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Angaben zur Veröffentlichung / Publication details:

Meixner, Uwe. 2022. "K. C. F. Krause: the combinatorian as logician." *European Journal for Philosophy of Religion* 14 (2): 131–58. https://doi.org/10.24204/ejpr.2022.3589.

K. C. F. KRAUSE: THE COMBINATORIAN AS LOGICIAN

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Abstract. In a time which it is not amiss to term "the Dark Ages of logic", Karl Christian Friedrich Krause stayed not only true to logic but actually did something for its advancement. Besides making systematic use of Venn-diagrams long before Venn, Krause — once more taking his inspiration from Leibniz — propounded what appears to be the first completely symbolic systematic representation of logical forms, strongly suggestive of the powerful symbolic languages that have become the mainstay of logic since the beginning of the 20th century. However, Krause's limits in logic are also clearly visible: Krause's method in logic is, in the main, not axiomatic; it is combinatorial (in other words, it consists in systematically producing finite lists of logical laws, following some organizational principle). More importantly, Krause remained entirely within the confines of traditional syllogistics (his flirt with "quantification of the predicate" notwithstanding), neglecting propositional logic and, of course, first-order relational terms.

At a time when logic was at a very low point in its evolution as a discipline of human knowledge — which, in view of its highwater mark in the Middle Ages, is one of the more ironic outcomes of the Enlightenment — Karl Christian Friedrich Krause was among the few philosophers who not only respected logic but also made a non-negligible contribution to it. In spite of this fact, neither W. and M. Kneale nor J. M. Bochenski mention him in their respective histories of logic.[1]

The very probable explanation of Krause's respect for — or rather, love of — logic is that he was not only a philosopher but also an able mathematician — a mathematician interested in *combinatorics*. Together with his friend Ludwig Joseph Fischer, he even published a textbook on combinatorics and

1 W. Kneale and M. Kneale, *The Development of Logic* (Clarendon Press, 1988) (third reprint as a paperback, first published in 1962); J. M. Bochenski, *Formale Logik* (Alber, 1962) (first published in 1956).

pp. 131–158
DOI: 10.24204/EJPR.2022.3589
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EUROPEAN JOURNAL FOR
PHILOSOPHY OF RELIGION
Vol 14, No 2 (2022)

arithmetic.[2] The first section of this book is on *pure* combinatorics (and seems to be the part of the book which is most *Krause*, and least *Fischer*). One of the central tasks of pure combinatorics is this: "lawfully to represent all possible orderings (reorderings, *permutationes*) of n members (elements)".[3] Krause (& Fischer) addresses this task assiduously, but, presumably because pure combinatorics is independent of arithmetic,[4] he does not come round to the *Distribution Principle* which is central to *the combination* of combinatorics and arithmetic:

> If N is the number of items to be distributed, and K the number of positions that the elements are to be distributed on (repetitions allowed), then the number of (possible) distributions of the items on the positions is equal to N^K (N to the power of K).

Two very important applications of this principle are the following: (1) How many ordered pairs can be formed with N items (repetition allowed)? *Answer*: We have two positions on which N items are to be distributed; therefore (according to the Distribution Principle), N^2 ordered pairs can be formed with N elements. (2) How many subsets are there of a non-empty finite set M with K elements?[5] *Answer*: We have K positions on which two items — "Yes", "No"– are to be distributed (always resulting in the complete specification of a subset of M); therefore (according to the Distribution Principle), there are 2^K subsets to a non-empty finite set with K elements.

Now, is there an application of the Distribution Principle to *logic*? There is indeed, and it is a very beautiful application — one that was still beyond the time of Krause, though, soon, it was to be not beyond the time of Boole and Frege: (3) How many N-adic *truth-functions* are there? *Answer*: An N-adic truth-function is a function that assigns a truth-value (T, F, or: 1, 0) to an or-

2 K. C. F. Krause, *Lehrbuch der Combinationlehre und der Arithmetik [Textbook of Combinatorics and Arithmetic]* (Arnold'sche Buchhandlung, 1812).

3 *Combinationlehre*, 6. All quotations in this paper have been translated into English by me; all emphases are already in the original.

4 *Combinationlehre*, XXXII.

5 Note that combinatorics cannot be used to answer the question of how many subsets there are of the empty set, or of how many subsets there are of an infinite set M. But the result that is obtainable by combinatorics for non-empty finite sets can be extended to the empty set and also to the infinite sets: in the first case, the number of subsets is 2^0; in the latter cases, the number of subsets is $2^{c(M)}$, where c(M) is the transfinite cardinal number of (the elements of) M.

dered sequence of truth values, one with N places. According to the Distribution Principle, there are 2^N ordered sequences of truth-values with N places; taking these sequences as positions, there are 2 *to the power of* 2^N assignments of truth-values to these positions, in other words: there are 2 to the power of 2^N N-adic truth-functions. Thus, there are 4 monadic truth-functions (*negation* is among them), 16 dyadic truth-functions (among them, *conjunction*, *disjunction*, and *material implication*), 256 triadic truth-functions, and so on.

As indicated, Krause is not interested in finding out how the number of the distributions of given items on given positions is related to the number of the items and the number of the positions; he is interested in generating complete lists of such distributions in a lawful fashion. "In a lawful fashion" means this: once you have grasped the principle, you simply apply it mechanically; you don't have to think; simply follow the rule and you can be sure that you will *systematically* generate the complete list of all the possible distributions of the items on the positions.

Can this interest in pure combinatorics be relevant for logic? Yes — as all know who ever wanted to construct a truth-table for a truth-functional formula with, say, five sentence-variables and wanted to make sure that they really list *all* the possible distributions of "T" and "F" on the five sentence-variables — and wanted to do so *without effort*, without waste of concentration. Moreover, it is an old dream of logicians after Leibniz, and before Gödel and Church, to have a procedure in their hands — for example, a procedure of combinatorics — with which they can mechanically produce *every* valid inference-law of a given branch of logic; such that all the intelligence needed for this is in the finding of the mechanical procedure, and they no longer need to invest any further intelligence. As we know today, due to the results of Gödel and Church, the dream of automatization and also the dream of completeness can only be realized within very narrow confines: truth-functional propositional logic can be completely automatized (or: mechanized), elementary predicate-logic, already, cannot be completely automatized, though it can still be completely presented in axiomatic form; the logic of classes (and already a part of it: arithmetic) cannot even be completely presented in axiomatic form.

I. KRAUSE AND SYLLOGISTICS

Karl Christian Friedrich Krause did not dream the dream just mentioned. His aims in logic were much more modest. He had before him a finite logical theory (with a long tradition): syllogistics, and his research project consisted in the question: What can combinatorics do for syllogistics? The answer is: It can do much for the *presentation* of syllogistics.

In the preface of the mentioned textbook on combinatorics and arithmetic, Krause writes: "We wish, however, that the learner connect the study of logic with the study of this textbook, for which purpose the textbooks of *Fries* and *Kiesewetter* are recommendable, and perhaps also my *First Grounds of Historical* […] *Logic*[6] […] will be found useful, wherein especially syllogistics is presented with combinatorial completeness and explicated by means of a fitting schematism".[7] The quotation well indicates what was Krause's ambition in logic. Another quotation from a later book of his (which book sums up his views on logic, and does so especially beautifully on the three lithographic tablets attached to it) strikingly corroborates this finding: "Already *Leibnitz* drafted this combinatorially complete schematic presentation [of syllogistics], but did not carry it out; yet his still extant manuscript contains more on it than has already been printed. I carried out this idea [of combinatorially complete schematic presentation] in my First Grounds [of Historical Logic], printed in 1803. Several older and newer treatises on logic have taken up this presentation in parts: for the elucidation of particular cases; but not at all completely, and not at all in the required combinatorial method, in the way it was done in my textbook [of 1803: *First Grounds of Historical Logic*]. I here [namely, at the end of *Outline of the System of Logic as a Philosophical Science*] present the main tablet (in lithographic print) that can already be found there [in the textbook of 1803], augmented by the addition of the valid *inference-forms* according to the Scholastic designations".[8]

6 K. C. F. Krause, *Grundriss der historischen Logik [First Grounds of Historical Logic]* (Gabler, 1803).

7 Krause, *Lehrbuch der Combinationlehre und der Arithmetik [Textbook of Combinatorics and Arithmetic]*, XXII-XXIII.

8 K. C. F. Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]* (1828), 128–129. Krause also notes on p. 128 of *Abriss* that his *Grundriss* of 1803 contains an exposition of syllogistics that is more detailed than any other known to him.

How can combinatorics serve the *presentation* of syllogistics? Krause has two ways of presenting syllogistics. The first way consists in the perspicuous use of combinatorics, the second way consists in the systematic use of quasi-geometrical diagrams. On the basis of his second way of presenting syllogistics, it is fair to say that what are called "Venn-diagrams" today should really be called "Krause-diagrams", for Krause systematically employed the quasi-geometrical representation of the conceptual relations involved in syllogisms long before John Venn was even born in 1834: he did so already in the *Grundriss* of 1803.[9] However, I shall leave Krause's second way of presenting syllogistics aside and will concentrate entirely on his first way. (In doing so, I shall only in part use Krause's own symbols.)

Consider three general terms, schematically represented by the letters "S", "P", and "M". All three of them are assumed to be non-empty: they are each taken to apply to something. Each of them may be either a pluri-general or a uni-general term, where *uni-general terms* are taken to be general terms that apply to precisely one thing (like "identical with Socrates") and *pluri-general terms* are taken to be general terms that apply to more than one thing (like "man"). (Note that for many purposes *singular terms* — like "Socrates" — can be logically equivalently replaced by the corresponding uni-general term; in the case of "Socrates", it is the uni-general term "identical with Socrates".)

Consider, then, three propositions, the first involving only P and M and a certain (logical) connective, the second involving only S and M and a certain connective, the third involving only P and S and a certain connective. Thus, the first proposition may have the form "M_P" or the form "P_M"; the second proposition may have the form "S_M" or the form "M_S"; the third proposition may have the form "S_P" or the form "P_S". In each case, "_" indicates the place of the connective involved in the proposition.

Now, of all the 6^3 (that is, 216) possibilities to arrange the six described proposition-schemata in an inference-figure with two premises (and one conclusion) consider *only* the following four inference-figures, the *syllogistic* inference-figures:

9 Krause did so, to repeat, in a *systematic* way, unlike Leibniz and Euler who used diagrams for the representation of syllogistically relevant matter even before Krause. Kneale and Kneale, *The Development of Logic*, 349–350 (on Euler), 420–421 (on Euler and Venn); and see Bochenski, *Formale Logik*, 304 (on Leibniz and Euler), 305–306 (on Venn).

First Figure: M_P, S_M → S_P
Second Figure: P_M, S_M → S_P
Third Figure: M_P, M_S → S_P
Fourth Figure: P_M, M_S → S_P

And consider *only* the following four connectives (in a wording — and therefore interpretation — that is strongly suggested by Krause's texts; for more on this, see section 3):

a: "is in its total extension". (Thus, "S a P" is to be read as "S is in its total extension P", or, in other words, as "Every S is P".)

e: "is in its total extension *not*". (Thus, "S e P" is to be read as "S is in its total extension not P", or in other words, as "No S is P".)

i: "is in a [non-empty, proper or improper] part of its [that is, the first term's] total extension". (Thus, "S i P" is to be read as "S is in a part of its total extension P", or, in other words, as "Some S is P".)

o: "is in a part of its total extension *not*". (Thus, "S o P" is to be read as "S is in a part of its total extension not P", or, in other words, as "Some S is not P".)

The logical relationships between the four propositions that can be formed by putting one of the four connectives between any (arbitrary, but non-empty) general terms X and Y (in this order: first X, then Y) are the following (traditionally presented by the so-called "square of oppositions"):

X a Y and X e Y are *contrary* to each other.[10]

X a Y and X o Y are *contradictory* to each other, and X e Y and X i Y are *contradictory* to each other.

X i Y is *subaltern* to X a Y, and X o Y is *subaltern* to X e Y.[11]

10 That X a Y and X e Y are *contrary* to each other means that they cannot both be true. Note that the contrariness of X a Y and X e Y can only be maintained without exceptions because all general terms considered in syllogistics are taken to be *non-empty*.

11 In other words: X a Y logically implies X i Y, and X e Y logically implies X o Y. Note that this is only true because all the general terms considered in syllogistics are taken to be *non-empty*. (For "No unicorn is two-horned" does not logically imply "Some unicorn is not two-horned", and "Every unicorn is one-horned" does not logically imply "Some unicorn is one-horned"; but "unicorn" is an *empty* general term, and syllogistics, therefore, leaves it out of consideration.)

X i Y and X o Y are *sub-contrary* to each other.[12]

It should also be noted that X i Y and Y i X, and X e Y and Y e X, are logically equivalent, whereas X a Y and Y a X, and X o Y and Y o X, are not.

Finally, consider for each of the four syllogistic inference-figures the 4^3 (that is, 64) inference-forms obtainable from it by distributing the four syllogistic connectives (a, e, i, o) on the three open slots in it. Which of these inference-forms are *logically valid*? The answer to this question is perfectly well-known, and Krause was certainly not the first to answer it completely. Krause provides the following listing:[13]

For the First Figure:	aaa, eae, aii, eio [, aai, eao]
For the Second Figure:	eae, aee, eio, aoo [, eao, aeo]
For the Third Figure:	aai, eao, iai, aii, oao, eio
For the Fourth Figure:	aee, aai, iai, eao, eio [, aeo]

Five comments:

(1) The logically valid syllogistic inference-forms — the logically valid syllogisms — are given in abbreviated form: around each vowel in each triple of vowels the consonants appropriate for the syllogistic inference-figure in question are to be supplemented.

(2) The logically valid syllogisms with weak ("subaltern") conclusion besides the strong conclusion, which syllogisms Krause omitted from his listing (but certainly was perfectly aware of), have been added by me in square brackets.

(3) Krause observes that "*eio* is the only mode common to all figures".[14] This is not true if also the logically valid syllogisms with weak conclusion besides the strong conclusion (S i P besides S a P, S o P besides S e P) are drawn into consideration. By arranging the *modes*

12 That X i Y and X o Y are *subcontrary* to each other means that they cannot both be false. Note that the subcontrariness of X i Y and X o Y can only be maintained without exceptions because all general terms considered in syllogistics are taken to be *non-empty*.

13 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 131.

14 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 133.

of logically valid syllogisms (indicated by the above triples of vowels) according to their commonality among the figures (which arranging Krause did not do), one obtains:

eao: I, II, III, IV

eio: I, II, III, IV

aai: I, III, IV

aii: I, III

aee: II, IV

aeo: II, IV

eae: I, II

iai: III, IV

aaa: I

aoo: II

oao: III

(4) Krause further observes that the "premises *ii*, *ee*, *ie*, *io*, *oi*, *eo*, *oe*, *oo* do not yield an inference in any figure".[15] What he means to say by this (and what he says is true) can be seen from the above arrangement of *modes*; for that arrangement shows that the *premise-modes* (obtainable from the *modes* by omitting the third member) aa, ae, ai, ao, ea, ei, ia, oa *do yield* a valid inference-form in at least one of the four syllogistic inference-figures, and that they are *all* of the premise-modes that do that. (The listed premise-modes — eight listed by Krause, eight listed by me — are, indeed, all of the 4^2, that is, 16, combinatorially possible ones.) As is seen, at least one of the premises of a syllogism must be general (its connective being "a" or "e") and at least one positive (its connective being "a" or "i") if the syllogism is to be logically valid; if none of its premises is general or none is positive, then the syllogism is not logically valid. The necessary condition of syllogistic logical validity just stated is, however, not also a sufficient one; for although

15 Ibid.

in the premise-mode *ie* the first premise is positive (and particular), the second general (and negative), no syllogism (in any figure) with this premise-mode is logically valid.

(5) Moreover, Krause states[16]

(a) that in the logically valid inference-forms of the First Figure, *a*, *e* occur in the first premise, *a*, *i* in the second premise, and *a*, *i*, *e*, *o* in the conclusion;

(b) that in the logically valid inference-forms of the Second Figure, *a*, *e* occur in the first premise, *a*, *i*, *e*, *o* in the second premise, and *e*, *o* in the conclusion;

(c) that in the logically valid inference-forms of the Third Figure, *a*, *i*, *e*, *o* occur in the first premise, *a*, *i* in the second premise, and *i*, *o* in the conclusion;

(d) that in the logically valid inference-forms of the Fourth Figure, *a*, *i*, *e* (but not *o*) occur in the first premise, *a*, *i* in the second premise, and *i*, *e*, *o* (but not *a*) in the conclusion.

These assertions are all true, though (d) is not quite as complete as it could be: there is a valid inference-form in the Fourth Figure, in the second premise of which *e* occurs (in fact there are two such inference-forms) — a small mistake, no doubt by mere oversight (the bane of all combinatorians).

Krause asserts that all rules of inference (*alias*: all logically valid syllogistic inference-forms, all logically valid syllogisms) are included in a table he presents: it is a table that contains precisely all and only the information given in (4) and (5) above;[17] he asserts that those rules of inference can all be "developed as particular theorems" from that table.[18] He cannot mean by this that they can be *deduced from* ("read off") that table. For example, in the Second Figure aoo is a logically valid mode. This cannot be deduced from (5)(b), not

16 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 133; here I present the content, and not word-by-word translations into English, of Krause's formulations.

17 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 132–133.

18 Ibid., 132.

even together with the information in (4). Rather, one will have to go through the combinations allowed by (5)(b) and check in each case whether one has two judgments "from which a third according to the form of thinking (vi formae) — in accordance with the law of *conditionality* in consequence of *ground* and *causality*, and solely in view of the *pure essence* (according to the dictum de omni et nullo […]), without any further determination of perception — follows (consequitur, concluditur) in lawful form (in forma legitima)".[19]

This last quotation adequately illustrates Krause's view on the ultimate source of logical cognition. It is a view that (details aside) everyone will have to follow who does not regard the truths of logic either as a (very, very general) empirical matter, or a matter of mere convention, or as a matter of a science-convenient mixture of the purely conventional and the empirical.[20] The quotation puts Krause in the company of Frege and Husserl (especially the latter), who believed in a non-conventionalist, a realist — and particularly in Husserl's case: a *perceptual* — apriority[21] of logic. Krause writes: "This schematic representation [of judgments, including their representation by quasi-geometrical diagrams] merely serves as explication by examples; the matter itself must be perceived purely proto-scientifically (purely intellectually), and every assertion must be grasped independently of any schemata, and must be shown and proved in the perception of essence".[22]

The second to last quotation suggests that Krause assigns a central role (for syllogistics) to the so-called *dictum de omni et nullo*. In view of his interpretation of the syllogistic connectives (see above), Krause must interpret the *dictum de omni et nullo* (which, supposedly, goes back to Aristotle[23]) in the following way: Whatever is affirmed / denied of X in its total extension is affirmed / denied of X in every (non-empty) part of its total extension. This is certainly true ("purely essentially" true). But does syllogistics follow from

19 Ibid., 128.

20 The three mentioned views on the truths of logic are the three possible *empiricist* views. They have been widely held in the 19th and 20th century.

21 How can something be known *a priori* and yet *perceptually*? Answer: The perception in question must be non-sensory; the prototype of such perception was introduced by Plato.

22 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 112.

23 Kneale and Kneale, *The Development of Logic*, 79.

it? Or how does the *dictum* help to justify the logical validity of syllogisms? For example,

M a P, S a M → S a P (put mnemotechnically: *barbara*)

M e P, S a M → S e P (put mnemotechnically: *celarent*)

are (the most famous) logically valid syllogisms (in the First Figure, modes: aaa, eae). How do they follow from the *dictum*? Or how can the *dictum* be applied to justify their logical validity? Instead of splitting one's head about this and instead of delving into the derivation of the logically valid syllogisms from a very few basic ones, which topic (syllogistic reduction) occupies much of Krause's attention, it is better to turn to a matter which in contrast to what has been considered in this paper so far (setting aside, however, Krause's *systematic* use of quasi-geometrical diagrams) serves to demonstrate the originality of Krause's contribution to logic, although that contribution is, in the end, seen to be still rather closely connected to syllogistics and to be not as considerable as it seems at first sight.

II. KRAUSE AND THE FORMS OF JUDGMENT

Krause's *Outline of the System of Logic as a Philosophical Science* contains at its end three consecutive folded tablets in lithographic print: Tablet I, II, and III. Tablet II and III (part of their contents can already be found in Krause's *First Grounds of Historic Logic* of 1803) are filled with annotated systematic quasi-geometrical diagrammatic presentations (with the ambition of combinatorial completeness) of syllogistic relationships. As already indicated, I shall not examine these Venn-diagrams *(longtemps) avant la lettre*.

Tablet I includes, as Subtablet II, the "Tablet of All [Possible] Relationships of [the] Two Members of a [Two-membered] Judgment". I shall concentrate on examining this tablet: Subtablet II of Tablet I;[24] for it completely *contains* what is in my view Krause's most notable contribution to logic.

24 Subtablet I on Tablet I is the "Tablet of All Self-Perceptions (Concepts)". Subtablet III on Tablet I is the "Tablet of the Relationships of Judgments regarding Opposition". The contents of Subtablet III is simply Krause's version of the *Square of Oppositions*, already considered above. I entirely leave out of consideration Subtablet I, the contents of which is more of a metaphysical than a logical nature.

In the subtitle of the *Outline* (on its title page) Krause speaks of a "new schematic designation of the forms of *judgments*". Now, the "new schematic designation of the forms of judgments" is *only part* of Krause's contribution to logic. It is, however, *his most notable contribution*: it is the first *completely symbolic — completely formalized — systematic representation of judgments* (in syllogistics and its close vicinity); thus, it points towards the completely symbolic systems of modern logic.[25] Still, Krause's contribution to logic is not only a contribution to *the form of logic* (to its *symbolic presentation*, and as such, indeed, more important than his contribution to its *diagrammatic presentation*), it is also a contribution to *the content of logic*. As will be pointed out below (see section 3), standard histories of logic include (not Krause but) two other logicians who — apparently independently of Krause, in the one case many years later, in the other case just one year earlier (as measured by dates of publication) — hit on the same innovation of content (known as "quantification of the predicate") that Krause introduced (yet, Krause does not seem to have put it to much use). Though one can never be quite certain in these matters (given the oblivion which is the lot of much writing on logic, even if published), Krause can probably *at least* be given priority for the systematic innovation of form (if not also for the innovation of content): that is, for the systematic completely symbolic, completely formalized representation of judgments (the first *idea* of such a representation, however, comes from Leibniz).

Subtablet II on Tablet I bears the title "Tablet of All Relationships of Two Members of a Judgment".[26] Subtablet II is subdivided into 6 areas. In each of these *areas* Krause lists 16 forms of judgment, in one area even 18. What follows is a description of the syntax of his formalism (Krause's own, very brief description is on the bottom of the left side of Tablet I) — and of the syntax of my transliteration of that formalism into a more tractable shape:

As term-variables, Krause uses "*a*" and "*b*"; I shall use "X" and "Y". Term-variables can be replaced by general terms (and by pseudo-terms).

As term-modifiers, Krause uses "*o*" (read: "omne" or "all") and "*q*" (read: "quoddam" or "some"); Krause also speaks of "generality" or "totality" with reference to "*o*", and of "parthood" with reference to "*q*". I shall use the same

25 However, one must not claim too much for it. In expressive power, it is still very far away from Frege's *Begriffsschrift*.

26 A facsimile of Tablet I is available at the end of this paper.

letters in italics as term-modifiers (but whereas Krause writes, for example, "*qa*", I shall write "*q*X"). Note that the result of applying a term-modifier to a general term is not a general term but a *pseudo*-term (to which neither "*o*" nor "*q*" is applicable). Thus, while "man" is a general term, "*q*man" (read: "some man", or: "a part of the total extension of *man*") and "*o*man" (read: "every man", or: "the total extension of *man*") are not general terms; they are only pseudo-terms. (The modifiers "*o*" and "*q*" are indeed *quantifiers*, but they are *pseudo-term-forming* quantifiers, *not* sentence-forming quantifiers, unlike the quantifiers in modern predicate logic.)

For *negation*, Krause uses a symbol that eerily reminds one of the modern negation-symbol "¬" (could it be that it originates with Krause?). Whatever is syntactically negated — the syntactical argument of negation — is put by Krause *below* his negation-symbol; I shall put the syntactical argument of negation *behind* the negation-symbol "¬" (as is usual nowadays). Krause has as syntactical arguments of *negation*: general terms, the pseudo-terms that are generable from general terms by "*o*" and "*q*", and his symbol for *relationship* ("Verhältnis" — "relationship" — is the semantically unspecific, purely formulary word used by Krause). Note that the result of (syntactically) negating a general term is again a general term, the result of negating a pseudo-term again a pseudo-term. Note also *that the negation of a general term is just as much presupposed to be non-empty as the general term itself*. (The presupposition of non-emptiness for *all* general terms is easily overlooked by modern logicians, who no longer make that presupposition; but sometimes it must be taken into account in order to confirm the universal truth of a formula Krause considers to be universally true.)

For symbolizing *relationship*, Krause uses a right angle, with its horizontal arm at the bottom of the line, and its vertical arm going upward on the left (in the perspective of the reader). I shall use instead the capital Greek letter "Γ". With Krause, the syntactical arguments of *relationship* stand to the left and right of his relationship-symbol; with me, they stand to the immediate left and right of "Γ". Clearly, Krause's symbol for *relationship* (and my "Γ") are stand-ins for certain two-place (and second-order!) predicates.[27] Thus, the result of (syntactically) negating that symbol (or my "Γ") are the (syntactical) negations of *those* two-place predicates.

27 Those predicates are second-order because they form sentences with two *general terms* (and *not* with two *singular terms*).

Krause has not only a symbol for negation, he also has a symbol for *affirmation*. It can be obtained from his negation-symbol by turning that symbol clockwise by 180 degrees (the hook is then pointing upward on the left, whereas before — before "¬" is turned by 180 degrees to the right — it was pointing downward on the right). Whatever is syntactically affirmed — the syntactical argument of affirmation — is put by Krause below his affirmation-symbol. Krause has as syntactical arguments of *affirmation*: general terms, the pseudo-terms that are generable from general terms by "*o*" and "*q*", and his symbol for *relationship*. I, however, shall simply symbolize affirmation by *the empty sign* (as is usual nowadays, affirmation being ubiquitous[28]). This measure of transliteration will make Krause's formalism considerably less complex.

Consider, then, the *first area* of Subtablet II of Tablet I (that is, of Krause's first lithographic tablet in the *Abriss* [*Outline*]). It has the heading "All possible cases, without regard to totality or parthood". For some reason, Krause listed the forms of judgments in this area *twice over*: one listing of eight items with the order of arguments <*a*, *b*>, and one listing of eight items with the order of arguments <*b*, *a*>. The distinction of order is, of course, important; but a note to the effect of "and the same again, with reverse order of arguments" would have sufficed. Probably he just liked to have all the possibilities marshalled before his eyes (a passion — or quirk — not uncommon among combinatorians). I, however, will only present the <X, Y>-list, not also the <Y, X>-list, attaching a note that takes care also of the latter list:

["All possible cases, without regard to totality and parthood"]

XΓY	XΓ¬Y
X¬ΓY	X¬Γ¬Y
¬XΓY	¬XΓ¬Y
¬X¬ΓY	¬X¬Γ¬Y

and the forms of judgment obtainable from the above 8 forms by putting "Y" in the place of "X", and "X" in the place of "Y".

Now, from the above, one would expect Krause to move on to listing, for example, the 64 forms of judgment "with totality or parthood" that can be obtained by replacing "X" by "*o*X" or "*q*X", and "Y" by "*o*Y" or "*q*Y", in the

28 Even if one negates something, one affirms its negation.

above (explicitly or implicitly) listed 16 forms of judgment "without regard to totality and parthood". For example, from $X\Gamma Y$ there can be obtained in this way: $oX\Gamma oY$, $oX\Gamma qY$, $qX\Gamma oY$, $qX\Gamma qY$; and from $\neg X\neg\Gamma\neg Y$ there can be obtained: $\neg oX\neg\Gamma\neg oY$, $\neg oX\neg\Gamma\neg qY$, $\neg qX\neg\Gamma\neg oY$, $\neg qX\neg\Gamma\neg qY$. But no, Krause does no such listing. In each of the remaining five areas of Subtablet II, he does, of course, list forms of judgment: 18 under the heading "First Case"; 16 under the heading "Second Case"; 16 under the heading "Third Case"; 16 under the heading "Fourth Case"; and 16 under the heading "Fifth Case". But among the forms of judgment he lists, there are *only two* (!) "with totality or parthood" both for the subject and for the predicate: $oX\Gamma oY$ and $oY\Gamma oX$, and both forms fall under "First Case" (and are responsible for the increased number of items under that heading). All the other listed forms of judgment are "with totality or parthood" *only for the subject-term*, not also for the predicate-term. Krause has (as transliterated by me):

"First Case"

(1) $oX\Gamma oY$ (6) $\neg qX\neg\Gamma Y$

(2) $oX\Gamma Y$ (7) $\neg qX\Gamma\neg Y$

(3) $qX\Gamma Y$ (8) $qX\neg\Gamma\neg Y$

(4) $\neg oX\neg\Gamma Y$ (9) $qX\neg\Gamma\neg Y$[29]

(5) $\neg oX\Gamma\neg Y$

and the forms of judgment obtainable from the above 9 forms by putting "Y" in the place of "X", and "X" in the place of "Y" (yielding, for example, $oY\Gamma oX$ from $oX\Gamma oY$).

29 This identical repetition of the formula that comes before in the list is in the original.

"Second Case"

(1) *o*XΓY	(5) *o*X¬Γ¬Y	(1′) *q*YΓX	(5′) *q*YΓ¬X
(2) *q*XΓY	(6) *q*X¬Γ¬Y	(2′) *q*Y¬ΓX	(6′) *q*Y¬Γ¬X
(3) ¬*q*XΓY	(7) ¬*q*XΓ¬Y	(3′) ¬*o*Y¬ΓX	(7′) ¬*q*YΓ¬X[30]
(4) ¬*q*X¬ΓY	(8) ¬*q*X¬Γ¬Y	(4′) ¬*q*Y¬ΓX	(8′) ¬*q*Y¬Γ¬X

✕[31]

"Third Case"

(1) *q*XΓY	(5) *q*XΓ¬Y	(1′) *o*YΓX	(5′) *o*Y¬Γ¬X
(2) *q*X¬ΓY	(6) *q*X¬Γ¬Y	(2′) *q*YΓX	(6′) *q*Y¬Γ¬X
(3) ¬*o*X¬ΓY	(7) ¬*o*XΓ¬Y	(3′) ¬*q*YΓX	(7′)¬*q*YΓ¬X
(4) ¬*q*X¬ΓY	(8) ¬*q*XΓ¬Y[32]	(4′) ¬*q*Y¬ΓX	(8′) ¬*q*Y¬Γ¬X

30 As will become apparent below, the forms of judgment as listed under "Second Case" ought to be *the diagonal images* (see note 31) of the forms of judgment as listed under "Third Case", with "Y" and "X" having switched places. Therefore, "(7′) ¬*q*YΓ¬X" under "Second Case", which is false under the assumption belonging to that heading, had best be replaced by "(7′) ¬*o*YΓ¬X", which is true under that assumption, just like "(7) ¬*o*XΓ¬Y" under "Third Case", of which formula "(7′) ¬*o*YΓ¬X" is the (correct) diagonal image, is true under the assumption belonging to that latter heading. (What the *assumptions* in question are — the assumptions "belonging to the headings" — will become apparent below.)

31 If one matches the (N)-form (for N = 1, …, 8) on the left side of "Second Case" with the (N′)-form on the right side of "Third Case", and the (N′)-form on the right side of "Second Case" with the (N)-form on the left side of "Third Case", one will observe that the two forms are the same — with "X" and "Y" having switched places. This diagonal imaging of the "Third Case"-forms by the "Second Case"-forms (and vice versa) ought to be perfect, since the two lists of forms are, in fact, based on assumptions inverse to each other (as will become apparent below). As things are presented by Krause, however, there are two imperfections: one already pointed out in note 30, the other to be pointed out in note 32.

32 In view of what is said in note 31, "(8′) ¬*q*Y¬Γ¬X" under "Second Case" should really be "(8′) ¬*q*YΓ¬X", which would be the diagonal image of "(8) ¬*q*XΓ¬Y" under "Third Case". The alternative correction — sticking with "(8′) ¬*q*Y¬Γ¬X" under "Second Case" and replacing "(8) ¬*q*XΓ¬Y" under "Third Case" by "(8) ¬*q*X¬Γ¬Y" — is not recommendable; the reason for this is given in note 37.

"Fourth Case"

(1) $qX\Gamma Y$	(5) $qX\Gamma\neg Y$
(2) $qX\neg\Gamma Y$	(6) $qX\neg\Gamma\neg Y$
(3) $\neg qX\Gamma Y$	(7) $\neg qX\Gamma\neg Y$
(4) $\neg qX\neg\Gamma Y$	(8) $\neg qX\neg\Gamma\neg Y$

and the forms of judgment obtainable from the above 8 forms by putting "Y" in the place of "X", and "X" in the place of "Y".

"Fifth Case"

(1) $oX\neg\Gamma Y$	(5) $oX\Gamma\neg Y$
(2) $qX\neg\Gamma Y$	(6) $qX\Gamma\neg Y$
(3) $\neg qX\Gamma Y$	(7) $\neg qX\Gamma\neg Y$
(4) $\neg qX\neg\Gamma Y$	(8) $\neg qX\neg\Gamma\neg Y$

and the forms of judgment obtainable from the above 8 forms by putting "Y" in the place of "X", and "X" in the place of "Y".

There are some puzzling features of Subtablet II. If Krause really just wanted to list all possible relationships (or rather, forms of relationship) of the two members of a two-membered judgment, but wished to avoid "quantification of the predicate" (with the exception of $oX\Gamma oY$ and $oY\Gamma oX$), why didn't he simply list all 64 relationships that remain, according to his formalism, when "quantification of the predicate" *is avoided*? They are the following relationships (or forms of relationship):

$qX\Gamma Y$	$qX\Gamma\neg Y$	$oX\Gamma Y$	$oX\Gamma\neg Y$
$qX\neg\Gamma Y$	$qX\neg\Gamma\neg Y$	$oX\neg\Gamma Y$	$oX\neg\Gamma\neg Y$
$\neg qX\Gamma Y$	$\neg qX\Gamma\neg Y$	$\neg oX\Gamma Y$	$\neg oX\Gamma\neg Y$
$\neg qX\neg\Gamma Y$	$\neg qX\neg\Gamma\neg Y$	$\neg oX\neg\Gamma Y$	$\neg oX\neg\Gamma\neg Y$
$q\neg X\Gamma Y$	$q\neg X\Gamma\neg Y$	$o\neg X\Gamma Y$	$o\neg X\Gamma\neg Y$
$q\neg X\neg\Gamma Y$	$q\neg X\neg\Gamma\neg Y$	$o\neg X\neg\Gamma Y$	$o\neg X\neg\Gamma\neg Y$
$\neg q\neg X\Gamma Y$	$\neg q\neg X\Gamma\neg Y$	$\neg o\neg X\Gamma Y$	$\neg o\neg X\Gamma\neg Y$
$\neg q\neg X\neg\Gamma Y$	$\neg q\neg X\neg\Gamma\neg Y$	$\neg o\neg X\neg\Gamma Y$	$\neg o\neg X\neg\Gamma\neg Y$

and the forms of judgment obtainable from the above 32 forms by putting "Y" in the place of "X", and "X" in the place of "Y". If "Γ" is replaced by "is" — that is, by the "is" which expresses the second-order relation of *subsumption* — and "¬Γ" by "is not", then the above are just the 64 *broadly syllogistic forms of judgment*.

Krause did not list these 64 forms. He listed only forms with affirmative subject-term (that is, with non-negated "X" or "Y" in subject-position), and of those forms he did not list all. He *did* list all of the (16) *q*-forms with affirmative subject-term under "Fourth Case". The *o*-forms with affirmative subject-term are treated by him in a different way: they are distributed among the five "Cases" ("Fourth Case" excepted), *with repetitions*, and — what is more disturbing — some of them are not listed at all!

[I]: *o*XΓY, ¬*o*X¬ΓY, ¬*o*XΓ¬Y, and their Y-for-X-duplicates.

[II]: *o*XΓY, *o*X¬Γ¬Y, ¬*o*Y¬ΓX.[33]

[III]: ¬*o*X¬ΓY, ¬*o*XΓ¬Y, *o*YΓX, *o*Y¬Γ¬X.

[IV]: No *o*-forms.

[V]: *o*X¬ΓY, *o*XΓ¬Y, and their Y-for-X-duplicates.

X-Y-forms *not listed*: ¬*o*XΓY and ¬*o*X¬Γ¬Y; Y-X-forms *not listed*: ¬*o*YΓX and ¬*o*Y¬Γ¬X.

The solution of the puzzle may seem to be that Krause did not simply list forms of judgment, but forms of judgment with affirmative subject-term which are (universally) *true*, given a certain specification of Γ (of *relationship*) and given a certain assumption, fulfilled by the general terms X and Y, out of a group of five possible assumptions. The relevant specification of Γ is this:"Γ" is to be read as (the subsumptive) "is". The relevant five assumptions are the five main possibilities — exclusive of each other and together exhaustive — in which the extension of X and the extension of Y (both taken to be non-empty) may stand to each other:

33 [II] should also include ¬*o*YΓ¬X (after having put it in the place — (7′) — where Krause, *presumably erroneously,* has ¬*q*YΓ¬X under "Second Case"; see notes 30 and 31).

("First Case") The extension of X and the extension of Y are identical.

("Second Case") The extension of X is properly included in the extension of Y.

("Third Case") The extension of Y is properly included in the extension of X.[34]

("Fourth Case") The extension of X and the extension of Y properly overlap each other.

("Fifth Case") The extension of X and the extension of Y have no common element.[35]

Indeed, these are the five cases Krause has in mind (as can be seen from the diagrams with which he illustrates the five cases); but the forms of judgment listed under the headings matching the cases do certainly not all turn out to be true under the assumption that belongs to the list they are in if "Γ" is read as "is". Those listed under "First Case" do, in fact, all turn out to be true; but under "Second Case", we have, for example, ¬*q*XΓY, in other words: "No[36] X is Y", which is *false* if the extension of X (always presupposed to be non-empty) is properly included in the extension of Y; or ¬*q*Y¬ΓX, in other words: "No Y is not X", which is also false *if the extension of* X *is properly included in the extension of* Y (which is the "Second Case"-assumption). Under "Third Case", correspondingly, we have ¬*q*YΓX and ¬*q*X¬ΓY, in other words: "No Y is X" and "No X is not Y", which are both false *if the extension of* Y *is properly included in the extension of* X (which is the "Third Case"-assumption). Under "Fourth Case", ¬*q*XΓY — "No X is Y" — is again a falsity, given the "Fourth Case"-assumption: that the exten-

34 The nature of the assumption for "Second Case" and of the assumption for "Third Case" adequately explains why the forms of judgment listed under "Second Case" ought to be *the diagonal images* (see notes 30 and 31) of the forms of judgment listed under "Third Case", with "Y" and "X" having switched places. That nature also explains why the second half of the list under "Second Case" and the second half of the list under "Third Case" is not simply a repetition, with "Y" and "X" having switched places, of the respective first half — as it is in the other three cases.

35 The distinction of the five cases and of the corresponding five term-relationships (one of which must obtain, and no two of which can obtain together, between general terms X and Y) cannot well be said to be *Krausian* advances in logic; Kneale and Kneale, *The Development of Logic*, 350, has it that the five relationships were already known to Boethius, and in 1816/17 a French mathematician, J. D. Gergonne, built a new system of syllogistics on the basis of those relationships; see Kneale and Kneale, *The Development of Logic*, 350–352.

36 "No X" is (logically equivalent to) "not some X".

sion of X and the extension of Y *properly overlap* each other (if these extensions are represented by circles, then the circles *cut* each other). Finally, under "Fifth Case", we have ¬*q*X¬ΓY and ¬*q*XΓ¬Y, in other words: "No X is not Y" and "No X is not-Y", both of which are false, given the "Fifth-Case"-assumption.

In order to remedy this situation, it might be proposed that Krause reads "¬*q*X" as "*q*¬X" and "¬*o*X" as "*o*¬X"; that for him "¬*q*X" just says the same thing as "*q*¬X", namely, *not* the same thing as "not some X", *not* the same thing as "no X", but the same thing as "some not-X"; and that for him "¬*o*X" just says the same thing as "*o*¬X", namely, *not* the same thing as "not every X", but the same thing as "every not-X". If one reads "¬*q*X" as "some not-X" and "¬*o*X" as "every not-X", then, indeed, it turns out that the forms of judgment listed under each of the five cases are true, given the truth of the assumption belonging to the respective case (and given the reading of "Γ" as the "is" of subsumption) *and* provided "(7′) ¬*q*YΓ¬X" and "(8′) ¬*q*Y¬Γ¬X" under "Second Case" are replaced by "(7′) ¬*o*YΓ¬X" and "(8′) ¬*q*YΓ¬X", as proposed in notes 30 and 32 (on the basis of the fact described in note 31).[37] Thus, the hypothesis that Krause, too, read his formulas in the indicated (quite counter-syntactical) way is very probably true. But was Krause *really* intellectually blind to some of the distinctions which his formalism allows him to make and which *should*, in truth, be made? The conclusion that this is indeed the case can hardly be avoided, considering that in the *Grundriss* he already has the modifiers "*o*" and "*q*", but never ever puts a negation-symbol (in the *Grundriss* it is "–" ["minus"]) in front of them.[38] He did no service to clarity by departing from that policy in the *Abriss*, given that he,

37 Note that "(8′) ¬*q*Y¬Γ¬X" is false under the assumption belonging to "Second Case" even if "¬*q*Y" is understood as "*q*¬Y". It is, therefore, highly recommendable to replace it by "(8′) ¬*q*YΓ¬X", thus diagonally matching "(8) ¬*q*XΓ¬Y" under "Third Case". "(7′) ¬*q*YΓ¬X", on the other hand, is true under that same assumption if "¬*q*Y" is understood as "*q*¬Y"; but "¬*o*YΓ¬X" is true under that same assumption, too, if "¬*o*Y" is understood as "*o*¬Y", *and it is the logically stronger assertion* (for "*o*¬YΓ¬X" is logically stronger than "*q*¬YΓ¬X"). It is, therefore, recommendable to replace "(7′) ¬*q*YΓ¬X" by "(7′) ¬*o*YΓ¬X", thus diagonally matching "(7) ¬*o*XΓ¬Y" under "Third Case".

38 Thus, in the *Grundriss*, we find formulas and inference-forms like the following (Krause's variables being replaced by mine, the modifiers being put in italics): *o*Y + X, (*q* – Z) + Y → (*q* – Z) + X [p. 291]; (*o* – Y) – Z, *o*X – Y → *o*X – Z [p. 292]. (The brackets are in the original; Krause's vertical presentation of inference-forms has been transliterated into a horizontal one.) Krause writes: "*o* symbolizes 'all', *q* 'some', + 'is', – 'is not'. " (*Grundriss*, 268; the simple quotation marks have been inserted by me.) He seems unaware that these reading-instructions do not address the negation-sense of "–" in "(*q* – Z)" and "(*o* – Y)".

very probably, stuck in the *Abriss* (as he did in the *Grundriss*) to quantification of the negated (term) and did not consider the negation of the quantified (term). Krause's formalism, it seems, is more logically advanced than Krause himself.

I turn to another matter, which, however, may well give one the *same* idea (just expressed).

III. THE STRENGTH AND THE LIMITS OF KRAUSE'S FORMALISM

Although Krause's formalism is entirely prepared for it, and although it is likely that Krause was to a considerable extent aware of the possibility of "quantification of the predicate", Krause avoids it almost entirely, the *sole* exceptions being $o\text{X}\Gamma o\text{Y}$ and $o\text{Y}\Gamma o\text{X}$ under "First Case". One wonders what might be the reason for this waiving of a large part of the — potential — expressive strength of his formalism.

The idea of "quantification of the predicate" is usually credited to William Hamilton (see, for example, Kneale and Kneale, *The Development of Logic,* 352–354), who, in 1860, distinguished eight forms of judgment with quantified predicate; Hamilton says to have considered the affirmative ones as early as 1833, the negative ones some seven years later (see Kneale and Kneale, *The Development of Logic,* 354). However, already in 1827 George Bentham (Jeremy Bentham's nephew) also distinguished eight forms of judgment with quantified predicate (see Bochenski, *Formale Logik,* 306–307). Here are the eight forms from each of the two British thinkers, presented as matching each other, and in the middle between them *the forms of those forms* in Krause's formalism (as transliterated by me):

Bentham:	*Krausian formalism*:	*Hamilton*:
X in toto = Y ex parte	$o\text{X}\Gamma q\text{Y}$	All a is some b.
X in toto ≠ Y ex parte	$o\text{X}\neg\Gamma q\text{Y}$	Any a is not some b.
X in toto = Y in toto	$o\text{X}\Gamma o\text{Y}$	All a is all b.
X in toto ≠ Y in toto	$o\text{X}\neg\Gamma o\text{Y}$	Any a is not any b.
X ex parte = Y ex parte	$q\text{X}\Gamma q\text{Y}$	Some a is some b
X ex parte ≠ Y ex parte	$q\text{X}\neg\Gamma q\text{Y}$	Some a is not some b.
X ex parte = Y in toto	$q\text{X}\Gamma o\text{Y}$	Some a is all b.
X ex parte ≠ Y in toto	$q\text{X}\neg\Gamma o\text{Y}$	Some a is not any b.

Evidently, Bentham and Hamilton offer two different interpretations of "quantification of the predicate", two different conceptual "fillings" of that part of the Krausian formalism that is relevant to their *formally* identical idea (which part of the Krausian formalism is only a small part of the total Krausian formalism, and even only a small part of the Krausian formalism with "quantification of the predicate"). Hamilton's interpretation, as it is formulated by him, is hard to understand; it is treated very harshly by Kneale and Kneale, *The Development of Logic*, 353–354, and it certainly seems confused (it certainly *is* confusing). Bentham's interpretation is much easier to get a hold of. In fact, Krause's and Bentham's interpretations are presumably identical. For corroboration — I consider only Krause's side –, remember that the heading of the first area in Subtablet II of Tablet I is this: "All possible cases, without regard to totality and parthood", in other words, without "*o*" — Krause calls this modifier "generality" or "totality" — and "*q*" — Krause calls this modifier "parthood". This does, of course, suggest Bentham's "in toto" and "ex parte". Recall also the "is in its total extension"/"is in a part of its total extension"-reading of the four syllogistic connectives (traditionally designated by "a", "e", "i", "o"); see section 1. That reading is strongly suggested by the following quotation from Krause: "*According to quantity, regarding the members,* the judgments are *general* (or better: […] totality-judgments); or *particular* (or better: […] parthood-judgments)",[39] after which quotation Krause introduces the four syllogistic connectives by the Latin saying: "Asserit a, negat e, sed universaliter ambo; asserit i, negat o, sed particulariter ambo". (For better readability, I have inserted the commas.)

We can take it that the Bentham-Krause interpretation of the 8 forms of judgment with "quantification of the predicate" which are so far under consideration is the following:

$o\mathrm{X}\Gamma q\mathrm{Y}$: the total extension of X *is* [identical to] a part of the total extension of Y.

$o\mathrm{X}\neg\Gamma q\mathrm{Y}$: the total extension of X *is not* [identical to] a part of the total extension of Y.

$o\mathrm{X}\Gamma o\mathrm{Y}$: the total extension of X *is* the total extension of Y.

$o\mathrm{X}\neg\Gamma o\mathrm{Y}$: the total extension of X *is not* the total extension of Y.

39 Krause, *Abriss des Systemes der Logik als philosophischer Wissenschaft [Outline of the System of Logic as a Philosophical Science]*, 111–112.

*q*XΓ*q*Y:	a part of the total extension of X *is* a part of the total extension of Y.
*q*X¬Γ*q*Y:	a part of the total extension of X *is not* a part of the total extension of Y.
*q*XΓ*o*Y:	a part of the total extension of X *is* the total extension of Y.
*q*X¬Γ*o*Y:	a part of the total extension of X *is not* the total extension of Y.

So far, so good. "Γ" is here being read as the "is" of identity, and no longer as the "is" of subsumption; but, as will soon be seen, the underlying relationship is, in fact, still *subsumption*.

Note that the above list of interpretations remains entirely valid if "X" is replaced by "¬X", or "Y" by "¬Y". "Quantification of the predicate"-judgments with *negative general terms* as constituents are already taken care of (as are, of course, all forms obtainable from the above by switching the positions of "X" and "Y"). However, from this list we get no idea of *how*, for example, *o*XΓ¬*q*Y or ¬*o*XΓ¬*q*Y is to be interpreted. Very likely, Krause did not recognize judgments with *negative pseudo-terms* in predicate-position — or, indeed, in subject-position — as cases to be interpreted in their own right, given his — very likely — semantic assimilation of "¬*o*X" and "¬*o*Y" to "*o*¬X" and "*o*¬Y", and of "¬*q*X" and "¬*q*Y" to "*q*¬X" and "*q*¬Y". This may be the reason why he omitted all forms of judgment that have "¬*o*Y", "¬*o*X", "¬*q*Y", "¬*q*X" *after* "Γ" from further consideration: they do not appear on Subtablet II.

But why are all of the above listed forms of judgment as well as their Y-X-inverses, all of which forms have an *affirmative pseudo-term* in predicate-position, *also* omitted by Krause from further consideration: they do not appear on Subtablet II (with the sole exception of *o*XΓ*o*Y and *o*YΓ*o*X)? Again, one can only surmise *the reason why*: Krause may well have recognized

(1) that "the total extension of X *is* a part of the total extension of Y" says nothing else than "X is in its total extension Y", in other words, nothing else than "X a Y" (formalized: *o*XΓY, where "Γ" stands for the "is" of subsumption); and that "the total extension of X *is not* a part of the total extension of Y" says nothing else than "X is in a part of its total extension not Y", in other words, nothing else than "X o Y" (formalized: *q*X¬ΓY);

(2) that "a part of the total extension of X *is* a part of the total extension of Y" says nothing else than "X is in a part of its total extension Y", in other words, nothing else than "X i Y" (formalized: *q*XΓY); and that "a part of the total extension of X *is not* a part of the total extension

of Y" (which is *not* logically equivalent to the *propositional negation* of "a part of the total extension of X *is* a part of the total extension of Y"!) says nothing else than "X is in a part of its total extension not Y", in other words, nothing else than "X o Y" (formalized: $q\mathrm{X}\neg\Gamma\mathrm{Y}$);

(3) that "a part of the total extension of X *is* the total extension of Y" says nothing else than "Y is in its total extension X", in other words, nothing else than "Y a X" (formalized: $o\mathrm{Y}\Gamma\mathrm{X}$); and that "a part of the total extension of X *is not* the total extension of Y" says that the total extension of X and the total extension of Y are not the same singleton-set; which is the case if, and only if, (the general terms) X and Y are not uni-general terms that apply to the same thing.[40]

Thus, Krause may well have concluded that "quantification of the predicate" with affirmative pseudo-terms (whether having an affirmative or a negative general term as their kernel) is an unnecessary complication of the logical apparatus *already (and traditionally) available* (whereas "quantification of the predicate" with negative pseudo-terms is, for Krause, not a separate case, but subsumable under "quantification of the predicate" with affirmative pseudo-terms, namely, such affirmative pseudo-terms as have a negative general term as their kernel). However, Krause apparently did not think so in the case of "the total extension of X *is* the total extension of Y" (formalized: $o\mathrm{X}\Gamma o\mathrm{Y}$; see "First Case") and — presumably — not in the case of "the total extension of X *is not* the total extension of Y" (formalized: $o\mathrm{X}\neg\Gamma o\mathrm{Y}$). But, in fact, the two phrases are no exceptions to the verdict "Unnecessary!"; for "the total extension of X *is* the total extension of Y" says nothing else than "X is in its total extension Y, and Y is in its total extension X", in other words, "X a Y and Y a X" (formalized: $o\mathrm{X}\Gamma\mathrm{Y} \wedge o\mathrm{Y}\Gamma\mathrm{X}$); and "the total extension of X *is not* the total extension of Y" says nothing else than "X o Y or Y o X" (formalized: $q\mathrm{X}\neg\Gamma\mathrm{Y} \vee q\mathrm{Y}\neg\Gamma\mathrm{X}$).

40 Uni-general terms that *do* apply to the same thing are, for example, "identical to 16" and "identical to the positive square root of 256". Their applying to the same thing is logically equivalent to: *every* (non-empty, proper or improper) part of the total extension of "identical to 16" is the total extension of "identical to the positive square root of 256"; in other words: *no* part of the total extension of "identical to 16" is not the total extension of "identical to the positive square root of 256"; in other words: *it is not the case* that *a* part of the total extension of "identical to 16" is not the total extension of "identical to the positive square root of 256". Generalizing from this example, it is clear that "a part of the total extension of X is not the total extension of Y" amounts to "(the general terms) X and Y are not uni-general terms that apply to the same thing".

A serious shortcoming of Krause's formalism has now — just now — become very clearly visible: Krause did not integrate the truth-functional propositional operators into his formalism. Like all "syllogisticians", Krause makes us of logical relationships that involve central truth-functional propositional operators: propositional conjunction and disjunction, and propositional negation; but these operators are "anonymous", as is indicated by his failing to formally represent them *as truth-functional propositional operators*, and Krause has no more than a rudimentary theory of them. Fortunately for Krause's formalism, *propositional negation* can in many cases — but certainly not in all — be logically equivalently expressed by using other types of negation, types of negation that Krause's *formalism* (though apparently not in every instance *Krause himself*) recognizes: term-negation (¬X), pseudo-term-negation (for example, ¬*o*X) and predicate-negation (¬Γ: the negation of any of the two-place predicates indicated by "Γ"). For example, the propositional negation of *q*XΓY in its syllogistic interpretation as "a part of the total extension of X is Y", which is ¬(*q*XΓY) (as *we* can formalize it today, moving *just one step* beyond Krause's formalism): "it is not the case that a part of the total extension of X is Y", is, in fact, neither representable by *q*X¬ΓY nor by *q*XΓ¬Y; but, fortunately, it is still representable (logically equivalently) by ¬*q*XΓY ("no part of the total extension of X is Y"). That is, ¬(*q*XΓY) is thus representable *if and only if* "¬*q*X" as "no X" (or "not some X") is *semantically* distinguished from "*q*¬X": "some not-X". Unfortunately, Krause failed to make this distinction in the interpretation of his formalism (very probably, as we have seen).

As far as logical content is concerned, Krause stays within the boundaries of syllogistics, his (rather fruitless) flirt with "quantification of the predicate" notwithstanding.[41] The forms of judgment that he *formally* (*symbolically*) reckoned with are, very likely, not even *the* 64 *broadly syllogistic forms of judgment* (see the previous section), but only half of them, because, as we have seen, Krause very likely took "¬*q*X" and "¬*q*Y" in the sense of "*q*¬X" and "*q*¬Y", and "¬*o*X" and "¬*o*Y" in the sense of "*o*¬X" and "*o*¬Y". (It follows that he took "¬*q*¬X" and "¬*q*¬Y" in the sense of "*q*¬¬X" and "*q*¬¬Y", and therefore as logically equivalent with "*q*X" and "*q*Y"; and that he took "¬*o*¬X"

41 It seems he never speaks of "quantification of the predicate", not by using these words (that is, the German equivalent of "quantification of the predicate") and not in any other way. Attributing the idea to him is largely based on the expressive capacities of Krause's formalism of 1828.

and "¬*o*¬Y" in the sense of "*o*¬¬X" and "*o*¬¬Y", and therefore as logically equivalent with "*o*X" and "*o*Y".) Very likely, Krause in 1828 (in the *Abriss*, four years before his death) ultimately took into account just the 32 *narrowly syllogistic forms of judgment* he already considered in 1803 (in the *Grundriss*). Consider:

*q*XΓY	*q*XΓ¬Y	*o*XΓY	*o*XΓ¬Y
*q*X¬ΓY	*q*X¬Γ¬Y	*o*X¬ΓY	*o*X¬Γ¬Y
q¬XΓY	*q*¬XΓ¬Y	*o*¬XΓY	*o*¬XΓ¬Y
q¬X¬ΓY	*q*¬X¬Γ¬Y	*o*¬X¬ΓY	*o*¬X¬Γ¬Y

and the forms of judgment obtainable from the above 16 forms by putting "Y" in the place of "X", and "X" in the place of "Y". If "Γ" is replaced by "is" (the "is" of subsumption) and "¬Γ" by "is not", then the above are just the 32 *narrowly syllogistic forms of judgment*.

Krause's true innovation concerns logical form, not logical content; his innovation of logical form is the completely symbolic, completely formalized representation of judgments (not to mention his systematic use of quasi-geometrical diagrams in logic). He could have made a rather momentous advance in logical content, even within syllogistics, by defining the (first-order) *sentence-forming existential quantifier* (as applicable not to predicates, but to general terms): ∃X := existent i X, or in the relevant filling of Krause's formalism: *q*existent is X, in other words: *existent* is in a part of its total extension X, in other words: a part of the total extension of *existent* is [subsumable under] X (that is, "Some existent is X"). With propositional negation at his hands (he had it *at* his hands, but he did not really have it *in* his hands), Krause might conceivably have gone on to define the (corresponding) *sentence-forming general quantifier*: ∀X := ¬∃¬X. Following the logical equivalences, ∀X comes down to "¬*q*existent is ¬X", or logically equivalently: "*o*existent is X", that is: "the total extension of *existent* is [subsumable under] X", or "*existent* is in its total extension X", or "existent a X" (that is, "Every existent is X"). Other, alternative definitions of ∃ and ∀ can be given within syllogistics, too: by using the general term "entity" or "thing" instead of "existent". (For these three general terms, an exception might be made regarding the rule, adhered to by Krause, that with a general term also its negation is presupposed to be nonempty; for this presupposition entails that at least one item is a nonentity, and that at least one item is a non-thing, and that at least one item is a non-existent. Especially the first-mentioned consequence seems unacceptable.)

Thus, sentence-forming quantifiers (not just term-forming quantifiers) are no problem for sophisticated syllogisticians. Singular propositions are no problem, either: "Socrates is a man" simply amounts to "identical-with-Socrates i man" (or put in the Krausian way: "*q*identical-with-Socrates is man"). What is really intractable, even for sophisticated syllogisticians, and had to await the advent of Frege and of (full-fledged) first-order predicate logic (which addressed what had so far been ignored: *first-order relational predicates and sentence-forming quantification applied to them*), are logically valid inferences like the following: "Something is such that everything has it as a cause [or: purpose, part, …] → Everything is such that it has something as a cause [purpose, part, …]". Or logically equivalently: "Something is such that it has nothing as a cause [purpose, part, …] → Nothing is such that everything has it as a cause [purpose, part, …]".

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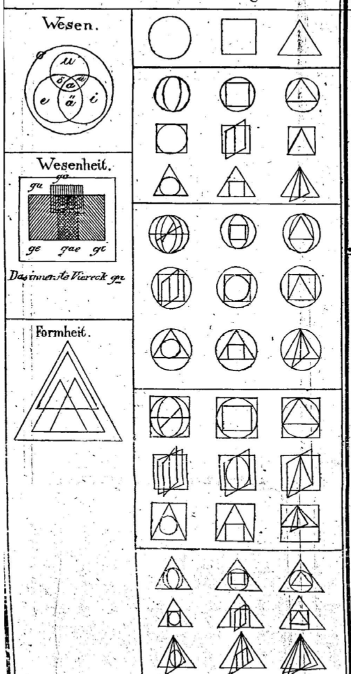
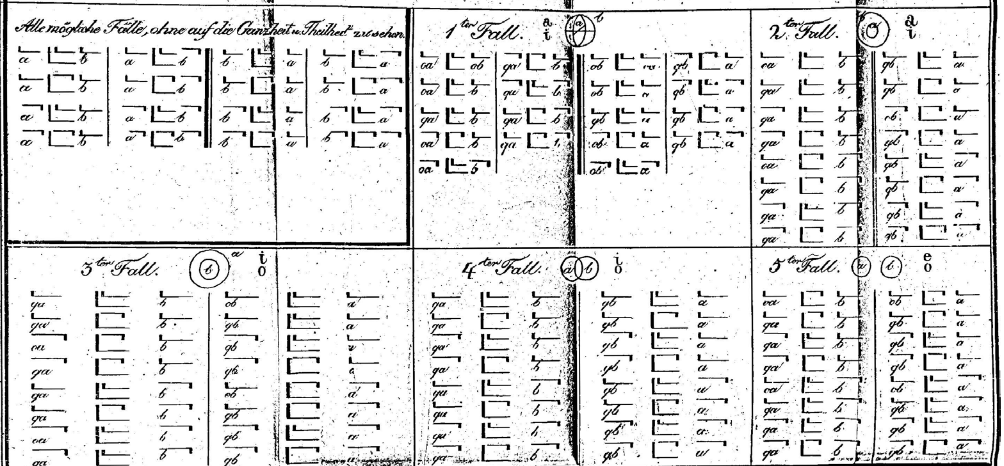
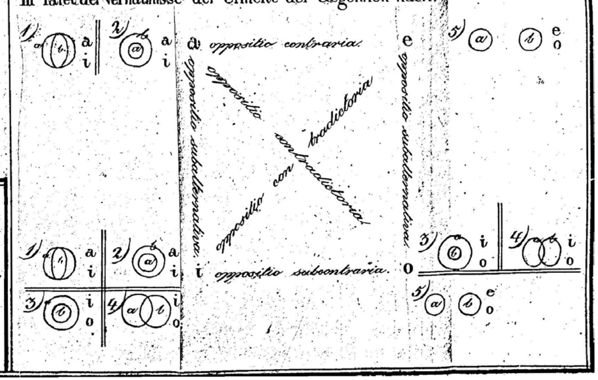
I Tafel aller Selbschaunisse (Begriffe)



II Tafel aller Verhältnisse zweier Glieder des Urtheiles.



III Tafel der Verhältnisse der Urtheile der Gegenheit nach.



Zeichenerklärung.

⌊__ Verhältniß. └─ Bejaheit. ─┐ Verneinheit. a└─ ein bejahiges Glied. a─┐ ein verneiniges Glied.
⌊__ ein bejahiges Verhältniß. ⌊__ ein verneiniges Verhältnis. o omne, Allgemeinheit;
q quoddam; Theilheit.

Die neben den schematischen Kreisen stehenden Buchstaben a, e, i, o, u deuten wie gewöhnlich, die Qualität und Quantität der Urtheile a ⌊__ b und b ⌊__ a an, welche der Form nach (vi formae) durch die schematisch abgebildeten Verhältnisse von a zu b, mitgegeben sind nach der Regel: {a allgemein / i besonders} bejahig und {e allgemein / o besonders} verneinig.