted from [41] are used to interpolate values for the store space problem. Within their product allocation model, they account for product level data and consider substitution and space elasticity. Within their store model, they use values for a predefined number of options between the upper and lower bound of shelf space for a category generated with the product allocation model. They interpolate between these options in order to determine a share of the total store space per category. Even though they use an advanced product allocation model, their approach does not account for divisions and different shelf types. Further, the numerical examples in this paper are very limited. They show a numerical study containing nine categories, which is not in line with practical needs (e.g., more than 50 categories with over 10,000 items are considered by retailers).

Related literature on product allocation. Following first studies on the effect of product allocation on sales and profits (e.g., [14,15]), Hansen and Heinsbroek [2] present the first product allocation model with non-linear elements and a space-elastic demand function. Subsequently, Corstjens and Doyle [3] and Zufryden [42] extend the decision model by considering cross-elasticity and other demand and cost effects. Borin et al. [43] present a product allocation model that integrates assortment decisions and lost sales. In addition to the shelf space of a category, Urban [23] also consider the backroom as additional inventory space. Irion et al. [41] incorporate a detailed cost function for the replenishment process. An additional decision aspect is introduced by Hübner and Schaal [44]. Their work considers backroom space. Besides similar demand functions, all these publications model shelf space as a one-dimensional input parameter. Other approaches such as those of [45–49] provide the possibility of considering several equal (one-dimensional) shelf levels. These publications demonstrate the increasing importance of considering different shelf space options. In

Table 1
Overview of related main literature.

Literature	Decision variables				Shelf inventory[c]	Shelf types per cat.[d]	Max. size of test class[e]	
	Facings	Assortment[a]	Cat.size	Div.size[b]				
Campo et al. [4]	–	–	✓	–	–	–	(c) 17	(i) –
Irion et al. [41]	✓	(✓)	✓	–	✓	–	(c) 9	(i) n.a.
Ghoniem et al. [38]	–	–	✓	–	–	equal	(c) 140	(i) –
Ghoniem et al. [39]	–	–	✓	–	–	equal	(c) 210	(i) –
Flamand et al. [6]	–	–	✓	✓	–	equal	(c) 210	(i) –
Flamand et al. [7]	–	–	✓	–	–	equal	(c) 800	(i) –
Ozgormus and Smith [8]	–	–	✓	–	–	equal	(c) 25	(i) –
Borin et al. [43]	✓	✓	–	–	–	–	(c) 1	(i) 6
Urban [23]	✓	✓	–	–	✓	–	(c) 1	(i) 6
Yang [45]	✓	(✓)	–	–	–	–	(c) 1	(i) 10
Hwang et al. [46]	✓	–	–	–	✓	–	(c) 1	(i) 4
Hansen et al. [47]	✓	–	–	–	–	–	(c) 1	(i) 100
Irion et al. [1]	✓	(✓)	–	–	✓	–	(c) 1	(i) 6
Geismar et al. [30]	✓	–	–	–	–	–	(c) 1	(i) n.a.
Bianchi-Aguiar et al. [32]	✓	–	–	–	–	–	(c) 1	(i) 240
Zhao et al. [48]	✓	–	–	–	✓	–	(c) 1	(i) 100
Hübner and Schaal [44]	✓	–	–	–	✓	–	(c) 1	(i) 2000
Hübner and Schaal [49]	✓	–	–	–	✓	–	(c) 1	(i) 200
Düsterhöft et al. [50]	✓	–	–	–	✓	–	(c) 1	(i) 300
Hübner et al. [51]	✓	–	–	–	✓	–	(c) 1	(i) 200
This paper	✓	✓	✓	✓	✓	individual	(c) 60	(i) 16,000

– means not applicable / not considered in model
[a] Assortment decision on a product level: ✓incl. substitution for listed products; (✓) without substitution effects.
[b] Consideration of different aggregation levels of store space.
[c] Minimum/maximum inventory (e.g., representation quantity) or considering replenishment needs.
[d] When more than one category is considered and store space allocation accounts for shelf elements; equal: all categories have the same shelf type; individual: categories have individual shelf types.
[e] Maximum number of categories (c) and/or resulting number of items (i) in a single test instance.

line with this, Düsterhöft et al. [50] and Hübner et al. [51] demonstrate the impact of considering different approaches to model shelf sizes and present models that consider three-dimensional shelves and optimize shelf layout. We refer to [5,9,10] for more detailed reviews.

2.3. Summary and scope of this paper

Table 1 summarizes related literature. Present literature addressing problems on a store level primarily focuses on finding the most attractive location for each product group within a retail store. To do so, aggregated values of category groups or divisions are used and other demand effects such as impulse-buying are in the focus of their work. Detailed data on a product level on the other hand are neglected. As the sequence of divisions and category groups within a store is usually a global decision that is valid for all a retailer's stores, these models are more applicable on an abstracted strategic planning level (i.e., store layout planning). It is important to consider exact product values and category-specific shelf types for the concrete sizing of divisions and categories. None of the existing approaches on store-wide problems provide a solution that incorporates and combines category sizing with the consideration of category-specific shelf types and profits driven by product allocation decisions. The integration of these decisions is important to obtain a detailed solution of category and division sizes due to their interrelation.

The majority of papers addressing product allocation define the number of facings for a given number of products for a specific category. In all the papers, the available shelf space is considered as an input parameter and cannot be modified within these models. This implies the problem that product allocation is based on the predetermined category space and can only be as good as the determination of the shelf size parameter itself. The majority of product allocation models do not apply different shelf elements among the items considered, which is a prerequisite for solving the holistic store-area decision problem. Further, they do not account for subsets on the product, category and division limits.

3. Model and solution approach

This section introduces the formal representation of the store-wide shelf space allocation problem. After discussion of the formal model we introduce our solution approach that uses a problem decomposition to address the problem.

3.1. Decision problem, general model and model complexity

Notation. Table 2 summarizes the notation for the store-wide shelf space allocation problem.

Sets, decision variables and parameters. Let I be the set of items $i, i \in I$, which comprises all available items across all categories C, with $c, c \in C$. The subset $I_c \subseteq I$ indicates the items i that belong to category c. As items can be delisted, we divide the set of all items of a category I_c into listed items $(I_c^+, I_c^+ \subseteq I_c)$ and delisted items $(I_c^-, I_c^- \subseteq I_c)$, such that $I_c^+ \cup I_c^- = I_c$ and $I_c^+ \cap I_c^- = \emptyset$. Further, each category c belongs to exactly one division $d, d \in D$, denoted by the subset $C_d \subseteq C$. The total store space R is divided into the space allowance for all divisions d, where each division has minimum and maximum space requirements $U_d^{\min}$ and $U_d^{\max}$. Retailers use different shelf elements for each category (e.g., regular shelves, high racks, chilled or freezing compartments). Each category c has exactly one type of such shelf element. The store space required by one shelf element of each category is denoted by r_c. The total store space R and store space per shelf element r_c is usually measured as a one-dimensional value (e.g., in running meters). Retailers need to define the number of shelf elements y_c $(y_c > 0)$ for every category $c, c \in C_d$, within each division d. The store space consumed by all categories (across all divisions) may not exceed the given total

Table 2
Notation for the store-wide shelf space allocation model.

Indices	
C	Set of categories c within the store, $c \in C$
D	Set of divisions d within the store, $d \in D$
C_d	Subset of categories c belonging to division $d, c \in C, d \in D$
I	Set of available items (products) i within the store, $i \in I$
I_c	Subset of items i belonging to category $c, i \in I, c \in C$
I_c^+ (I_c^-)	Subset of listed (delisted) items i belonging to category $c, i \in I, c \in C$
Store- and shelf-space-related parameters	
$E_c^{\min}$ ($E_c^{\max}$)	Minimum (maximum) number of shelf elements for category c
R	Total store space for assigning shelf elements
s_c	Available shelf space for product allocation per shelf element of category c
r_c	Store space required for one shelf element of category c
$U_d^{\min}$ ($U_d^{\max}$)	Minimum (maximum) store space for division d
Product-related parameters	
a_i	Space required for allocating one facing of item i
g_i	Sales units behind one facing of item i
m_i	Net margin of one unit of item i
$T_i^{\min}$ ($T_i^{\max}$)	Minimum (maximum) shelf quantity of item i
α_i	Basic demand of item i
β_i	Space-elasticity of item i
γ_{ji}	Substitution rate from delisted item j to listed item i
δ_i	Total demand of item i
Decision variables	
x_i	Number of facings of item i
y_c	Number of shelf elements of category c

store space R, so it needs to be ensured that $\sum_{c \in C} r_c \cdot y_c \le R$. In doing this, one implicitly decides on the store space allocated to each division with $\sum_{c \in C_d} y_c \cdot r_c, \forall d \in D$. There are also limits to the number of shelf elements y_c for each category c, with $E_c^{\min} \le y_c \le E_c^{\max}$ and $E_c^{\min} \ge 1$, as at least one shelf element per category is necessary. Besides the consumption of available store space per shelf element and category r_c, each shelf element is associated with an available shelf space, denoted as s_c. This indicates the space available to allocate facings of items $i, i \in I_c$, to one shelf element of category c. Using the number of shelf elements per category y_c, the shelf space per category S_c is denoted by $S_c = s_c \cdot y_c,\ \forall c \in C$. With respect to shelf space consumption of items it is sufficient to consider the space consumption of one facing of an item i, denoted by a_i, and the number of facings allocated to an item, indicated by the integer variable $x_i, x_i \in \mathbb{N}_0$. In the event that zero facings have been assigned to an item ($x_i = 0$), it is delisted (i.e., $i \in I_c^-$). The facing-related space consumed by all listed items $i, i \in I_c^+$, of a category c can then be calculated by $\sum_{i \in I_c^+} a_i \cdot x_i$, and needs to be equal or smaller than the available shelf space per category, i.e., $\sum_{i \in I_c^+} a_i \cdot x_i \le S_c$. Retailers fill up shelves to the maximum possible units. The number of facings x_i is therefore decision relevant, whereas the number of units behind one facing g_i is derived by shelf and item depth. The parameter g_i is uniquely defined for each item and depends on the item depth and the shelf depths of one element of the item-related category c. The total shelf quantity of each item i is determined accordingly by $q_i = x_i \cdot g_i$. The shelf quantity needs to lie within a minimum representation quantity $T_i^{\min}$ and a maximum inventory reach $T_i^{\max}$, i.e., $T_i^{\min} \le q_i \le T_i^{\max},\ \forall i \in I_c^+, c \in C$.

Objective function. Retailers pursue the objective of maximizing the total store profit Ω by selecting the optimal number of shelf elements y_c across all categories c, and the corresponding optimal number of facings x_i across all items i, represented by the vectors $\bar{x}$ and $\bar{y}$, with $\bar{x} = \{x_1, x_2, \ldots, x_{|I|}\}$ and $\bar{y} = \{y_1, y_2, \ldots, y_{|C|}\}$. The objective function Ω can thus be formulated as follows.

$$\max\ \Omega(\bar{x}, \bar{y}) = \sum_{i \in I} m_i \cdot \delta_i(x_i) \tag{1}$$

The total profit of an item is calculated as the product of its total demand δ_i and its net margin m_i per sales unit. The total demand δ_i of an item i is a composite function of the basic demand α_i, the demand dependent on space-elasticity β_i, and the out-of-assortment substitution γ_{ji} from delisted items $j, j \in I_c^-$ to listed items $i, i \in I_c^+$. The basic demand α_i represents the retailer's demand forecast for an item that is independent of the number of facings (cf. [2,5,9]). The forecast may be based on historical sales, but may also incorporate further marketing effect. In our context, the basic demand α_i of an item i already incorporates the location effect within the store as each item belongs to one category and each category is assigned to one division. The location of the division and of each category within a division is predetermined. The same holds true for effects from shelf positioning. For example, price segments or brand blocks define the location of products on shelf levels (e.g., items that belong to the economy segment are usually positioned on the bottom level). The higher the visibility of an item, the higher its demand [14,15,25,26]. The item visibility increases with the number of facings x_i. In accordance with prior research (cf. e.g., [2,41]), the facing-dependent demand rate is a polynomial function of the number of facings x_i and the space-elasticity β_i (with $0 \le \beta_i \le 1$).

The assortment size $|I_c^+|$ of each category c depends on the available shelf space (i.e., number of shelf elements y_c) and the selected number of facings x_i across items. If fewer shelf elements are available, total shelf space decreases and it may not be possible to list all items. It may also be more profitable to delist less profitable items to increase the number of facings for more profitable items. We assume that if item j is delisted, customers substitute a certain share of the basic demand α_j of item j with an alternative item i, to compensate for the lack of item j in the assortment. The maximum quantity that can be substituted cannot be higher than the basic demand as the space-elastic demand in the case of $x_i = 1$ corresponds to the basic demand. Additionally, we follow the usual assumption that substitution takes place across one round only (cf. e.g., [34,52]). Assortment size reductions therefore result in a shift of demand (substitution) from delisted items $j, j \in I_c^-$ to other listed items $i, i \in I_c^+$, expressed as substitution rate γ_{ji}, and in lost sales, expressed as $1 - \sum_{j \in I_c^-} \gamma_{ji}$ (see e.g., [34,53]). The substitution rate γ_{ji} can be estimated, for example, proportional to the demand share of an item i of the total demand (see e.g., [9,10]). Given our strategic planning problem and the decision variables x_i and y_c, the demand function δ_i of an item i can be formulated as denoted in Eq. (2).

$$\delta_i(x_i) = \alpha_i \cdot x_i^{\beta_i} + \sum_{j \in I_c^-} \alpha_j \cdot \gamma_{ji} \quad \forall i \in I_c^+, c \in C \tag{2}$$

The item unit margin m_i corresponds to sales price minus purchase costs, replenishment costs and further related costs (e.g., for listing). The replenishment costs depend on the ratio of demand to shelf quantity. Whenever the shelf quantity q_i of an item i is not sufficient to cover the demand $\delta_i(x_i)$ of an item i, additional replenishment from the backroom has to be performed, which decreases the margin m_i.

Model complexity. Product allocation problems belong to the class of knapsack problems that are known to be NP-hard [54]. The combinatorial complexity of such problems increases very rapidly with the number of products considered and the shelf space allotted. The possible combinations for allocating $|I_c|$ products to a given shelf space S_c can be calculated using $Y(I_c, S_c) = \binom{|I_c|+S_c-1}{S_c}$. Assuming instances with $|I_c| = 50$ items and space for 100 units ($S_c = 100$), this results in $6.7 \cdot 10^{39}$ possible combinations of one category. Each of these combinations results in different substitution settings and thus a different demand among the items that would need to be factored in. Furthermore, in our case we consider up to 80 categories with hundreds of items each. Defining the shelf space S_c for each category is part of the decision problem.

To summarize, our model combines the decisions on assortment sizes and product allocation with store spacing while taking into account the interdependency of these decisions. The model presented is a non-linear integer problem (NLIP) due to the mutual dependency of the decision variables. A special case and reduced problem (i.e., the product allocation) is already known to be an NP-hard problem. A combination of product allocation with the decisions on store-wide shelf space allocation additionally increases the size of the combinatorial problem significantly. In this case, only limited data sets of minor sizes can be solved, and hence an efficient solution approach is required.

3.2. Solution approach

The central aspect of our problem is determining the size of each category. This decision is based on a bottom-up profit calculation of each possible category size with a product allocation model. We present a tailored solution approach, Store-Wide Shelf Space optimization (SWISS optimization), which determines the optimal shelf space per category based on the profit contribution of possible shelf sizes. Fig. 4 represents the strategy of our solution approach. SWISS optimization uses a decomposition of the store-wide shelf space allocation model into two subproblems: the *Product Allocation Model (PAM)* and the *Store Area Model (SAM)*. First, the PAM is solved to determine an optimal assortment and product allocation for each possible shelf space configuration. This means that for each category a solution of PAM is obtained for each possible shelf size of this category. The individual profit contributions of these shelf configurations obtained by the PAM are then used as input parameters for the SAM, which determines optimal category sizes. We implemented the SWISS optimization in a Java framework. To do so, first a preprocessing step is required where input data and model-specific values for the PAM and SAM are calculated. Using the preprocessing, we are able to connect both subproblems in our solution approach, while each model can be solved using the CPLEX solver. In the following we detail the respective subproblems PAM (Section 3.2.1) and SAM (Section 3.2.2) as well as the complete SWISS solution algorithm (Section 3.2.3).

3.2.1. Solving the product allocation model for all category shelf size combinations

We use the PAM to obtain a profit value for the different shelf sizes of a category depending on the number of shelf elements used. The number of possible shelf elements for category c can be formulated as the set $E_c = \{E_c^{\min}, E_c^{\min} + 1, \ldots, E_c^{\max}\}$, where $E_c^{\min}$ and $E_c^{\max}$ represent the lower and upper limits for each category. The shelf size S_{ce} of category c is then denoted by the number of shelf elements $e, e \in E_c$, and the corresponding shelf space s_c, i.e., $S_{ce} = e \cdot s_c$. We introduce Π_{ce} as the profit contribution of category c when e shelf elements are allocated to it. That means the profit contribution has to be calculated $|E_c|$ times for each category, resulting in $\sum_{c \in C} |E_c|$ combinations.

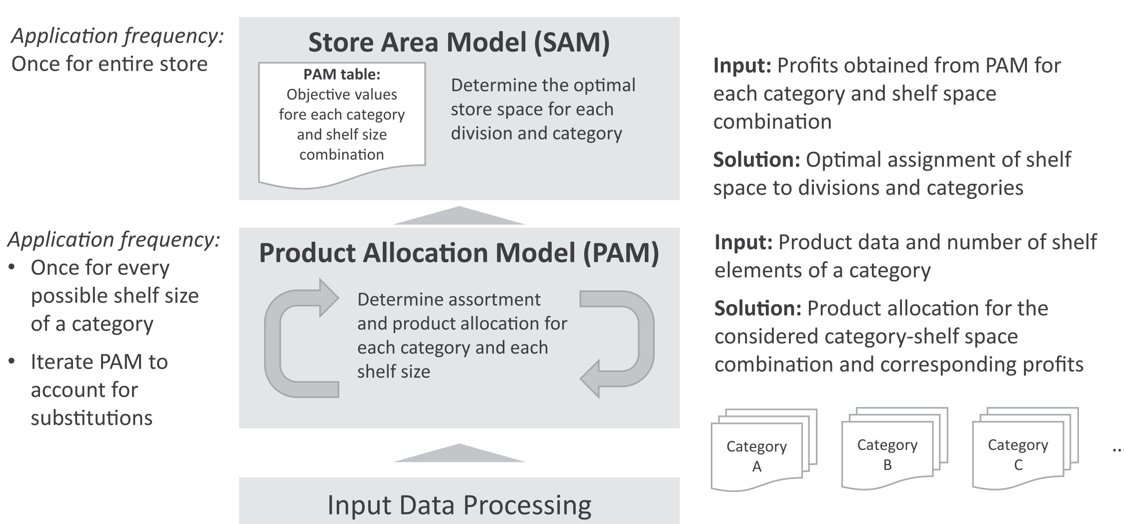

Fig. 4. Bottom-up strategy of SWISS optimization.

We leverage the fact that facings can only have integer values and formulate the PAM as a binary integer program (BIP). Using the maximum shelf quantity $T_i^{\max}$ for item $i, i \in I$, we can calculate the upper limit for the number of facings $K_i^{\max}$ of each item i using $K_i^{\max} = \lceil \frac{T_i^{\max}}{g_i} \rceil$. Similarly, $K_i^{\min}$ is calculated using $T_i^{\min}$. We then apply this to define the set K_i of possible facings for item i, with $K_i = \{0, K_i^{\min}, K_i^{\min}+1, \dots, K_i^{\max}\}$, where 0 is included to enable zero facings, which is equivalent to delisting item i. Furthermore, the number of facings $k, k \in K_i$, assigned to an item $i, i \in I_c$, is denoted by the binary decision variable x_{ik}. The demand values δ_{ik} for each item listed i ($i \in I_c^+$) and each possible facing k are calculated by

$$\delta_{ik} = \alpha_i \cdot k^{\beta_i} + \sum_{j \in I_c^-} \alpha_i \cdot \gamma_{ji}. \tag{3}$$

This is identical to Eq. (2), and can be found in related problems (see e.g., [44]). The profit contribution π_{ik} of each item i and k facings is computed accordingly by $\pi_{ik} = m_i \cdot \delta_{ik}$. The central benefit of this formulation is a priori determination of individual limits $K_i^{\min}$ and $K_i^{\max}$ for each item $i, i \in I_c$, as well as the consideration of the individual category-related boundaries within the sets E_c. Both significantly reduce computational efforts. The PAM can be formulated as follows to determine the profit contribution Π_{ce} for each category c when e shelf elements are used:

$$max\ \Pi_{ce}(x_{ik}) = \sum_{i \in I_c} \sum_{k \in K_i} \pi_{ik} \cdot x_{ik} \tag{4}$$

subject to

$$\sum_{k \in K_i} x_{ik} = 1 \ \forall \ i \in I_c \tag{5}$$

$$\sum_{i \in I_c} \sum_{k \in K_i} k \cdot a_i \cdot x_{ik} \le S_{ce} \tag{6}$$

$$x_{ik} \in \{0, 1\} \ \forall \ i \in I_c, k \in K_i \tag{7}$$

The objective function (4) determines the number of facings x_{ik} for each item $i, i \in I_c$ such that the total profit Π_{ce} is maximized for the given category c and number of shelf elements $e, e \in E_c$. Constraints (5) ensure that the binary variable x_{ik} is set active for only one number of facings k for each item $i, i \in I_c$. With Constraints (6) it is ensured that the available shelf space S_{ce} is not exceeded by the sum of all items allocated. Finally, the decision variable x_{ik} is defined as binary by Constraints (7).

3.2.2. Store area model

The second part of our decomposition is the SAM, which addresses the central aspect of our work, i.e., the store space allocation to divisions and categories. It is therefore the superordinate problem within the SWISS optimization. SAM uses the profits calculated with PAM. In detail, SAM uses the profits Π_{ce} generated across all categories $c, c \in C$, and all corresponding options of shelf elements $e, e \in E_c$, and determines the optimal composition of shelf space across categories that generates the highest overall store profit Ω. As the total store space R is indicated in shelf meters, the linear intake of the shelf elements of a category on the floor is sufficient (i.e., the consumed shelf meters per shelf). We therefore define r_{ce} for the shelf meters consumed by category c when e shelf elements are used. We introduce the binary variable y_{ce}, indicating the number of shelf elements e allocated to category c. The SAM can then be formulated as follows.

$$max\ \Omega(y_{ce}) = \sum_{c \in C} \sum_{e \in E_c} \Pi_{ce} \cdot y_{ce} \tag{8}$$

subject to

$$\sum_{e \in E_c} y_{ce} = 1 \ \forall \ c \in C \tag{9}$$

Algorithm 1 Pseudo code of SWISS optimization.

1: Input: Sets of divisions D, categories C, items I, and facings K_i ($i \in I$)
2: Precalculation of parameters
3: Set individual limits $K_i^{\max}$ and $K_i^{\min}$ for each item i
4: **for** each category $c \in C$ **do**
5: Set $E_c = \{e \in \mathbb{N} | E_c^{\min} \le e \le E_c^{\max}\}$
6: Set $\ell = 0$ and $\Pi_{ce}^{(\ell)} = 0$ and calculate $\delta_{ik}^{(\ell)}$ assuming $I_c^{+(\ell)} = I_c$ and $I_c^{-(\ell)} = \emptyset$
7: **for** each number of shelf elements $e \in E_c$ **do**
8: Set $\ell = \ell + 1$, set $\delta_{ik}^{(\ell)} = \delta_{ik}^{(\ell-1)}$, solve PAM$^{(\ell)}$ to obtain $x_{ik}^{(\ell)}$, and determine $I_c^{+(\ell)}$ and $I_c^{-(\ell)}$
9: Update demand $\delta_{ik}^{(\ell)} = \alpha_i \cdot k^{\beta_i} + \sum_{j \in I_c^-} \alpha_i \cdot \gamma_{ji}$ for each item $i \in I_c^{+(\ell)}$ and set $\delta_{ik}^{(\ell)} = 0$ for each item $i \in I_c^{-(\ell)}$
10: If $|\Pi_{ce}^{(\ell)} - \Pi_{ce}^{(\ell-1)}| \le \epsilon$ holds true, stop and return Π_{ce}, else go to line 8
11: **end for**
12: **end for**
13: Input Π_{ce} from PAM and respective store space of shelves r_{ce} and solve SAM
14: **return** Assigned store space to each division $d \in D$ and category $c \in C$ and resulting store profit Ω

$$\sum_{c \in C} \sum_{e \in E_c} r_{ce} \cdot y_{ce} \le R \tag{10}$$

$$U_d^{\min} \le \sum_{c \in C_d} \sum_{e \in E_c} r_{ce} \cdot y_{ce} \le U_d^{\max} \ \forall \ d \in D \tag{11}$$

$$y_{ce} \in \{0, 1\} \ \forall \ c \in C, e \in E_c \tag{12}$$

Within the objective function (8) the total profit of the store Ω is maximized by summing up the chosen shelf elements times the respective profits across all categories. Constraints (9) ensure that exactly one number of shelf elements is assigned to each category. The total store space is respected using Constraints (10). Moreover, the upper and lower bounds for divisions are set in Constraints (11). Finally, the decision variable y_{ce} is defined as binary in Constraint (12).

3.2.3. SWISS optimization

Algorithm 1 summarizes the solution approach for SWISS optimization that combines PAM and SAM.

PAM determines the profits for all possible shelf sizes of each category Π_{ce} ($c \in C, e \in E_c$) (see lines 3 to 12 in Algorithm 1). The Demand Function (3) results in a non-linear model due to the substitutions. We therefore apply an iterative approach to solve a linearized PAM for each possible allocation of shelf elements $e, e \in E_c$, and the corresponding shelf size S_{ce}. This means that we precalculate the substitution demand (see second term of (3)) – using the assortment decisions of the previous PAM iteration – and then update the complete demand (incl. substitution effects) after each iteration with the newly obtained assignment (see details below). In this way it possible to solve the BIP with CPLEX, but it has the tradeoff that the actual demand is always lagging one iteration behind. We repeat the iterations until the objective value no longer changes. Hübner and Schaal [55] demonstrate that this is an efficient approach to include substitutions. We detail the iterative approach of PAM that comprises an initialization and three further steps in the following:

- *Step 0 – Initialization:* We introduce the index ℓ to count the number of iterations. For the initialization of PAM, the demand for all items of a category $i, i \in I_c$, is calculated assuming that all items are included in the assortment ($I_c^+ = I_c$). This means that there is no substitution effect (as all items are listed, $I_c^- = \emptyset$) and the demand function reduces to $\delta_{ik}^{(0)} = \alpha_i \cdot k^{\beta_i}$. In this way the demand can be precalculated for all items i and possible facings $k,\ k \in K_i$.

- *Step 1 – Solving PAM:* In each iteration ℓ we set the demand for each item equal to the demand obtained in the previous (initial) iteration, i.e., $\delta_{ik}^{(\ell)} = \delta_{ik}^{(\ell-1)}$. With this demand, we then precalculate the item profit π_{ik} for every item and every possible facing and solve the PAM$^{(\ell)}$ as denoted by Formulas (4)–(7). The solution of PAM provides the number of facings $x_{ik}^{(\ell)}$ for each item, and with that a solution for the current assortment, i.e., it determines the sets $I_c^{+(\ell)}$ and $I_c^{-(\ell)}$ for listed and delisted items.
- *Step 2 – Demand update:* The solution of PAM$^{(\ell)}$ (Step 1) is now used to update the demand values. In detail, using the current assortment solution, i.e., the sets $I_c^{+(\ell)}$ and $I_c^{-(\ell)}$, we can now update the demand $\delta_{ik}^{(\ell)}$ using Eq. (3). This means in particular that the substitution demand is updated using the information of delisted items in the current iteration. We therefore add the substitution demand ex post to the current solution by updating the demand for all items listed $i, i \in I_c^+$. The updated demand $\delta_{ik}^{(\ell)}$ is then used for the next iteration (see Step 1).
- *Step 3 – Stop criteria:* In the final step of each iteration ℓ, we compare the solution obtained to that of the previous iteration. If the change in profits between iterations lies below the limit ϵ (i.e., $|\Pi_{ce}^{(\ell)} - \Pi_{ce}^{(\ell-1)}| \le \epsilon$), the algorithm stops and returns the profit Π_{ce} for this category shelf element combination. Otherwise, Steps 1 and 2 are repeated.

Once the profit values Π_{ce} are available for all categories $c \in C$ and related shelf elements $e \in E_c$ the SAM can be solved with Formulas (8)–(12). The SAM provides the optimal segmentation of store space, i.e., the division and category sizes are determined.

4. Numerical analysis

This section analyzes the efficiency and effectiveness of SWISS optimization. First we present a case study that shows the improvement potential for retailers using real-world data. To generalize these findings, we further apply tests on a large set of simulated data. Here we show the effectiveness of the planning approach chosen and demonstrate the performance of the two-step approach suggested. Furthermore, we examine the runtime dependent on varying problem sizes and parameters to derive additional managerial insights.

SWISS optimization is implemented within a Java applet that calls the IBM ILOG CPLEX Optimization Studio 12.6.2.0 in order to solve the models PAM and SAM. The experiments have been computed on a Windows 8 64 bit computer with 16 GB RAM and an Intel(R) Core(TM) i5-6440HQ CPU with 2.6 GHz. The runtime presented in this section refers to the complete processing times of SWISS optimization, from data input to solution output. This includes the preprocessing and precalculation steps as well as the application of PAM and SAM. We set the stop criteria of PAM at $\epsilon = 0.2\%$. This means the PAM iterations stop if the profit delta between two iterations is below this value.

4.1. SWISS optimization applied in practice

The SWISS optimization was tested in a case study with one of Europe's biggest hypermarket chains. We had access to a test supermarket located in Eastern Europe and the corresponding data of each category. In our application we consider modern trade formats such as supermarkets and hypermarkets. These markets follow comparable settings across Europe and as such no differences between markets in Western or Eastern Europe apply. The test set included the food categories. The retailer had excluded non-food divisions and service counters from the scope. In total, the data set comprises three divisions with 53 categories and between 20 and 1445 items per category. The divisions are regular dry food (A), dairy products (B) and self-service shelves for fresh meat (C). Division A contains 44 categories, B six categories and C three categories, see Appendix B for a detailed list of the single categories and the corresponding number of products. A total of 11,628 items are considered. An individual minimum shelf quantity $T_i^{\min}$ of 6 days of supply and a maximum shelf reach $T_i^{\max}$ of 70 days is considered for each item listed. Product margins and demand figures are subject to a non-disclosure agreement. The data set further includes item dimensions to determine the space consumption of a facing a_i as well as the possible number units behind each facing g_i for each product i. Space elasticity is set at $\beta_i = 0.17$ (see [25]). In the case study, substitution is assumed to be zero ($\gamma_{ji} = 0, \forall j \in I_c^-$) due to the retailer's request to compute a lower bound of the profit potential and low substitution rates assumed by the retailer. On a category level, for each category a minimum and maximum number of shelf elements $E_c^{\min}$ and $E_c^{\max}$ is provided that is part of the retailer's master data and usually defined taking into consideration several impacts (e.g., marketing strategy, layout guidelines, purchasing contracts, and logistics planning). The categories are assigned to either a regular shelf or a chilled shelf. All shelves across categories in division A are regular shelves. Within division B some categories are displayed in regular shelves while others need to be placed within chilled shelves. Finally, all categories of division C are assigned to chilled shelves. The dimensions of a regular shelf are $133 \times 180 \times 57$ cm and for a chilled shelf $125 \times 200 \times 80$ cm. The available store space R for these three divisions, measured in running meters, is 886 meters. Further, we received data on the current number of shelf elements. The data provided did not contain the information about the actual product allocation in the store. To obtain a benchmark we therefore applied our PAM for each category using the current number of shelf elements. This means that the benchmark is based on the assumption that the product allocation of the status quo within each category is already optimal (in terms of PAM) for the current number of shelf elements. The results of this analysis provide a lower bound of the actual improvement potential using SWISS. More specifically, it represents the minimal profit improvement for the retailer when applying SAM, while the positive effect of PAM on product allocation cannot be singled out.

We identify the impact of optimizing the category sizes across all divisions of the store in Table 3. The minimum profit improvement is 3.2%. The runtime of SWISS optimization amounts to 830 s.

For division A, a profit increase of 1.9% is achieved, which results exclusively from the reallocation of shelf elements across categories, while the total space assigned to division A decreases. For example, the number of shelf elements increases by three shelf elements for Category 4 and the profit contribution increases by 11.9 currency units (CU), whereas the number of shelves is reduced by four elements for Category 5 and the profit contribution decreases by 5.6 CU. As a result, total profit increases by 6.3 CU, while the total number of shelf elements decreases by one for division A. We see significant improvements of 8.6% and 6.6% for divisions B and C. On the one hand, the total number of shelf elements assigned to these divisions increases up to $U^{\max}$, which has a straightforward positive effect on each division's profit. On the other hand the allocation of shelf elements to categories is optimized across divisions B and C as explained for division A.

Table 4 presents further details on a category level. There have been changes in the number of shelf elements and profits in 41 out of 54 categories. The reduction of space of a certain category

Table 3
Impact of SWISS optimization on a division level.

	U_d^{min}	U_d^{max}	Current no. shelf elements	SWISS	
				No. shelf elements	Profit impact by SAM [a]
Division A	342	707	607	573	1.9%
Division B	44	75	44	75	8.6%
Division C	10	23	18	23	6.6%
Total	396	805	669	671	3.2%
Regular shelves		617	585		
Chilled shelves		52	86		

[a] Profit improvement by SAM with SWISS optimization representing the lower bound on profit potential.

Table 4
Case study: Impact of SAM on category level.

Categories		Category values	Change in %		
Unchanged	Changed		Min.	Avg.	Max.
13	41	Profit	13.0	+3.2	+25.2
		Shelf elements	60.0	+18.1	+200.0

and the related demand losses are outperformed by increasing the profits of categories receiving more space. The positive average increase of shelf elements of +18.1% highlights a tendency to increase smaller categories, which results in high percentage increases. The absolute number of shelf elements within the store is only increased by 0.3% (see Table 3).

4.2. Performance tests and generalization of results with simulated data

The analysis of performance considers computations times, objective values and solution structures. Randomly generated data instances are employed that are informed by the case study data. We therefore leverage the case study to generate additional instances to generalize our findings.

Data generation. We use instances ranging from 20 to 80 categories and 50 to 200 items per category. The minimum number of shelf elements assigned to a category is randomly chosen (following a uniform distribution) between one and five for categories with 50 items and between two and eight for categories with 200 items. The range for the required space is then chosen using the same limits, resulting in the total range, i.e., the lower and upper bound for shelf elements. For instance, if the minimum number of shelf elements has been set at two for a test set with 50 items, the range of space requirement for the category is again chosen randomly between one and five elements. Assuming that the range was set at 3, the total range of possible shelf elements is given by the lower bound two and the upper bound five ($E_c = [2, 3, 4, 5]$). We consider three different shelf types to map different shelf layouts within the stores (e.g., regular shelf, chilled shelf and freezer). The corresponding dimensions (width $\times$ height $\times$ depth) observed in practice for the different types are as follows (in cm): type 1: $133 \times 180 \times 40$, type 2: $125 \times 160 \times 50$ and type 3: $140 \times 60 \times 70$. The shelf types are randomly assigned to each category such that an average of 70% of the categories are defined by shelf type 1, 20% by type 2 and 10% by type 3. Exactly one shelf type is selected for each category. The lower and upper limits for the shelf quantity of an item are defined in accordance with the case study and ensure at least 6 days and at most 70 days of supply. We further assume that case packs are allocated to the shelves, i.e. each facing contains several sales units next to each other. The dimensions of case packs are randomly distributed at 7–30 cm for the width, at 10–40 cm for the height, and at 5–40 cm for the depth. The number of sales units within one case pack ranges between 4 and 24 pieces. The margin for each item follows a triangle distribution and ranges between 0.4 and 1.1 of the purchasing price of a product, with a mode of 0.8. Finally, the base demand of items is generated using a gamma distribution with the parameters $p = 8$ and $b = 3$ (i.e., using the probability density function $f(x) = \frac{1}{\Gamma(p)} b^p x^{p-1} e^{-bx}$). These distributions resemble the data structure that we have found in our case data. The space elasticity parameter is again set at $\beta_i = 0.17$ for each product (cf. [25]). If not stated otherwise, we use an aggregated substitution rate for items $i, i \in I_c^-$ with $\gamma = \sum_{j \in I_c^+} \gamma_{ij} = 0.50$. That means 50% of the demand of a delisted item is substituted and 50% is lost. Thereby the substitution demand is equally split across all listed items. The corresponding store size for each instance is calculated by considering the shelf spaces of all corresponding categories with their minimum and maximum number of shelf elements. All categories are assigned to five divisions according to the following proportion: 25% of categories are assigned to divisions 1, 2 and 3 each, while 15% are assigned to division 4 and the remaining 10% of categories belong to division 5. The lower and upper limits for the shelf space of each division are 20% and 30% of the total store space for divisions 1, 2 and 3, 10% and 20% for division 4, and finally 5% and 15% for division 5.

Computational times of SWISS for varying problem sizes. The computational effort of SWISS optimization is assessed by examining eight instance classes of increasing size, from 1000 to 16,000 items. We analyze 10 instances with the given specifications for each class. Table 5 summarizes the average computation times across the instances for each test class. Even for the largest class of instances with 16,000 items (80 categories, 200 items per category), SWISS requires only 599 s on average. The maximum runtime amounts to 665 s, which is still a reasonable time considering that we are dealing with a tactical decision problem and in particular as the PAM is solved iteratively and called upon multiple times to incorporate substitution effects. An average of 99.9% of the entire computation time is consumed by the PAM. As we show, the runtime decreases significantly when substitution effects are not considered.

Impact of decomposition. In this experiment we analyze the efficiency of our two-step approach. To do this, we compare our approach to two alternative approaches that solve the problem in a single step. The alternative approaches represent a product allocation model, where the shelf space refers to the total store space and the items considered refer to the whole assortment of the store. In other words, the PAM is executed for the complete store, i.e., across all categories. The alternatives are denoted as $\text{PAM}_{AllInOne}$ and $\text{PAM}_{AllInOne}^{Cat.limits}$. The latter includes category limit constraints by taking into account the space consumption of all items of a category, while the first neglects these limits. We further applied some simplifications to these one-step approaches. In detail, as it is common for product allocation models we decide on the shelf space assigned to individual items, not on the number of shelf elements of a category, nor do we consider dif-

Table 5
Runtime analysis of SWISS optimization for varying problem sizes.

Number of categories $\|C\|$	Number of items per category $\|I_c\|$	Total number of items $\|I\|$	Store space R [m]	Runtime [s] with substitution Min.	Avg.	Max.	Runtime [s] without substitution[a] Min.	Avg.	Max.
20	50	1000	130	14	16	18	9	10	11
20	200	4000	240	122	154	192	63	82	106
40	50	2000	260	29	34	39	15	18	24
40	200	8000	450	278	311	350	144	160	181
60	50	3000	390	47	52	55	23	31	71
60	200	12,000	700	399	484	534	201	239	272
80	50	4000	520	80	87	91	33	37	42
80	200	16,000	900	550	599	665	287	338	398

[a] Runtime without substitution and only one PAM iteration.

Table 6
Solution of SWISS optimization compared to alternative approaches.

	SWISS	$PAM_{AllInOne}^{Cat.limits}$	$PAM_{AllInOne}$
Avg. runtime [s]	69	73	44
Min. profit delta[a]	–	+ 0.11%	+ 1.05%
Avg. profit delta[a]	–	+ 0.14%	+ 1.07%
Max. profit delta[a]	–	+ 0.17%	+ 1.14%

[a] Delta in objective value compared to SWISS solution.

ferent shelf types. We further do not include substitution effects. These simplifications are necessary due to the increased complexity of a one-step approach. The runtime increases exponentially and causes memory issues for larger instances. For the result comparison, the number of shelf elements per category are calculated ex-post based on the obtained space allocation decisions for corresponding items of a category. We use 10 data sets containing 15 categories and 200 products per category, which results in 3000 items. Further, we define two different shelf types with different widths: type 1 (90 cm width) and type 2 (133 cm width). The height and depth of the shelf types are set at 180cm and 40cm in both cases due to the computational limitations of $PAM_{AllInOne}$.

Table 6 shows the average performance across all instances. Table 7 presents details of the results of a sample instance for each category and assigned shelf space. It shows that the integer requirement for the shelf elements is not kept in both benchmarks, and that $PAM_{AllInOne}$ violates the minimum and maximum number of shelf elements in 6 out of 15 categories. Even though runtime seems to be very low for both variants, our tests resulted in memory issues whenever the $PAM_{AllInOne}$ had to compute larger data sets or to include substitution.

As there are no category size limits, $PAM_{AllInOne}$ provides a 1.1% higher total profit on average. It can be seen as an upper bound for the profits and indicates the profit impact of category size limitations when identical shelves across categories are applied. However, in $PAM_{AllInOne}$ solutions, all items are treated as items of one big category, and shelf space is thus spread arbitrarily across items. That also means that items of different categories share the same shelves, which is unwanted and non-feasible in retail practice. In contrast, SWISS optimization and $PAM_{AllInOne}^{Cat.limits}$ respect the given category limits. This results in a 0.1% marginally higher profit than SWISS. Yet only SWISS optimization provides feasible and optimal results by determining integer values for the number of shelf elements. The implementation of minimum / maximum restrictions within $PAM_{AllInOne}^{Cat.limits}$ results in a number of shelf elements that lies between these bounds. This means that fractional parts of shelf elements are assigned to categories, which also leads to non-feasible solutions. In conclusion, the results of SWISS optimization are close to the solutions of a model without limits ($PAM_{AllInOne}$) and fractional solutions ($PAM_{AllInOne}^{Cat.limits}$), which provide an upper bound for possible profits. The implications of these results are even more impressive when considering that differing shelf dimensions per category have not been considered, and that it was only possible to solve limited data sets of up to 3000 items.

Impact of detailed product allocation. A core aspect of SWISS optimization is the product allocation. The allocation of products has to be applied for varying category sizes to approximate the corresponding profit contributions of each category setting. It is consequently essential that the product allocation can be solved time-efficiently to ensure short processing times of SWISS optimization. We use PAM as a streamlined product allocation model in our approach. It focuses on the essential parts of product allocation and approximates profits for each category without using sophisticated extensions to mirror exact shelf dimensions and characteristics. In this section we analyze if a more sophisticated product allocation model can improve the results of SWISS optimization.

We introduce an extended SWISS optimization approach, denoted as $SWISS^{ext}$. $SWISS^{ext}$ follows the same algorithmic structure as SWISS, but relies on an extended product allocation model. In detail, the extended allocation model incorporates the determination of the number and position of shelf elements, exact stacking restrictions within the shelf level dimensions and a detailed allocation of products to shelf levels. It is therefore in line with state of the art product allocation models that consider multiple shelf dimensions (see e.g., [50]) and enable a detailed planning of planograms for retailers. The mathematical formulation of the extended allocation model is provided in Appendix A. Please note that $SWISS^{ext}$ causes memory issues for any larger data sets with more than 5 categories and 50 products per category due to the extended product allocation model. The analysis is therefore limited to this scope.

Table 8 presents the results of a comparison between SWISS and $SWISS^{ext}$. The runtime analysis demonstrates a significant increase when $SWISS^{ext}$ is used. On average, the runtime increases by a factor of 150, with a maximum increase by a factor of 570. This resembles tremendous growth in runtime, especially considering the very limited data size.

We further analyze whether the high computational effort of the more sophisticated model is justified with better solution quality. The objective of our approach is to determine the optimal shelf space for categories and divisions. We therefore need to evaluate whether the store space solutions generated within the SWISS are a good basis for the actual product allocation in stores, which takes place after the strategic planning of store space and its assignment to categories. For this purpose we first solve the store space allocation problem with SWISS and $SWISS^{ext}$ and apply the advanced product allocation model PAMiSD of [51] afterwards. The right part of Table 8 shows that the SWISS optimization reaches an average of 99.8% of the solution quality using $SWISS^{ext}$ when both are post-optimized with the detailed shelf optimization model PAMiSD. Tak-

Table 7
Example: number of shelf elements for different solution approaches.

Category data			Optimization with		
Category (shelf type)	Minimum shelf elements	Maximum shelf elements	SWISS shelf elements	$PAM_{AllInOne}$ shelf elements	$PAM^{Cat.limits}_{AllInOne}$ shelf elements
C1 (type 1)	2	7	7	9.59	6.99
C2 (type 1)	5	11	11	10.84	10.49
C3 (type 1)	3	12	12	10.99	10.53
C4 (type 1)	10	18	10	8.54	10.00
C5 (type 1)	7	12	11	9.81	9.07
C6 (type 1)	10	14	12	10.27	10.00
C7 (type 1)	3	10	10	9.94	9.55
C8 (type 1)	2	5	5	10.16	4.99
C9 (type 1)	8	13	13	10.77	9.91
C10 (type 1)	6	12	12	10.39	9.92
C11 (type 2)	8	15	9	7.29	8.01
C12 (type 2)	3	8	8	6.85	6.60
C13 (type 2)	10	14	10	6.58	10.00
C14 (type 2)	9	16	9	6.51	9.00
C15 (type 2)	6	9	7	5.73	6.00

Table 8
Comparison of SWISS with PAM and PAM^{plus}, with $|C|$=5 and $|I_c|$=50.

Instance	Runtime [s]			Obj. value of SWISS in % of $SWISS^{ext}$ [a]
	SWISS	$SWISS^{ext}$	Δ in %	
1	2	240	+ 14,010	99.7
2	2	1285	+ 57,613	99.4
3	2	263	+ 11,005	98.9
4	2	187	+ 9518	99.8
5	2	255	+ 13,077	99.9
6	2	86	+ 5401	100.0
7	2	156	+ 9314	99.9
8	2	188	+ 10,320	100.0
9	3	481	+ 16,156	100.0
10	2	163	+ 8406	99.8
Average	2	330	+ 15,481	99.8

[a] Both with ex-post optimization using PAMiSD of [51].

Table 9
Impact of substitution rates on solution structure.

	Aggregated substitution rate γ				
	0.00	0.25	base = 0.50	0.75	1.00
Change in total profit, in %	2.5	1.3	–	+1.4	+3.0
Avg. assortment size[a], in %	78	77	77	75	73
Avg. min. assortment size[a], in %	42	40	38	31	27
Avg. max. assortment size[a], in %	89	89	89	90	92
Avg. share of categories changed[b], in %	41.8	27.5	–	31.2	58.0
Max. increase[c], abs.	5	5	–	4	6
Max. decrease[c], abs.	2	2	–	5	7

[a] Average (minimum, maximum) ratio of listed items to total assortment per category.
[b] Categories with changed shelf space compared to base case.
[c] Maximum increase (decrease) of shelf elements compared to base case.

ing into account the runtimes of $SWISS^{ext}$ and the need to process larger data sets, we can state that PAM provides a sufficient approximation of profits for product allocation within the SWISS optimization for store space allocation decisions.

Impact of substitution. Product substitution has an impact on demand and thus on the facing assignment for each item. We therefore analyze the impact of substitution with different magnitudes as shown in Table 9. We use test instances with $|C| = 60$ and $|I_c| = 200$ for this analysis. A change in substitution rates affects total store profits. If the substitution rate is doubled from 50% to 100%, the profit increases by 3.0% in our setting. The other way around, if no substitution is assumed, profits decrease by 2.5%. In our base setting an average of only 77% of the total assortment of a category is listed, as space is very limited. This means a large proportion of products are delisted and gives substitution greater importance. To this end, this analysis provides a rough indication and upper bound of the total profit impact for such a strong assortment reduction. In the case of increased store space, broader assortment sizes can be expected and the impact of substitutions may be lower.

Substitution also has a significant impact on solution structure. This can be seen by assortment and category changes. Assortments are reduced by 4 percentage points on average when the substitution rate is doubled. This equals a delisting of 480 items of the store. The impact decreases for lower substitution rates. With respect to categories, an average of 42% receive a different number of shelf elements when no substitution is considered. The category size is very sensitive to the substitution rate, leading to the conclusion that substitution has a major impact on the solution quality and accuracy in the context of store space allocation.

Impact of store size. Store size is a central aspect of our problem as it sets the limits for the optimization. We therefore analyze the impact of the store size R on store space allocation decisions. Again, the test class with $|C| = 60$ and $|I_c| = 200, \ \forall c \in C,$ is used. With less space available, the competition between categories and items within categories becomes more intense. On the other hand, more available store space offers more options that have to be considered, i.e., more possible combinations of space allocated for each category. As a consequence, the possible store profit depends on R, and by analyzing different store sizes we provide insights on the impact of potential store dimensioning decisions. We use our base case with 700 shelf meters and show the effect of possible extension and reductions. The number of items and categories remains unchanged. Please note that store space can be reduced by no more than -30% to ensure minimum quantities $T^{\min}$.

Table 10 summarizes the results. The runtime of SWISS is robust against changes of R. The results show that the possibility of extending the store space to achieve increasing profits is limited. It may be uneconomical to extend the store space by +30% when at the same time profits only increase by +3.5%. The average assortment size increases with increasing store size and vice versa. Again, the changes are limited and the entire assortment is not listed even in very large stores. In the -30% setting, a disproportionate reduction of the average assortment can be observed. The average minimum assortment size undergoes an even higher reduction in this setting. This is induced by a total store space that becomes so small that each category is forced to its minimum size and thus – as minimum shelf quantities need to be considered for listed items – only a very small assortment can be chosen.

Table 10
Impact of varying store space R.

	Average change compared to base setting, in %				
			Assortment[a]		
Store space R	Runtime	Profit	Avg.	Min.	Max.
30%	2.5	10.2	16.9	39.5	1.1
20%	2.5	5.4	7.8	11.6	0.0
15%	4.2	3.7	5.2	4.7	0.0
10%	4.2	2.3	2.6	2.3	0.0
5%	2.5	1.1	0.0	2.3	0.0
base = 700 m	239	40,729	154/200	86/200	188/200
+5%	0.8	+0.9	+2.6	0.0	+1.1
+10%	2.1	+1.8	+3.9	0.0	+2.2
+15%	2.1	+2.5	+3.9	0.0	+3.4
+20%	1.3	+3.1	+5.2	0.0	+5.6
+30%	2.5	+3.5	+6.5	0.0	+7.9

[a] Delta in average (min/max) ratio of listed products to total assortment per category.

Table 11
Impact of division limits with evenly distributed categories.

Scenario	Fixed limit	Range limit	Without limit
Avg. division shares			
- Division 1	25.0%	21.4%	19.9%
- Division 2	25.0%	21.2%	20.1%
- Division 3	25.0%	22.6%	21.3%
- Division 4	15.0%	19.8%	19.7%
- Division 5	10.0%	15.0%	18.9%
Avg. delta in obj. value[a]	1.51%	–	+0.28%
Avg. runtime [s]	264	280	245

[a] Delta in objective value compared to base scenario with range limits.

Impact of division limits. Our concluding test analyses the impact of different settings for division limits. Retailers apply divisions to define locations and roles for categories. Certain space limits therefore apply for marketing or image reasons. We consider three different settings for the division limits. The standard setting (as given above with min./max. values for each range, denoted as range limit) is extended by a scenario with fixed limits, i.e., the size of each division is fixed in advance, and by a scenario without any predefined limits. Within the fixed limit scenario, the division shares are fixed at 25% for divisions 1, 2 and 3, 15% for division 4, and 10% for division 5 of total store space R. In the scenario without limits, the choice of the division sizes is unrestricted. We further need to relax category affiliation, as otherwise the number of categories is too restrictive for the decision on division sizes. To do so, we alternate the data setting to one where the share of categories per division is evenly distributed (i.e., 20% of total categories per division). We use the same data set as for the store analysis above. Table 11 summarizes our findings. Imposing fixed limits decreases the profit by 1.51%, whereas without division limits the profit can only be increased by 0.28%. Furthermore, the share of store space for single divisions changes significantly when there are no limits, or a limit range is specified. For instance, the share of division 5 increases from 10% (fixed limits) to 15% (range limits) and 18.9% (no limits). This shows that the store space allocation decisions are sensitive to the given division limits. The determination of division limits should therefore be part of the optimization problem by providing limits for the actual minimum and maximum sizes required.

5. Conclusion

Our work presents a hierarchical approach for the space allocation to categories. We consider the total store space and find a partition of available space across categories and divisions to maximize the overall store profit. Our approach links store layout planning with product allocations. It provides a basis for product allocation decisions which is only successful if the shelf space allocated for each category is reasonable. At our case company the assignment of shelf space to categories had up to the start of our study mostly been driven by "gut feeling" decisions without taking into account the actual profit potential of each category and the superordinate division. In literature, the majority of publications focus on the product allocation problem. This means that the available shelf space for each category is assumed to be an exogenous input parameter that has been defined in advance. Consequently, publications are lacking that consider the definition of available space per category. Prevailing publications with a store-wide focus determine category sizes but neglect product allocation decisions and corresponding category profit changes. There is consequently no work in our problem context that provides a model for a detailed category sizing together with numerical experiments to analyze the impact of these decisions within a practical application. We solve the store-wide shelf space allocation problem using SWISS optimization, which consists of two solution steps to determine store space allocation: (1) the PAM approximates the profit contribution of each category for all possible shelf space allocation options; (2) the SAM selects the optimal combination of category sizes while respecting the restrictions of the corresponding divisions. Using preprocessing and iterations, both models can be implemented as BIP and solved using CPLEX. This fact increases the attractiveness of our approach in practice, where understandable and efficient solution methods are required. The efficiency of our approach is shown in various experiments for practical relevant problem sizes. Further, we show that our PAM approach is a sufficient approximation tool for profits per category, and thus provides a solid basis for additional instore planning steps. Finally, we demonstrate the practical use of our approach in a case study with a major European retailer. We show that our approach is able to improve the given planning situation by more than 3%.

This paper closes an existing gap in literature and provides an efficient planning approach for practitioners. However, there are still numerous possibilities for future research projects. As mentioned above, we use an approximation for the detailed product allocation to obtain profits per category. In our setting, we show that a more sophisticated product allocation model does not improve overall performance, and that only small instances are solvable when more complex models are used. Nevertheless, one has to bear in mind that the PAM serves as approximation of profits per category for a chosen shelf size. The results obtain from SWISS build the basis for the subsequent shelf space allocation. The SWISS solutions can be used as input for more detailed and granular shelf space allocation models for each category where shelf space is an input parameter (see e.g., [5,45,51]). The iterative application of the super- and subordinate planning approaches would be worth investigating. In line with this, the models could be enriched by stochastic demand, seasonal demand, and demand effects caused by promotions or item pricing, as these are valuable avenues for further research in this area (see [6]). Our approach aims at a space allocation that maximizes category profit and with this the corresponding store profit. We therefore optimize the problem form a retailer perspective. A different point of view would be to consider the perspective of a manufacturer in the context of "category captainship" (cf e.g., [11,56]). A study that addresses all the relevant subjects of negotiation between manufacturers and retailers (such as assortment, prices and shelf space) would be a valuable contribution. Lastly, our approach determines the share of shelf space for each category and division, while we assume the number and location of both categories and divisions as given. Our problem could be further extended to decide on the sequence of categories within a division, or the overall arrangement of categories and divisions within stores.

CRediT authorship contribution statement

Manuel Ostermeier: Conceptualization, Methodology, Validation, Formal analysis, Writing - original draft. **Tobias Düsterhöft:** Methodology, Software, Formal analysis, Investigation, Visualization, Data curation, Writing - original draft. **Alexander Hübner:** Conceptualization, Validation, Funding acquisition, Project administration, Supervision, Writing - review & editing, Resources.

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Appendix A. Extended product allocation model

The objective function of the PAM^{plus} in (A.1) contains a parameter π_{iklh} that provides the precalculated value for item i with k facings on shelf level l with a stacking height of h. The binary decision variable x_{iklh} corresponds to 1 if item i is assigned with k facings on shelf level l which has a stacking height h. Within the extended demand function $\hat{D}_{iklh} = \alpha_i \cdot k^{\beta_i} \cdot n_{ih}^{\delta_i} \cdot \epsilon_l$ additionally n_{ih} provides the number of items stacked one above the other combined with a vertical space-elasticity δ_i and a shelf level dependent demand factor ϵ_i. Further replenishment costs are considered within $RC_{iklh} = max[0; (\hat{D}_{iklh} - q_{iklh}) \cdot RV]$, where, in the event that the total shelf quantity q_{iklh} of an item i with k facings and a height dependent number of items stacked n_{ih} is smaller than the demand $\hat{D}_{iklh}$, replenishments have to be executed with a cost factor RV per item. The profit contribution of an item i with k facings on level l and height h is then denoted by $\pi_{iklh} = \hat{D}_{iklh} \cdot M_i - RC_{iklh}$. Restriction (A.2) links the decision variable x_{iklh} with an auxiliary variable y_{lh} for the determination of shelf levels and heights using a sufficiently large number (BigM). Restrictions (A.3) and (A.4) define that only one solution is permitted for each item i, and that each shelf level l has to be defined precisely with one stacking height h. The height of a specific shelf rack P_c of a category c is considered in restriction (A.5), where GP corresponds to the granularity of possible height adjustments and $H^{\min}$ is the minimum stacking height a shelf level must take into account. All shelf levels determined on a shelf rack with their individual stacking heights h must not exceed the total height P_c of the shelf rack. The shelf space S_{ce} of category c with e shelf elements for each level l is considered in (A.6) with the number of facings k of all items i and their individual product dimensions a_i. In line with PAM, upper and lower bounds for the number of facings $K_{ih}^{\min}$ and $K_{ih}^{\max}$ are given in Restrictions (A.7). Finally restrictions (A.8) and (A.8) define the decision variables x_{iklh} and y_{lh} as binary.

$$max\ \hat{\Omega}(x_{iklh}) = \sum_{i\in I_c}\sum_{k\in K_c}\sum_{l\in L}\sum_{h\in H} x_{iklh} \cdot \pi_{iklh} \tag{A.1}$$

subject to

$$\sum_{i\in I_c}\sum_{k\in K_c} x_{iklh} - \text{BigM} \cdot y_{lh} \le 0 \ \forall l \in L, h \in H \tag{A.2}$$

$$\sum_{k\in K_c}\sum_{l\in L}\sum_{h\in H} x_{iklh} = 1 \ \forall\ i \in I_c \tag{A.3}$$

$$\sum_{h\in H} y_{lh} = 1 \ \forall\ l \in L \tag{A.4}$$

$$\sum_{l\in L}\sum_{h\in H} y_{lh} \cdot h \cdot GP + H^{\min} = P_c \tag{A.5}$$

$$\sum_{i\in I_c}\sum_{k\in K_c}\sum_{h\in H} k \cdot x_{iklh} \cdot a_i \le S_{ce} \ \forall\ l \in L \tag{A.6}$$

$$K_{ih}^{\min} \ge \sum_{k\in K_c}\sum_{l\in L}\sum_{h\in H} k \cdot x_{iklh} \le K_{ih}^{\max} \ \forall\ i \in I_c \tag{A.7}$$

$$x_{iklh} \in \{0, 1\} \ \forall\ i \in I_c, k \in K_c, l \in L, h \in H \tag{A.8}$$

$$y_{lh} \in \{0, 1\} \ \forall\ l \in L, h \in H \tag{A.9}$$

Appendix B. Case study categories

Divison A	Regular dry food
Categories	Number of items
Backery products, desserts	242
Bags, foils	70
Beer	159
Bread	173
Sandwich spread	164
Lemonade	84
Oil, vinegar	165
Ready meals	52
Fish tinned food	112
Ready sauces and spice mixtures	65
Meat tinned food	73
Tinned vegetables	247
Face and body care	1,445
Spices	277
Hair care	825
Medicine cabinet	77
Dog food	149
Toiletries	220
International food	47
Coffee	264
Cat food	168
Baby food	440
Cosmetics	72
Cake	321
Cereals	162
Snacks organic	35
Tinned fruits	30
Cleaning agents	610
Rice and potatoes	165
Juice	191
Salty snacks	254
Sparkling wine	44
Chocolate	379
Ice tea	146
Liquor	326
Sweet snacks	338
Tea	130
Pasta	184
Washing agents	293
Water	47
Wine	459
Dips	251
Sugar, Salt, Flour	151
Candy	183

Divison B	Dairy products
Categories	Number of items
Convenience chilled	29
Eggs	20
Delicacies chilled	87
Cheese self service chilled	361
Diary products white line chilled	385
Diary products cheese unchilled	108
Divison C	**Self service chilled**
Categories	Number of items
Meat self service chilled	46
Chicken self service chilled	44
Sausages self service chilled	269

Supplementary material

Supplementary material associated with this article can be found, in the online version, at doi:10.1016/j.omega.2021.102425.

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