Optimized planning of nursing curricula in dual vocational schools focusing on the German health care system

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A B S T R A C T

We investigate a problem in vocational school planning for nurses in countries with a dual vocational system, which closely combines theoretical and practical education and is highly regulated by federal legislation. The apprentices rotate through vocational school-blocks followed by assignments to hospital units, where they receive practical education. This program is regulated in high detail. Hospital units offer some slots for apprentices but expect just enough apprentices to be trained and educated. We create two mixed-integer programming models to optimally solve the underlying planning problems of (1) scheduling classes to theoretical and practical education blocks and (2) assigning apprentices to hospital units. The first model determines the number and length of school- and work-blocks on a class level, where its result is input to the second model, which finds individual unit-assignments for every apprentice fulfilling detailed curriculum requirements. Furthermore, it tries to exploit the units' educational capacities as well as possible. To solve the second model, we develop a heuristic decomposition procedure that enables good feasible solutions in short time. Our computational study is based on real-world data of our cooperation partner and provides valuable insights for management. The dataset consists of manually created schedules over the full 3-year program horizon and information on hospital units and their respective capacities. We test different parameter settings for our heuristic procedure and how they influence solution quality and runtime. Finally, we test, if students can be enabled to request individual vacations and evaluate benefits and drawbacks of different degrees of flexibility.

1. Introduction

The immanent shortage of nurses is one of the most urgent problems in the provision of hospital services in most industrialized countries. Finding well-trained nurses, therefore, is a big challenge for hospitals and a critical factor in treating patients effectively, especially when wages are, as in many European countries, regulated by labor union agreements and employers are not able to provide financial incentives. Besides recruiting senior nurses, who are currently very hard to find (German Federal Employment Agency, 2020), hospitals might think about training new qualified personnel on their own by providing young people with an apprenticeship program. In most developed countries vocational programs are strictly regulated and usually mean sharing time between learning at school and training in a company. In the European Union, 23 countries have a legal framework for apprenticeships following the above principles (European center for the Development of Vocational Training, 2018). Though we will focus on the framework of dual vocational education, which is mostly encountered in Germany and other German-speaking countries, the developed models and algorithms can easily be applied to all forms of vocational programs that require to differentiate between practical and theoretical education as two different entities of the program.

In Germany, Austria, Switzerland, and some other countries, a major part of professional nursing education is not carried out at universities and colleges but in a dual vocational training system. The latter closely combines theoretical and practical education and is highly regulated by federal legislation (German Federal Ministry of Education & Research, 2019). In order to become a licensed practical nurse ("Pflegefachkraft") in Germany, young people usually have to finish ten years of school education. To join an apprenticeship program, a student must first find an employer, where practical training will be performed at. Theoretical education can take place in a specialized professional school, generally run by the local government. Large hospitals may take this opportunity to educate personnel tailored for their special needs in a privately-run school. Please note that vocational training should

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not be compared to any residency program (common in the U.S.) where the latter is meant to support recent graduates' transition into clinical practice (Meyer Bratt, 2013).

Providing a superior apprenticeship program gives hospitals a competitive advantage in finding new personnel by promoting their vocational education. The organization of such a program can be very challenging for large hospitals since it has to incorporate requirements of several stakeholders into the program. For young people, a plannable, reliable, and well-organized apprenticeship program is important and a key factor in finding enough apprentices. Units of the hospital offer educational capacities by providing slots for the apprentices, so they can receive practical education in different medical subjects required to complete their program. Since the apprenticeship programs have to comply with a strict and complex curriculum enforced by the local government, students have to reach educational goals over the course of the program. Finally, students (unlike pupils in Elementary or High School) are entitled to plan individual days of vacation, by federal law.

Combining all of the above factors, private vocational school principals face a difficult task: they have to satisfy wishes of teachers, apprentices, and hospital units, regarding lecture timetables, reaching educational goals, and a constant supply with apprentices to receive education in the hospital's units. These conflicting goals lead to an enormous effort necessary to create sufficient educational schedules, determining alternating phases of school education and practical education for every class. Additionally, unit assignments for every individual apprentice have to be created over the full program horizon, complying with the imposed detailed curriculum and allowing students to formulate individual vacation requests.

The purpose of this paper is to describe and classify the dual vocational apprenticeship system and aims to identify other literature solving similar or related problems. We provide models and algorithms to efficiently solve the underlying planning problems and introduce a real-world case. This paper contributes two Integer Programs (IPs) ranging into the strategic/tactical and tactical/operational planning levels. The first IP decides on the duration of alternating blocks of practical and theoretical education for every class. The second model uses information of the previous level and specifies the practical education blocks by assigning students to hospital units subject to individual vacation requests. The overarching goal of our optimization model is the optimal use of educational capacities by using a linearization of the quadratic objective function. Since the second model is not tractable within a reasonable runtime limit, we develop an iterative solution algorithm based on an intuitive decomposition. In our computational study, we consider a real-world case of a dual vocational school in Germany. In particular, we show that automated planning can improve supply of students for medical units by 40% compared to manually created plans. Furthermore, we provide a system coping with individual requests instead of class-wise off-days (i.e., individual vacation requests). Our computations evaluate benefits and drawbacks of such degrees of flexibility and give insights into the consequences of different school holiday calendars. Finally, the paper provides insights into parameter selection for models and algorithms regarding runtime and solution quality. The presented model and solution process can be transferred and applied to a variety of other professions relying on the dual educational system, like many other health professions in Germany and Europe. As all of these educational programs share the same general dual system, most elements of our optimization models (generic elements) can still be used and only some program-specific elements (side constraints) have to be exchanged.

The remainder of this paper is organized as follows. First, we classify this problem by previous literature in terms of problem description and properties of the underlying optimization models. We will then formally describe the educational system and underlying concepts to the problem in Section 3. To solve the given problems, we create two IPs and introduce necessary notation, as well as a linearization approach. In Section 3.2, we discuss our developed solution algorithm and evaluate it in Section 4 using real-world data of a large university hospital in south Germany. We close in Section 5 by summarizing our findings and giving possible directions on future research in terms of solution algorithms.

2. Literature review

The domain of academic planning and education received much attention in recent years (Johnes, 2015). Researchers focused for example on university course and high school timetabling (Bettinelli, Cacchiani, Roberti & Toth, 2015; Burke, Mareček, Parkes & Rudová, 2012; Mühlenthaler, 2015), exam timetabling (Burke & Bykov, 2016; Burke, Pham, Qu & Yellen, 2012), or student grouping problems (Fan, Chen, Ma & Zeng, 2011; Gallego, Laguna, Martí & Duarte, 2013). Also, a strategic view on the educational system has been applied regarding questions of public funding (Cobacho, Caballero, González & Molina, 2010) and quality measurement with DEA (see Johnes (2015) or Johnes (2006) for an extensive overview) or using a portfolio approach (Jessop, 2010).

Our problem of optimizing school operations, though, is not related to any of the mentioned topics for several reasons. We want to divide the planning horizon into blocks of arbitrary length, covering given curriculum requirements. In contrast to this, in course timetabling, one is interested in finding cyclic weekly assignments of lectures to rooms and time periods while taking capacities, teachers' availabilities, and curriculum conflicts into account. Exam timetabling models assign all given exams to a set of timeslots and rooms while ensuring no student has to take more than one exam at the same time and room capacity is not exceeded. Usually, additional soft constraints regarding conflicts are considered as well. Also, the planning horizon is very different from our problem setting. Finally, the Curriculum Design Problem must be solved before school operations start and is, therefore, a preliminary problem (Johnes, 2015).

With regards to vocational schools, only few papers were published. Hua Chen, Tso Lin and Tau Lee (2004) deal with a preliminary step in vocational education planning, namely selecting the best set of partner companies, when initializing a vocational training program. Most other papers approach the topic on a higher level: they deal with questions of curriculum design, on pedagogical principles, or even on general advantages and disadvantages of a vocational education system for national economies (Shavit & Muller, 2000). To our knowledge, there is no research dealing with the problem of operational planning at vocational schools in the area of operational research. Therefore, we reveal relations of the two problem levels to other popular fields of research and give an overview of existing literature. The most evident special property of both problem levels is their block structure, requiring a special set of constraints, as first described in Bowman (1959). These can also be found in literature on shift or task scheduling (Brunner, Bard & Kolisch, 2009; Volland, Fügener & Brunner, 2017), medical resident scheduling (Cohn, Root, Kymissis, Esses & Westmoreland, 2009; Kraul, 2020), and in multi-period assignment problems (Bhadury & Radovilsky, 2006) in general. Just recently, Akbarzadeh and Maenhout used a decomposition-based heuristic (Akbarzadeh & Maenhout 2021a) and developed a branch-and-price algorithm (Akbarzadeh & Maenhout, 2021b) to determine sequences of assignments in medical student schedules. Their model is based on a network flow formulation. The problem can be classified as resident scheduling and is unlike our approach, as it, for example, does not differentiate between school and work blocks.

As we pointed out, all of the above papers deal with only one of the problem levels in planning vocational school operations. But other problem classes exhibit a structure similar to the vocational planning problem: first, in *crew scheduling*, the planning problem is decomposed into two consecutive planning levels, in a similar way. There, in a first decision level, employees are paired to crews. In the consecutive step, these crews are assigned to flight legs, such that flights can be operated and each crew member starts and ends duty at their respective home airport. The problem considered there is distinguishable because transferred to the problem of vocational planning the first level decision would be to decide when flights take place. Second, in university course timetabling, Vermuyten, Lemmens, Marques and Beliën (2016) enhanced a decomposition approach of Burke, Mareček, Parkes and Rudová (2010) to reduce complexity. They use a first optimization model to fix some of their decisions while ensuring feasibility and delegate some additional and more detailed goals and variables to a second model. As a consequence, both decomposed models become easier to solve than a single monolithic model but feasibility can still be guaranteed. As our partner school, too, handles the planning process in two consecutive steps, it implicitly follows this approach. Another instance of such a combination of decisions on two planning levels is Horn, Jiang and Kilby (2007), as they help the Royal Australian Navy both schedule when patrol boats leave their ports for off-shore mission or on-shore maintenance and assign crews to missions or on-shore trainings in two consecutive planning steps. Still, their model treats crews as a whole in all planning levels; opposed to our application, where classes are uncoupled to single students on the second level. Solutions link activities of boats and crews by a common set of activities. The authors use simulated annealing and specially developed heuristics to solve the problems consecutively since an integrated integer linear programming approach failed due to the size and complexity of the problem.

In conclusion, there is no pertinent literature on our problem setting. Neither in academic planning nor personal planning literature appropriate models combining both planning steps do exist. Therefore, we classified both planning levels separately into existing literature. With regards to related and follow-up planning problems, please notice that planning holidays and vacations is delegated to the strategic level and is therefore not part of our decision problem. Finally, the generation of duty rosters at hospital units is clearly a follow-up decision to our problem.

3. Problem description and modelling

The vocational school planning problem arises due to the special structure of professional education, namely the dual vocational training system, traditionally used in Germany and other German-speaking countries, and recently introduced in South Korea (Blossfeld & Stockmann, 1998; Ji-Eun, 2014). There, apprenticeship programs combine theoretical education and on-the-job training in a dual system. Young people are required to find a company, which takes responsibility for organizing such a program for its apprentices. All on-the-job trainings are usually performed at this employer. Professional schools can either be organized publicly by a body of the local state government or privately by one or more employers. In both cases, theoretical and practical subjects and learning objectives are defined in detail in curriculum requirements and will be evaluated during and at the end of the apprenticeship by an independent government committee. The vocational school, employer, and apprentice are jointly responsible for meeting the stated requirements, although vocational school principals take the leading role in the underlying planning process. (German Federal Ministry of Education & Research, 2019)

The curriculum requirements include detailed information relevant for both theoretical and practical education. For the schools, it regulates subjects and teaching volume for every semester of education. For apprentices (i.e., employees at the hospitals), it defines a total working volume for the whole program horizon. All professional education programs using the dual system implement this general structure and have issued individual curriculum requirements. The following ideas and models can be applied to any profession requiring the assignment of apprentices to specific workplaces during practical training. We will differentiate between generic elements and program specific side constraints when developing our models to broaden their applicability. For the healthcare apprenticeship programs in focus, the curriculum requirements map each medical unit to at least one medical subject group. They further state detailed requirements on the number of hours a student is assigned to such subject groups. Additionally, it can impose further rules on the number of units within a group, which a student has to be assigned to. (Bavarian Ministry of Education & Culture, 2005)

Further restrictions to the problem are the educational capacities provided by both the vocational school and the employers. The schools have only limited capacities on classrooms and teachers available so that only a certain number of classes can be schooled at the same time. Medical units provide training capacity in terms of experienced or qualified personnel, that students will accompany during their practical education. Also, capacity might be limited in terms of interventions or actions that are relevant to students' education from one unit to another. Due to organizational reasons, both school and work blocks should have a mandatory minimum and maximum duration.

In state-run vocational schools, public school holidays apply, so students are assigned to work at their employer. They can request vacation during work blocks by their employer directly, following the Federal Leave Act as any other employee in Germany. It guarantees 24 days of vacation or four weeks (the number of off-days might be extended in labor union agreements; for the healthcare sector it is usually extended to six weeks) and to choose their vacation freely over the course of the year. For students at privately run vocational schools this situation can be significantly different: schools may declare *holidays*[1] (that may or may not follow public school holidays) and all students of a class are off during this time, i.e. they are neither at school nor assigned to work during this period.

Since students can also act as employees towards their employer, the Federal Leave Act does also apply to them. Therefore, students should also be eligible for a free choice of off-days. At our partner institution, students and employers resolved this conflict with a simple agreement: schools may impose some school holidays for whole classes that cannot be influenced by students. But students are eligible to choose their remaining days of *vacation*[2] freely during work block assignments.

According to our literature review, the first- and second-level decisions of the operational vocational planning problem are – following the classification of Pentico (2007) – *multi-period bottleneck assignment problems with side constraints of agents to jobs*. In terms of vocational training in the healthcare sector, these are assignments of classes to multiple periods of theoretical or practical education, as well as students to different medical units. Additionally, block constraints and coverage constraints are applied.

We give an example of the underlying planning process. Fig. 1 shows a snapshot of the first 52 periods (e.g., weeks) of the plan-

[1] From now on, holidays will refer to class-wise off-days determined by school

[2] From now on, vacation will refer to student-individual off-days during practical education blocks

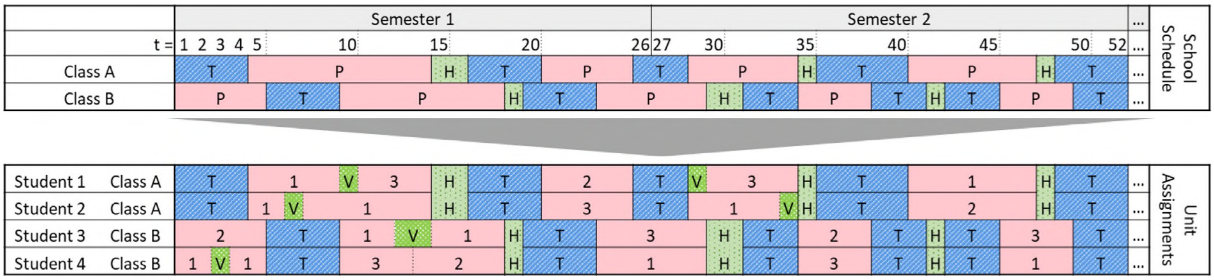

Fig. 1. Example of School Schedule and Unit Assignments.

ning horizon in detail for two classes (School Schedule) and four students (Unit Assignments). The whole horizon is evenly divided into semesters of 26 periods. In the depicted timeframe, classes must be scheduled to 28 periods of theoretical education (“T”), 20 periods of practical education (“P”), and four periods of holidays (“H”) within a year of 52 periods (weeks). All theoretical blocks should have a minimum consecutive length of 3 periods and a maximum of 5 periods. Practical education blocks should be between 4 and 10 periods long. For example, Class A is scheduled for theoretical education for the first four periods, followed by ten weeks of practical education. All students of this class are on holidays in periods $t = 15$ and $t = 16$. The result of this first planning step is called the ***School Schedule*** which is used to cope with the complexity and to enable coordination between schools, employers, and students. The School Schedule is created by the principals (see upper part of Fig. 1). It determines the start and duration of school and work blocks for every class and ensures a minimum number of school hours over the planning horizon to fulfill this curriculum requirement. These blocks are in line with a fixed holiday schedule, predetermined by schools. In addition, it should comply with mandatory block lengths and schooling capacities.

Based on the fixtures of the School Schedule, students are able to file their individual vacation requests. It is agreed that such requests must be limited to times of practical education, i.e. work blocks. All this information is then combined to create **Unit Assignments** in a consecutive (second) planning step (see lower part of Fig. 1). This means, whenever a class is scheduled to a block of practical education (also called a work block), all respective students must be assigned to a unit of the hospital. These assignments must respect curriculum requirements for every student, regarding medical subjects, seniority restrictions, and mandatory assignment durations. Of course, these work blocks will be interrupted by periods of individual vacation, if requested by the respective student. For the units, assignments should obey their respective maximum educational capacity and not waste any educational resources by assigning too few students. If this is not possible, deviations should be equally distributed among all units. Overall, this process ensures students can fulfill all imposed curriculum requirements to be able to become licensed practical nurses. It helps the hospital to use its educational capacities as well as possible while fulfilling all requirements imposed by federal legislation and state requirements. In the lower part of Fig. 1, the second planning step is depicted. In Unit Assignments, individual students are assigned to hospital units (represented by numbers) during all practical education blocks of their respective classes. The class structure is relaxed in this planning step: periods of theoretical education and holidays are identical for every student of the same class, but (medical) Unit Assignments during practical education are individual for every student. In the example, every student must spend at least 10 weeks on “Unit 1”, 5 weeks on both “Unit 2” and “Unit 3”. Focusing on student 3, they are assigned to “Unit 2” during Class B’s first block of practical education in periods $t = 1$ to $t = 5$. A theoretical block (for Class B) of four periods, i.e. $t = 6$ to $t = 9$, follows. Please note, student 4 has the same theoretical education block, since both students are in the same class B. As the corresponding assignment of student 2 shows, practical education blocks that are at least twice the minimum length (in this example: 6 periods), can be divided into assignments to multiple units. Additionally, every student can choose one individual period of vacation (“V”) during practical education blocks. For instance, student 4 chose a period of vacation in $t = 2$. So, the block of practical education in “Unit 2”, ranging from $t = 1$ to $t = 3$ is suspended and resumed afterward. This still complies with minimum block lengths, as we treat this as a single assignment of 3 periods.

To summarize, the vocational school planning problem is solved in two consecutive, inter-dependent planning steps, namely the School Scheduling Problem and the Unit Assignment Problem (see Fig. 2). It mainly concerns the tactical planning level but spans into both the strategic and operational level, due to the time horizon and differing levels of detail in the problems. The static class holiday calendar (strategic level) is deemed as fixed and given for both planning levels. The duty rostering (operational level) is performed for every hospital unit independently after (medical) Unit Assignments have been published. Therefore, it is a downstream problem to us.

3.1. IP models for school scheduling and unit assignments

To tackle the two planning steps accordingly and to be in line with the suggested process, we propose two IPs. The first IP model, called **School Scheduling Model (SSM),** represents the first-level decision on class-wise assignments to either school or work blocks. It primarily incorporates requirements regarding the students’ curriculum on theoretical education and school capacity. The second IP model, called **Unit Assignment Model (UAM)**, corresponds to the subsequent decision of assigning students to units during their respective work blocks. It focuses on the units’ demands and remaining curriculum requirements on practical education. The overall objective of both models is to minimize deviations from given minimum and maximum educational capacities imposed by the hospital’s units, i.e. medical departments with its units.

Sets

$t \in T$	Set of periods
$s \in S$	Set of students
$a \in A$	Set of (medical) units
$c \in C$	Set of classes

Parameters

$b^{MinWork}$, $b^{MaxWork}$	Min. and max. length of a work block
$b^{MinSchool}$, $b^{MaxSchool}$	Min. and max. length of a school block
$h^{WorkWeek}$, $h^{SchoolWeek}$	Number of hours gained in one period of work or school block

We divide the planning horizon into a set of periods $t \in T$. Further, we introduce sets of all students $s \in S$, classes $c \in C$, and medical units $a \in A$. The model has additional information

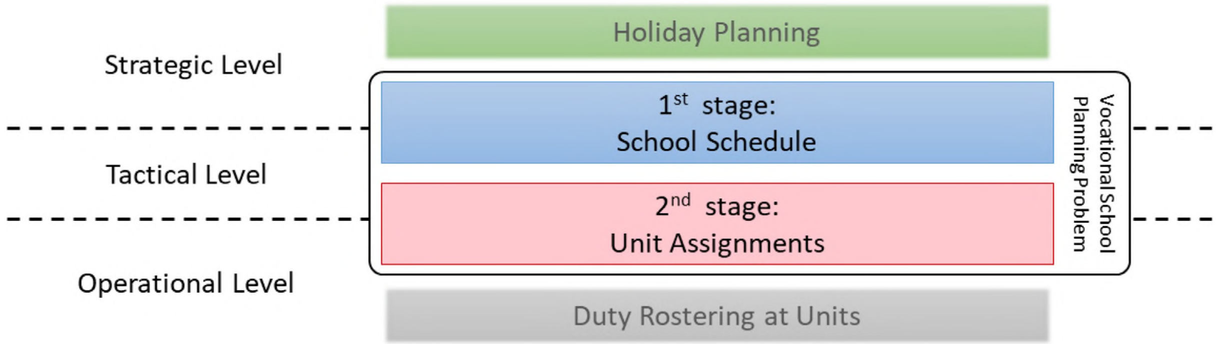

Fig. 2. Classification of the Vocational School Planning Problem into Planning Levels.

on the minimum and maximum length of school (work) blocks $b^{MinSchool}$ ($b^{MinWork}$) and $b^{MaxSchool}$ ($b^{MaxWork}$).

School scheduling model (SSM)

The SSM will optimize the School Schedule for all classes over their respective remaining course of the apprenticeship program. It decides on the start and length of work and school blocks while guaranteeing to comply with all curriculum requirements on school education, the previously fixed global holiday calendar, and classroom capacities. Since it is not possible to optimize the main objective (i.e. minimizing under- and overstaffing of students during practical education; the objective function of UAM) on this planning level, we use an auxiliary objective: this is to smoothen supply of students (in particular, the number of students assigned to a working block) over the semesters. To achieve this effect, SSM maximizes the minimum number of students assigned to a work block over all periods of a semester in (1.1). In a preliminary study, we tested several different leveling approaches and evaluated their effects on quality measures of UAM. These additional objectives include maximizing the number of work blocks over the whole planning horizon (1.1b), imposing a target staffing level and penalizing positive and negative deviations in the number of students assigned to work in every period (1.1c), and one that was inspired by the Value-at-Risk concept (1.1d). The latter is an advanced variant of (1.1) that allows some proportion α of outliers, which will not be considered for the objective value. In this paper, we only present the best objective (1.1) of these alternatives but provide additional results and insights to all tested objectives in Appendix C.

Sets

$p \in P$	Set of semesters
$t \in T_c^{ForceSchool}$	Set of periods class c is required to be in school
$t \in T_p^{Semester}$	Set of periods in semester p

Parameters

n_c	Number of students in class c
t_{ct}^{Off}	1, if class c not available in period t (either class has finished or has not yet started the program, or class is on holidays), 0 otherwise
r_t	Number of classrooms available in period t
$h_{cp}^{TotalSchool}$	Number of hours class c has to be assigned to school in semester p

Decision variables

$w_{ct} \in \mathbb{B}$	1, if class c is assigned to work in period t, 0 otherwise
$v_{ct} \in \mathbb{B}$	1, if class c is assigned to school in period t, 0 otherwise
$w_{ct}^{Start} \in \mathbb{B}$	1, if class c starts a new work block in period t, 0 otherwise
$v_{ct}^{Start} \in \mathbb{B}$	1, if class c starts a new school block in period t, 0 otherwise
$w_p^{Min} \in \mathbb{N}$	Minimum number of students assigned to work in any period of semester p

We divide the planning horizon into a set of semesters $p \in P$, which represent logical stages in the apprenticeship program. The semesters are determined by the start of the program and dates of midterm and final examinations as defined in the curriculum. For every class exists a predefined set of periods $T_c^{ForceSchool}$, in which a school assignment (i.e., $v_{ct} = 1$) is required. Additionally, we assign every period exclusively to a semester p and all such periods are element of $T_p^{Semester}$. In Germany, they span September to March and vice versa. This also implies, that the number of classes and therefore the number of students in the system is constant throughout the semester but is likely to change between successive semesters since some classes will finish their apprenticeship program and others might start.

Parameter n_c represents the number of students in each class c. The educational capacities of the school might also change over time and are given as classroom capacities r_t for each period t. It might also be influenced by absences of teachers or examination dates for some classes, which require multiple teachers to administer a single class. Parameter $h_{cp}^{SchoolReq}$ gives the number of hours a class c has to be assigned to school over the course of a semester p and is derived directly from the imposed curriculum requirements.

$$\max \hat{w}^{Min} = \sum_{p \in P} w_p^{Min} \tag{1.1}$$

s.t.

$$\sum_{c \in C} n_c \cdot w_{ct} \geq w_p^{Min} \qquad \forall\ p \in P,\ t \in T_p^{Semester} \tag{1.2}$$

$$v_{ct} + w_{ct} = 1 - t_{ct}^{Off} \qquad \forall\ c \in C,\ t \in T \tag{1.3}$$

$$v_{ct} = 1 \qquad \forall\ c \in C,\ t \in T_c^{ForceSchool} \tag{1.4}$$

$$\sum_{t \in T_p^{Semester}} v_{ct} = \left\lceil \frac{h_{cp}^{SchoolReq}}{h^{SchoolWeek}} \right\rceil \qquad \forall c \in C,\ p \in P \tag{1.5}$$

$$\sum_{c \in C} v_{ct} \leq r_t \qquad \forall\ t \in T \tag{1.6}$$

$$v_{ct} - v_{c,t-1} \leq v_{ct}^{Start} \qquad \forall\ c \in C,\ t \in T \tag{1.7}$$

$$v_{ct}^{Start} \leq v_{ct'} \qquad \forall\ c \in C,\ t,\ t' \in T : t \leq t' < t + b^{MinSchool} \tag{1.8}$$

$$\sum_{t'=t}^{t+b^{MaxSchool}} v_{ct'} \leq b^{MaxSchool} \qquad \forall\ c \in C,\ t \in T \tag{1.9}$$

$$w_{ct} - w_{c,t-1} \leq w_{ct}^{Start} \qquad \forall\ c \in C,\ t \in T \tag{1.10}$$

$$w_{ct}^{Start} \leq w_{ct'} \qquad \forall\ c \in C,\ t,\ t' \in T : t \leq t' < t + b^{MinWork} \tag{1.11}$$

$$\sum_{t'=t}^{t+b^{MaxWork}} w_{ct'} \leq b^{MaxWork} \qquad \forall\ c \in C,\ t \in T \tag{1.12}$$

$$w_{ct},\ v_{ct}, w_{ct}^{Start},\ v_{ct}^{Start} \in \mathrm{B} \quad \forall\ c \in C,\ t \in T \tag{1.13}$$

$$w_p^{Min} \in \mathrm{N} \qquad \forall\ p \in P \tag{1.14}$$

We maximize the minimum number of students assigned to a work block for any semester in the planning horizon in objective (1.1). Constraints (1.2) determine the minimum number of students w_p^{Min} assigned to a work block in any semester p. The constraints (1.3) enforce a valid school curriculum for every class by assigning it to either a work or school period whenever the class is available. There are only two exceptions to that rule, both modeled using parameter $\mathrm{t}_{\mathrm{ct}}^{\mathrm{off}}$. First, the parameter may indicate structural information of the apprenticeship program, as class c may have finished the program or has not yet started into the program in period t. In both cases, the model must not assign a school nor a work block. Second, the parameter may indicate a holiday period for all students of class c. We want to emphasize, holidays are scheduled by the principal before the School Schedules are being created (as compared to vacation that is requested by individual students and handled on the second planning stage in UAM). Therefore, this information is no decision for the model, as Fig. 2 illustrates. Please note, a holiday block (no matter how long) ends the previous school or work block and forces the start of a new one afterward. Arranging two periods of holidays not consecutively but too close together (distance $< \min(b^{MinSchool}, b^{MinWork})$) will make the problem infeasible due to mandatory block lengths. Constraints (1.4) force a school assignment if required so by the school due to examination period or due to external fixtures. Finally, the curriculum requires an exact number of school block assignments in each semester p (see constraints (1.5)). Constraints (1.6) correspond to the teaching capacity of the school and limits the number of classes to r_t for any period t. Constraints (1.7) to (1.12) form the block structure of both school and work blocks. The first set of constraints triggers indicator variables v_{ct}^{Start} if the class c starts a school block in period t. These indicator variables are used in constraints (1.8) to enforce a minimum block length as well as in constraints (1.9) to impose a maximum block length. Constraints (1.10) to (1.12) work accordingly for work blocks. The definition of the decision variables is given in (1.13) and (1.14).

The above SSM can easily be applied to other professional dual educational programs, as it considers the overall program structure, alternation of blocks, and aggregated class assignments. All constraints and decisions can be considered as generic model elements. Though, it might be necessary to adapt the anticipating objective (1.1) to the respective profession.

Unit assignment model (UAM)

After fixing the School Schedule via SSM and receiving (eventual) individual vacation requests, we can optimize Unit Assignments for all students during their respective work blocks. We minimize the over- and understaffing of medical units subject to fulfilling curriculum requirements for every individual student. The model decides to which medical units the students are assigned during their classes' work block. If the respective work block is long enough, it additionally decides if and when a second medical unit is assigned during the same work block. Please note, parameters $\bar{w}_{ct}$ represent the solution value of the corresponding decision variables w_{ct} in SSM.

Sets

$s \in S_c$	Set of students in class c
$g \in G$	Set of medical subject groups

Parameters

$\bar{w}_{ct}$	1, if period t is part of a work block for class c
h_{st}	1, if student s takes vacation in period t
$d_{at}^{Min}, d_{at}^{Max}$	Lower/Upper bound of target staffing level of unit a in period t
$t_{ca}^{From},\ t_{ca}^{To}$	First/Last possible period of assignment of a student of class c to unit a
$\omega^-,\ \omega^+$	Weights for missing/exceeding target staffing level on any unit
$h^{TotalWork}$	Total number of working hours required
$h_g^{GroupTotal}$	Total number of working hours required in subject g
h_g^{Min}	Total number of working hours required on any unit of subject g to fulfill additional requirements
$r_g^{MinUnits}$	Number of units in subject g the additional requirement has to be fulfilled
q_{ag}	Share of subject g learned when assigned to unit a

Decision variables

$x_{sat} \in \mathrm{B}$	1, if student s is assigned to unit a in period t
$x_{sat}^{Start} \in \mathrm{B}$	1, if student s starts a work block on unit a in period t
$u_{sat} \in \mathrm{B}$	1, if student s takes vacation during work on unit a in period t
$z_{sga} \in \mathrm{B}$	1, if student s meets requirements of subject group g on unit a
$\Delta_{at}^-, \Delta_{at}^+ \in \mathrm{N}$	Number of students missing/exceeding target staffing level of unit a in period t

To model the relation of students to classes, we additionally introduce sets of students S_c for a given class c. These subsets form a family of disjoint sets of S, such that $\cup_{c\in C} S_c = S$ and $S_{c_1} \cap S_{c_2} = \emptyset\ \forall c_1, c_2 \in C : c_1 \neq c_2$. Also, we define medical subject groups $g \in G$, which represent learning objectives stated in the curriculum requirements. Every unit a is assigned to at least one of these medical subject groups g, according to the expected educational content a student can acquire when working there. Parameter $\bar{w}_{ct}$ indicates when class c is scheduled to a work block in period t. Note that school blocks and holidays need not be differentiated on the second planning level. Both are subsumed as $\bar{w}_{ct} = 0$. Parameter h_{st} indicates an individual vacation request for student s and period (week) t. Due to the basic initial agreement of vocational schools and students – no vacation during school blocks – the relation $h_{st} \leq \bar{w}_{ct}$ $(\forall\ c \in C,\ s \in S_c,\ t \in T)$ must hold for the input data. Parameters d_{at}^{Min} and d_{at}^{Max} take the desired staffing levels of all medical units (obviously $0 \leq d_{at}^{Min} \leq d_{at}^{Max}$ must hold). If the number of assigned students is within the limits, i.e. $\sum_{s\in S} x_{sat} \in [d_{at}^{Min}; d_{at}^{Max}]$, then no penalty is invoked. Deviations from these interval limits of desired staffing levels will be quadratically penalized with weights ω^- and ω^+ in the objective (2.1), respectively. Finally, there is a set of parameters corresponding to curriculum requirements on Unit Assignments. First, $h^{TotalWork}$ gives the total number of hours any student is required to receive practical education throughout the course of their apprenticeship program. For the subject groups $g \in G$, additional rules can apply: students must fulfill some hours $h_g^{GroupTotal}$ working in medical units of the respective subject group g. Further, they must work for a minimum of h_g^{Min} hours in each of at least $r_g^{MinUnits}$ medical units. Finally, parameter q_{ag} represents the share of expected learning achievements for subject group g based on the assignment to medical unit a. Note that $\sum_{a\in A} q_{ag} = 1$ holds for all groups g and $q_{ag} \in [0; 1]$. Binary variables x_{sat} indicate whether a student s is assigned to unit a in period t. Similar to SSM, x_{sat}^{Start} is used to generate a block structure for the Unit Assignments. Decision variables u_{sat} decide in which medical units the students will take their vacation. We need the variables to guarantee the block structure of Unit Assignments during work blocks. Binary variables z_{sga} indicate for each student whether or not curriculum requirements for subject groups and medical Unit Assignments are met. Finally, integer variables Δ_{at}^- and Δ_{at}^+ count how many students are missing to meet the minimum staffing levels d_{at}^{Min} or exceeding maximum staffing levels d_{at}^{Max}, respectively.

$$\min \hat{\Delta}^2 = \sum_{a\in A}\sum_{t\in T} \omega^- \cdot \left(\Delta_{at}^-\right)^2 + \sum_{a\in A}\sum_{t\in T} \omega^+ \cdot \left(\Delta_{at}^+\right)^2 \tag{2.1}$$

s.t.

$$\sum_{a\in A} x_{sat} = \bar{w}_{ct} \cdot (1 - h_{st}) \qquad \forall\, c \in C,\; s \in S_c,\; t \in T \tag{2.2}$$

$$x_{sat} = 0 \qquad \forall\, c \in C,\; s \in S_c,\; a \in A,\; t \in T : t^{From}_{ca} > t \vee t > t^{To}_{ca} \tag{2.3}$$

$$\sum_{a\in A} \sum_{t\in T} q_{ag} \cdot x_{sat} \geq \frac{h^{TotalGroup}_{g}}{h^{WorkWeek}} \qquad \forall\, s \in S,\; g \in G \tag{2.4}$$

$$\sum_{t\in T} q_{ag} \cdot x_{sat} \geq \frac{h^{Min}_{g}}{h^{WorkWeek}} \cdot z_{sga} \qquad \forall\, s \in S,\; g \in G,\; a \in A \tag{2.5}$$

$$\sum_{a\in A} z_{sga} \geq r^{MinUnits}_{g} \qquad \forall\, s \in S,\; g \in G \tag{2.6}$$

$$x_{sat} + u_{sat} - x_{s,a,t-1} - u_{s,a,t-1} \leq x^{Start}_{sat} \qquad \forall\, s \in S,\; a \in A,\; t \in T \tag{2.7}$$

$$x^{Start}_{sat} \leq x_{sat'} + u_{sat'} \qquad \forall\, c \in C,\; s \in S_c,\; a \in A,\; t, t' \in T : t \leq t' < t + b^{MinWork} \tag{2.8}$$

$$\sum_{a\in A} u_{sat} = h_{st} \qquad \forall\, s \in S,\; t \in T \tag{2.9}$$

$$x_{s,a,t-1} + u_{s,a,t-1} + x_{s,a,t+1} + u_{s,a,t+1} \geq u_{sat} \qquad \forall\, s \in S,\; a \in A,\; t \in T \tag{2.10}$$

$$\sum_{s\in S} x_{sat} + \Delta^{-}_{at} \geq d^{Min}_{at} \qquad \forall\, a \in A,\; t \in T \tag{2.11}$$

$$\sum_{s\in S} x_{sat} - \Delta^{+}_{at} \leq d^{Max}_{at} \qquad \forall\, a \in A,\; t \in T \tag{2.12}$$

$$x_{sat}, x^{Start}_{Sat}, u_{sat} \in \mathrm{B} \qquad \forall\, s \in S,\; a \in A,\; t \in T \tag{2.13}$$

$$z_{sga} \in \mathrm{B} \qquad \forall\, s \in S,\; g \in G,\; a \in A \tag{2.14}$$

$$\Delta^{-}_{at}, \Delta^{+}_{at} \in \mathrm{N} \qquad \forall\, a \in A,\; t \in T \tag{2.15}$$

The objective function (2.1) of UAM is to minimize the weighted quadratic sum of staffing level violations to use educational capacities as efficiently as possible and maintain quality of supervision during the program. A quadratic function is used to avoid strong violations for single medical units. Since quadratic integer programs are generally hard to solve (Nemhauser & Wolsey, 2010), we linearize objective function (2.1), as we show in Appendix A. We tested two additional min-max-approaches for the objective in a preliminary study. First, in objective (2.1b) we identify for every period t an individual unit a with the strongest positive and negative deviation, respectively. Second, in objective (2.1c) we identify corresponding global maximum over- and understaffing instead. Both alternative objectives will minimize the difference of these max-values. These alternatives were clearly outperformed by objective (2.1) in this preliminary study. We provide more insights and numerical results in Appendix D.

In constraints (2.2), we assign every student to a medical unit during assigned work blocks by SSM. Students cannot be assigned to some (medical) units during some parts of their apprenticeship program. This might be due to educational rules defined by the school (like aligning theoretical and practical education) or due to seniority rules imposed by a medical unit (e.g., an intensive care unit can only educate very senior students). Therefore, constraints (2.3) do only allow assignments to units within a class-individual time window from period t^{From}_{ca} to t^{To}_{ca}. The following block of constraints (2.4) to (2.6) forms the curriculum requirements stated by the local authorities. Constraints (2.4) ensure enough assignments to fulfill the total required number of hours for every subject group g. Constraints (2.5) in combination with constraints (2.6) force a minimum number of h^{Min}_{g} assigned hours for any subject group g to be performed in a minimum number of $r^{MinUnites}_{g}$ medical units a corresponding to g. The decision variables z_{sga} indicate if the student s has enough assigned hours at a single medical unit a corresponding to subject group g.

The block of constraints (2.7) to (2.10) forms the block structure of medical Unit Assignments. Constraints (2.7) trigger indicator variable x^{Start}_{sat}, if the student is assigned to (medical) unit a in period t but is not assigned to the same unit in the previous period $t-1$. Constraints (2.8) ensure a minimum length of $b^{MinWork}$ periods for any block of Unit Assignments. Note, a constraint ensuring maximum block length is not necessary, because this rule is implicitly imposed by the structure (i.e. maximum length) of work blocks in SSM given by the parameters $\bar{w}_{ct}$.

Without any extensions, individual vacation (encoded in h_{st}) can easily cause infeasibilities when it is requested too close to the beginning or the end of a work block. To avoid these problems, we introduce additional decision variables u_{sat}, which are strongly related to x_{sat}. Constraints (2.9) force that every period of individual vacation is assigned to a medical unit as well. Therefore, in combination with (2.7), (2.8), and (2.10) any intermediate periods of (individual) vacation do not end the current medical unit assignment block. So, the model is guaranteed to be feasible, with regards to any individual vacation.

Finally, the last two sets of constraints (2.11) and (2.12) force decision variables Δ^{-}_{at} and Δ^{+}_{at} to take negative or positive deviations from the intervals of minimum and maximum target staffing levels for any medical unit a and period t, respectively. Decision variable definitions are given in (2.13) to (2.15).

To adapt the UAM to another profession one will have to differentiate between the generic model elements and program specific side constraints. Clearly, constraints regarding the general structure of the program (2.2, 2.7, 2.8, 2.11, and 2.12) are generic. Decisions on individual student assignments and the objective (2.1) are also generic for any dual professional program. In contrast, restrictions on seniority (2.3), medical subject groups (2.4, 2.5), detailed hourly requirements (2.6), and vacation (2.9, 2.10) are problem-specific side constraints for vocational schools for nursing. These will have to be reworked or replaced in order to apply the model to another profession based on the respective curriculum requirements.

3.2. Decomposition based matheuristic solution approach

In order to translate the current manual planning process and the stated requirements into a solution approach based on the newly introduced IP models of Section 3.1, we use a hierarchical approach. Therefore, we fix decisions taken in SSM to find optimal solutions for UAM.

In Fig. 3, we visualize the relation between the SSM and UAM and show how parameters and decision variables propagate through different steps of the planning problem. In a preliminary

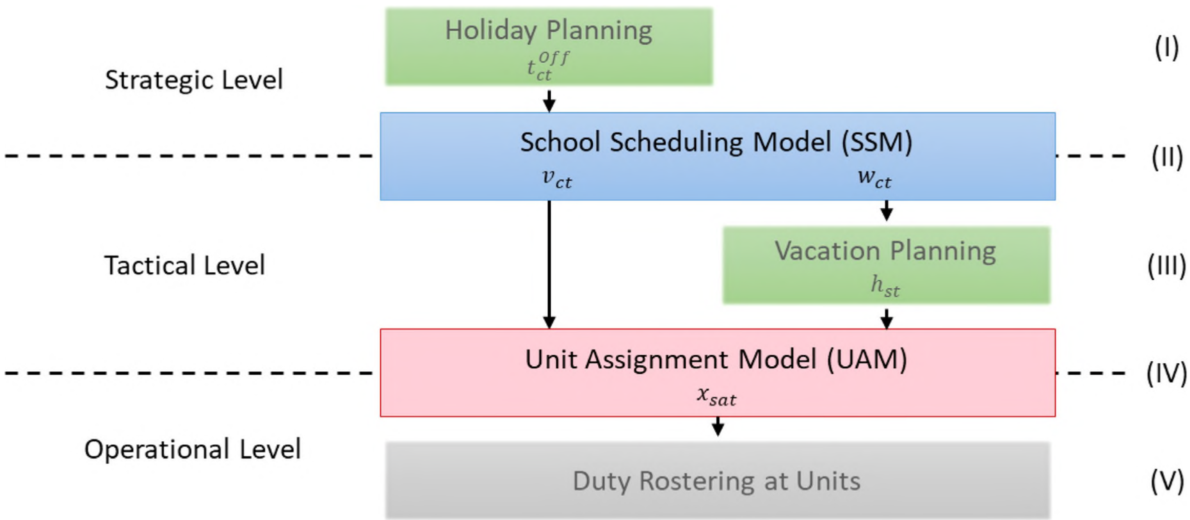

Fig. 3. Information- and Control-Flow between Planning Problems.

step (I), the principal determines school holidays for all classes, which may be some proportion or all of the total vacation entitlement of the students. This information serves as input (t_{ct}^{Off}) to the SSM. Then SSM determines the periods of school education (v_{ct}) and the periods of practical education, i.e. work blocks (w_{ct}) in step (II). The latter serves as input to both vacation requests of individual students and UAM. For the former, any student files a request for the remaining vacation entitlement (depending on how many periods of holiday have previously been fixed for the respective class in step (I)) during periods of practical education, i.e., work blocks, in step (III). This information is represented by the parameter h_{st}. For the latter (i.e., UAM), periods of practical education and vacation requests are the foundation of medical Unit Assignments for every individual student in step (IV). Finally, the medical Unit Assignments serve as necessary input to any following duty rosters in step (V), which is out of scope for this piece of research.

Due to the enormous size, solving UAM is intractable for standard solvers. Therefore, we reduce the size and complexity of the model by a decomposition. In general, such models can be decomposed either by time or by agent (in our case by student). For our model, the former means to solve UAM for all students but only for some periods. Applying such a rolling horizon approach may result in infeasibility, in particular, due to constraints (2.4) to (2.6). Therefore, we decompose our model by agents, i.e. students. We solve UAM for only a subset of students but over the full planning horizon. This allows us to ensure feasibility of the model, i.e., the curriculum, throughout the whole solution process. The decomposed models are connected to each other solely through the objective function (2.1). We can easily take previous solutions into account by in- or decrementing d_{at}^{Min} and d_{at}^{Max} accordingly. The solution algorithm is given as pseudocode in Fig. 4. First, the holiday calendar is created and SSM is solved with standard software (l.1 to l.3). Subsequently, students can formulate their individual vacation requests based on the SSM solution (l.4). Then, we decompose the set of all students S into a partition of M disjoint and nonempty subsets $S_1', S_2', S_3', \ldots, S_M'$ (l.5). This means every student is contained in exactly one subset S_m'. There are three strategies for how the subsets can be composed (homogenous, heterogenous, random), partially based on similarity of students. We evaluate these compositions in the computational study (see Section 4.4). When forming homogenous groups, we try to assign students with similar prerequisites, regarding school schedule, seniority, and achieved progress in the curriculum, to the same subset. This usually means that students of the same class will belong to the same subset. For a heterogenous decomposition, we try to merge students from different classes into the same subset. Finally, we use a random generator for these assignments to subsets.

As we strongly rely on a standard mathematical programming solver to find feasible solutions to the decomposed subproblems, we categorize our approach as Matheuristic which is generally defined as an application of mathematical programming within heuristics (Maniezzo, Boschetti & Stützle, 2021). Our solution procedure consists of main iterations (from l.7 to l.15) and sub-iterations (from l.9 to l.13). For any sub-iteration, we fix the decision variables associated with students not considered ($s \notin S_m'$) by their previous assignments. Note, if such assignments (of previous sub-iterations) do not exist, we simply do not take them into account. Finally, we increment m to complete the corresponding sub-iteration (l.12). After UAM has been solved M times, we enter the next main-iteration (l. 7) and re-solve UAM for all subsets M trying to improve the current solution. We repeat the whole process for a predefined number of C main iterations and finally report the solution for UAM (l.16), which can easily be found by combining the last solution of every decomposed UAM.

With this algorithm, we are able to find local optimal solutions to the vocational school planning problem as defined in Fig. 3. We show the effectiveness of our heuristic solution approach in the following experimental study, in particular in Section 4.4. Furthermore, we will discuss the influence of different parameter settings for the algorithm as well.

4. Experimental study

In this section, we conduct an experimental study to verify the two IPs as well as our solution algorithm for the second IP. We show that using optimization models to plan vocational school operations can minimize violations of educational capacities in the hospital. We further show that reorganizing the holiday system to a more flexible system of granting individual vacation requests enables better (medical) Unit Assignments. Our partner hospital and its vocational school provide us with real-world data from previous years as well as for upcoming school years. The study is structured as follows: first, we describe and analyze the data set, define several planning scenarios, and present the characteristics of the imposed curriculum. Second, we define multiple performance measures to evaluate the quality and present solutions for both planning levels (SSM and UAM). Third, we test several parameter settings for the solution algorithm introduced in Section 3.2.

```
use Holiday Calendar as t_{ct}^{Off}
solve SSM
use SSM solution as \bar{w}_{ct}
use Vacation Requests as u_{st}
decompose S into disjoint and nonempty subsets S'_1, S'_2, S'_3, ..., S'_M
c = 1
while c <= C do
   m = 1
   while m <= M do
         solve UAM for subset S'_m
         update solution
         m = m+1
   end while
   c = c+1
end while
report final solution of UAM
```

Fig. 4. Pseudocode of Vocational School Planning Problem decomposition algorithm.

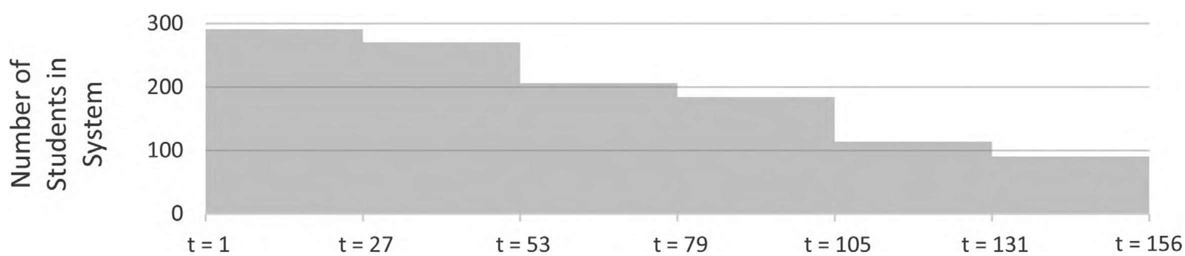

Fig. 5. Students in the system.

4.1. Input data analysis

Our partner's vocational school consists of twelve classes for Licensed Practical Nurses with four classes per year of the apprenticeship program. In total, 291 students are currently enrolled (Fig. 5). Each class consists of around 25 students on average. Since classes start and end on multiple dates of a year, the number of students in the system varies systematically. To account for the full program duration of all current classes, our planning horizon consists of 3 years or 156 weeks. The current holiday calendar is very rigid, as all off-days are centrally planned and granted class-wise before the first planning step (see step (I) in Fig. 3).

Fig. 6 explains the timeline of classes entering and leaving the system. Classes indicated by dashed bars are not considered any more or not yet, as they left the system before or will enter for future planning processes. Classes are labeled by the year entering the apprenticeship program and a letter code indicating a start at the first or second possible date in that year (e.g. Class 19B enters the program on the later date in 2019). Milestones indicate when classes may enter and leave the system, and are identical with start and end of semesters.

In the second level of the problem (UAM), students can be assigned to 62 medical units in the hospital itself or at additional external partners. The largest unit offers a range of 8 to 20 practical educational spots per period, the smallest do not offer any regular capacity but accept students if necessary. In total, there is a minimum requirement of 160 and a maximum of 308 educational spots to be filled in every period. Unit capacities are displayed in Fig. 7, where the lower bar indicates d_{at}^{Min} and upper bar d_{at}^{Max} of any individual unit, respectively. For our case study, they do not change over time.

The curriculum imposed by the responsible Bavarian Ministry of Education and Culture (2005) requires theoretical education of 2100 hours or 53 periods over the whole program horizon for every class. For practical education, a total of 2500 hours or 63 periods is required. According to their respective specialization and training content, units are assigned to one or more of four medical subject groups. At the end of the program, students must prove education of a given number of hours in every such subject group. Additionally, for some of the groups complex rules apply: these can be (a) at least 80 hours in every unit of the group or (b) visiting two out of five units in a group for at least 60 hours each. Given the most basic imposed curriculum requirements, any student can be assigned to practical education for at most 94 periods. With an average of 293 students in the apprenticeship program at any point in time, on average 170 students (compared to a minimum requirement of 160) can be assigned to practical education. Adding further requirements to these rough calculations, it becomes apparent that (at least for some periods) it will not be possible to fill all demanded medical Unit Assignments.

Parametrization

We limit the scope of all following analyses to the first 52 periods (weeks) of the planning horizon, so $\hat{t} = 52$ and $\hat{T} = \{t \mid t \in T : t \leq \hat{t}\}$. According to best practice of our partner school, we set the required block lengths of theoretical education $b^{MinSchool} = 3$ weeks

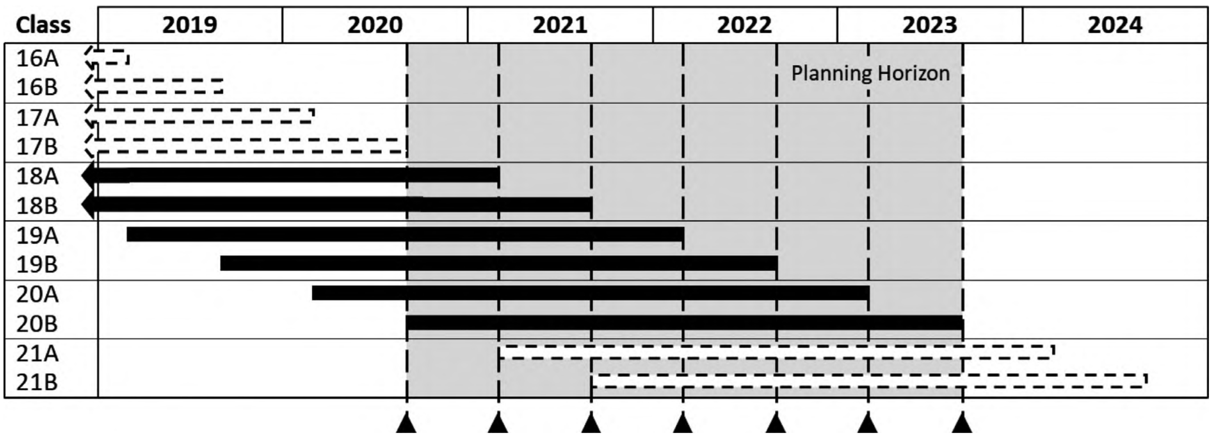

Fig. 6. Timeline of the vocational planning problem.

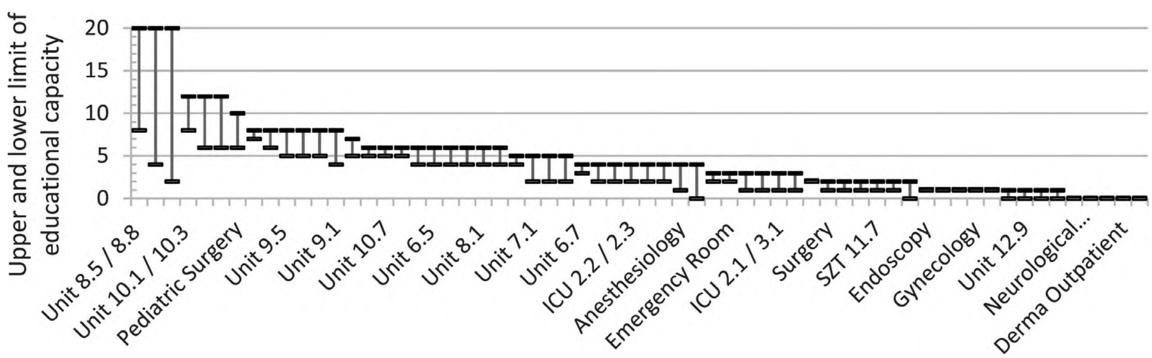

Fig. 7. Unit's educational capacities.

and $b^{MaxSchool} = 5$ weeks and of practical education to $b^{MinWork} = 4$ weeks and $b^{MaxWork} = 10$ weeks. The school can schedule classes to $r_t = 7$ rooms all year except for periods 19 to 22 where $r_{19} = \ldots = r_{22} = 0$ due to final examinations, and for periods 39 and 40 ($r_{39} = r_{40} = 0$) where no classes may be at school. For the UAM, we set objective function weights to $\omega^- = 1$ and $\omega^+ = 0.5$, since our partner hospital is trying to achieve a balanced result but puts more emphasis on avoiding understaffing.

All experiments are conducted using CPLEX 12.9.0 to solve all models on a standard personal computer with a multicore 2.6 gigaHertz processor and 8 gigaByte of RAM. The solution algorithm was implemented in OPL Script using the IBM ILOG Optimization Studio.

4.2. Plan quality: comparing with an expert

We evaluate the two levels of our solution algorithm separately, introduce quality measures, and visualize solutions of all model outputs. For these instances, all off-days are granted as class-wise holidays. Please note, the second level can only be evaluated meaningfully when the first level has also been solved. First, we want to prove that the optimization models lead to better utilization of educational capacities and can minimize penalties. Therefore, we use manual plans created by an expert (Instance EXP) for both problem levels, which are currently used in real-word and have been provided by our partner school. These plans contain assignments for about 70% of all periods. We use UAM and SSM to compute the missing 30%. We compare these results with plans computed completely by our models (Instance OPT) while keeping all parameters untouched.

Results for SSM

To evaluate the quality of the solutions, we use three performance measures for SSM output. The (1) objective function value $\hat{w}^{Min}$, which is the sum of minimum assignments per semester w_p^{Min} within the evaluation horizon. Additionally, we measure the (2) number of total assignments $\hat{w} = \sum_{t \in \hat{T}} w_t$ and its respective (3) standard deviation $\hat{\sigma} = \sqrt{Var(w)}$. $\hat{w}^{Min}$ is capable of measuring the degree of student supply and its even timely distribution at the same time. $\hat{\sigma}$ can solely measure the latter. Finally, $\hat{w}$ will be used for model and scenario validation in the upcoming experiments.

When evaluating the manual assignments of Instance EXP, it is clear that in most periods in the evaluation horizon, demand of students cannot be met. As seen in Fig. 8, the number of students assigned to practical education (solid black line) is often below the minimum number of students required which is 160 (base line). The area (red or green) visualizes the difference and indicates understaffing (red) or overstaffing (green). The demand can only be met during and around summer break ($19 \le t \le 22$) and Christmas holidays ($39 \le t \le 40$) and some adjoint periods. School closings are indicated by grey shadings. As the following experiments show, this leads to an unfavorable initial situation for UAM.

Although Instance OPT has many periods with understaffing as well, it is able to smoothen supply between the different periods. In reality, this makes the situation for units much more predictable. Also, the (red) area below the graph's baseline is smaller, meaning that stronger violations will be reduced in the second level (see Fig. 9).

When comparing both solutions, it is clear that model-based planning (Instance OPT) clearly outperforms manual planning (Instance EXP) in all relevant performance measures. The minimum number of students assigned w_p^{Min} can be increased in all semesters. The total number of work block assignments remains the same ($\hat{w} = 7055$) due to constraints (1.5), which force an exact number of school blocks. However, the standard deviation could be slightly reduced, as Table 1 shows. *Instance OPT* was solved to optimality within 15 minutes by CPLEX. Two alternative objective

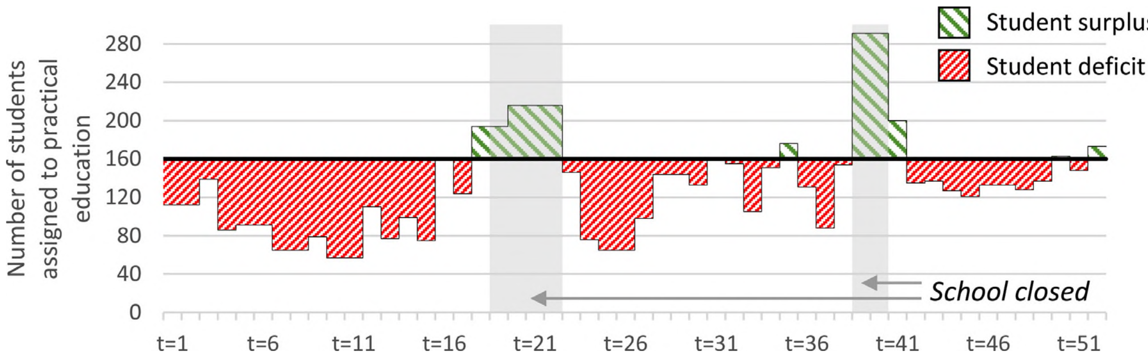

Fig. 8. Students assigned to practical education by SSM for Instance EXP.

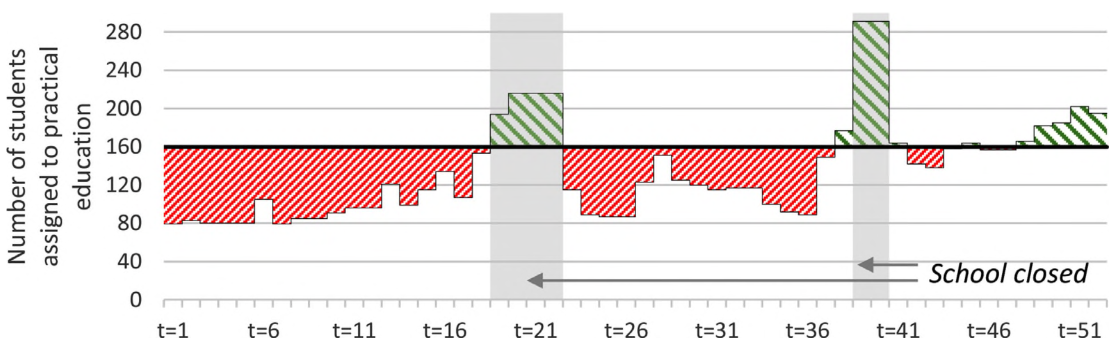

Fig. 9. Students assigned to practical education by SSM for Instance OPT.

Table 1
Performance measures for SSM for Instances EXP and OPT.

Instance	$\hat{w}^{Min}$	$\hat{w}$	$\hat{\sigma}$
EXP	138 (100%)	7,055	52.2
OPT	168 (123%)	7,055	51.3

Table 2
Performance Measures for UAM for Instances EXP and OPT.

Instance	$\hat{\Delta}^2$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
EXP	7394.0 (100%)	3246	121.9	1903	5	3
OPT	4794.5 (65%)	2516	73.1	1672	5	2

functions, that were evaluated in preliminary tests, could also outperform *Instance* EXP. We refer to Appendix C for detailed results of these tests.

Results for UAM

For UAM we use five different performance measures to evaluate the solution quality. The objective measures the (1) weighted sum of squared violations $\hat{\Delta}^2$ over the evaluation horizon. Additionally, we report the (2) (unweighted and unsquared) sum of deviations $\hat{\Delta} = \sum_{t\in\hat{T}} \hat{\Delta}_t = \sum_{a\in A}\sum_{t\in\hat{T}}(\Delta^-_{at} + \Delta^+_{at})$, the (3) absolute number of violated units per period in the evaluation horizon $\hat{c} = |\{(a,t)|a \in A,\ t \in \hat{T} : \Delta^-_{at} > 0 \vee \Delta^+_{at} > 0\}|$. The (4) largest violations $\hat{\Delta}^{-Max} = \max_{a\in A,\ t\in\hat{T}}(\Delta^-_{at})$, $\hat{\Delta}^{+Max} = \max_{a\in A,\ t\in\hat{T}}(\Delta^+_{at})$, and (5) $\hat{\sigma} = \sqrt{Var(\hat{\Delta})}$ complement the evaluation. We use these measures to identify different aspects of over- and understaffing for the medical units. We use $\hat{\Delta}^2$to incorporate severity and to promote a timely fair and an even distribution of violations among units. All remaining measures neglect at least one of these aspects, so we can better observe the single dimensions of our objective. E.g., unweighted and unsquared sum of deviations $\hat{\Delta}$ can measure the supply of students isolatedly. We parameterize our algorithm with $C = 3$ main iterations and $M = 10$ subgroups using the heterogenous decomposition approach. This choice is based on preliminary testing and is evaluated in Section 4.4.

We now use the SSM output of Instances EXP and OPT as input to UAM and evaluate the severity of violations for single units. Comparing manual (Fig. 10) to optimized assignments (Fig. 11) you can see that both the number of violations – as the total area of the graph suggests –, as well as the severity of these violations – as the shift to lighter colors indicates –, are significantly reduced. Also, fluctuation from period to period can be reduced strongly.

Interestingly, in the expert solution (Instance EXP) some periods exist, where some units are understaffed whilst some others are overstaffed. These situations occur systematically during school closings and before final examinations, as the principals have to make sure all students completed required working hours for all subject groups. To do so, some unfavorable assignments have to be performed right before the end of the apprenticeship program, which leads to this situation of simultaneous under- and overstaffing. The optimized solution can avoid such situations. All these properties are confirmed by the performance measures in Table 2. Squared violations $\hat{\Delta}^2$, sum of deviations $\hat{\Delta}$, and count of violated units $\hat{c}$ are dramatically reduced. Also, the maximum overstaffing $\hat{\Delta}^{+Max}$ can be reduced by one. Solutions to both instances were computed within 3 hours of runtime with our decomposition heuristic. Please note that we tested additional objective functions that clearly outperformed manual solutions. These objectives are trying to minimize either the global or the unit-individual maximum of over- and understaffing. However, these objectives did not outperform the quadratic objective function (2.1) for any performance measures. Again, we refer to Appendix D for detailed results.

4.3. Effects of class-wise holidays and individual vacation

In a second evaluation, we are going to analyze how the underlying holiday calendar and increasing flexibility for individual

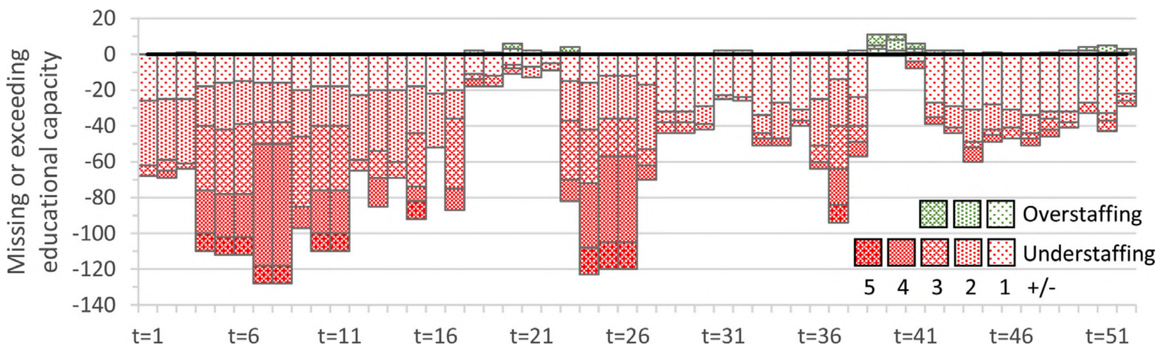

Fig. 10. Violated capacity restrictions by UAM for Instance EXP.

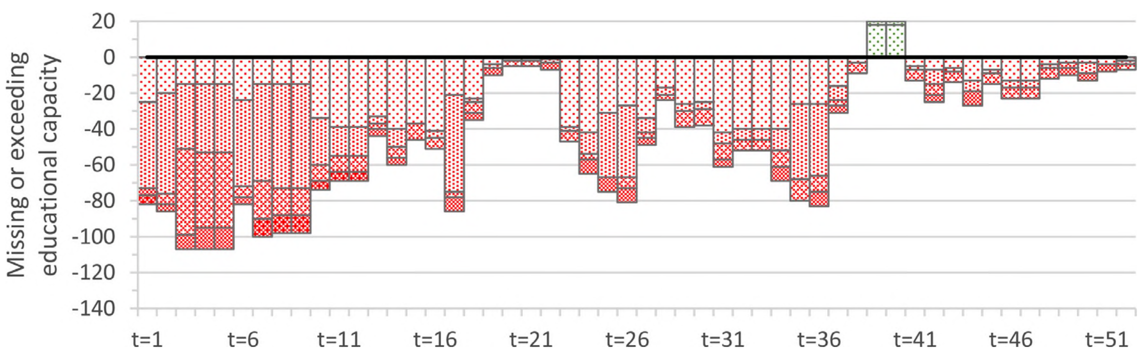

Fig. 11. Violated capacity restrictions by UAM for Instance OPT.

vacation influences plan quality at the second level. Therefore, we define several additional instances. The first one has a rigid holiday system (Instance HOLIDAY-A), meaning that all off-days are granted class-wise in the first planning level. Consequently, it is comparable to the situation as in Instance OPT, but with a different underlying holiday schedule. Further instances increase the number of periods, where off-days are not granted by class but individually per student in UAM. Instance HOLIDAY-B assigns 1/3, Instance HOLIDAY-C assigns 2/3, and Instance HOLIDAY-D assigns all vacation periods individually.

The new holiday calendar tries to level the number of classes that are off due to holidays over the whole planning horizon and to reflect the framework induced by commonly accepted rules within the vocational school and the hospital. This includes that no classrooms are available during final examinations in summer and no class-wise holidays should be granted during and around Christmas. Our suggestion was accepted by the principals and may be used in future years. Our partner school is currently assigning all off-days as class-wise holidays. Therefore, our dataset cannot contain individual vacation requests h_{st}. We generate them based on random distributions that incorporate peaks of requests at some periods (like for summer or skiing vacations), and such that blocks of two or more consecutive off periods are likely. With the increasing number of periods of individual vacation entitlement, new requests are added, while previous requests are unchanged. We assume all requested periods of vacation are granted, as we want to evaluate the unbiased effects of individual vacations. In reality though, vocational schools can decline such requests to achieve even better staffing levels.

Results

For these tests, we first optimize SSM with the respective (rigid) holiday schedule. The resulting schedules are then used to generate vacation requests, as described above. Finally, both become input to UAM. Since the influence of the holiday calendar can be evaluated best on the latter level of the problem (i.e. UAM), we do not discuss results of SSM here. These results can be found in Appendix B. We first compare Instances OPT and HOLIDAY-A, as they both have a rigid and fixed but different holiday calendar. So, we can identify effects of more evenly spread holiday blocks. All results are given in Table 3 and the graphical representations can also be found in Appendix B. We use the same performance measures and add column two which indicates the share of class-wise and individual requests. It is clear, that the holiday calendar in Instance HOLIDAY-A is superior in all performance measures, as it can reduce the weighted squared deviations $\hat{\Delta}^2$, the sum of deviations $\hat{\Delta}$, and the number of violations $\hat{c}$ significantly. Please note, proportional change of $\hat{\Delta}^2$ is always set in ration to the solution of Instance EXP ($7394.0 \triangleq 100\%$).

In the next analysis, we use the superior holiday calendar and gradually reduce the share of fixed class-wise holidays in exchange for more student-individual vacations (see column two in Table 4). Our results prove that granting some individual off-days, as in Holiday-B, is clearly beneficial for the system, as both squared and sum of violations improve strongly compared to Holiday-A. The maximum level of overstaffing $\hat{\Delta}^{+Max}$ could even be reduced

Table 3
Performance measures for UAM for different holiday schedules.

Instance	H | V	$\hat{\Delta}^2$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
OPT	1 | 0	4794.5 (65%)	2516	73.1	1672	5	2
Holiday-A	1 | 0	3585.0 (48%)	2103	39.6	1586	5	2

Table 4
Performance measures for UAM granting more individual vacation.

Instance	H | V	$\hat{\Delta}^2$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
Holiday-A	1 | 0	3585.0 (48%)	2103	39.6	1586	5	2
Holiday-B	2/3 | 1/3	3025.5 (41%)	1849	34.4	1437	5	1
Holiday-C	1/3 | 2/3	3240.0 (44%)	1814	51.1	1264	5	1
Holiday-D	0 | 1	2993.0 (40%)	1811	36.2	1386	4	1

Table 5
Performance Measures for UAM for different parameter settings with 10 student subgroups (Results for random decompositions are mean values of 5 runs).

ID	C	M	S	runtime (s)	$\hat{\Delta}^2$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
1	1	10	hetero	3589	3158.5 (43%)	1851	34.1	1651	4	1
2	2	10	hetero	5882	3085.0 (42%)	1865	35.0	1443	5	1
3	**3**	**10**	**hetero**	**7103**	**3025.5 (41%)**	**1849**	**34.4**	**1437**	**5**	**1**
4	1	10	homo	3716	3775.5 (51%)	2332	34.7	1895	3	1
5	2	10	homo	6360	3392.0 (46%)	1923	37.6	1640	4	1
6	3	10	homo	8594	3334.0 (45%)	2064	37.8	1743	4	1
7	1	10	rand	3670	3339.7 (45%)	2013	34.0	1753	3	1
8	2	10	rand	5934	3296.5 (45%)	1928	35.7	1606	4	1
9	3	10	rand	7846	3270.1 (44%)	1985	35.0	1592	5	1

by one. When increasing flexibility even more (Holiday-C), some drawbacks might occur in terms of weighted squared violations $\hat{\Delta}^2$. Nevertheless, full flexibility (Holiday-D) gives the best objective value and can reduce maximum understaffing $\hat{\Delta}^{-Max}$ by one. As a consequence, we clearly recommend using a leveled holiday schedule and allowing some degree of individualization (i.e. granting individual vacation requests). All solutions were computed within 3 hours of runtime with our solution algorithm and parameter settings as described above.

4.4. Factorial study on matheuristic solution algorithm parameters

Subsequently, we want to evaluate the algorithm's sensitivity on the choice of parameters used when solving UAM with the algorithm shown in Section 3.2. We use the setup as in Instance Holiday-B, as we deem it the most likely case to be implemented at our partner school. In our solution algorithm, we can influence three different properties: the number of main iterations C, the number of subsets of students M, and the way to decompose the set of students S into disjoint subsets $S'_1, S'_2, \ldots, S'_M$. For the latter, we can decide to form homogenous groups, heterogenous groups, and a random (re-)assignment after every main iteration, as described in Section 3.2.

Preliminary tests showed that for $M \leq 5$, the solver was not able to find feasible solutions for a majority of tested instances. Also, in iterations $C > 3$, no progress could be observed in any of the tested instances and we, therefore, limit our reports to at most 3 main-iterations.

Results

The results show two main findings, which lead us to the conclusion that using $C = 3,\ M = 10$, and a heterogenous decomposition is the best of our tested alternatives (see Table 5). For this setting, both weighted squared violations $\hat{\Delta}^2$ and absolute number of violated units $\hat{c}$ is minimal (ID 3).

First, we look at the effects of C, the number of main iterations. Consider the first three rows in Table 5, where an increase of C from 1 to 3 subject to constant parameter settings is reported. By increasing C, we can see improved performance ($\hat{\Delta}^2$ decreases from 3158.5 to 3025.5; ID 1 to ID 3) due to performed re-assignments at the costs of longer runtimes. For any further increase of main iterations ($C \geq 4$), no additional progress can be made. Similar results can be seen for homogenous groups (ID 4 to ID 6) and random groups (ID 7 to ID 9) (Table 6).

Second, we evaluate different group decompositions. When using a homogenous (ID 6) instead of heterogenous (ID 3) decomposition, the similarity of students within the same subgroup S'_m increases. According to the detailed inspection of medical Unit Assignments, the algorithm is not able to exploit educational capacities as well as before. The relevant measures (especially $\hat{\Delta}^2$ from 3025.5 to 3334.0) increase significantly. However, we can see that individual assignments become more similar to each other for students of the same class. This means, it is possible to promote perceived fairness within a class, which might be favorable in some situations. Finally, when using a random decomposition (ID 9), the performance increases compared to homogenous groups (ID 6) but is still worse than heterogenous groups (ID 3).

Table 6
Performance Measures for SSM granting more individual vacation.

Instance	H | V	$\hat{w}^{Min}$		$\hat{w}$	$\hat{\sigma}$
HOLIDAY A	1 | 0	173	125%	7055	40.9
HOLIDAY B	2/3 | 1/3	230	167%	7476	37.8
HOLIDAY C	1/3 | 2/3	231	167%	7897	38.5
HOLIDAY D	0 | 1	231	167%	8318	40.7

Third, to complement the analysis, we investigate the influence of the number of subsets M. Choosing this parameter too small results in intractable UAMs at sub-iterations. Choosing it too large reduces optimization potential. We tested different values for M. For instance, $M = 5$ was not solvable, while $M = 20$ resulted in significant runtime reduction, but could not achieve better performance, as $M = 10$. As a result, a balanced setting for M is crucial. We recommend $M = 10$ for our application.

4.5. Managerial insights

Summarizing the findings of our computational study, we can first conclude that model-based scheduling is able to generate feasible solutions quickly and avoid systematic violations. Therefore, it is clearly superior to manual scheduling. Second, we enable vocational school principals to allow students to file individual vacation requests, as the model-based approach can incorporate them into a feasible schedule. We could show that some share of these individual requests is even beneficial for the overall plan quality. More flexibility comes at the cost of wasting some small proportion of educational capacities but will noticeably improve student satisfaction. Finally, we evaluate the sensitivity of our solution algorithm. We show that 3 main iterations are enough to achieve a local optimal solution. By choosing different sets of parameters, planners can prioritize runtime over solution quality or can ensure that assignments of students of the same class, with comparable progress, or same seniority are similar. The latter will help to promote a feeling of fairness and of objective decisions during the planning process.

Overall, the model-based approach will improve reliability for hospital units by reducing variability of assignments. Besides, the IPs provide managers of educational institutions with a tool able to evaluate the optimal number of students that can participate in an apprenticeship program and an opportunity to test and evaluate different holiday schedules easily. Bottlenecks of educational capacities (e.g. units' minimum and maximum capacities or the number of available classrooms and teachers) can be identified and consequences can be examined. The conducted analysis focused on the German dual vocational system, but can easily be adapted to meet the requirements of any vocational education system that combines practical and theoretical education in alternating blocks. Especially in the health care sector, several professions also require a combination of theoretical and practical education in Germany. Among these are, occupational therapists, physiotherapists, and orthoptics with prescribed practical education of at least 1700, 1600, and 2800 hours, respectively. Practical education is always the responsibility of the school, which also specifies the trainees' areas of work. Further examples are the training for physiotherapy, massage, podiatry, midwifery and maternity nursing, and emergency paramedics but can also be found outside of healthcare: for example, vocational education for dieticians requires 1400 practical hours.

Across Europe, a variety of professional education programs exist. All of these are built on different traditions, program durations, school types, and academic levels. Besides these differences, all programs of the nursing profession have mandatory practical education in common. In Austria, education in this profession is delivered by universities of applied sciences. In these programs, students have extensive clinical work placements in partner hospitals and nursing homes each semester. In Switzerland and France, the proportion of practical education in such a program is as high as 50%. Finally, in non-European countries such as the US, all Registered Nurse (RN; this is the equivalent level of education compared to the German case) programs require extensive skills lab or clinical hours.

Despite all the differences that professional education systems in Europe and the US have in terms of type of the program, duration, form of teaching, degree, and school organization, practical training is an essential part for healthcare professions. Therefore, although the German system is special in some ways, it can act as a prototype and role model for most other countries, especially for healthcare professions. On the other hand, this dual system is widely used in other professions in Germany.

5. Summary and outlook

In this paper, we describe the professional vocational education system and focus our application on the dual system of Germany. We discuss, why special planning problems arise within both privately and publicly organized vocational schools and show that this problem has never been investigated in scientific operations research literature. The substantial number of externally enforced rules, conflicting goals of stakeholders, and two interdependent planning levels make this a particularly challenging problem for school principals. We propose two IP models to optimize the decisions for the two planning levels consecutively. Since it is not possible to find an integrated solution in reasonable runtime with standard software, we develop a heuristic decomposition algorithm capable of finding local optimal solutions. In our experimental study, we prove functionality of our models and algorithms and evaluate them against schedules manually created by experienced school principals. We show that our solutions are superior in all relevant performance measures. Furthermore, we conduct a factorial experiment on parameterization of the algorithm.

Our paper offers several opportunities for future research. In terms of model extensions, UAM can be adapted to other domains of apprenticeship programs than healthcare as well as to other related types of apprenticeship systems. We expect other side constraints will become necessary due to different structures of curriculum requirements, while the general framework of the model will remain unchanged. We strongly expect models for specific professions to share a set of general constraints, such that our decomposition idea and algorithm will still be able to solve them efficiently. However, a theoretical investigation and complexity analysis of the general structure might give some valuable insights for designing efficient algorithms. Such a general model will consist of only the key features of the dual training system, namely determining blocks of theoretical and practical education and assigning students to individual workplaces. We expect this reformulation will help to open the model to a broader range of application scenarios. For SSM, other ideas of leveling supply with students might be of interest. If units' educational capacities vary over time, some other objective functions may be better suited. In terms of methodological extensions, this paper raises the question of how (optimal) SSM solutions influence (optimal) UAM solutions. Generating valid lower bounds efficiently for the integrated problem might be of interest. For instance, an advanced column generation application might be used. To enable easy real-world applicability, we considered currently enrolled students in our computational study. This leads to classes leaving the system and to a decreasing number of students over time. Therefore, anticipating assignments of future students might have a big impact on plan quality and plan stability. One might incorporate existing schedules, by fixing some or all decision variables. Using these fixtures in line with all constraints, our algorithm guarantees to find feasible assignments for all classes and students which results in a successful completion of the vocational program. In a more sophisticated approach, future students can be anticipated by the model, if the number of students leaving and entering the system does differ significantly. Finally, models might be able to anticipate unforeseeable absences of students (e.g., long-term illness, drop-outs) and avoid unused resources by adding stochasticity to the problem (i.e., stochastic student demand).

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Appendix A

To linearize the quadratic terms in (2.1) we introduce additional sets and variables. To model all possible values of Δ_{at}^{-} and Δ_{at}^{+} we introduce additional sets N^{-} and N^{+}. New binary variables δ_{atn}^{-}, δ_{atn}^{+} are added to the model, indicating if exactly n students are missing or exceeding target staffing levels. Note that the choice of elements of N^{-} and N^{+} limit feasible solutions of the linearized model since these will implicitly be the new domain of variables $\Delta_{at}^{-} \in \{0\} \cup N^{-}$ and $\Delta_{at}^{+} \in \{0\} \cup N^{+}$. So, to not artificially constraint the solution space of the original model, the sets must be chosen as $N^{-} = \{1, 2, \dots, \max(d_{at}^{Min})\}$ and $N^{+} = \{1, 2, \dots, \max(\sum_{c \in C} n_c \cdot \bar{w}_{ct}) - \min(d_{at}^{Max})\}$. Since the cardinality of these sets determines the number of additional variables, it is a critical factor for practical solvability. It might make sense to limit the range of N^{+} to a set of realistic values like $N^{+} = \{1, 2, \dots, 10\}$. Please keep in mind, this will cut off feasible (but probably very bad) solutions but might help solve the model in reasonable time.

$n \in N^{-}, N^{+}$	Slack/surplus of target staffing levels
$\delta_{atn}^{-},\ \delta_{atn}^{+} \in \mathbb{B}$	1, if exactly$n$students are missing/exceeding target staffing level of unit$a$in period$t$

With these additional elements, the original objective function can easily be linearized as shown in objective function (2.1′).

$$\min \sum_{a \in A} \sum_{t \in T} \sum_{n \in N^{-}} n^2 \cdot \omega^{-} \cdot \delta_{atn}^{-} + \sum_{a \in A} \sum_{t \in T} \sum_{n \in N^{+}} n^2 \cdot \omega^{+} \cdot \delta_{atn}^{+} \tag{2.1′}$$

For this modeling device to work, additional constraints must be introduced. Constraints (2.16) and (2.18) establish the relation between the integer variable Δ_{at}^{-} and binary variable δ_{atn}^{-} or Δ_{at}^{+} and δ_{atn}^{+}, respectively. The combination with constraints (2.17) and (2.19) forces exactly one of the new binary variables to be one if staffing levels for unit a cannot be met in period t. If the staffing level is met, all of the variables will take the value zero.

$$\Delta_{at}^{-} = \sum_{n \in N^{-}} n \cdot \delta_{atn}^{-} \qquad \forall\ a \in A,\ t \in T \tag{2.16}$$

$$\sum_{n \in N^{-}} \delta_{atn}^{-} \le 1 \qquad \forall\ a \in A,\ t \in T \tag{2.17}$$

$$\Delta_{at}^{+} = \sum_{n \in N^{+}} n \cdot \delta_{atn}^{+} \qquad \forall\ a \in A,\ t \in T \tag{2.18}$$

Table 7
Evaluation of different objectives for SSM including effects on UAM performance.

SSM Objectives		SSM		UAM								
		$\hat{w}^{Min}$		$\hat{w}$	$\hat{\sigma}$	$\hat{\Delta}^2$		$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
(1.1)	EXP	138	(100%)	7055	52.2	7394.0	(100%)	3246	121.9	1903	5	3
(1.1)	OPT	168	(123%)	7055	51.3	4794.5	(65%)	2516	73.1	1672	5	2
(1.1b)		141	(102%)	7055	67,3	5770.0	(78%)	2773	92.1	1817	8	4
(1.1c)		153	(109%)	7055	51.8	5013.5	(68%)	2388	78.2	1751	6	3
(1.1d)	$\alpha = 0.95$	120	(87%)	7055	57.9	10,979.5	(148%)	3621	120.6	2088	6	3
(1.1d)	$\alpha = 0.85$	109	(79%)	7055	56.1	13,041.0	(176%)	4022	137.4	2211	6	3
(1.1d)	$\alpha = 0.75$	101	(73%)	7055	61.5	14,863.0	(201%)	4058	161.5	2354	6	3
(1.1d)	$\alpha = 0.65$	112	(81%)	7055	57.5	7.958.5	(107%)	3311	120.6	1938	5	3

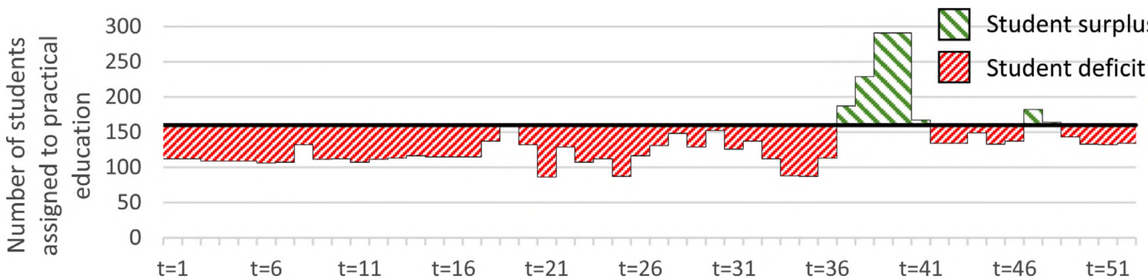

Fig. 12. Students assigned to practical education by SSM for Instance HOLIDAY-A.

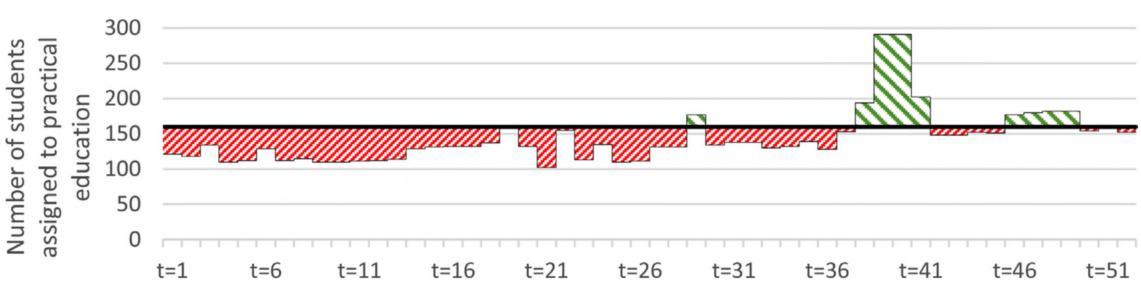

Fig. 13. Students assigned to practical education by SSM for Instance HOLIDAY-B.

$$\sum_{n \in N^+} \delta^+_{atn} \leq 1 \qquad \forall\ a \in A,\ t \in T \tag{2.19}$$

$$\delta^-_{atn},\ \delta^+_{atn} \in \mathrm{B} \qquad \forall\ a \in A,\ t \in T,\ n \in N \tag{2.20}$$

Appendix B

In Section 4.3, we solely reported performance measures of the second level UAM to evaluate the four instances with increasing degree of individuality regarding vacation requests. Of course, both problems were solved and we report the performance measures of SSM here in Table 7. When not granting any individuality at all (*HOLIDAY-A*), UAM has the fewest degrees of freedom, as all periods of school holidays are fixed. Therefore, the number of assignments $\hat{w}$ is smallest of all instances. When some periods of class-wise holidays are transformed into individual vacations, the model is relaxed, as these periods may now be declared a work block. In total, there are 1263 (= 8318 − 7055) student-individual off-periods. After some individual vacation is allowed (*HOLIDAY-B*) UAM may assign 1/3 of these additional periods. These additional degrees of freedom can improve $\hat{w}^{Min}$ by 33%. For any further relaxations (Instances *HOLIDAY-C* and *HOLIDAY-D*), no significant improvement can be made in terms of $\hat{w}^{Min}$. For these three instances visualizations of results are similar (Fig. 12 Fig. 13, Fig. 14, Fig. 15).

We already reported performance measures for UAM on the holiday instances in Section 4.3. Here, we additionally present the visualization of these assignments (Fig. 16, Fig. 17, Fig. 18, Fig. 19).

Appendix C

We will evaluate the relation between solution quality of UAM and different forms of SSM objectives. Therefore, we define three additional targets and associated constraints. For all of those, we will evaluate their performance based on the optimal solution found with the proposed UAM algorithm.

Maximize minimum number of working students

The original objective of SSM as defined in our manuscript seeks to maximize the minimum number of students receiving practical education. This leads to a leveling over all periods $t \in T_p^{Semester}$ within a semester p. Constraints are used to limit decision variables w_p^{Min} to the appropriate values.

$$\max \sum_{p \in P} w_p^{Min} \tag{1.1}$$

$$\sum_{c \in C} n_c \cdot w_{ct} \geq w_p^{Min} \qquad \forall\ p \in P,\ t \in T_p^{Semester} \tag{1.2}$$

Maximize number of working students

This very simple goal minimizes the sum over all work blocks and does not incorporate any relations regarding time, in contrast to objective (1.1). It solely seeks to schedule as many work blocks as possible. Therefore, differentiating semesters is not required and

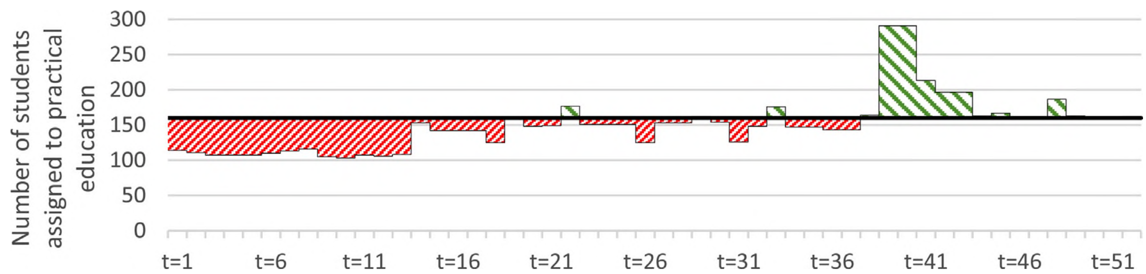

Fig. 14. Students assigned to practical education by SSM for Instance HOLIDAY-C.

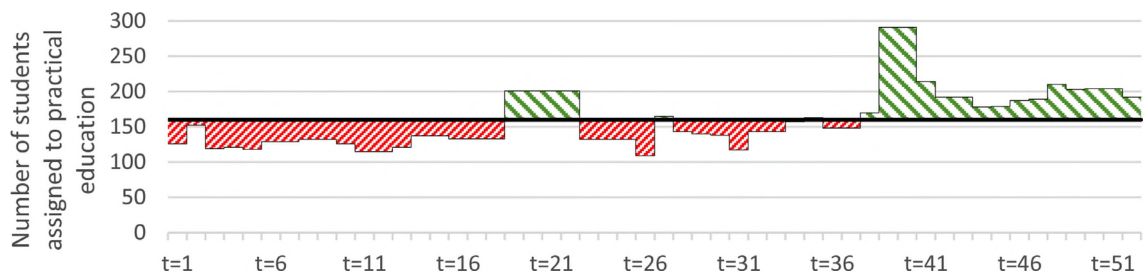

Fig. 15. Students assigned to practical education by SSM for Instance HOLIDAY D.

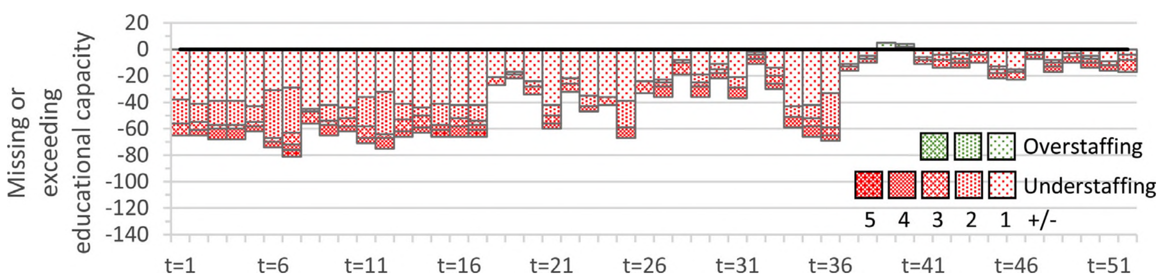

Fig. 16. Violated capacity restrictions by UAM for Instance HOLIDAY-A.

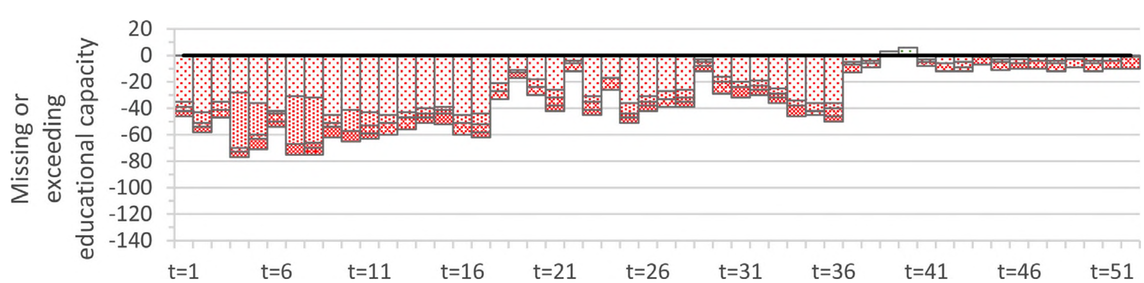

Fig. 17. Violated capacity restrictions by UAM for Instance HOLIDAY-B.

a leveling does not take place. Please note, constraints (1.5) already impose objective (1.1b) implicitly.

$$\max \sum_{c \in C} \sum_{t \in T} n_c \cdot w_{ct} \tag{1.1b}$$

Minimize deviations from target level

A more advanced alternative objective is to define a desired staffing level $\bar{w}_t$ subject to penalizing any deviations from that level. Here, leveling over all periods of the planning horizon will occur. This will require a new set of variables $\Delta_t \geq 0$. A new set of constraints (1.15) completes the goal-programming approach.

$$\min \sum_{t \in T} \Delta_t \tag{1.1c}$$

$$\bar{w}_t - \sum_{c \in C} n_c \cdot w_{ct} \leq \Delta_t \qquad \forall\ t \in T \tag{1.15}$$

Maximize value at risk (VaR)

Finally, we propose the very sophisticated method of using a Value at Risk (VaR) approach. This objective seeks to maximize the minimum number of students receiving practical education during a semester p. In contrast to objective (1.1), we neglect the $\lceil (1-\alpha) \cdot |T_p^{Semester}| \rceil$ periods with smallest supply of students working for every semester p in this VaR approach. This decision variable $VaR_p \in \mathbb{N}$ takes the number of working students of the $\lfloor \alpha \cdot |T_p^{Semester}| \rfloor$ worst period of semester p, because exactly $\lfloor \alpha \cdot |T_p^{Semester}| \rfloor$ of all constraints (1.16) are tight (see constraints (1.17)). By using different levels of α several different solutions can be computed. Note that for this extension to work, $M \geq \sum_{c \in C} n_c$ must hold.

$$\max \sum_{p \in P} VaR_p \tag{1.1d}$$

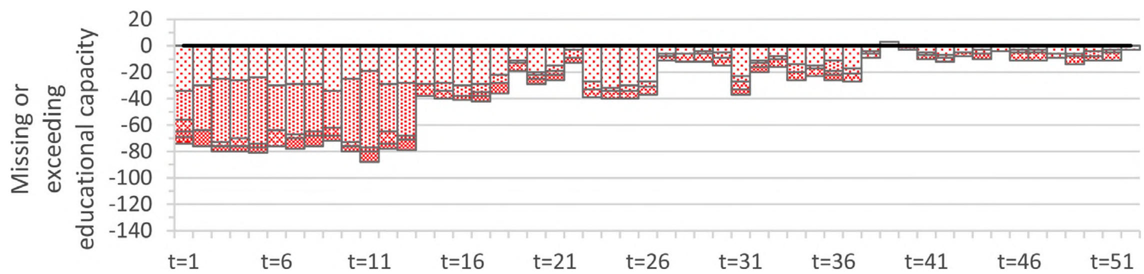

Fig. 18. Violated capacity restrictions by UAM for Instance HOLIDAY-C.

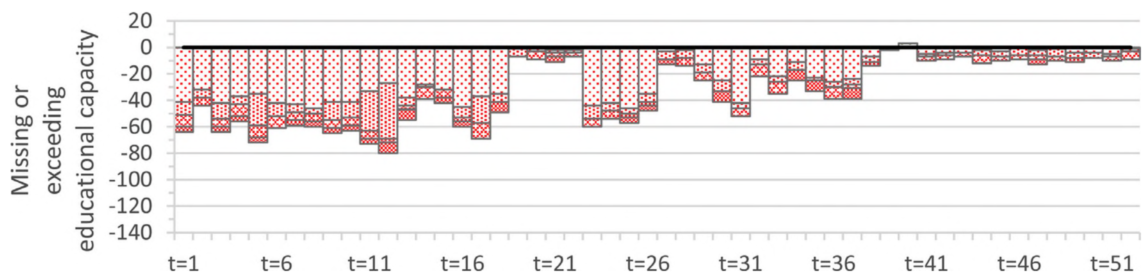

Fig. 19. Violated capacity restrictions by UAM for Instance HOLIDAY-D.

$$\sum_{c \in C} n_c \cdot w_{ct} \geq VaR_p - M \cdot y_t \qquad \forall\ p \in P,\ t \in T_p^{Semester} \tag{1.16}$$

$$\sum_{t \in T_p^{Semester}} y_t \leq \alpha \cdot \left| T_p^{Semester} \right| \qquad \forall\ p \in P \tag{1.17}$$

We used all the above extensions of SSM and created an optimal SSM with respect to the chosen objective. In a subsequent step, the UAM was solved according to our algorithm with an identical set of parameters. We evaluate the quality of the SSM based on the weighted sum of squared deviation $\hat{w}^{Min}$ and the standard deviation $\hat{\sigma}$ of the resulting UAM. In Table 7, we summarize performance measures for the four tested objectives. For the VaR-approach, we further tested four different levels of α. We compare all values with the value of objective (1.1). Please note, the evaluation of (1.1) is equivalent to Instances EXP and OPT in Table 1.

Results

Using objective (1.1b) for the first stage of the problem (i.e. SSM), the resulting school schedule is significantly different from results of Instance OPT. These unit assignments have very distinctive peaks of negative violations. Furthermore, the supply of students is delivered in waves, meaning that periods with great supply follow periods with little supply and vice versa. The performance indicators reflect this behavior: the sum of weighted squared violations increases by 20%, the standard deviation even more. As we deem these results very unfavorable, we discard this objective and refuse to discuss results in more detail.

For objective (1.1c) a different behavior can be observed. Here, peaks of supply can be strongly reduced. It achieves a high number of periods with just enough or more students than the total demand, as the objective suggests. Please note, the mean number of students working over the evaluated time horizon $\hat{w}$ is exactly equal. So, for the next planning step – UAM – the same number of assignments can be made but in a different timely order. When analyzing the resulting unit assignments, we found that objective (1.1c) leads to a rather small number of absolute violations $\hat{\Delta}$, but some very strong single violations.

Finally, for the VaR-approach and objective (1.1d), no clear relation between parameter α and the quality of the resulting assignments can be seen. The best tested setting $\alpha = 0.65$ results in very strong periods of overutilization followed by blocks of underutilization. This explains the tremendous increase in deviation. All other tested settings of this approach are practically useless and are outperformed by manual planning.

In conclusion, we deem the first objective to be the most suitable for the problem since it clearly achieves the lowest sum of weighted squared violations $\hat{\Delta}^2$ as well as the lowest deviation $\hat{\sigma}$ of all alternatives. We could also show that the standard deviation of assigned students $\hat{\sigma}$ of SSM is an efficient proxy for the quality of the resulting unit assignments, too.

Table 8
Violated capacity restrictions by UAM for objective (2.1b).

Instance	$\hat{\Delta}^2$	$\hat{\Delta}$	$\hat{\sigma}$	$\hat{c}$	$\hat{\Delta}^{-Max}$	$\hat{\Delta}^{+Max}$
EXP	7394.0 (100%)	3246	121.9	1903	5	3
OPT	4794.5 (65%)	2516	73.1	1672	5	2
Objective (1.1b)	5157.0 (70%)	2611	72.7	2611	5	3
Objective (1.1c)	5968.5 (81%)	2703	86.1	2.703	6	3

Appendix D

We will provide additional evaluation and performance measures for alternative objectives for UAM. First, in objective (2.1b) we identify for every period t an individual unit a with the strongest positive and negative deviation Δ_{at}^{+} and Δ_{at}^{+}, respectively. Second, in objective (2.1c) we identify corresponding global maximum over- and understaffing $\hat{\Delta}^{+Max}$ and $\hat{\Delta}^{-Max}$ instead. We will discuss the findings and compare solutions with instance OPT, which is using objective (2.1). All performance measures are reported in Table 8.

$$\min\left(\sum_{t \in T} \left(\omega^{-} \cdot \max_{a \in A} \left(\Delta_{at}^{-} \right) + \omega^{+} \cdot \max_{a \in A} \left(\Delta_{at}^{+} \right) \right) \right) \tag{2.1b}$$

$$\begin{aligned} &\min\left(\omega^{-} \cdot \max_{a \in A,\ t \in T} \left(\Delta_{at}^{-} \right) + \omega^{+} \cdot \max_{a \in A,\ t \in T} \left(\Delta_{at}^{+} \right) \right) \\ &\quad = \min\left(\omega^{-} \cdot \hat{\Delta}^{-Max} + \omega^{+} \cdot \hat{\Delta}^{+Max} \right) \end{aligned} \tag{2.1c}$$

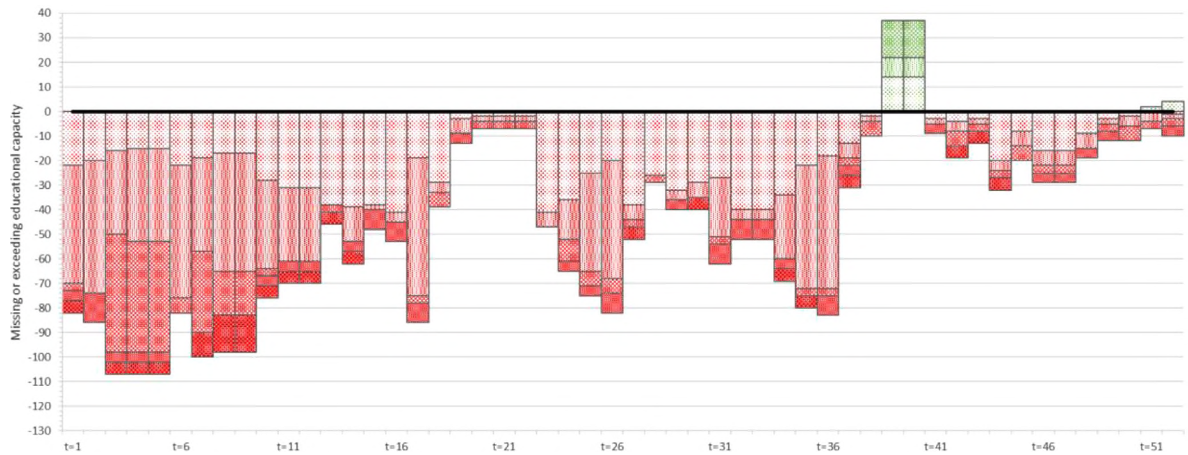

Fig. 20. Violated capacity restrictions by UAM for objective (2.1b).

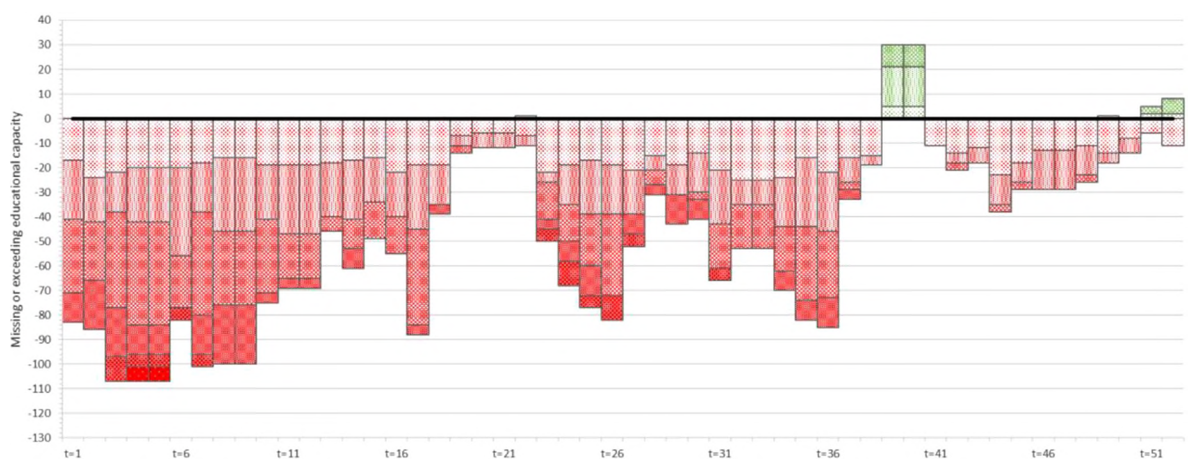

Fig. 21. Violated capacity restrictions by UAM for objective (2.1b).

Evaluating objective (2.1b). The resulting Unit Assignments of objective (2.1b) have a very similar structure, to those of Instance OPT. We can observe the same periods with peaks in over- and understaffing (Fig. 20), as we used identical School Schedules. The peak at $t = 39,\ 40$ itself got significantly bigger, meaning that students could not be distributed amongst units as well as before. For the other periods, we see a shift to darker colors, indicating that the severity of violations increased, while the overall area of the chart was only changing slightly. These two observations mean, the total number of violations ($\hat{\Delta}$) did not change (significantly) but some medical units experience stronger understaffing ($\hat{\Delta}^2$) than before. Concluding, this type of objective is also suitable for the UAM and will find solutions that will improve the current planning procedure.

Evaluating objective (2.1c). Analyzing Unit Assignments created with objective (2.1c), reveals the myopic nature of our solution algorithm. We see (Fig. 21) a heavy shift to stronger violations and even a maximum understaffing ($\hat{\Delta}^{-Max}$) of 6 in periods $t = 5,\ 6$. This becomes even more unintuitive, as this value is explicitly part of the objective function. These effects can be explained by the structure of the solution algorithm: with only a subset of students available, the solver is not able to improve the objective value in single sub-iterations. Therefore, a large number of symmetric solutions exists, but no progress can be made between sub-iterations.

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