Reply

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The effects of financing rules in pay-as-you-go pension systems on the life and the business cycle

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Abstract: Empirically, revenues of public pension systems are more volatile than expenditures. Therefore, the question arises how the social security authority should buffer its revenues and adjust its contributions over the business cycle. This paper studies the corresponding effects on the life cycle of households and the business cycle in a large-scale overlapping generations model. In particular, the labor supply is endogenous and takes the intertemporal links between contributions and pension benefits into account. Sluggish adjustments of contribution rates that are implemented by adjusting a financial buffer stock both stabilize an economy and decrease the volatility of lifetime utilities of most workers and retirees, in contrast to sole adjustments of contribution rates. However, changes of consumption, capital income, or lump sum taxes, which aim to balance public pension budgets, improve the allocation of aggregate risk across cohorts for people up to an age of at least 71 years.

Keywords: Overlapping Generations, Pay-As-You-Go Pension Systems, Financing Rules, Business Cycle, Life Cycle, RBC-Model

JEL Classification: H55, E21, E30

1 Introduction

Revenues and pension benefits in pay-as-you-go (PAYG) pension schemes depend mainly on labor earnings which fluctuate over the business cycle. Fig. 1 shows the associated development of the cyclical component of employees' compensation and GDP for Germany from 1991:1 to 2019:4. Both variables vary substantially and, as a consequence, contribution rates, pension benefits and/or the stock of fi-

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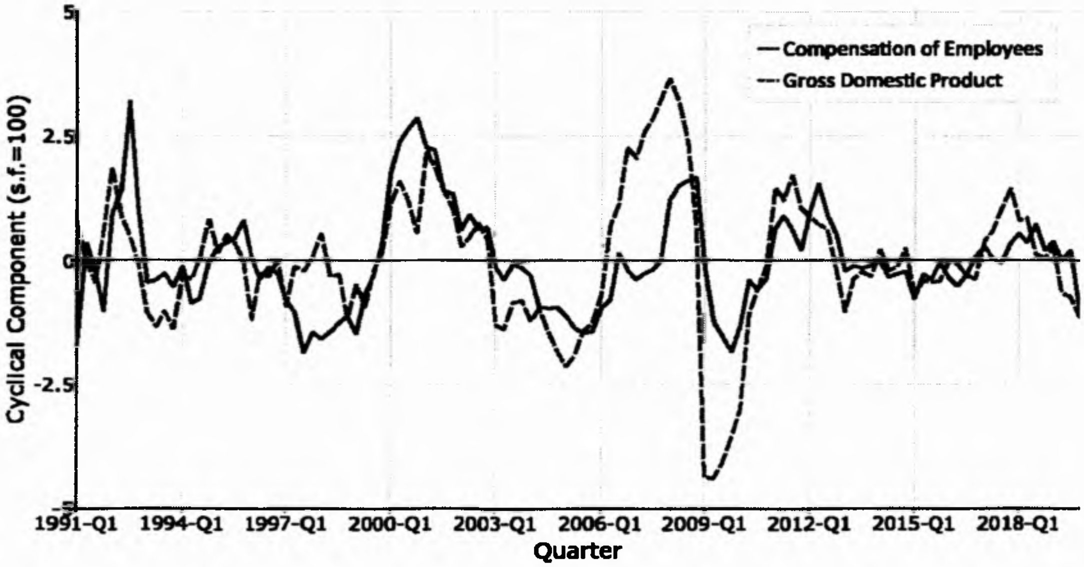


Figure 1: The Cyclical Component of Employees' Compensation and GDP (real, logged, and hp-filtered with weight 1600, s. f. = scaling factor, source: Federal Statistical Office 2020).

nancial assets in a PAYG system have to be adjusted so that its budget is balanced. These adjustments affect the intergenerational allocation of income and aggregate risk which in turn influences the consumption smoothing behavior of households. In addition, households experience several economic booms and recessions over their life cycle such that these financing rules with regard to short-term economic fluctuations over the business cycle also have very important impacts on household welfare in the long term. For example, the social security authority could only adjust the contribution rates. Such a financing rule shifts macroeconomic risks to younger generations since it increases the volatility of net wages of workers and holds pension benefits constant. Workers are, however, better able to deal with higher economic risks by changing their labor supply and savings rates in response to macroeconomic shocks, whereas retirees can only adjust their savings rate. Is such a financing rule preferable or should a government rather find another way to keep the budget of its PAYG scheme balanced?

In this paper, I study the effects of different financing rules for potential financial surpluses in PAYG systems on the business cycle and the age-specific consumption smoothing behavior of households in a large-scale real business cycle model with overlapping generations. In particular, I find that sluggish adjustments of contribution rates that are implemented by adjusting a buffer stock of financial assets of a PAYG system both stabilize an economy and help to decrease the volatility of (remaining) lifetime utilities of retirees and most workers, in contrast to solely complete adjustments of contribution rates. Such a policy reduces the distortionary effects of labor taxation and also allows a more flexible accumulation of wealth over the life cycle, which helps future retirees to hedge better against macroeconomic shocks over the business cycle. However, changes of consumption, capital income, or lump sum tax are able to improve the allocation of aggregate risk among households who are below an age of at least 71 years, even though they imply higher volatilities of aggregate variables.

The most closely related papers to mine are Thøgersen (1998), Wagener (2003), Krueger and Kubler (2006), Harenberg and Ludwig (2019), and Hasanhodzic and Kotlikoff (2018). Thøgersen (1998) studies the effects of PAYG pensions programs on the intergenerational allocation of risk and welfare. He finds that defined contribution rates imply a lower income risk and higher ex-ante welfare across generations. In contrast, Wagener (2003) shows that different PAYG schemes are not comparable in an ex ante perspective due to different information sets and decisions over the life cycle. From an ex post perspective, he concludes that fixed replacement rates are preferable to defined contributions. They improve intergenerational risk-sharing and induce higher utility levels. Both studies, however, assume that labor supply is completely inelastic and exclude general equilibrium effects since all prices (respectively, their probability distributions) are fully exogenous. Moreover, the framework of these models with only two overlapping cohorts implies that the implemented shocks should rather be interpreted as longer-lasting shocks in contrast to business-cycle shocks. Moreover, Krueger and Kubler (2006) study the introduction of an unfunded social security system in a stochastic model with nine overlapping generations and aggregate risk. They find that claims in PAYG pensions programs strengthen the risk-sharing between generations and that the introduction of a PAYG financed social security system can be welfare-improving, if the associated crowding-out effect of capital is negligible. Harenberg and Ludwig (2019) address a similar research question by means of an overlapping generations model with aggregate and idiosyncratic risk. They show that introducing a PAYG system with a minimum pension results in pronounced long-run welfare gains. Furthermore, Hasanhodzic and Kotlikoff (2018) investigate the effects of the bond market and social security on generational risk in a model with overlapping generations and aggregate risk. The authors conclude that both channels improve the allocation of risk between generations. These three studies, however, also assume that the age-specific labor supply is exogenously given. As a consequence, workers are more exposed to economic shocks since they are not able to smooth their utility over their life-cycle by adjusting their labor supply, which in turn affects risk sharing across generations.

This study extends the aforementioned research and examines the age-specific impacts of different financing rules in PAYG systems which keep the social security budget balanced over the business cycle. It therefore abstracts from long-term demographic trends[1] to isolate more clearly the potential effects of the interplay between short-term economic fluctuations and these financing rules on the welfare of households. The model used in this study methodically builds on Ríos-Rull (1996) and Heer and Maußner (2012). In particular, it is simulated on a quarterly basis with 260 different generations, takes general equilibrium effects into account, and labor supply is endogenous. Workers can, therefore, adjust their labor supply in response to changes in factor prices, where they also take the inter-temporal link between contributions and pension benefits in the PAYG system explicitly into account. Moreover, I calibrate the model to the German economy using both macroeconomic data and age-specific data from the German Socio-Economic Panel (2019) such that the model also roughly replicates relevant life-cycle patterns.

The rest of this paper is organized as follows. Section 2 describes and explains the model, which I calibrate in Section 3. The resulting steady state is discussed in Section 4, while Section 5 studies the effects of different financing rules in pay-as-you-go systems on aggregate variables and the consumption smoothing behavior of households over the business cycle. Section 6 concludes.

2 The model

In this section, I present a model with overlapping generations and aggregate uncertainty, where the period length is set to one quarter. Households optimize their expected lifetime utility, firms maximize profits, a government collects tax revenues for government consumption, and a PAYG system transfers resources across generations. Moreover, I assume that each household consists of one adult so that the terms "household" and "individual" have the same meaning and are interchangeable in this model.

2.1 Demographics

Each year, a new cohort is born and its size ψ_s is constant at age $s = 1$ (corresponding to a real life age of 26 years). Households live at most T quarters, where they

1 For example, Heer et al. (2020) examine the long-term sustainability of PAYG systems in 14 European countries and in the U.S by means of an overlapping generations model with distortionary taxation on labor and demographic changes. They conclude that the planned payments of pension benefits to retirees can not be met by tax revenues in European countries before 2050.

work up to an age of T_w quarters and enter retirement at an age of T_r quarters. I hereby assume that households can also work after reaching the retirement age such that the condition $T_w \geq T_r - 1$ always holds. In addition, each s-year old household survives from age s to $s + 1$ with an exogenously given probability of ϕ_s, where $\phi_0 \equiv 1$. Thus, the mass of households ψ_{s+1} at age $s+1$ evolves according to $\psi_{s+1} = \phi_s \psi_s$. For simplification, I normalize the total mass of living households $\sum_{s=1}^{T} \psi_s$ to one.

2.2 Households

A household at age $s = 1$ in period t maximizes the following discounted expected lifetime utility U_t with respect to consumption c_t^s, labor supply n_t^s, and savings in the form of capital goods k_{t+1}^{s+1}:

$$U_t = E_t \sum_{s=1}^{T} \beta^{s-1} \left(\prod_{j=1}^{s} \phi_{j-1} \right) \left[u(c_{t+s-1}^s, n_{t+s-1}^s) + (1 - \phi_s)\, b(k_{t+s}^{s+1}) \right], \tag{1}$$

where $n_t^s \in [0,1]$ for $s \leq T_w$ and $n_t^s \equiv 0$ for $s > T_w$. Moreover, the specification of the first instantaneous utility function $u(c_t^s, n_t^s)$ follows Trabandt and Uhlig (2011),

$$u(c_t^s, n_t^s) = \frac{1}{1-\eta} \left[(c_t^s)^{1-\eta} \left(1 - \frac{\gamma_0^s (1-\eta)}{1 + 1/\gamma_1} (n_t^s)^{1+1/\gamma_1} \right)^{\eta} - 1 \right], \tag{2}$$

and the second instantaneous utility function $b(k_{t+s}^{s+1})$ introduces a warm-glow bequest motive in a similar way to that used in De Nardi and Yang (2014):

$$b(k_{t+s}^{s+1}) = \frac{\gamma_3}{1-\eta} \left[(k_{t+s}^{s+1})^{1-\eta} - 1 \right]. \tag{3}$$

These preferences feature a constant Frisch elasticity of labor supply γ_1 and a constant intertemporal elasticity of substitution $1/\eta$. The age-specific parameter γ_0^s and the variable γ_3 control the labor supply and the strength of the bequest motive in the steady state of the model.

Households at age $s = 1$ are born without assets and accumulate a stock of capital $k_{t,j}^s$ over their life cycle.[2] Their capital earns the real interest rate r_t and

2 Please note that it is not possible to include a risk-free and risky asset with an endogenous portfolio choice into my model since I need a second order approximation around the steady state for this problem. However, the model has 1795 variables so that this solution method is not feasible. Moreover, most studies about PAYG systems and aggregate risk, see also my review

depreciates at the rate δ. The net labor income of workers depends on the real wage w_t, the age-specific productivity e^s, the income tax rate τ_t^g, and the contribution rate τ_t^p for the PAYG system, where pension benefits $pben_t^s$ are only paid to retired agents.[3] The government collects all accidental bequests and transfers them lump-sum back to the households in the form of tr_t. The variables τ_t^k and τ_t^c denote the capital income tax rate and the statutory value-added tax (VAT) rate levied on consumption goods, respectively. The budget constraint of a s-year old household in period t is given by

$$\begin{aligned}
(1+\tau_t^c)\,c_t^s + k_{t+1}^{s+1} &= \left[1+(r_t-\delta)\left(1-\tau_t^k\right)\right]k_t^s + (1-\tau_t^p-\tau_t^g)\,w_t e^s n_t^s + \\
&\quad tr_t, \quad \text{for } s < T_r, \\
(1+\tau_t^c)\,c_t^s + k_{t+1}^{s+1} &= \left[1+(r_t-\delta)\left(1-\tau_t^k\right)\right]k_t^s + (1-\tau_t^p-\tau_t^g)\,w_t e^s n_t^s + \\
&\quad tr_t + pben_t^s, \quad \text{for } T_r \le s \le T_w, \\
(1+\tau_t^c)\,c_t^s + k_{t+1}^{s+1} &= \left[1+(r_t-\delta)\left(1-\tau_t^k\right)\right]k_t^s + tr_t + pben_t^s, \\
&\quad \text{for } s > T_r,
\end{aligned} \tag{4}$$

with $k_t^1 \equiv 0$, $n_t^s \equiv 0$ for $s > T_w$, and $pben_t^s \equiv 0$ for $s < T_r$. The pension entitlements $pent_t^s$ depend on average lifetime labor earnings before a household enters retirement at age $s = T_r$ and an exogenously given replacement ratio ζ. For ease of notation, I also introduce the parameter θ_t which is equal to one in the steady state and can be adjusted by the social security authority such that it controls the effective replacement ratio $\zeta\theta_t$ in period t outside the steady state. Thus, pension benefits are represented by

$$pben_t^s = \theta_t\, pent_t^s, \tag{5}$$

where pension entitlements can be expressed as

$$pent_t^s = \begin{cases} \dfrac{\zeta}{T_r-1}\displaystyle\sum_{i=1}^{T_r-1} w_{t-i}e^{T_r-i}n_{t-i}^{T_r-i}, & \text{for } s = T_r, \\ pent_{t-s+T_r}^{T_r}, & \text{for } s > T_r. \end{cases} \tag{6}$$

of the literature, assume that labor supply is exogenous so that the respective models can be solved numerically. In my model, labor supply is endogenous and, as a consequence, I have to introduce this simplifying assumption because of computational reasons. Furthermore, I abstract from borrowing constraints since I linearize the model around a deterministic steady state.

3 The model features a perfect capital market without borrowing constraints such that private pensions accounts and private savings are perfect substitutes. For this reason, I abstract from private pensions since private pension accounts would only crowd out voluntary private savings without having any effects on the results. See also Fehr (2000).

The representative first-order conditions that solve the optimization problems of households consist of the aforementioned budget constraints (4) and

$$\lambda_t^s = \frac{\partial u(c_t^s, n_t^s)}{\partial c_t^s} \frac{1}{1+\tau_t^c},$$

$$\lambda_t^s = \beta\phi_s E_t \left\{\lambda_{t+1}^{s+1}\left[1+(r_{t+1}-\delta)\left(1-\tau_{t+1}^k\right)\right]\right\} + (1-\phi_s)\frac{\partial b(k_{t+1}^{s+1})}{\partial k_{t+1}^{s+1}},$$

$$0 = \frac{\partial u(c_t^s, n_t^s)}{\partial n_t^s} + (1-\tau_t^p-\tau_t^g)\, w_t e^s \lambda_t^s + E_t\left\{\sum_{a=T_r}^{T}\beta^{a-s}\left(\prod_{j=s+1}^{a}\phi_{j-1}\right)\lambda_{t+a-s}^a \frac{\zeta\theta_{t+a-s} w_t e^s}{T_r-1}\right\},\quad \text{for } s < T_r,$$

$$0 = \frac{\partial u(c_t^s, n_t^s)}{\partial n_t^s} + (1-\tau_t^p-\tau_t^g)\, w_t e^s \lambda_t^s,\quad \text{for } T_r \le s \le T_w. \tag{7}$$

The variable λ_t^s denotes the Lagrange multiplier.

2.3 Production

Aggregate output Y_t is characterized by a Cobb-Douglas production function,

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}. \tag{8}$$

The variables N_t and K_t denote aggregate labor and capital, respectively. Moreover, the stochastic technology level Z_t follows a standard AR(1) process: $\ln Z_t = \rho \ln Z_{t-1} + \epsilon_t$, where $\epsilon_t \sim N(0, \sigma^2)$. The corresponding profit maximization under perfect competition implies zero profits and that factor rewards equal their marginal products,

$$w_t = (1-\alpha) Z_t \left(\frac{K_t}{N_t}\right)^{\alpha}, \tag{9}$$

$$r_t = \alpha Z_t \left(\frac{K_t}{N_t}\right)^{\alpha-1}. \tag{10}$$

2.4 Social security & government

The social security authority collects contributions at the rate τ_t^p of gross labor incomes of workers and holds a buffer stock of financial assets F_t which is invested in capital goods. Moreover, the age-specific public pension entitlements $pent_t^s$ adjust over time since they depend on gross pre-retirement earnings and the steady state replacement ratio ζ according to equation (6). These entitlements give in turn the pension benefits $pben_t^s = \theta_t pent_t^s$ that can be adjusted in the short run by the variable θ_t, as also described in equation (5). Thus, the budget of the PAYG system is given by

$$F_{t+1} = \tau_t^p w_t N_t + (1 + r_t - \delta) F_t - Pben_t, \tag{11}$$

where

$$Pben_t = \theta_t Pent_t, \tag{12}$$

$$Pent_t = \sum_{s=T_r}^{T} \psi_s \, pent_t^s, \tag{13}$$

$$N_t = \sum_{s=1}^{T_w} \psi_s e^s n_t^s. \tag{14}$$

In order to describe the dynamics of the variables τ_t^p, F_{t+1}, and θ_t around the steady state, I follow a similar approach as in Galí et al. (2007) for fiscal policy rules and specify the following financing rule in the PAYG scheme,

$$F_{t+1} - F = \omega_F S_t, \tag{15}$$

$$\theta_t Pent_t - Pent_t = \omega_R (1 - \omega_F) S_t, \tag{16}$$

with

$$S_t = \tau^p (w_t N_t - wN) + (R_t F_t - RF) - (Pent_t - Pent) \tag{17}$$

and $\omega_F, \omega_R \in [0, 1]$. Variables without a time index denote the corresponding steady state values. The expression $R_t \equiv 1 + r_t - \delta$ in equation (17) defines the gross interest rate. Let us assume that the PAYG administration keeps the parameter θ_t, which controls the effective replacement ratio $\zeta\theta_t$, and the contribution rate constant outside of the steady state of the model, $\theta_t = \theta$ and $\tau_t^p = \tau^p$. Then, the term S_t measures both the level and the absolute change of potential revenue surplusses in the PAYG system since S is equal to zero in the steady state. The budget of the PAYG, however, must be balanced in every period according to equation (11) such that the social security authority either has to adjust the stock of financial assets F_{t+1}, the parameter θ_t, or the contribution rate τ_t^p in order to spend the laissez-faire revenue surplus S_t. Thus, the exogenous parameters ω_F and ω_R in equation (15) and (16) control the adjustments of financial assets and effective replacement ratios over the business cycle. If, for example, a positive productivity shock results in a positive revenue surplus, then the parameters $\omega_F = 1$ and $\omega_R = 0$ imply that the PAYG authority will fully invest S_t in the stock of financial assets F_{t+1}. In contrast, if one sets the parameters $\omega_F = 0$ and $\omega_R = 0.5$, then 50 % of the surplus S_t will finance an increase of current pension benefits $\theta_t Pent_t$ and the remaining 50 % will reduce the contribution rate τ_t^p.

For simplicity, I follow Heer et al. (2017) and assume that all accidental bequests are collected by the government and transferred as lump-sums to the household sector. Government spending G_t is financed by tax revenues, where C_t denotes aggregate consumption.[4] This implies

$$\begin{aligned} tr_t &= \sum_{s=1}^{T}(1-\Phi_{s-1})\psi_{s-1}\left[1+(r_t-\delta)\left(1-\tau_t^k\right)\right]k_t^s, \\ G_t &= (r_t-\delta)\,\tau_t^k(K_t-F_t)+w_t\tau_t^g N_t+\tau_t^c C_t. \end{aligned} \tag{18}$$

2.5 Equilibrium

In a general equilibrium, individual and aggregate behavior must be consistent. Thus, the following conditions have to be satisfied for all t,

$$N_t = \sum_{s=1}^{T_w}\psi_s e^s n_t^s, \tag{19}$$

$$K_t = \sum_{s=1}^{T}\psi_{s-1}k_t^s + F_t, \tag{20}$$

$$C_t = \sum_{s=1}^{T}\psi_s c_t^s, \tag{21}$$

such that the goods market clears:

$$Z_t N_t^{1-\alpha}K_t^{\alpha} = C_t + I_t + G_t, \tag{22}$$

where $I_t = K_{t+1} - (1-\delta)K_t$.

4 In the macroeconomic literature, it is usually a standard assumption that government spending is modeled as pure waste and does not affect the utility of households at all. See, for example, Galí et al. (2007), Khan and Reza (2017), or Uhlig (2010). However, it is also possible to assume that government spending enters the utility function in an additive separable way and that only the fluctuations of government spending outside of the steady state, $G_t - G$, are pure waste. For simplicity, I dispense with this assumption.

3 Calibration

I calibrate the model on a quarterly basis for the German economy and linearize the model around the steady state.[5] For the reader's convenience, I use the real life age in years in contrast to the quarterly age index s in the discussions and figures hereinafter.

Households enter the economy at age 26 and work up to an age of $T_w = 79$, where T_w and the age-specific parameter γ_0^s are set so that the model replicates the smoothed pattern of average age-specific labor supply of individuals in the German Socio-Economic Panel (2019) for the years 2016 to 2018. In addition, I use the same data source to calculate the median of real hourly earnings of individuals for every age in the sample 2016–2018 as an approximation for the productivity profiles e^s. For simplification, the productivity e^s at age 26 is normalized to one. The parameter γ_3 is set to 14.46 so that the net worth of the oldest household at age 90 is equal to the scaled real median net worth at that age from the SOEP sample 2017.[6] The survival probabilities ϕ_s also refer to the year 2017 and are taken from the Federal Statistical Office (2020). Moreover, I set the discount factor β equal to 1.00 such that the real rate of return on capital, $r_t - \delta$, equals a value of 4 % which describes the long term average according to Busl and Seymen (2013). With respect to the Frisch labor supply elasticity, I choose $\gamma_1 = 2.15$ in order to roughly match a relative volatility of aggregate hours to aggregate output of 0.63 for the sample 1991:1 to 2019:4. This value is in line with the macro-economic literature, which often uses Frisch elasticities between 2 and 4.[7] Furthermore, I choose a standard value of 2 for the parameter η implying an intertemporal elasticity of substitution of 0.5.

The tax rates are equal to $\tau^c = 0.15$, $\tau^k = 0.23$, and $\tau_g = 0.23$ such that $\tau_g + \tau^p = 0.41$, as in Trabandt and Uhlig (2011). Throughout the simulations, I assume that the government itself does not change these tax rates so that government consumption is always given by $G_t = (r_t - \delta)\tau^k K_t + w_t \tau^g N_t + \tau^c C_t$. With respect to the production technology, I use values estimated by Flor (2014) for the German economy. The production elasticity of capital is equal to $\alpha = 0.34$ and the depreciation rate δ equals 1.7 %. Moreover, the autocorrelation parameter for technology shocks is set to $\rho = 0.83$ and the corresponding standard deviation of innovations is equal to $\sigma = 0.0082$, where Flor (2014) takes both capital and labor as factor inputs into account for the calculation of the Solow residual.

5 In particular, I use the solution methods described in Chapters 9 and 10 in Heer and Maußner (2009) and modified codes of the provided CoRRAM package (see www.wiwi.uni-augsburg.de/vwl/maussner/dge_buch/dge_book_2ed/downloads_2nd).

6 The SOEP collects data on individual net worth only every five years.

7 See, for example, Peterman (2016).

Regarding the PAYG system, households start receiving pension benefits at age $T_r = 64$ according to Deutsche Rentenversicherung Bund (2020) for the year 2017. The replacement ratio ζ of pensions relative to average pre-retirement earnings is set to 40 % and taken from Kluth and Gasche (2015). Moreover, I assume that the stock of financial assets F is equal to aggregate (quarterly) pensions entitlements *Pent* in the steady state and set the parameter θ, which controls the effective replacement ratio $\zeta\theta_t$ in period t, equal to one, respectively.[8] The resulting stationary contribution rate amounts to $\tau^p = 18.4\,\%$, which is very close to its empirical counterpart of 18.70 % for the year 2017.[9] With respect to financing rules for additional revenue surpluses in the PAYG system, I distinguish between six cases that I will discuss in the following sections[10]:

- **Case 1:** This is the benchmark case, where I assume that the PAYG authority seeks to keep the contribution rate τ_t^p as constant as possible and does not adjust the effective replacement ratio $\zeta\theta_t$ over the business cycle.[11] For that reason, I set the parameter $\omega_R = 0$ and $\omega_F = 0.95$.[12]
- **Case 2:** The social security authority chooses $\omega_R = 1$ and $\omega_F = 0$ so that only the effective replacement ratio $\zeta\theta_t$ fluctuates over the business cycle.
- **Case 3:** The social security authority only adjusts the contribution rates, $\omega_R = \omega_F = 0$. Hence, the stock of financial assets and the effective replacement ratio stay constant.
- **Case 4:** The social security authority gets the additional option to adjust only the value added tax rate τ_t^c. In this case, it gets the amount $(\tau_t^c - \tau^c)C_t$ from the government so that the budget constraint of the PAYG scheme, see equation (11), changes to $F = (\tau_t^c - \tau^c)C_t + \tau^p w_t N_t + (1 + r_t - \delta)F - Pent_t$ since $\tau_t^p = \tau^p$, $\theta_t = \theta$, and $F_t = F$.
- **Case 5:** The social security authority is allowed to change only the capital income tax rate τ_t^c. Then, it gets the amount $(\tau_t^k - \tau^k)(r_t - \delta)(K_t - F_t)$ from

8 The financial buffer stock only amounts to 1 % of aggregate capital in the steady state. Therefore, the opportunity costs of maintaining a financial buffer stock as well as the effects on the marginal products of labor and capital are negligible.

9 See Deutsche Rentenversicherung Bund (2020).

10 Alternatively, the social security authority could also adjust the retirement age T^R or the maximum working age T^W. However, it is very unlikely that a social security authority is able to use these tools in the short-term to smooth economic fluctuations over the business cycle.

11 In Germany, the PAYG administration adjusts the contribution rates when the size of the reserve fund (Nachhaltigkeitsrücklage) exceeds (undershoots) the monthly expenditures by 150 (20) percent, see §158 SGB VI.

12 This calibration still ensures local stability around the steady state and allows to approximate this form of financing. Then, the PAYG administration mainly changes its stock of financial assets and dampens the adjustments of contribution rates.

the government so that its budget constraint is represented by $F = (\tau_t^k - \tau^k)(r_t - \delta)(K_t - F_t) + \tau^p w_t N_t + (1 + r_t - \delta) F - Pent_t$ since $\tau_t^p = \tau^p$, $\theta_t = \theta$, and $F_t = F$.

- **Case 6**: A lump sum tax Tax_t, which is equal to zero in the steady state, is levied on all households to cover the total surplus in the PAYG system. This scenario implies that the term Tax_t enters all individual budget constraints in equation (4). Moreover, the new budget constraint of the PAYG scheme is given by $F = Tax_t + \tau^p w_t N_t + (1 + r_t - \delta) F - Pent_t$ since the total mass of the population is normalized to one, $\tau_t^p = \tau^p$, $\theta_t = \theta$, and $F_t = F$.

4 Steady state

Fig. 2 presents the behavior of households over the life cycle in the steady state. The consumption profile in the upper left panel increases until an age of 59 years and roughly follows a hump-shaped pattern which is qualitatively consistent with empirical evidence provided by Kluge (2011). The labor supply, as displayed in the upper right panel, replicates the smoothed empirical profile due to the age-specific calibration of the parameter γ_0^s. It increases until an age of 45 and falls monotonously thereafter. Moreover, the lower left panel shows the net worth of households over the life cycle, which roughly matches the pattern of its empirical counterpart. Nevertheless, the net worth of young households is a little bit too high in comparison to the data. The age-specific efficiency profile is displayed in the lower right panel. It increases rapidly between the ages of 26 and 33 years and then remains relatively constant until an age of 60 years. Thereafter, it decreases and remains at this level from the age of 70 onwards.

5 Effects over the business cycle

In this section, I study how financing rules in PAYG systems affect aggregate variables and the consumption smoothing behavior of households. For Case 1 to 3, Fig. 3a presents the associated impulse responses of aggregate variables to a positive one-time productivity shock of one standard deviation in period $t = 2$.[13] The first two rows in Fig. 3a show the benchmark case. A technology shock increases output, labor supply, consumption, and investment. The real interest rate rises

13 The variables are expressed as percentage deviations from steady state.

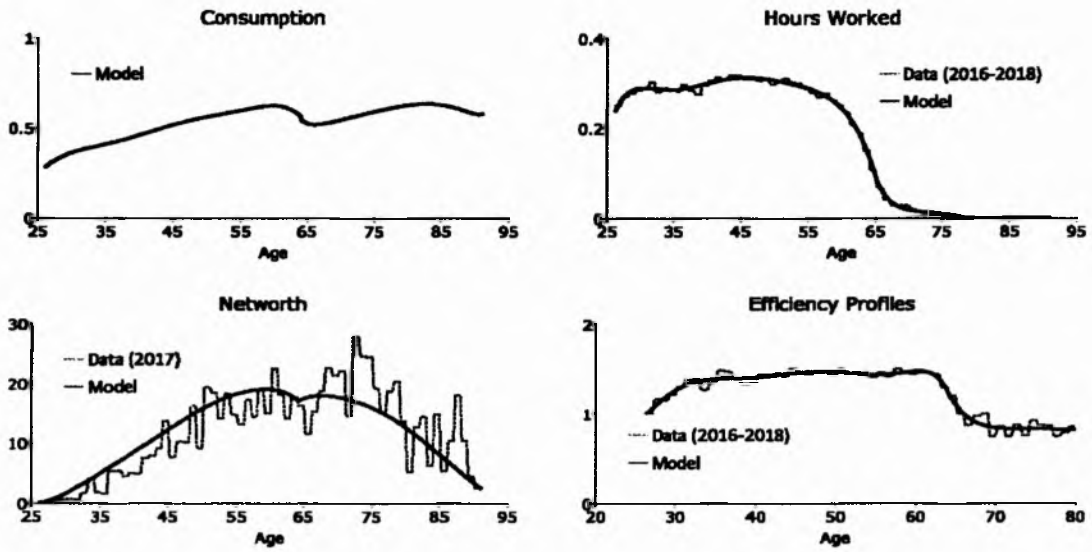


Figure 2: Steady-State Behavior of Households.

due to the increase in productivity and labor supply. Moreover, both the rise of average productivity and the increase of the stock of capital in the subsequent periods dominate the negative effects of labor supply increases on the marginal product of labor so that the real wage rate also rises, as illustrated in the upper right panel of Fig. 3a. Furthermore, the increase in labor incomes leads to financial surpluses in the PAYG system and growing pension entitlements. For Case 1, the panels of the second row show, in particular, that the PAYG administration mainly invests these surpluses in financial assets in order to dampen the reduction in contribution rates.

The impulse responses for Case 2 and 3 are plotted in the last four rows of Fig. 3a. Overall, the behavior of aggregate variables in Case 2 is almost identical in comparison with Case 1, while the amplitudes of impulse responses in Case 3 are a little bit more pronounced.[14] For example, output Y_t increases on impact by 1.41 % in Case 1, 1.42 % in Case 2, and by 1.78 % with respect to Case 3, whereas the contribution rates decline by 0.07 %, 0 %, and 1.83 % in Cases 1 to 3, respectively. The economic intuition for these results is straightforward. On the one hand, pronounced adjustments of contribution rates in Case 3 induce stronger distortionary effects on individual labor supply decisions and, therefore, result in larger fluctu-

14 For ease of comparison between Cases 1 to 6, Fig. A1 in the Appendix displays the absolute deviations of output, labor, investment, the real wage, and the real interest rate with respect to the benchmark case.

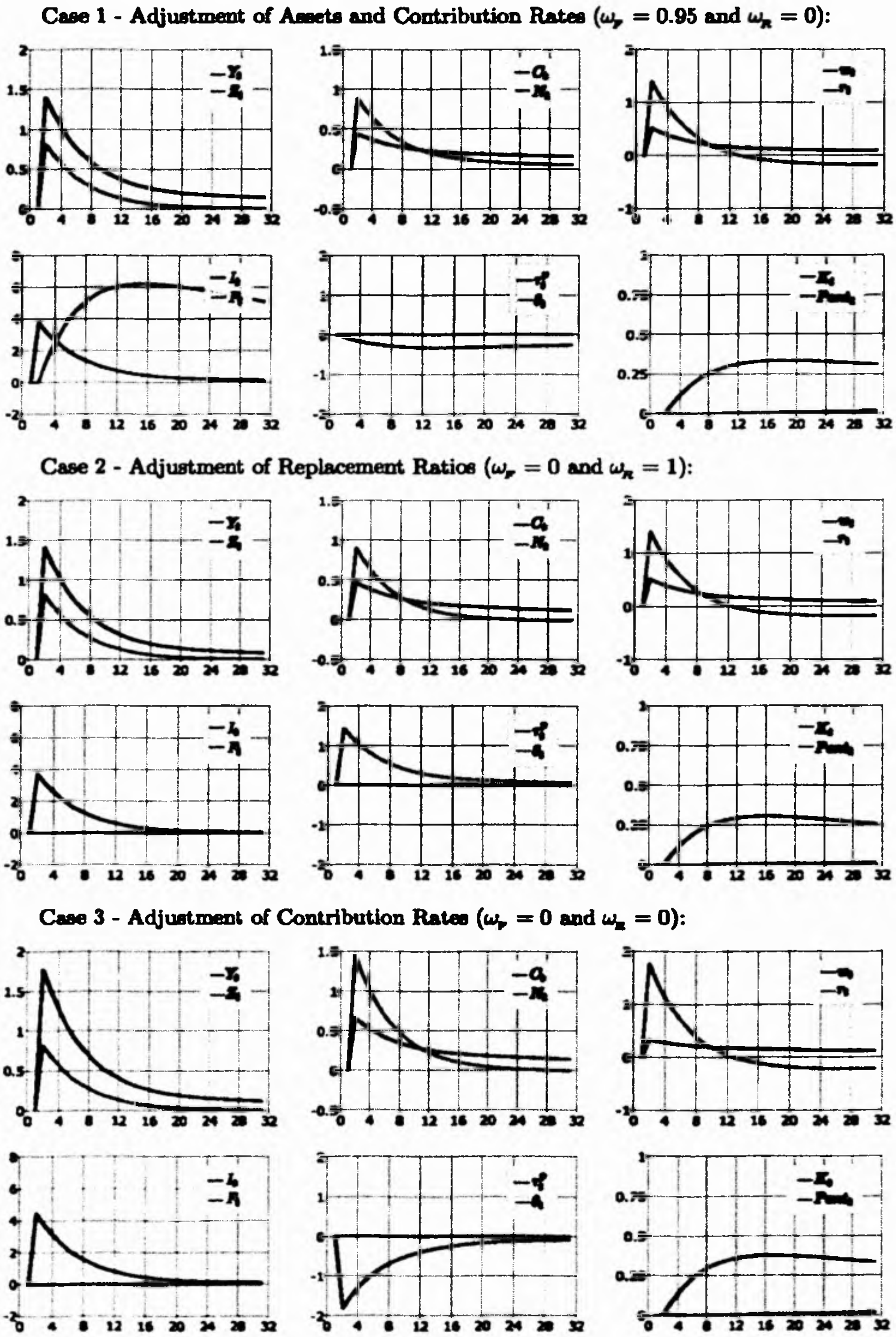
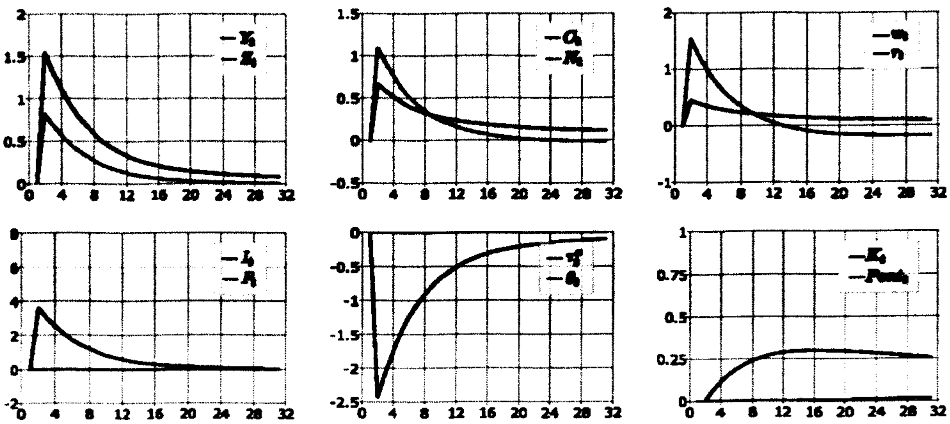
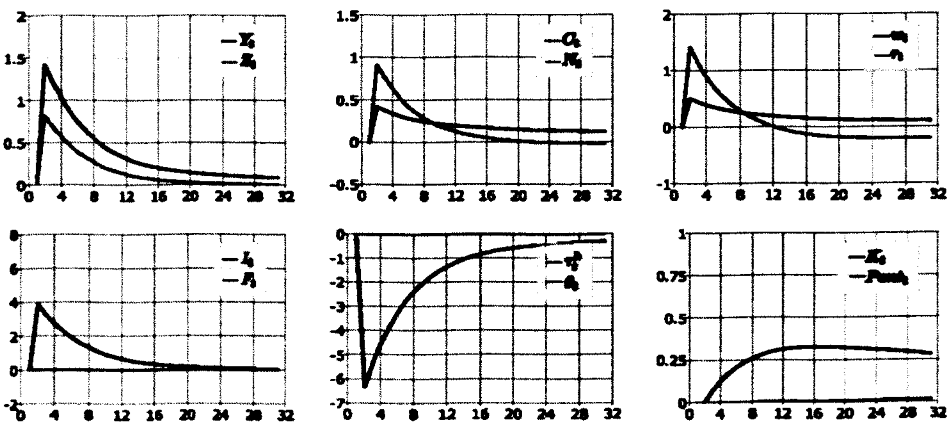


Figure 3a: Impulse Responses of Aggregate Variables (ordinate: percent deviations, abscissa: periods).

Case 4 - Adjustment of Value Added Tax Rates:



Case 5 - Adjustment of Capital Income Tax Rates:



Case 6 - Adjustment of Lump Sum Taxes:

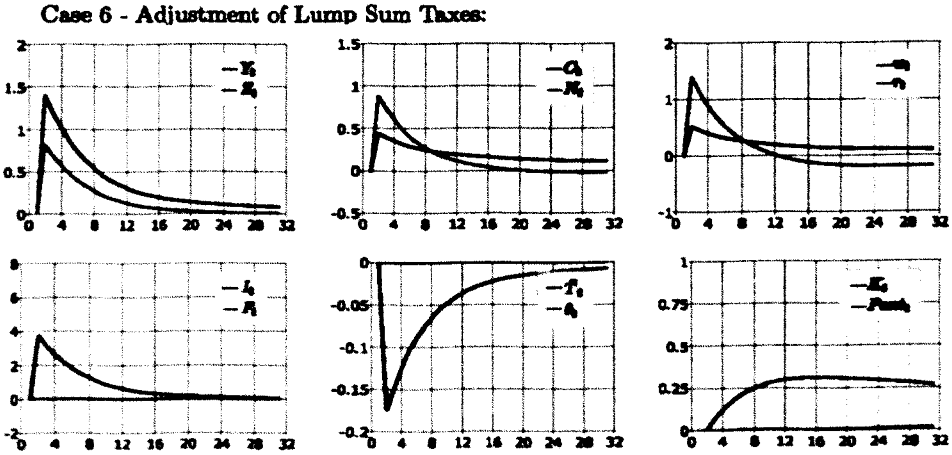


Figure 3b: Impulse Responses of Aggregate Variables (ordinate: percent deviations, abscissa: periods).

ations of aggregate labor supply and real output. On the other hand, the share of financial assets only amounts to 0.98 % of aggregate capital in the steady state. For that reason, the associated distortionary impacts of asset changes in a PAYG system in Case 1 on real factor prices are rather negligible so that the results in Case 1 and 2 are very similar. Thus, if the PAYG authority solely adjusts the contribution rates, it increases these distortionary effects, while the other financing forms help to stabilize the economy by keeping the contribution rates (almost) constant.

Fig. 3b presents the impulse responses of aggregate variables with respect to Case 4 to 6.[15] Changes of the consumption tax rate τ_t^c, see the first two rows of Fig. 3b for Case 4, amplify the distortionary effects on output and labor markets in contrast to adjustments of capital income or lump sum taxes, which are displayed in the last four rows of Fig. 3b. However, the amplitudes in Case 4 are somewhat less pronounced compared to Case 3, where the social security authority only adjusts the contribution rates. On impact, output Y_t increases by 1.54 % in Case 4, whereas the corresponding increase amounts to only 1.43 % and 1.41 % in Case 5 and 6. On the one hand, a decline in τ_t^c, see the second panel in the second row of Fig. 3b, implies an increase in the value of work since it rises the purchasing power of individuals. On the other hand, it also increases the value of work with respect to leisure. The associated distortionary effects outweigh the distortionary effects of capital income or lump sum taxation in Case 5 and 6, respectively.

Table 1, which displays the simulated standard deviations of aggregate variables and their empirical counterparts for the sample 1991:1 to 2019:4, also confirms the previous results. Financing rules, which try to keep the contributions rates mostly constant, imply lower volatilities of aggregate output, labor, and consumption. For example, the second column shows that the standard deviation of aggregate output amounts to 1.68 in Case 1 and increases by 27 % to 2.13 in Case 3, whereas it almost stays constant with regard to Case 2, 5, and 6. In addition, only value added tax adjustments imply higher standard deviations, which increase GDP volatility by 10.12 %. Comparing the benchmark model in the first two rows with empirical data in the last two rows, Table 1 shows that the benchmark model produces standard characteristics of business cycle volatilities which roughly match the data regarding output, labor, and investment. In particular, the relative volatility of labor with respect to output is equal to its empirical counterpart due to the calibration of the parameter γ_1 and the relative volatility of investment amounts to 2.71 compared to a value of 2.37 in the data. However, the relative

15 Please note that the lump sum tax Tax_t is equal to zero in the steady state of the model. For that reason, I plot the relative deviation with respect to steady state output instead.

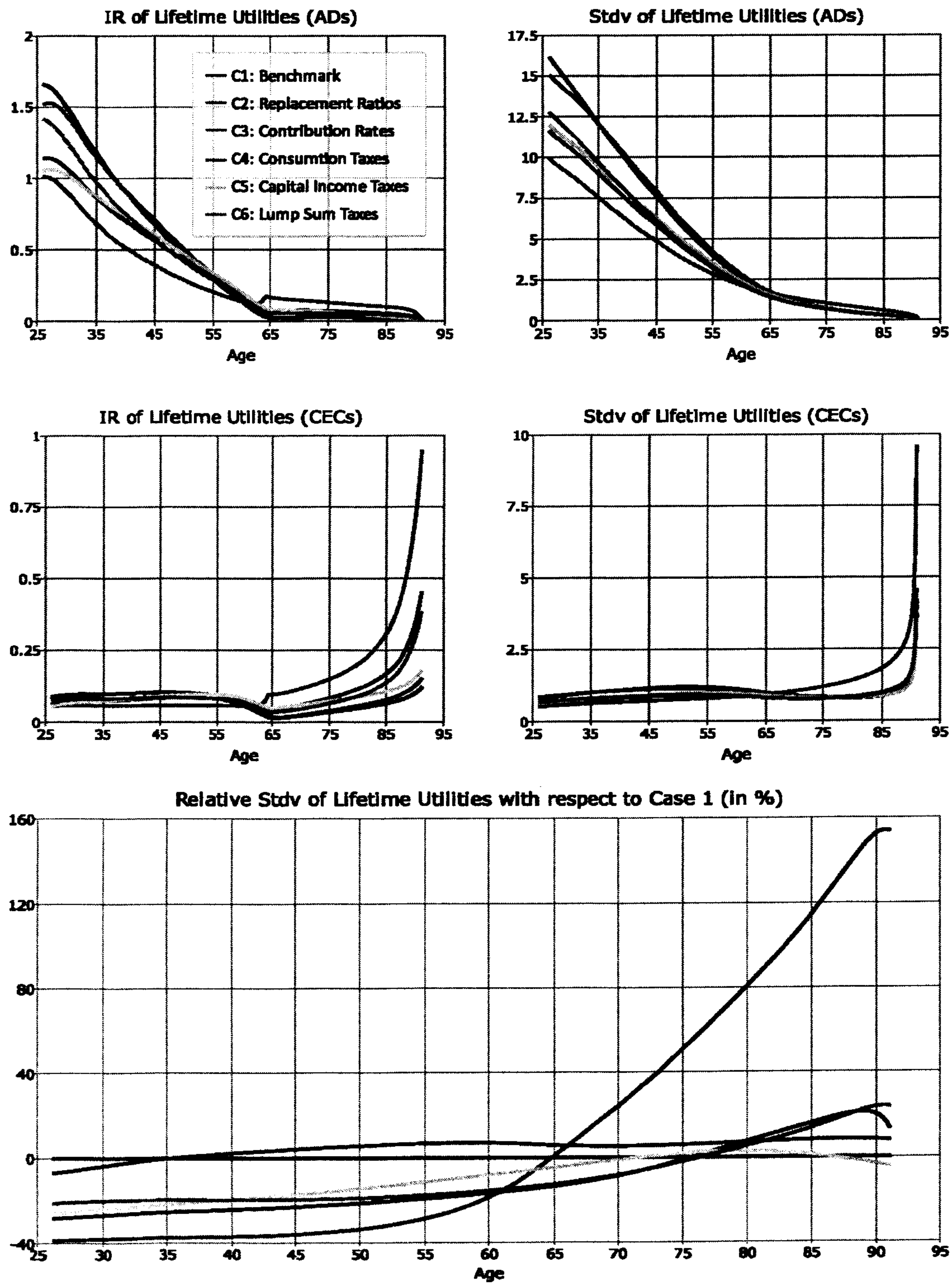


Figure 4: Impacts on Lifetime Utilities.

Table 1: Standard Deviations of Aggregate Variables (time series were logged and hp-filtered using a parameter of 1600 over 100,000 simulations with a period length of 116 quarters; in parentheses: relative deviations with respect to output; *sample: 1991:1-2019:4, own calculations).

	Y	N	I	C	w	τ^P
Case 1:	1.68	1.07	4.56	0.53	0.62	0.26
	(1.00)	(0.63)	(2.71)	(0.31)	(0.37)	(0.15)
Case 2:	1.70	1.09	4.51	0.56	0.61	0.00
Case 3:	2.13	1.75	5.42	0.79	0.39	2.19
Case 4:	1.85	1.32	4.40	0.80	0.53	0.00
Case 5:	1.71	1.10	4.73	0.50	0.61	0.00
Case 6:	1.69	1.07	4.55	0.53	0.62	0.00
Data*:	1.404	0.89	3.32	0.90	0.85	.
	(1.00)	(0.63)	(2.37)	(0.64)	(0.61)	.

Source: Federal Statistical Office (2020). Aggregate investment is measured by gross fixed capital formation and the wage is defined as total compensation of employees divided by total hours worked by employees. All aggregate variables are expressed in per capita terms and nominal variables were deflated with the implicit GDP deflator.

standard deviations of consumption and the real wage differ from their empirical values to a small extent, whereas the corresponding absolute deviations perform somewhat better.

Except for VAT adjustments, the previous findings clearly suggest that financing forms aiming to keep the contribution rates of a PAYG scheme (nearly) constant stabilize an economy. These financing rules, however, have different effects on the intergenerational allocation of risk and affect the consumption smoothing behavior of households over the life cycle. For this reason, I compute the impulse responses and standard deviations of (remaining) ex-post lifetime utilities of all households at ages 26 to 90. The impulse responses also refer to a positive one-time productivity shock of one standard deviation. In contrast, the volatilities are calculated in a simulation with 1,000,0000 periods with the same sequence of random productivity shocks for Case 1 to 6.[16] For the reader's convenience, I express the effects on lifetime utilities as absolute deviations (ADs) as well as consumption equivalent changes (CECs) in Fig. 4.[17]

16 Please note that Cases 1 to 6 share the same steady state. Therefore, the differences in welfare are driven exclusively by the dynamics outside of the steady state.

17 It is always possible to add a constant to the instantaneous utility functions (2) and (3) so that relative deviations of lifetime utilities cannot be meaningfully interpreted. Therefore, the consumption equivalent change, which describes the percentage variation of steady state con-

The upper left panel in Fig. 4 presents the impulse responses of lifetime utilities as absolute deviations from the steady state. Young households generally face the biggest increase in lifetime utilities since they can benefit from the effects of a positive productivity shock for the longest time. Moreover, the profiles decrease rapidly until households enter retirement at an age of 64 and remain almost constant thereafter as long as the PAYG authority does not adjust the replacement ratios, which result in the largest welfare increases among retirees. However, households are exposed to both positive and negative productivity shocks over their life cycle since these shocks follow an AR(1) process in my model. Thus, the volatilities of lifetime utilities, which are displayed in upper right panel of Fig. 4, are the more appropriate measure for welfare comparisons. The fluctuations in lifetime utilities of youngest households are again most pronounced and decline up to age 90 due to decreasing lifespans. Moreover, compared to the impulse responses, these profiles are smoother because the remaining lifetime utilities at age s also depend on the previously experienced shocks that affect the net worth at this age.

The left panel in the second row of Fig. 4 displays the impulse responses of lifetime utilities with respect to consumption equivalent changes. In Case 1 and Case 3 to 6, the CECs stay almost constant at young ages before they decrease among households close to retirement since these age groups benefit less from wage increases due to their lower labor supply. In the subsequent periods, the CECs increase until an age of 90 years. Moreover, the CECs stay also constant until an age of 62 and rise to 0.94 % at an age of 90 with respect to Case 2. Furthermore, the CECs are quite low because all current generations face only one productivity shock over their life cycle. For example, the CEC of households at age 26 amounts to only 0.08 % in Case 1. In contrast, the volatilities of consumption equivalent changes are much more pronounced over business cycles, as depicted in right panel of the second row in Fig. 4. In particular, the profiles of very old households increase exponentially in comparison to younger cohorts. On the one hand, the bequest motive becomes more and more important as people age due to lower survival probabilities. On the other hand, these bequests depend on the accumulated wealth, which in turn depends on the history of previously experienced shocks over the life cycle at a given age. As a consequence, the CECs are much more volatile among very old households.

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sumption that is equivalent to a given absolute change in lifetime utility, is usually the standard welfare measure in models with overlapping generations. The CEC takes remaining lifespans and time preferences into account so that it allows to draw more meaningful comparisons across generations, even though both welfare measures are equivalent to each other.

For ease of comparison, the third row in Fig. 4 displays the percentage deviations of lifetime volatilities with respect to the benchmark case.[18] Sole adjustments of replacement ratios in Case 2 imply very low fluctuations of lifetime utilities for workers but they are also associated with very high standard deviations after households enter retirement at an age of 64. For example, the standard deviation declines by 38.67 % at age 26 and increases by 153.81 % at age 90 in comparison to the benchmark case. Thus, this financing rule particularly allows young households to better smooth their consumption over the life cycle. In this case, the aggregate risk in the budget of the PAYG system is completely borne by retirees so that it minimizes the distortionary effects on labor supply over the business cycle. In contrast, changes of contribution rates, see Case 3, reduce the volatilities slightly until an age of 35 years with the largest decline of 6.87 % at age 26, while the values increase by a maximum of 8.33 % for older households. Moreover, the profiles for Case 4 to 6 show that changes of consumption, capital income, or lump sum taxes result in lower lifetime utility volatilities of households up to a age of 75, 71, and 76 years in comparison to Case 1. These declines are very pronounced among young households and amount, for example, to 26.76 %, 24.02 %, and 20.06 % at an age of 30 years in Case 4, 5, and 6. In contrast, adjustments of consumption or lump sum taxes increase the volatilities of older age groups by a maximum of 21.00 % and 23.89 %, respectively. Changes of capital income taxes, however, imply a maximum increase of only 2.80 %, where the standard deviations even decrease again slightly after an age of 86 years. Thus, these kinds of taxes allow more age groups to better smooth their consumption over time in comparison to Case 2 and 3.

6 Conclusion

The analysis in this paper shows how different financing rules for additional surpluses in a PAYG system affect aggregate variables and the consumption smoothing behavior of households over the business cycle. In the benchmark case, a financing rule that mainly adjusts the stock of financial assets of a PAYG scheme implies lower volatilities of lifetime utilities for households older than 35 years and stabilizes an economy over the business cycle, in contrast to complete adjustments of contribution rates. Moreover, sole adjustments of pension benefits result in very low fluctuations of lifetime utilities of workers since the PAYG authority completely shifts its aggregate risk to retirees. In this case, the associated

18 Please note that both welfare measures imply the same figure.

constant contributions rates do not distort the labor supply of workers, while retirees have to face very pronounced fluctuations in their lifetime utilities. Such a form of financing has, therefore, very detrimental consequences on the intertemporal welfare of retirees.

In addition, this paper also studies the effects of possible adjustments of consumption or capital income as well as lump sum taxes to keep the PAYG budget balanced. These financing forms imply much lower standard deviations of individual lifetime utilities up to an age of at least 71 years in comparison to the aforementioned financing rules. For example, changes of consumption, capital income, or lump sum taxes decrease the volatilities of lifetime utilities of 30 year old workers by 26.76 %, 24.02 %, and 20.06 % with respect to the benchmark case. Moreover, changes of capital income tax rates are only associated with slightly higher lifetime utility fluctuations between an age of 72 and 86 years, where the maximum increase amounts to 2.80 %. One should, however, be careful to use these welfare results for normative conclusions since no financing form studied in this paper strictly dominates the other.

Finally, it remains to be noted that different financing rules for revenue surpluses in a PAYG scheme imply very different and very pronounced effects on the allocation of aggregate risk across generations so that the welfare analysis about PAYG systems and aggregate risk should take these financing rules into account. Moreover, I have assumed that it is not possible to mix these different financing rules with respect to a weighted minimization of lifetime utility volatilities for different cohorts. A social security authority could, for example, target specific individuals (e. g. young vs. old or poor vs. rich individuals) and simultaneously adjust the contribution rates and the (effective) replacement ratios. Furthermore, labor supply is completely flexible over the life cycle but work restrictions may limit the abilities of workers to change their labor supply in response to economic shocks. These might be very interesting topics for future research.

Conflict of interest: The author declares that he has no conflict of interest.

Appendix A

Fig. A1 displays the impulse responses of output, labor, investment, the real wage, and the real interest rate as absolute deviations from the benchmark case. Evidently, adjustments of contribution rates (Case 3) imply the largest deviations for all variables except consumption in the first periods after a productivity shock.

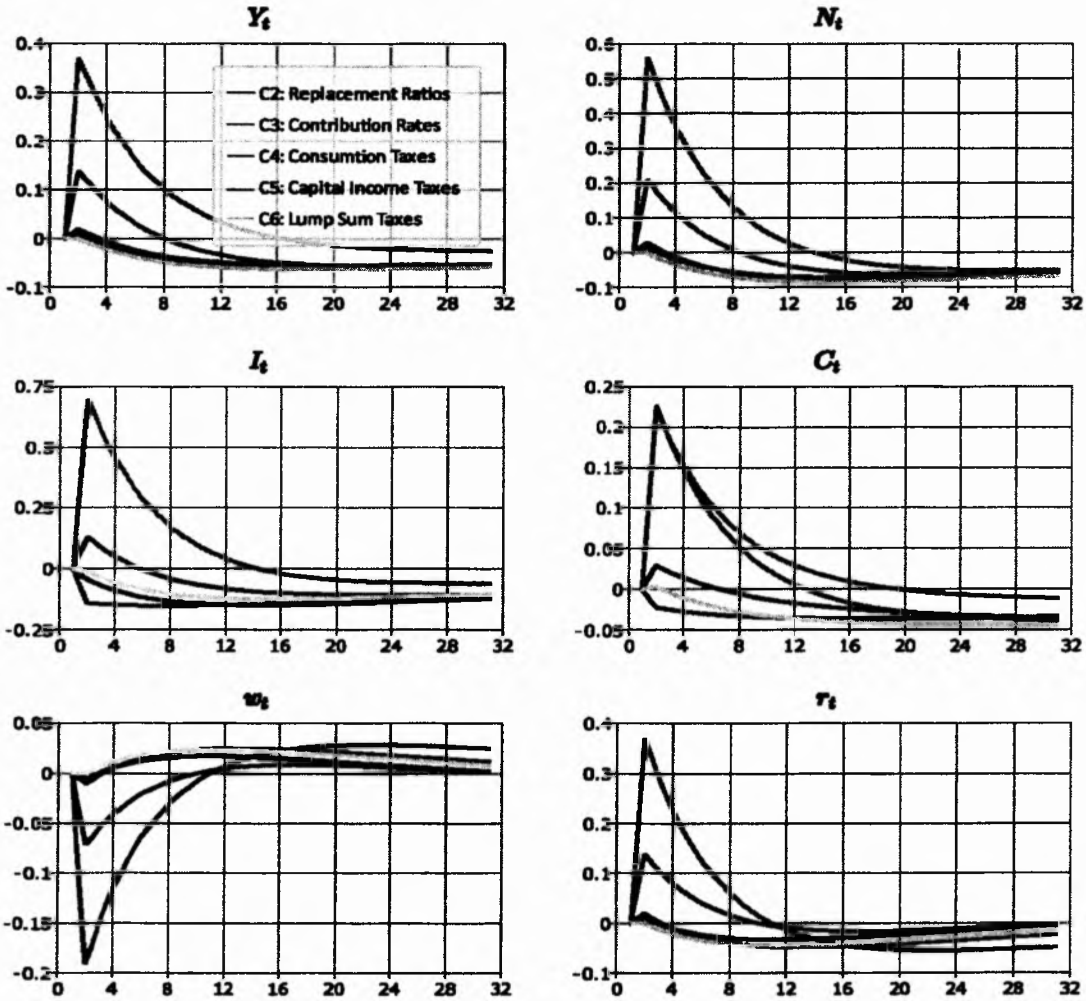

Figure A1: Impulse Responses of Aggregate Variables (ordinate: absolute deviations with respect to the benchmark case, abscissa: periods).

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