Characterizing Time Irreversibility in Disordered Fermionic Systems by the Effect of Local Perturbations

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We study the effects of local perturbations on the dynamics of disordered fermionic systems in order to characterize time irreversibility. We focus on three different systems: the noninteracting Anderson and Aubry-André-Harper (AAH) models and the interacting spinless disordered t-V chain. First, we consider the effect on the full many-body wave functions by measuring the Loschmidt echo (LE). We show that in the extended or ergodic phase the LE decays exponentially fast with time, while in the localized phase the decay is algebraic. We demonstrate that the exponent of the decay of the LE in the localized phase diverges proportionally to the single-particle localization length as we approach the metal-insulator transition in the AAH model. Second, we probe different phases of disordered systems by studying the time expectation value of local observables evolved with two Hamiltonians that differ by a spatially local perturbation. Remarkably, we find that many-body localized systems could lose memory of the initial state in the long-time limit, in contrast to the noninteracting localized phase where some memory is always preserved.

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Introduction.—The second law of thermodynamics imposes strong constraints on the time reversibility of nonadiabatic processes between thermodynamic states. However, the applicability of the second law requires, in general, ergodicity, which is absent for closed many-body localized (MBL) systems [1–13]. This has recently also been shown experimentally [14–18]. Consequently, after a nonadiabatic process, such MBL systems do not inherit a thermodynamic description, leading to a major question: To what extent does this breaking of ergodicity influence time reversibility?

In this work, we probe the time reversibility of closed quantum many-body systems with disorder. Specifically, we study the sensitivity of the nonadiabatic dynamics due to weak local perturbations in three fermionic systems including the Aubry-André-Harper (AAH) as well as the Anderson model with and without interactions. We characterize time reversibility by noticing that the faster the departure of the perturbed and unperturbed trajectories, the stronger the sensitivity of quantum motion and therefore the stronger time irreversibility. In this work, the quantification of the distance of the two time-evolved systems is based on two complementary measures: First, we study the sensitivity in terms of the Loschmidt echo (LE) [19,20], which quantifies the deviation on the basis of the full quantum many-body wave function; second, we introduce a quantity which measures the closeness of only local properties instead of global wave functions, by measuring the local densities, representing a less strict measure as compared to the LE. We find numerical evidence corroborated by analytical arguments that the various distinct phases of our fermionic models can be detected and characterized by studying the long-time dynamics of these measures. Our predictions can be tested experimentally, because both of the studied quantities are, in principle, experimentally accessible for ultracold atoms in optical lattices and trapped ions where signatures of MBL have been already observed recently [14–18]. In systems of ultracold atoms, local densities can be measured with the use of quantum gas microscopes [21,22] and LE by interferometric techniques [23–25]. The local control of trapped ions provides direct access to local densities, and LE has been already measured in recent experiments [26,27].

Models and methods.—We study the Hamiltonian

$$\hat{H} := -\frac{t_h}{2}\sum_{x=-(L/2)}^{(L/2)-2}\hat{c}_x^\dagger\hat{c}_{x+1} + \text{H.c.} + \sum_{x=-(L/2)}^{(L/2)-1} h_x\hat{\tilde{n}}_x + V\sum_{x=-(L/2)}^{(L/2)-2}\hat{\tilde{n}}_x\hat{\tilde{n}}_{x+1},$$

where $\hat{c}_x^\dagger(\hat{c}_x)$ is the fermionic creation (annihilation) operator at site x and $\hat{\tilde{n}}_x = \hat{n}_x - \frac{1}{2}$ with $\hat{n}_x = \hat{c}_x^\dagger\hat{c}_x$, L the system size, and $N = (L/2)$ the number of fermions.

We consider three different cases: (i) The noninteracting Aubry-André-Harper (AAH) model, obtained from $\hat{H}$ with $V = 0$, $t_h = 2$, and $h_x = W\cos(2\pi x\phi + \alpha)$, where $\phi = (1+\sqrt{5}/2)$; α is a random phase uniformly distributed in $[0, 2\pi]$. The AAH model has a metal-insulator transition at $W_c = 2$ (extended phase for $W \leq W_c$ and localized phase for $W > W_c$). The localization length close

to the transition diverges as $\xi_{\rm loc} \sim \log^{-1}(W/2)$ [28]. (ii) The noninteracting Anderson model [29], given by $V = 0$, $t_h = 1$, and $\{h_x\}$ independent random variables uniformly distributed in $[-W, W]$. In the Anderson model, all the single-particle eigenstates are exponentially localized, and $\xi_{\rm loc} \sim W^{-2}$ in the weak disorder regime [30]. (iii) The spinless disordered t-V chain, obtained from the Anderson model by turning on the interaction with $V = 1$. This t-V chain is believed to have a MBL transition at a critical disorder strength $W_c \approx 3.5$ [4,9,10,31] (extended or ergodic for $W < W_c$ and localized for $W > W_c$) at an infinite temperature.

To study a spatially local perturbation of the Hamiltonian $\hat{H}$, we define

$$\hat{H}_\epsilon := \hat{H} + 2\epsilon\hat{n}_0, \tag{1}$$

with $\epsilon > 0$. A central object studied in this work is the LE [32–37], which in related forms has already been studied in disordered systems [11,38–41]:

$$\mathcal{L}(t) := |\langle\psi|e^{it\hat{H}}e^{-it\hat{H}_\epsilon}|\psi\rangle|^2. \tag{2}$$

To understand how states deviate in their local properties if evolved with H and H_ϵ, we study the difference of the local density profile (DLDP) [41,42], defining

$$\mathcal{D}(t) := \sum_x |\delta\rho(x,t)| \tag{3}$$

with

$$\delta\rho(x,t) := \langle\psi|e^{it\hat{H}}\hat{n}_x e^{-it\hat{H}}|\psi\rangle - \langle\psi|e^{it\hat{H}_\epsilon}\hat{n}_x e^{-it\hat{H}_\epsilon}|\psi\rangle. \tag{4}$$

Moreover, we are interested in the long-time behavior of $\mathcal{D}(t)$, which quantifies the long-time relative temporal fluctuations

$$\mathcal{D}_\infty := \lim_{T\to\infty}\frac{1}{T}\int_0^T ds\mathcal{D}(s). \tag{5}$$

For the initial state $|\psi\rangle$, we choose a product state in the occupation basis $(\prod_{s=1}^N c_{2s}^\dagger|0\rangle)$ (charge density wave state), which is easy to realize in experiments [14]. The strength of the perturbation ϵ is set equal to 0.1, so $\epsilon < \{t_h, W, V\}$. We would like to emphasize that, in contrast to Refs. [41,42], we consider sudden quenches at infinite temperatures, which as we will show exhibit very different physics. The average over disorder is indicated with an overline [43], e.g., $\bar{\mathcal{D}}(t)$.

Noninteracting models.—In this section, we study the LE and the DLDP for the AAH and Anderson models. We compute the LE for these models using a free fermion technique [44], which permits us to inspect large system sizes for long times. Figures 1(a)–1(c) show the LE in the two phases of the AAH and in the Anderson model. In the extended phase of the AAH model ($W = 1.5$), the LE

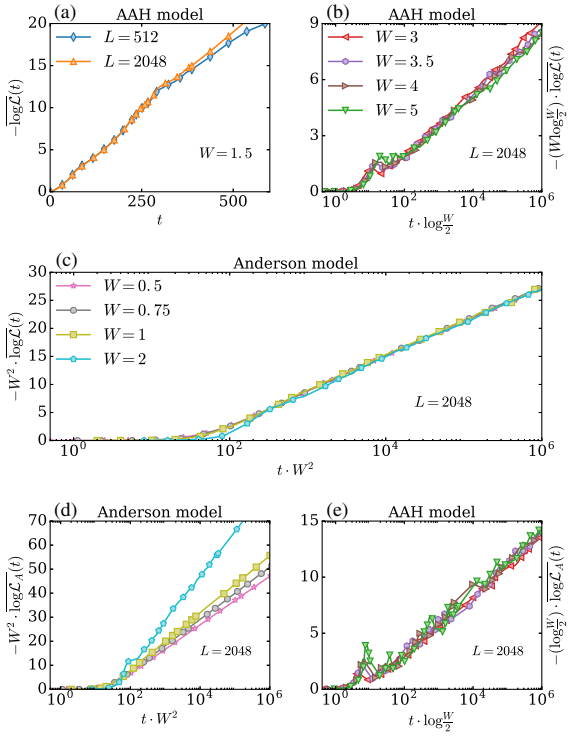

FIG. 1. (a),(b) Behavior of $-\overline{\log\mathcal{L}}(t)$ for the AAH model in the extended phase ($W = 1.5$) [$\mathcal{L}(t) \sim e^{-\Gamma t}$] and in the localized phase for several values of W [$\mathcal{L}(t) \sim t^{-\beta}$]. In the localized phase, t and $\mathcal{L}(t)$ have been properly rescaled to underline the time scale on which the decays starts and the behavior of the exponent of the algebraic decay β. (c) $-\overline{\log\mathcal{L}}(t)$ for the Anderson model for several values of W; here also a rescaling has been done on t and $\mathcal{L}(t)$. (d),(e) The approximate formula $\mathcal{L}_A(t)$ for the two noninteracting models and for the same values of W. The averages have been performed over 5000 random configurations of disorder.

decays exponentially as $\mathcal{L}(t) \sim e^{-\Gamma t}$, revealing the strong effect of local small perturbations. In the localized phase for both models (AAH and Anderson model), the LE decays algebraically in time as $\mathcal{L}(t) \sim t^{-\beta}$. Note that, in both phases, the long-time saturation value is exponentially small in system size, i.e., $\mathcal{L}(t \to \infty) \sim e^{-\eta L}$. Still, the two phases can be distinguished through the decay of the LE as a function of time.

For the localized phase, Figs. 1(b) and 1(c) also show the relation between the exponent β and the microscopic parameter of the Hamiltonian (W), with a good collapse of the curves. For the Anderson model, we observe $\beta \propto W^{-2}$, indicating that β is proportional to $\xi_{\rm loc}$ at least in the weak disorder limit. For the AAH model, we find the scaling $\beta \propto [W\log(W/2)]^{-1}$. Thus, β is again proportional to the localization length $\xi_{\rm loc}$ on approaching the metal-insulator transition to leading order. The rescaled time in the LE deserves particular attention: The time scale for the onset of the algebraic decay is proportional to the localization length, which on approaching the metal-insulator transition shifts to infinity in the thermodynamic limit.

We now present an analytical argument supporting the algebraic decay of the LE in the localized phase. In the Lehmann representation, the LE reads

$$\mathcal{L}(t) = \left| \sum_{n,m} \langle \psi | n \rangle \langle n | m_\epsilon \rangle \langle m_\epsilon | \psi \rangle e^{-it(E_n - E_m^{(\epsilon)})} \right|^2, \quad (6)$$

where E_n ($|n\rangle$) and $E_m^{(\epsilon)}$ ($|m_\epsilon\rangle$) are the eigenvalues (eigenvectors) of $\hat{H}$ and $\hat{H}_\epsilon$, respectively. The simple picture is that, in the localized phase, the local perturbation causes an exponentially weak dephasing of the energies of the unperturbed Hamiltonian with respect to the perturbed one, inducing the decay of the LE. The following approximations, which are equivalent to a first-order expansion in ϵ [45], permit us to estimate the behavior of the LE and relate the power-law exponent β to the localization length. We confirmed this relation close to the metal-insulator transition with exact numerics. First, we assume that the behavior of the LE is not affected by the choice of the initial product state. Our second approximation is that the perturbation affects only the eigenenergies but not the eigenstates.

The first approximation allows us to replace the overlap with the initial state $|\psi\rangle$ in Eq. (6) with a normalized trace over the entire Hilbert space. The second approximation implies $\langle n | m_\epsilon \rangle = \delta_{n,m}$. Finally, evaluating the energy difference $E_n - E_n^{(\epsilon)}$ using the first-order perturbation theory in ϵ, $E_n - E_n^{(\epsilon)} \approx \epsilon \langle n | \hat{n}_0 | n \rangle$, we can express the result in a closed form:

$$\mathcal{L}_A(t) = \prod_{j=1}^{L} \cos^2[\epsilon |\phi_j(0)|^2 t], \quad (7)$$

where $\{\phi_j(0)\}$ are the single-particle wave functions evaluated at the center of the chain. The subscript A underlines that this is an approximate formula. Since all single-particle eigenstates are exponentially localized, after an appropriate relabeling of the index j, we assume that $|\phi_j(0)|^2 \sim [e^{-(j/\xi)}/\xi]$. Thus, the only factors that contribute significantly are the ones where $\epsilon |\phi_j(0)|^2 t \approx 1$:

$$\mathcal{L}_A(t) \approx \prod_{j=1}^{\xi \log(\epsilon t/\xi)} \cos^2[\epsilon |\phi_j(0)|^2 t] \sim \left(\frac{\epsilon t}{\xi}\right)^{-c\xi}, \quad (8)$$

with $c > 0$. The last row of Figs. 1(c) and 1(d) shows the algebraic decay with time of the LE from Eq. (7) as $\mathcal{L}_A(t) \sim t^{-\beta_A}$ for the two models and several values of W. Surprisingly, despite being a perturbative expansion in ϵ, $\mathcal{L}_A(t)$ reproduces the algebraic decay of the LE also for long times. The exponents β_A and β have the same dependence on the microscopic parameter W in the vicinity of the critical point, namely, $\beta, \beta_A \sim W^{-2}$ as $W \to 0$ for the Anderson model and $\beta, \beta_A \sim \log^{-1}(W/2)$ as $W \to 2$ for the AAH model. Indeed, as shown in Figs. 1(d) and 1(e), β_A is proportional to the localization length ($\beta_A \propto \xi_{\rm loc}$). For the Anderson model, the deviation with increasing disorder strength W is just a sign that the perturbative expansion for $\xi_{\rm loc}$ is breaking down. Moreover, the approximate formula Eq. (8) describes well the rescaling of time, given by $t \to (\epsilon t/\xi)$.

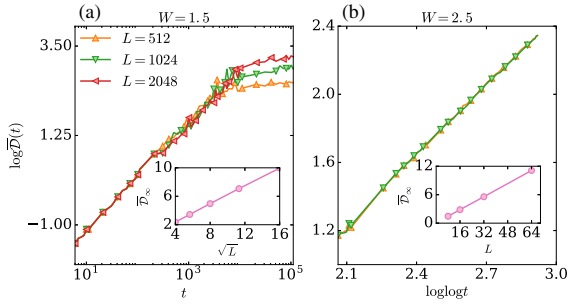

FIG. 2. $\bar{\mathcal{D}}(t)$ for the AAH model in the two phases for different L. (a) $W = 1.5$, $\bar{\mathcal{D}}(t) \sim t^\alpha$, while its inset shows $\bar{\mathcal{D}}_\infty$ as a function of L ($\bar{\mathcal{D}}_\infty \sim \sqrt{L}$). (b) $W = 2.5$, $\bar{\mathcal{D}}(t) \sim \log^\alpha t$, and its inset shows $\bar{\mathcal{D}}_\infty \sim L$. The averages have been performed over 2500 random configurations of disorder.

Next, we probe the effect of local perturbation on the dynamics of local observables by studying $\bar{\mathcal{D}}(t)$. Figure 2 shows $\bar{\mathcal{D}}(t)$ for two different values of W for the AAH model. In the extended phase with $W = 1.5$, $\bar{\mathcal{D}}(t)$ shows an algebraic growth with time, $\bar{\mathcal{D}}(t) \sim t^\alpha$, $\alpha \approx 0.6$ for $W = 1.5$. The saturation point in time of $\bar{\mathcal{D}}(t)$ is consistent with the scale $\sqrt{L}$ [inset, Fig. 2(a)] with the system size, indicating that in the long-time limit the average over index sites of the DLDP ($\bar{\mathcal{D}}_\infty/L$) relaxes algebraically with the system size [46].

In the localized phase, $\bar{\mathcal{D}}(t)$ has a loglike slow growth, $\bar{\mathcal{D}}(t) \sim \log^\alpha t$ with $\alpha \approx 1.3$ for $W = 2.5$, so the effect of local perturbations on the dynamics is exponentially slow in time. Moreover, $\bar{\mathcal{D}}_\infty \sim L$ [inset, Fig. 2(b)], so that the relaxation of $\bar{\mathcal{D}}_\infty/L$ never takes place.

Spinless t-V chain.—Having shown that the LE captures the salient features of the metal-insulator transition in the AAH model, we now study $\mathcal{L}(t)$ for the interacting spinless t-V chain that has a MBL transition. We perform the time evolution using full diagonalization for small system size $L \leq 16$ and using the Chebyshev integration technique [47] for larger L ($18 \leq L \leq 24$). Figure 3 shows the behavior of the LE for the interacting model for different values of disorder strength W. The enhanced decay compared with the noninteracting problem is also shown in Fig. 3. Nevertheless, in the localized phase, the LE still decays algebraically as in the localized phase of the noninteracting models. For $W = 6$, the function $-[\overline{\log \mathcal{L}}(t)/t]$ [inset, Fig. 3(c)] does not present any systematic dependence on the system size, indicating that the algebraic decay could be the asymptotic thermodynamic behavior. In the ergodic phase with $W = 1$, the LE decays at least exponentially with time, and the function $-[\overline{\log \mathcal{L}}(t)/t]$ does not decay for

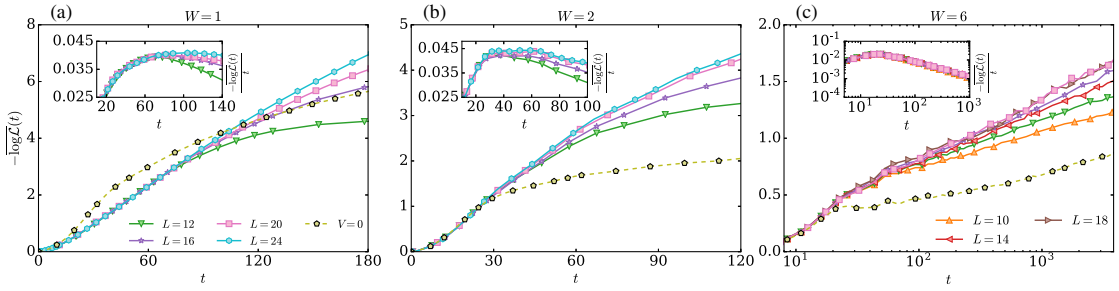

FIG. 3. The panels show $-\overline{\log \mathcal{L}}(t)$ for different values of disorder strength W. (a) The system is in the ergodic phase $W = 1$, and the LE decays at least exponentially fast with time. (b) An intermediate disorder strength $W = 2$, $-\overline{\log \mathcal{L}}(t)/t$ (inset) forms a plateau with time which is enlarging with the system size, showing that the range of times for which the LE decays exponentially fast is expanding. (c) The system is in the localized phase $W = 6$, and the LE decays algebraically with time. We also show the LE for the noninteracting case ($V = 0$) for the largest system size in each panel ($L = 24$ for $W = 1, 2$ and $L = 20$ for $W = 6$). The averages have been performed over 10^4 random configurations for system size $L \leq 14$ and 5000 for $L = 16$ and 2500 for larger system sizes.

times in which the decay of the LE is not affected by finite size effects [inset, Fig. 3(a)]. Figure 3(b) also shows an intermediate disorder value $W = 2$, at which the function $-[\overline{\log \mathcal{L}}(t)/t]$ develops a plateau with respect to t, like in the extended phase, after which a slower decay sets in. This plateau is enlarging with increasing system size, which may indicate that in the thermodynamic limit ergodicity will be completely restored and the LE will decay exponentially with t.

We now study the effects of perturbations in the dynamics of local observables by studying the DLDP. Figure 4 shows $\bar{\mathcal{D}}(t)$ in the interacting model for two values of W. We give evidence that the behavior of $\bar{\mathcal{D}}(t)$ in the ergodic phase for long times is drastically different from the noninteracting case: $\bar{\mathcal{D}}(t)$ is not a monotonic function of t [inset, Fig. 4(a)]. For short times, $\bar{\mathcal{D}}(t)$ grows to a maximum value from which it starts to decay to a finite L-dependent value. The nonmonotonic behavior is intimately connected with the thermalization of the system. Indeed, the long-time expectation values of local observables for thermal systems at an infinite

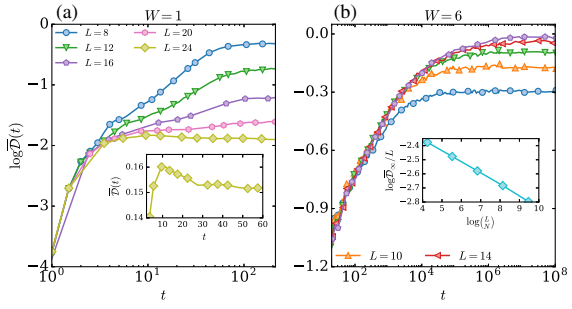

FIG. 4. $\bar{\mathcal{D}}(t)$ for the spinless disorder t-V chain for different L and two values of W. (a) $W = 1$; the inset shows $\bar{\mathcal{D}}(t)$ for $L = 24$ to underline its nonmonotonic dependence on t. (b) $W = 6$; the inset shows D_∞/L as a function of L, and it decays exponentially fast with L, $(\bar{\mathcal{D}}_\infty/L) \sim (\frac{L}{N})^{-\gamma}$. The averages have been performed over 10^4 random configurations for system size $L \leq 14$ and 5000 for $L = 16$ and 2500 for larger system sizes.

temperature should be unchanged if the system is locally perturbed. The average time in which the decay of $\bar{\mathcal{D}}(t)$ starts defines a time scale τ; this is roughly the time at which $\bar{\mathcal{D}}(t)$ changes concavity and starts to decrease. For times much larger than τ, the expectation value of a local observable is given by the expectation value over a many-body random state (eigenstate thermalization hypothesis at an infinite temperature), so that $|\delta\rho(x, t \gg \tau)| \sim (\frac{L}{N})^{-\gamma} \sim e^{-(\gamma \log 2)L}$.

In the localized phase, the finite size effects become more important, and for smaller system sizes it could seem that $\bar{\mathcal{D}}(t)$ [48] has an unbounded slow growth similar to the localized phases for the noninteracting models. However, a careful analysis shows that the saturation value is merely an exponential decay such as in the extended phase, consistent with $\bar{\mathcal{D}}_\infty \sim L(\frac{L}{N})^{-\gamma}$ [inset, Fig. 4(b)]. Compared with the ergodic phase, in the localized phase the exponent γ is small, so that, for the considered system sizes, the behavior of $\bar{\mathcal{D}}_\infty$ is dominated by the linear prefactor L. In the thermodynamic limit, we expect that the final shape will be similar to the one in the ergodic phase, so that $\bar{\mathcal{D}}(t)$ will eventually also decay with time at long times. Note that the time scale at which this decay will take place is extremely large; the limitation on the system size does not allow us to estimate an upper bound of the time scale τ, which leaves open the possibility that τ might shift to infinity with increasing L. The behavior of $\bar{\mathcal{D}}_\infty$ in the localized phase is reminiscent of the long-time "volume-law" saturation of the entanglement entropy $\mathcal{S}(t)$ after a quantum quench. The distinction between the ergodic and the MBL phase lies only in the numerical value of the prefactor in front of the saturation value of $S(t)$ [49,50], while the scaling with L is the same in both phases (volume law).

Conclusion.—In this work, we probed the effects of local perturbations on the dynamics of several disordered systems by studying the LE and the DLDP. First, with a combination of analytical arguments and exact numerical simulations, we showed that the LE in the localized phase decays algebraically in time. Furthermore, we found,

for the noninteracting models, that the exponent of the algebraic decay is proportional to the single-particle localization length, which diverges at the metal-insulator transition. In the extended phase, the LE decays exponentially fast with time. The faster exponential decay in the extended phase compared with the algebraic decay in the localized phase implies that time irreversibility is more strongly manifested in the extended phase than in the localized phase, at least for local perturbations. Second, we studied the DLDP for the same models, and we found that the long-time behavior saturates algebraically with the system size in the extended phase of the Aubry-André-Harper model, while it never relaxes for the noninteracting localized phase. For the DLDP in the spinless disordered t-V chain, the relaxation is exponential in system size in both phases: In the ergodic phase this is due to thermalization, while in the MBL phase it could be due to the interaction-induced dephasing mechanism, which also explains the long-time saturation values of the entanglement entropy after a quantum quench. The study of the change in the expectation values of local observables when the system is perturbed gives a different perspective concerning time irreversibility as opposed to the LE. Indeed, the long-time expectation value of local observables in a thermal system at an infinite temperature should be unchanged if the system is locally perturbed. We give numerical evidence that this also happens in the MBL phase.

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Note added.—Recently, we have become aware of related works on the LE in the MBL phase [51,52].

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