Persistent current of correlated, spin carrying electrons

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The consequences of electron spin on the persistent current I of strongly interacting electrons is studied in a one-dimensional ring that contains an impurity barrier. At zero temperature the current shows discontinuities and the total spin of the ground state oscillates with the flux. Strong electron–electron interaction enhances I, though it does not reach the value expected for a clean ring which itself is smaller than I for non-interacting electrons. In the limit of very strong interaction the response to small fluxes is always diamagnetic.

It is an easy exercise to show that a magnetic flux ϕ penetrating a ring of charge carriers imposes a phase to the boundary conditions for the azimuthal part of the wave function which causes a non-vanishing equilibrium expectation value of the circulating current [1]. Disorder on the ring, however, reduces the current of non-interacting electrons considerably [3]. This is in contrast to the observations [2] where magnitudes $\sim ev_{\rm F}/L$ ($v_{\rm F}$ is the Fermi velocity, L the circumference of the ring) were found which are by orders larger than expected. The one-dimensional models [4—9] provided valuable insight to the role of the electron–electron interaction. At least in continuous models for the ring the interaction can counteract to the reduction caused by the disorder [5].

A new aspect to our full understanding of interacting electrons in a random potential is the electron spin [7]. Already without interaction it modifies the relationship between particle number and the sign of the susceptibility at small fluxes [8]. At strong interactions the electron spin is central for the low energy excitations [10]. The 2^N-fold spin degenerate vibronic ground state of the electron Wigner crystal is split by quantum corrections.

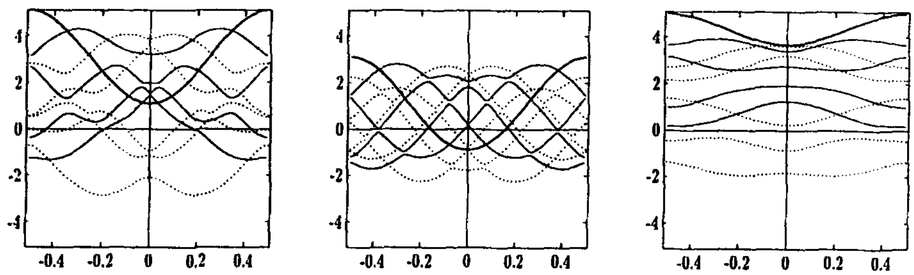
Here, the model

$$H = B\sum_{j=1}^{N}\left(-i\frac{\partial}{\partial\vartheta_j}-\phi/\tfrac{h}{e}\right)^2 + \frac{1}{2}\sum_{j,j'}w(|\vartheta_j-\vartheta_{j'}|)+\sum_j v(\vartheta_j) \qquad (1)$$

for N electrons on a ring will be investigated. $B=\hbar^2/2mL^2$ is the rotational constant of a mass m and ϑ_j are polar coordinates. The interaction $w(\vartheta)$ is assumed to depend only on the spatial separation between the electrons (like the Coulomb interaction) and therefore conserves the total electron spin S. Similar as in [9], the free overall rotation of the finite Wigner crystal is prevented by the impurity barrier, $v(\vartheta)$.

Within the pocket state approximation (1) becomes a lattice model with $N!$ sites in the N-dimensional configuration space [10] (which can be related to a Hubbard type model at half filling [11]). The splitting of the vibrational ground state is connected with the amplitudes for the crystallized electrons to interchange positions [10]. In the present situation the most important processes are the interchanges of two adjacent electrons on the ring (if the impurity barrier is between them this reduces the corresponding rate [12]) and the cyclic exchange involving all of the electrons along the ring. Only the latter amplitude for the collective circulation depends on the enclosed magnetic flux and determines therefore the persistent current being a derivative w.r.t. ϕ. All of these amplitudes can in principle be estimated within the multidimensional WKB approximation.

The consequences of the electron spin can be seen in Fig. 1 where the lowest eigenenergies $E(\phi)$ of $N=5$ electrons are shown. Repulsion occurs between levels of same S (same types of lines). In Fig. 1b and 1c the strength of the interaction and of the impurity are increased, respectively, compared to Fig. 1a. For strong impurity barrier (1c) the behaviour of the persistent current resembles the case without interaction : the ground state shows minimal spin ($S=1/2$) and the susceptibility is



a b c

Figure 1. : Energy levels versus the magnetic flux $\phi/\frac{h}{e}$ for $N = 5$ electrons. Thick solid lines : $S = 5/2$, thin solid lines : $S = 3/2$, dotted lines : $S = 1/2$. b corresponds to strong interaction and c to strong impurity compared to a .

paramagnetic ($N - 2 = 3$ cannot be divided by 4). The electron spin leads to qualitatively new behaviours :

i) The ground state spin depends on ϕ.

ii) Levels of different spins can intersect even in the presence of the impurity; this makes the current discontinuous at zero temperature.

iii) In the limit of strong interaction one can show [12] that the magnetic susceptibility becomes *diamagnetic*, irrespective of N. This is contrary to spinless electrons [4].

Two reasons for the impurity to diminish the current must be distinguished. Firstly, the repulsion between the energy levels (of same spin) is enhanced which reduces the slopes of $E(\phi)$ and therefore the current. Secondly, the potential barrier must be passed by tunneling which also limits the magnitude of the persistent current. Increasing interaction does (to first approximation) not affect the tunneling but reduces the level repulsion within given spin sectors. This is because the interaction reduces the rate for adjacent electrons to exchange places and the presence or absence of the impurity inbetween makes less difference. Thus, the interactions between electrons on a one-dimensional, continuous ring can enhance the persistent current [5], however, the current expected on a clean ring cannot be recovered even in the limit of infinite interaction strength.

This demonstrates that the electronic spin degree of freedom is another ingredient of qualitative importance that has to be considered for our understanding of the interplay between disorder and interaction. The persistent current becomes discontinuous at zero temperature. Always diamagnetic response is obtained in the limit of strong interaction due to the spin. This latter result can help to explain the experimental finding of a diamagnetic response in an ensemble of semiconducting rings [13] which is not possible by non-interacting or by spinless electrons.

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