A mixed truck and robot delivery approach for the daily supply of customers

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1. Introduction

Traffic congestion and pollution are growing problems in cities around the world. Home deliveries are contributing to this problem due to the increasing volume of online orders (Allen et al., 2018; Ishfaq, Defee, Gibson, & Raja, 2016; Wollenburg, Hübner, Trautrims, & Kuhn, 2018), in particular as many deliveries are still performed by diesel trucks. New concepts are needed to enable the projected growth of delivery volumes and prevent urban traffic from collapsing (Agatz, Fleischmann, & van Nunen, 2008; Hübner, Holzapfel, Kuhn, & Obermair, 2019; Orenstein, Raviv, & Sadan, 2019). While attended home deliveries are convenient for customers, they account for a large share of logistics costs (Hübner, Kuhn, & Wollenburg, 2016; Kuhn & Sternbeck, 2013). The complexity of planning deliveries is growing with entry restrictions in inner cities (e.g., ban of diesel engines) and the growing application of time windows to attended home deliveries. This increases customer service and reduces the number of failed deliveries, i.e., deliveries that are not accepted as customers are not at home. In addition, the COVID-19 pandemic has not only increased the home deliveries, but also created consumer preferences for deliveries without human interaction and challenged companies to protect their workforce.

Delivery by truck and robots is a promising approach to address these issues as well as to flexibly accommodate customers' time window preferences. Autonomous delivery robots (e.g., by Starship, 2019 and Marble, 2019) can transport a single parcel or grocery bag to customers for attended home delivery. They are designed to travel short distances at pedestrian speed. Due to their lower speed and limited range, delivery robots are combined with specialized trucks to enable a fast and efficient delivery process. This means that a truck transports the corresponding goods for delivery together with robots and releases the robots at dedicated drop-off locations for the actual home delivery. As there are many customer deliveries on a tour, the truck picks up robots from robot depots on the way. The robots return to the closest robot depot by themselves. Daimler (2019) has tested such a concept and has shown that it potentially decreases lead time and traffic. Baum, Assmann, & Strubelt (2019) predict that delivery robots will likely be introduced on a larger scale soon due to their low production costs and limited legal obstacles. Recent routing literature shows the suitability and cost efficiency of the combination of trucks and robots and

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provides methods for cost-optimal routing (Boysen, Schwerdfeger, & Weidinger, 2018; Ostermeier, Heimfarth, & Hübner, 2021).

Existing truck-and-robot (TnR) concepts with robot depots exclusively consider robots for final delivery to customers or make simplifying assumptions that limit the benefit of robot use. In practice, however, there are multiple reasons for deliveries requiring human interaction and therefore final delivery by a person. First, some customers may be unable or unwilling to interact with the robot and to retrieve the goods from it, such as elderly or disabled persons. Second, the delivery of some goods would be forbidden or risky via a robot. This includes valuables, drugs and hazardous substances such as cleansing agents, paint, pesticides, etc. Third, individual orders may be too bulky to fit into the robot compartment. This can be the case with some electronics, household and do-it-yourself products, and even groceries being delivered in bulk. According to Forbes (2019), 10–25% of Amazon deliveries could not be handled by aerial drones, whose size restrictions are similar to those of delivery robots. Up to one in four orders must therefore be delivered without the use of robots and completed by conventional delivery by truck and human driver. Moreover, even when an order is suitable for robot delivery, the possibility of choosing between truck or robot increases routing flexibility and may yield cost reductions.

In the related routing approaches for attended home delivery, the prevailing literature deals either with a vehicle routing problem (VRP) for truck delivery (e.g., Laporte, 2009; Toth & Vigo, 2001) or a TnR routing problem with delivery by robots (see e.g., Bakach, Campbell, & Ehmke, 2021; Boysen et al., 2018; Ostermeier et al., 2021). In these concepts only truck or robot deliveries are considered, ignoring requirements and the potential benefits of combining deliveries by robot and truck as described above. Existing publications on a combined concept (Chen, Demir, & Huang, 2021a; Chen, Demir, Huang, & Qiu, 2021b) are limited to sequential delivery actions by truck and robots (i.e., while the robots move, the truck is idle and vice versa). A new approach that provides the additional flexibility of parallel deliveries by truck and robots is therefore needed. We close this gap in literature by proposing the Mixed Truck and Robot (MTR) delivery concept, leading to the Mixed Truck and Robot Routing Problem (MTR-RP). This is a generalization of the TnR routing problem and determines which customers are supplied via truck, which customers are approached via robots, and how these deliveries are integrated into the delivery tour. In this application, the truck not only transports the robots to drop-off locations, but is also deployed for direct customer deliveries. This additional option increases the complexity of routing. As such, we solve the MTR-RP with a variant of General Variable Neighborhood Search (GVNS) that incorporates problem-specific insights into the operators. Furthermore, the MTR-RP is different from truck-and-drone concepts. First, only a small number of drones is used during a tour, whereas with MTR, the truck picks up multiple new robots along its way. Second, the drones return to the truck, whereas robots return to a depot.

The delivery concept with robots is innovative and we therefore first outline the detailed problem characteristics based on existing concepts and technology in Section 2. Section 3 discusses related literature and highlights the differences versus other last-mile delivery concepts. Section 4 presents the formal model of the MTR-RP. We detail our GVNS approach in Section 5. Section 6 presents numerical experiments to compare our approach to existing routing frameworks and to analyze the impact of the additional delivery mode by truck. Section 7 summarizes our findings.

2. Problem description

This section outlines how truck and robots are combined for attended home deliveries with time windows. Section 2.1 introduces the related technology on which the problem is based. We then describe the MTR delivery concept in Section 2.2.

Fig. 1. Specialized truck with freight containers and delivery robots (Mercedes-Benz Vans, 2016).

2.1. Technical properties of robots and customized trucks

Delivery robots navigate autonomously on sidewalks and bike lanes but can be remote controlled in the event of problems. To do so, most models rely on several cameras, map data and GPS. In addition, many robots use lidar, ultrasound and radar. For communication, LTE and WiFi are widely-used, at times also touch displays and speakers (Baum et al., 2019). The sensors enable autonomous driving and help prevent theft or vandalism. Recent studies show that robot technology is ready for industry applications. Starship (2019) reports successful tests in more than 80 cities worldwide, and Jaller, Otero-Palencia, & Pahwa (2020) discuss robot models that are already in use in the US and Europe. Baum et al. (2019) count 19 different models, of which the majority have already been tested in the field. According to their overview, most robots operate at pedestrian speed, i.e., at 6 to 8 km/h. The maximum range lies between 6 and 77 km (Jennings & Figliozzi, 2019). The payload varies from one parcel and 10 kg to 20 parcels and 70 kg. When a robot arrives at the delivery destination, customers are notified (e.g., via mobile phone) and can unlock the robot's compartment with a code to retrieve the order (Marble, 2019; Starship, 2019). This means that a customer has to be present to retrieve the parcel from the robot, and thus motivates the use of robots for attended home delivery with time windows. We henceforth apply that a delivery cannot occur before its time window and causes penalty costs if it occurs after the time window. Furthermore, as there is only one compartment for customer order retrieval, we also apply that each robot supplies only one customer on the robot tour.

Given the relatively low speed of robots, companies such as Daimler (2019) have developed customized trucks to transport them. Otherwise robots would have to drive the complete distance from the warehouse to the customer and back. In large delivery areas, this would imply long travel times, issues with lead times and meeting short-term time windows, and low robot utilization. The trucks transport robots to overcome larger distances (e.g., between the warehouse and city center) and release them at dedicated drop-off locations. This enables the efficient use of delivery robots, especially in urban areas. Trucks typically provide space for around eight robots on their floor and enable autonomous pick-up and drop-off via automatic doors and ramps. A shelf system above the floor can be used to carry goods for delivery and offers space for around 54 storage boxes (see, e.g., Mercedes-Benz Vans, 2016). It is only driving the truck and loading robots that remain manual tasks. Figure 1 shows a typical truck setup. For robot deliveries, the truck driver enters the front part of the cargo bay, retrieves

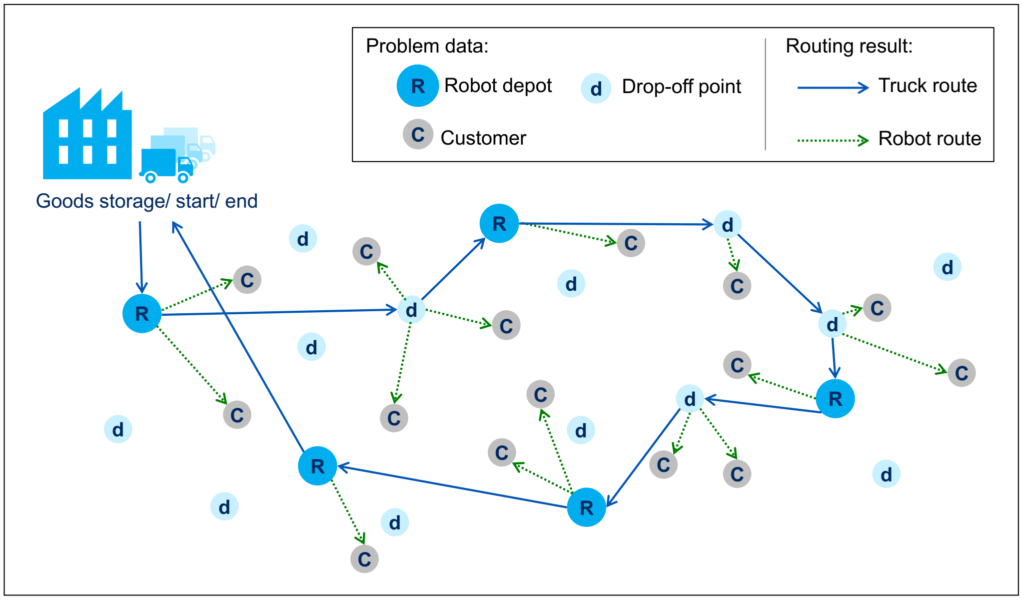

Fig. 2. TnR tour (with all deliveries by robots).

the goods from the shelf system, loads them into robots, and these then leave the truck via a ramp to the side. Direct deliveries by the driver (i.e., without a robot) can therefore easily be included in this system. These orders could be loaded to the rear of the shelf system, for instance, and when the driver arrives at the customer location, (s)he picks up the order from the back door and walks to the customer.

2.2. Concept of mixed truck and robot deliveries (MTR)

Conventional Truck-and-Robot. In line with Boysen et al. (2018) and Ostermeier et al. (2021), TnR is a system in which the delivery robots are transported by truck and therefore the times and locations of both vehicle types are coupled. The central element of this concept is that robots are carried by truck and dropped off close to customers (see Fig. 2). The distribution process therefore consists of a truck tour, visiting different robot drop-off locations (i.e., a location where the truck can safely stop and release robots onto the sidewalk, see solid arrows in Fig. 2), and robot tours visiting a single customer each (dotted arrows in Fig. 2). Some of these drop-off locations are so-called robot depots, where robots are stored and charged. Trucks can both pick up robots at robot depots for later drop-off or load and release robots directly for delivery without transporting them. The number of available robots per depot is limited. As we consider attended home delivery, the robots need to supply the customers within an agreed time window and after the delivery each robot returns to the closest robot depot (not displayed in Fig. 2 for sake of readability). At the depot (which consists only of an outdoor charging station and parking space), it is again charged and waits for the next delivery. Other drop-off points are spots where trucks can stop and release robots for delivery, but no robots are stored. This concept reduces the truck mileage and increases the driver's productivity, which makes it attractive from a cost and environmental perspective (Boysen et al., 2018; Ostermeier et al., 2021).

Mixed truck-and-robot concept. In the conventional TnR concept described above, the truck acts solely as a taxi for robots and does not deliver parcels directly to customers. However, some deliveries are not suitable for robot delivery and must be made by a delivery person. This is necessary for bulky goods that do not fit into the robot's compartment, and goods that must be handed over personally, such as valuables and drugs. A customer could also choose not to receive robot deliveries based on personal preferences or skills. In these cases, a direct truck delivery is indispensable. Please note that truck deliveries also have to happen within an agreed time window as they are part of the same service as robot deliveries (attended home delivery). Truck deliveries can be done by a separate delivery tour (as in prevailing truck-only concepts) or by employing the truck used for robot drop-off to directly approach those customers (as shown in Fig. 3). Using one truck for both delivery modes has the potential to reduce the fleet needed and the costs and emissions caused for serving a set of customers. Besides customers requiring truck delivery, there are customers who can be visited by either truck or robot. Visiting those customers by truck can in some cases further decrease costs as it may lead to shorter tours or reduce robot use and delays. Note that when the truck stops at a customer, it can launch robots to other customers from there as well. Finally, there may be a third set of customers requesting robot deliveries as in the basic TnR concept. These customers cannot be served by truck and no robots can be launched from there. As a consequence, we extend the existing TnR concept to account for both delivery types. The stops for truck delivery have to be integrated into the truck routes for dropping off robots (see solid arrows in Fig. 3). This complicates the search for optimal truck tours, since truck deliveries also have to take place within the designated time windows. Early arrivals at customer locations cause waiting times for the truck and late arrivals cause delay costs in the form of reduced future revenues (due to lower customer satisfaction) or the granting of rebates. The admission of additional truck deliveries therefore causes new dependencies and increases the problem complexity.

Decision problem structure. MTR routing requires simultaneous decisions on different routing problem aspects. To illustrate this, Fig. 4 shows the different vehicles' actions in a truck-and-robot tour over time. For the truck, it includes driving between the goods warehouse, robot depots and drop-off points and customers, as well as potential waiting time at customers. For the robot, it comprises travel time between drop-off points, customers and depots, and potential waiting time. For the truck, there is a mileage-based cost (mainly for fuel) and a time-based cost (for the driver's salary). These have to be considered separately since the truck might have to wait if it reaches a customer before the time

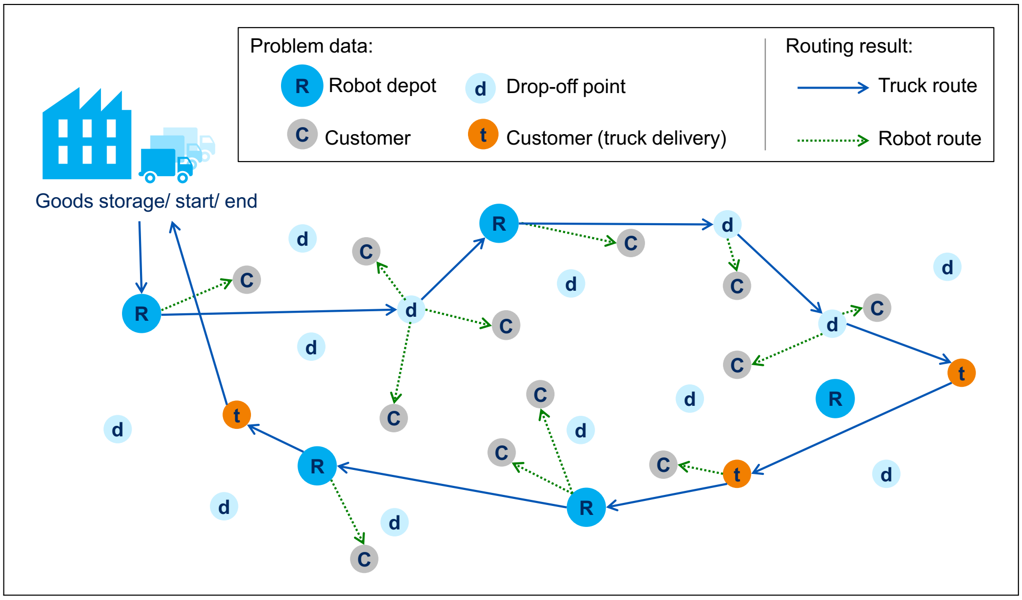

Fig. 3. MTR tour (incl. deliveries by truck).

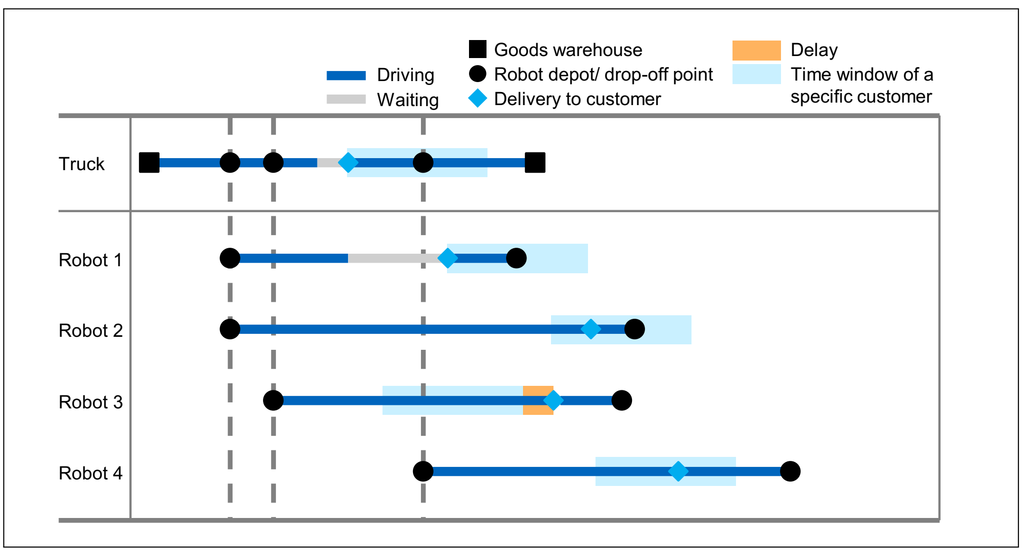

Fig. 4. Gantt chart of an MTR tour (example).

window (see diamond in the truck lane of Fig. 4). The robots start from a depot or drop-off point visited by the truck, drive to a customer and must also wait for the time window in the event of early arrival (see Robot 1 in Fig. 4). After the delivery, the robots return empty to the closest depot, i.e., we do not consider pickup requests in our setting. A time-based robot fee applies during this entire time. If an order arrives late (see Robot 3 in Fig. 4), a delay cost is incurred, consisting of a rebate granted to the customer or accounting for penalties for reduced customer satisfaction. A feasible solution must ensure all customers are served after the start of their respective time window by truck or robot, depending on the request. The decision problem at hand aims to minimize total delivery costs. To achieve this, it is necessary to define (i) which customers are served via truck, which via robot, (ii) which robot depot and drop-off locations are visited during the truck tour, (iii) in which sequence these locations are visited, and (iv) from which stop on the tour each robot delivery is started. The truck starts and ends at the goods warehouse, whereas a robot starts from either a depot or a drop-off location and, after meeting the customer, returns to the closest depot. Besides required travel times and synchronization of truck and robot actions, the decision is constrained by the number of robots available on the truck and in each robot depot.

3. Review of related literature

This section provides an overview of related routing approaches for robot-based deliveries. We first highlight the similarities and differences of related concepts, namely truck-and-drone delivery and delivery with covering options. These concepts share the idea of two vehicle types making deliveries together. Next, we provide a summary of robot routing literature, separated into hub-and-robot concepts and TnR concepts with and without depots. We conclude by highlighting the gap in related literature.

(i) Truck-and-drone delivery. Truck delivery supported by drones has received a lot of attention in recent publications (e.g., Agatz, Bouman, & Schmidt, 2018; Ulmer & Thomas, 2018; Sacramento, Pisinger, & Ropke, 2019). A truck visits customers to make deliveries and a drone serves other customers not visited by the

truck. Initially the truck transports the drone. While the truck stops to make a delivery, the drone can start with a parcel, serve one customer and meet the truck again at a later customer on the truck route. This can be repeated several times. Since every drone delivery starts at a customer served by the truck, the highest possible share of drone deliveries is 50% (Agatz et al., 2018; de Freitas & Penna, 2020; Ha, Deville, Pham, & Hà, 2018; Murray & Chu, 2015). Even for an extended scenario with up to four drones on the truck, solved by Murray & Raj (2020) and Moshref-Javadi, Hemmati, & Winkenbach (2020), the share of truck deliveries must remain above 20%. Murray & Raj (2020) further note that adding drones leads to diminishing marginal improvements, since too many drones cause long take-off and landing queues at the truck. The major differences between drone concepts and the robot concept considered are therefore the lower number of autonomous vehicles (drones), and their return to the truck instead of dedicated depots. The MTR concept has a higher potential to reduce truck mileage as a truck can launch multiple robots at each stop. Furthermore, the truck picks up additional robots during the tour from robot depots, whereas the pertinent applications in truck-and-drone routing rely on a given number of drones on the truck. A further difference is that there are many optional truck stops in the MTR concept (i.e., robot depots and drop-off points), whereas on truck-and-drone tours, only customers are visited. Routing approaches for truck-and-drone are as such not directly applicable to MTR since they rely on the fact that many customers need to be visited by truck and the truck does not have other (optional) locations to visit. Pertinent heuristics improve the solution of the traveling salesman problem (TSP) by reassigning customers to the drone (Agatz et al., 2018; de Freitas & Penna, 2020; Ha et al., 2018; Kitjacharoenchai et al., 2019; Murray & Chu, 2015; Murray & Raj, 2020; Sacramento et al., 2019). A detailed analysis of the differences between truck-and-drone and TnR is performed by Ostermeier et al. (2021). Alongside these differences, practical advantages of robots are their high safety level, robustness in any weather conditions and fewer regulatory obstacles due to slow driving instead of flying. These strengths could soon enable the large-scale practical application of delivery robots in cities (Baum et al., 2019). Lemardelé, Estrada, Pagès, & Bachofner (2021) compare truck-and-drone with delivery robots combined with consolidation centers by applying continuous approximation. They conclude that delivery robots are economically more attractive in dense urban areas and generally create less externalities than drones. In summary, delivery robots and drones are used in different setups (based on their strengths) and problem specifics. We refer to Otto, Agatz, Campbell, Golden, & Pesch (2018), Macrina, Di Puglia Pugliese, Guerriero, & Laporte (2020) and Li, Chen, Wang, & Bai (2021) for a detailed overview of the truck-and-drone concept and its challenges.

(ii) Delivery with covering options. Enthoven, Jargalsaikhan, Roodbergen, uit het Broek, & Schrotenboer (2020) introduce the two-echelon vehicle routing problem with covering options (2E-VRP-CO). In this last-mile delivery application, the truck on the first echelon can either deliver a parcel to a satellite location, from where cargo bikes bring it to the customers, or to a covering location (i.e., a parcel locker) from which nearby customers can pick up the parcel. Similar to the MTR-RP, the truck only needs to visit a subset of given potential locations, and the delivery type which makes the last mile has to be defined. The proposed solution approach relies on an Adaptive Large Neighborhood Search (ALNS) with tailored operators. Several aspects of our MTR-RP are more complex, however, despite the similarities. First, robots are applied for attended home delivery and thus have to meet time windows. Second, the robots are transported by truck and released at drop-off locations as part of our decision problem. This is not the case in a two-echelon setup. The MTR-RP is based on a high number of potential truck stops (in the same order of magnitude as the number of customers). In the two-echelon case, each potential truck stop has a fixed number of bikes available and there are only a few of these stops. Finally, both vehicle types of the MTR-RP can visit customers, whereas in the 2E-VRP-CO this is only possible for cargo bikes. These differences add dependencies to the truck schedule, as robots can only launch from a location while the truck is present and the truck must meet the customer's time window. Similarly, other two-echelon models fall short of characteristics required in the MTR-RP (Perboli, Rosano, Saint-Guillain, & Rizzo, 2018).

(iii) Hub-and-robot. The first concepts developed involving robots can be described as hub-and-robot. Their principle is that robots move between a fixed hub and customers. They do so independently of other means of transportation. Consequently, hubs have the ability to store goods and load the robots, which requires a more sophisticated infrastructure compared to the robot depots (i.e., charging stations) in the TnR or MTR case. Bakach et al. (2021) propose a mixed integer program (MIP) to allocate customers to hubs and robots. Their objective is to minimize the number of hubs and robot mileage required, while respecting the robots' maximum range. Poeting, Schaudt, & Clausen (2019b) and Poeting, Schaudt, & Clausen (2019a) optimally solve an MIP for truck tours visiting hubs and customers and a schedule of pendulum robot tours from these hubs to customers. Sonneberg, Leyerer, Kleinschmidt, Knigge, & Breitner (2019) minimize the costs of tours for robots with several compartments applying an MIP. Due to their nature, hub-and-robot systems do not consider mixed delivery but only robot deliveries paired with an existing hub infrastructure.

(iv) Truck-and-Robot without robot depots. The MTR-RP originates from TnR systems. These concepts constitute a more complex routing problem than hub-and-robot due to the coupling of truck and robot movements. Without depots, the truck has to wait for robots to return or meet robots later on the tour. This limits the routing decisions, in particular the distance travelled by robots. Jennings & Figliozzi (2019) and similarly Figliozzi & Jennings (2020), assess a TnR system based on continuous approximation and conclude that it has the potential to reduce truck mileage. They do not solve a specific routing problem, but estimate the system's performance based on average distances and speeds. Simoni, Kutanoglu, & Claudel (2020) propose a delivery mode similar to truck-and-drone, in which a robot leaves the truck at a customer location, makes one or two deliveries and meets the truck again at a later customer on the truck route. Accordingly, their solution approach relies on finding good TSP tours within a local search with adaptive perturbation and then optimally inserting robot tours with dynamic programming. Due to the limited speed of robots, a large share of customers is still served by truck and the reported savings potential of around 20% is lower than savings achieved by TnR with robot depots. Chen et al. (2021a) and Chen et al. (2021b) propose a concept in which a truck visits a customer, launches several robots to serve customers nearby and waits for their return. The authors propose a cluster-first-route-second approach (Chen et al., 2021b) and an ALNS framework for parallel clustering and routing (Chen et al., 2021a). Computationally, this concept is less complex than the MTR-RP, since it is necessary to keep the robot travel short (therefore the authors cluster customers based on location), the number of robots on the truck does not change and the number of potential stops is smaller. The disadvantage of this concept is that the truck has to wait for the slow robots to return. This results in savings of 4 to 17% compared to normal truck deliveries reported by Chen et al. (2021b), as op-

Table 1
Overview on related delivery concepts.

Concept	Synchronization		Mothership	Storage	No. of smaller	Usual share of
	Handover[1]	Return[2]			vehicles[3]	truck delivery
Two-echelon	-	- (D)	-	✓	any	0%
Hub-and-Robot	-	- (D)	-	✓	any	0%
Truck-and-Drone	✓	✓(MS)	✓	-	1–4	>20%
Truck-and-Robot						
- without depots	✓	✓(MS)	✓	-	1–10	20–80%
- with depots	✓	- (D)	✓	-	20–200	0–20%

[1] Handover to second transportation mode. [2] In brackets: return to mothership *MS* or depot *D*. [3] Such as drones, bikes, robots etc. ✓: part of the corresponding concept, -: not considered within the concept.

posed to more than 50% reported by Ostermeier et al. (2021) for a concept based on robot depots. We therefore consider the use of more robots and robot depots as a key enabler for an efficient TnR application.

(v) Truck-and-Robot with robot depots. To date, three publications explicitly deal with TnR routing involving robot depots. In the seminal paper, Boysen et al. (2018) introduce the idea of robot depots to eliminate truck waiting time and aim to minimize the number of delayed deliveries. The system analyzed consists of 40 customers and several depots and drop-off points. They solve the problem with a multi-start local search (LS) procedure and show that a TnR system with one truck can replace several traditional delivery vehicles while maintaining service quality. The authors do not incorporate truck deliveries in their approach nor do they provide a quantification of financial and environmental benefits. Some simplifications are assumed (e.g., unlimited robot availability at every depot). Alfandari, Ljubić, & de Melo da Silva (2021) build on this work by analyzing alternative delay measures and proposing a Branch-and-Benders-cut scheme for faster computation. Ostermeier et al. (2021) have extended the problem to account for limitations in robot availability at every depot and minimize total logistics costs, including both truck- and robot-specific costs. Again, the problem is restricted to robot delivery only, while direct truck deliveries are not considered. The authors propose a local search to deal with the increased complexity. In their experiments the concept reduces costs by up to 68% and truck mileage by up to 82% compared to classical truck delivery.

Research gap. Table 1 summarizes the key differences between concepts involving trucks and smaller vehicles. It shows whether the vehicles are synchronized (i) when handing goods over to the smaller vehicles and/or (ii) when the smaller vehicles return, (iii) whether the truck acts as a mothership (transporting smaller vehicles), and (iv) whether goods storage facilities exist in the network. The last two columns indicate the typical numbers of smaller vehicles involved in a delivery tour and the share of deliveries made by truck.

In summary, the MTR concept leads to a routing problem that requires problem-tailored solution approaches. Approaches for the concepts mentioned in paragraphs (i) to (iv) do not yet include the necessary specifics of the MTR-RP, in particular time windows, a large fleet of smaller vehicles transported and dropped off by truck and a selection of alternative delivery modes to the customer. For a more detailed review of last-mile delivery concepts we refer to Boysen, Fedtke, & Schwerdfeger (2021). There are only three publications on TnR routing involving robot depots and none of them enables mixed truck and robot deliveries (see Table 2). All publications dealing with this innovative last-mile delivery concept focus on robot deliveries, while the truck does not visit customers directly, but only stops at given drop-off locations. However, in a practical application the combination of both delivery modes is needed to ensure that all types of orders can be processed on the same truck tour to reduce costs. We therefore extend the existing literature by addressing the MTR-RP, in which truck deliveries are incorporated when required and a decision between truck and robot delivery is made if both modes are feasible. The corresponding decision model is presented in the next section.

4. Formulation of the MTR-RP

This section introduces the mathematical formulation of the MTR-RP. The notation used is summarized in Table 3.

The following sets form the basis of the MTR-RP. The set of customers C consists of three disjointed subsets: customers with mandatory truck delivery C^{m}, customers requiring robot delivery C^{r}, and customers for which the delivery mode is optional C^{o} (i.e., both truck and robot delivery are possible), with $C = C^{\mathrm{m}} \cup C^{\mathrm{r}} \cup C^{\mathrm{o}}$. Every customer $k \in C^{\mathrm{r}} \cup C^{\mathrm{o}}$ can (without loss of generality) be served by one robot, every customer $k \in C^{\mathrm{m}} \cup C^{\mathrm{o}}$ by the truck. The truck-and-robot infrastructure consists of a set of robot drop-off locations D, where the truck can start robots, and a set of robot depots R, where the truck can pick up and start robots. We further duplicate drop-off and depot locations to allow multiple visits of the same depot or drop-off point. This results in the duplicate sets $\hat{D}$ and $\hat{R}$. For clarity, we summarize all (duplicate) locations that can be visited by truck in $\hat{L} := C^{\mathrm{m}} \cup C^{\mathrm{o}} \cup \hat{D} \cup \hat{R}$. For every distinct location $a, a \in D \cup R$, we denote the set of its duplicates as $I_a, I_a \subset \hat{L}$, and the set of indices in I_a that are less or equal to $m, m \in I_a$, as I_a^m. The set I_a^m is required to keep track of the order in which

Table 2
Summary of existing literature on TnR routing involving robot depots.

Publication	Objective	Methodology	Aspects considered in modeling and optimization: Delays	Robot availab.	Costs	Truck delivery	Truck/robot selection
Boysen et al. (2018)	Number of late deliveries	Local search	✓	-	-	-	-
Alfandari et al. (2021)	3 different delay measures	Branch-and-Benders-cut	✓	-	-	-	-
Ostermeier et al. (2021)	Total costs	Local search	✓	✓	✓	-	-
This paper	Total costs	GVNS	✓	✓	✓	✓	✓

✓: considered, -: not considered.

Table 3
Notation of the MTR-RP.

Index sets	
C	Set of all customers $k \in C$
C^{m} (C^{r})	Subset of customers requiring truck (robot) delivery, with $C^{\mathrm{m}} \cup C^{\mathrm{r}} \subseteq C$
C^{o}	Subset of customers indifferent regarding truck or robot delivery, with $C^{\mathrm{o}} \subseteq C$
D (R)	Set of distinct robot drop-off points (robot depots)
$\hat{D}$ ($\hat{R}$)	Set of robot drop-off points (robot depots) including duplicates
$\hat{L}$	Set of all (duplicate) locations reachable by truck: $\hat{L} := C^{\mathrm{m}} \cup C^{\mathrm{o}} \cup \hat{D} \cup \hat{R}$
I_a	Set of duplicate indices $i, i \in \hat{D} \cup \hat{R}$, of one distinct location $a, a \in D \cup R$
I_a^m	Set of elements $i \in I_a$ with $i \leq m$
Problem parameters	
d_k	Deadline for customer $k, k \in C$
K	Maximum robot capacity of a truck
r_a	Initial amount of available robots in location $a, a \in R$
γ ($\bar{\gamma}$)	Start (end) position of the truck, with $\gamma, \bar{\gamma} \notin \hat{L}$
δ	Initial number of robots aboard the truck
ϵ_k	Length of time window of customer k
$\lambda_{i,j}$	Distance between locations i and $j, i, j \in \hat{L}$
$\vartheta^{\mathrm{t}}_{i,j}$	Truck travel time from location i to location $j, i, j \in \hat{L}$
$\vartheta^{\mathrm{r}}_{i,k}$	Robot travel time from location $i, i \in \hat{L}$, to customer $k, k \in C$
$\vartheta^{\mathrm{b}}_{k}$	Robot travel time from customer k back to the closest robot depot
Cost parameters	
c^{l}	Cost of delay per time unit
c^{d}	Cost of truck per distance unit
c^{t} (c^{r})	Cost of truck (robot) per time unit
Decision variables	
$s_{i,j}$	Binary: 1, if truck travels from location i to location j; 0 otherwise
$x_{i,k}$	Binary: 1, if customer k is supplied by a robot from location i; 0 otherwise
Auxiliary variables	
t_i	Arrival time of truck at location i
q_i	Number of robots aboard the truck after visiting location i
e_i	Number of robots taken out of depot location $i, i \in \hat{R}$
ν_k	Delay of delivery for customer k
w_k	Waiting time for robot at customer k

duplicates are visited and to enforce the constraint on available robots after every visit.

The truck starts in γ (e.g., a goods warehouse, $\gamma \notin \hat{L}$) with δ robots on board and has a maximum capacity of K robots ($\delta \leq K$). It is already loaded with the goods to be delivered. In every robot depot $a, a \in R$, there is an initial number of robots r_a available for use by the truck but the depot capacity is not limited with respect to the return of robots. Every customer $k, k \in C$, has a delivery time window defined by a deadline d_k and the time window length ϵ_k. The delivery comprises one customer order (possible consisting of multiple items) and cannot take place before the customers' time window starts (i.e., not before $d_k - \epsilon_k$). In this case truck or robot waiting time applies. If it occurs after the deadline, delay costs at the rate of c^{l} are incurred. The distance between locations i and j is denoted by $\lambda_{i,j}$, the resulting travel times by $\vartheta^{\mathrm{t}}_{i,j}$ for the truck and $\vartheta^{\mathrm{r}}_{i,k}$ for the robots. We further denote the robot travel time from customer k, back to the closest depot as $\vartheta^{\mathrm{b}}_{k}$. Note that the costs of the robots' return to the closest depot is a parameter for each customer supplied by robot as the closest depot is known in advance. Any processing time for loading and unloading is added to these times. We introduce the dummy end location $\bar{\gamma}$ (typically equal to the starting location, $\bar{\gamma} \notin \hat{L}$) to track total truck time. This is necessary since the truck may have to wait to meet a time window for delivery. The total truck time that is needed to assess truck usage costs is thus the arrival time at the end node $\bar{\gamma}$, indicated by $t_{\bar{\gamma}}$. The time-based cost rate of the truck is denoted as c^{t} and the distance-based cost rate c^{d}. A time-based machine rate c^{r} is assumed for the use of robots. It is incurred while loading the robot, its travel to the customer, waiting for the beginning of the time window (if necessary), unloading by the customer, and the return to the closest depot.

In the course of minimizing total costs, we further define the following decision variables. The binary variable $s_{i,j}$ indicates whether the truck travels from location i to location j or not. The binary variable $x_{i,k}$ defines whether customer k is supplied by robot from location i, i.e., whether a robot travels from i to k. To track feasibility and costs of a solution, the following auxiliary decision variables are needed. The variable t_i defines the arrival time of the truck at location $i, i \in \hat{L}$, and q_i the quantity of robots aboard the truck when leaving the location. The quantity of robots taken out of depot $i, i \in \hat{R}$ (i.e., loaded on the truck or directly started towards a customer) is defined by e_i. For every customer k, ν_k indicates the duration of delay (in the event of late arrival) and w_k the robot waiting time (in the event of early arrival). We then formulate the MTR-RP as follows.

$$\min F(S, X, T, V, L, E, W) = \\ = c^{\mathrm{t}} t_{\bar{\gamma}} + \sum_{i \in \hat{L} \cup \{\gamma\}} \sum_{j \in \hat{L} \cup \{\bar{\gamma}\}} c^{\mathrm{d}} \lambda_{i,j} s_{i,j} + \sum_{i \in \hat{L}} \sum_{k \in C^{\mathrm{r}} \cup C^{\mathrm{o}}} c^{\mathrm{r}} (\vartheta^{\mathrm{r}}_{i,k} + \vartheta^{\mathrm{b}}_{k}) x_{i,k} \\ + \sum_{k \in C} (c^{\mathrm{l}} \nu_k + c^{\mathrm{r}} w_k) \tag{1}$$

subject to

$$\sum_{i \in \hat{L}} x_{i,k} + \sum_{i \in \hat{L} \cup \{\gamma\}} s_{i,k} = 1 \qquad \forall k \in C^{\mathrm{o}} \cup C^{\mathrm{m}} \tag{2}$$

$$\sum_{i \in \hat{L}} x_{i,k} = 1 \qquad \forall k \in C^{\mathrm{r}} \tag{3}$$

$$\sum_{k \in C} x_{j,k} \leq M \sum_{i \in \hat{L} \cup \{\gamma\}} s_{i,j} \qquad \forall j \in \hat{L} \tag{4}$$

$$\sum_{j\in\hat{L}} s_{\gamma,j} \leq 1 \tag{5}$$

$$\sum_{i\in\hat{L}\cup\{\gamma\}} s_{i,j} = \sum_{i\in\hat{L}\cup\{\bar{\gamma}\}} s_{j,i} \qquad \forall j\in\hat{L} \tag{6}$$

$$t_\gamma = 0 \tag{7}$$

$$t_j \geq t_i + \vartheta^{\mathrm{t}}_{i,j} - M(1-s_{i,j}) \qquad \forall j\in\hat{L}\cup\{\bar{\gamma}\};\, i\in\hat{L}\cup\{\gamma\} \tag{8}$$

$$t_k \geq d_k - \epsilon_k \qquad \forall k\in C^{\mathrm{m}} \tag{9}$$

$$t_k \geq d_k - \epsilon_k - M(1-\sum_{i\in\hat{L}\cup\{\gamma\}} s_{i,k}) \qquad \forall k\in C^{\mathrm{o}} \tag{10}$$

$$q_\gamma = \delta \tag{11}$$

$$q_j \leq q_i + e_j - \sum_{k\in C} x_{j,k} + M(1-s_{i,j}) \qquad \forall i\in\hat{L}\cup\{\gamma\};\, j\in\hat{R} \tag{12}$$

$$q_j \leq q_i - \sum_{k\in C} x_{j,k} + M(1-s_{i,j}) \qquad \forall i\in\hat{L}\cup\{\gamma\};\, j\in\hat{D}\cup C^{\mathrm{m}}\cup C^{\mathrm{o}} \tag{13}$$

$$v_k \geq t_k - d_k \qquad \forall k\in C^{\mathrm{m}}\cup C^{\mathrm{o}} \tag{14}$$

$$v_k \geq t_j + \vartheta^{\mathrm{r}}_{j,k} - d_k - M(1-x_{j,k}) \qquad \forall k\in C^{\mathrm{r}}\cup C^{\mathrm{o}},\, j\in\hat{L} \tag{15}$$

$$w_k \geq (d_k-\epsilon_k) - t_j - \vartheta^{r}_{j,k} - M(1-x_{j,k}) \qquad \forall k\in C^{\mathrm{r}}\cup C^{\mathrm{o}},\, j\in\hat{L} \tag{16}$$

$$t_i \leq t_j \qquad \forall a\in R;\, i,j\in I_a : i\leq j \tag{17}$$

$$\sum_{h\in\hat{L}\cup\{\gamma\}} s_{h,i} \geq \sum_{h\in\hat{L}\cup\{\gamma\}} s_{h,j} \qquad \forall a\in R;\, i,j\in I_a : i\leq j \tag{18}$$

$$r_a - \sum_{i\in I_a^m} e_i \geq 0 \qquad \forall a\in R;\, m\in I_a \tag{19}$$

$$s_{i,j}\in\{0,1\} \qquad \forall i\in\hat{L}\cup\{\gamma\};\, j\in\hat{L}\cup\{\bar{\gamma}\} : i\neq j \tag{20}$$

$$s_{i,i} = 0 \qquad \forall i\in\hat{L} \tag{21}$$

$$x_{i,k}\in\{0,1\} \qquad \forall i\in\hat{L};\, k\in C^{\mathrm{r}}\cup C^{\mathrm{o}} \tag{22}$$

$$x_{i,k} = 0 \qquad \forall i\in\hat{L};\, k\in C^{\mathrm{m}} \tag{23}$$

$$e_i\in\mathbb{Z} \qquad \forall i\in\hat{R} \tag{24}$$

$$t_i \geq 0 \qquad \forall i\in\hat{L}\cup\{\bar{\gamma}\} \tag{25}$$

$$q_i\in\{0,\ldots,K\} \qquad \forall i\in\hat{L} \tag{26}$$

$$v_k, w_k \geq 0 \qquad \forall k\in C \tag{27}$$

The objective function (1) minimizes total costs. The first term considers the cost of truck time (at cost rate c^{t}). It comprises the total truck time including travel time between locations and potential waiting time if customers are approached too early. The second term covers the truck's distance costs (at cost rate c^{d}). The third term comprises the robot costs dependent on associated travel times to the customer and back to the closest depot (at cost rate c^{r}). The last term of the objective function sums up the cost of possible delayed deliveries (cost rate c^{l}) and robot waiting times across all customers. Constraint (2) ensures exactly one visit by either truck or robot for every customer $k\in C^{\mathrm{o}}\cup C^{\mathrm{m}}$. Similarly, constraint (3) ensures that each customer who requires a robot delivery is visited by exactly one robot. Constraint (4) states that robots can only be launched from stops that are actually visited by truck. Constraint (5) defines that the truck only leaves once from the starting point, and (6) ensures that if the truck reaches a location, it must also leave it. Constraints (7) and (8) determine the truck arrival time at every stop based on travel times. This also prevents a second visit to the same (duplicate) stop. Constraint (9) ensures that a required truck delivery is not made before the respective time window and (10) does so for optional truck deliveries in case they are made by truck (and not by robot). The following constraints (11), (12) and (13) handle the number of robots aboard the truck when leaving the starting point, a depot or any other location, respectively. Constraint (14) defines the delay for customers receiving truck delivery. Constraints (15) and (16) define the delay and waiting time for customers receiving robot deliveries. Constraints (17) and (18) ensure without loss of generality that duplicates of the same location are visited in ascending order of their index. This fact is then used by constraint (19) to track the robot stock in every depot and to ensure that the stock is ≥ 0. Finally, the variable domains are defined by constraints (20) to (27).

The MTR-RP extends the classical TnR problem, i.e., without truck deliveries, in several ways: Some customers must be served by truck, others can be. This means that the total number of robots started (tracked by (11), (12) and (13)) is not predetermined but part of the decision problem. Moreover, total truck time is no longer based merely on the legs $s_{i,j}$ traveled since the truck may have to wait for the beginning of a time window ((9) and (10)). We need to determine the usage time of a truck instead by using the return time to the warehouse $t_{\bar{\gamma}}$, and add the term $t_{\bar{\gamma}}c^{\mathrm{t}}$ to the objective function. Since the optimal $t_{\bar{\gamma}}$ is determined via the recursive constraints (8), (9) and (10), this is computationally expensive even for small instances.

5. Solution approach

The MTR-RP generalizes the NP-hard TnR routing problem and therefore constitutes an NP-hard optimization problem by itself (see Boysen et al., 2018). Since even small instances cannot be solved exactly, we propose a tailored solution approach, denoted as *MTR heuristic*, that is based on a GVNS framework (see Hansen & Mladenović, 2001; Mladenović & Hansen, 1997). GVNS conducts several rounds of Variable Neighborhood Descent (VND) with a shaking step between them. The general steps of a GVNS are shown in Algorithm 1.

Basic Variable Neighborhood Search (VNS) formulations (of which VND is a special case) have been used successfully for many variants of routing problems (e.g., de Freitas & Penna, 2020; Henke, Speranza, & Wäscher, 2015; Kovacs, Golden, Hartl, & Parragh, 2014;

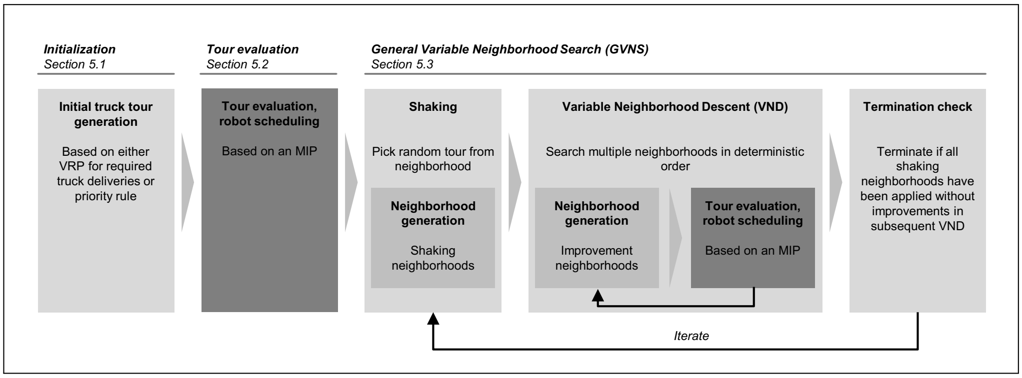

Fig. 5. Structure of the MTR heuristic proposed.

Algorithm 1 GVNS framework (adapted from Hansen & Mladenović (2018)).

Require: Starting solution π_s
$\pi_{best} = \pi_s$; // best solution found
$k_s = 1$ // shaking neighborhood
while $k_s \leq$ number of shaking neighborhoods **do**
$\pi_{current}$ = random(shakeneighborhood(π_{best}, k_s)) // shake tour
$\pi_{VND} = VND(\pi_{current})$ // perform VND to improve tour
if $Z(\pi_{VND}) < Z(\pi_{best})$ **then**
$\pi_{best} = \pi_{VND}$
$k_s = 1$ // restart GVNS with new best tour
else
$k_s = k_s + 1$ // continue with next shaking neighborhood
end if
end while
return π_{best}

Pia & Filippi, 2006) as they provide a high degree of flexibility and can be tailored to the given problem specifics. The key benefit of GVNS for this application (compared to local search approaches previously used for TnR, see Boysen et al., 2018 and Ostermeier et al., 2021) is that complete neighborhoods are evaluated in a defined order. This is necessary for finding improvements as the objective function is sensitive to small changes in the truck route. The robot scheduling depends on the available truck stops and therefore changing a single stop may have a significant impact on overall costs. Evaluating all possibilities for a certain change operation consequently ensures the best option is found. Furthermore, defining an order of assessed neighborhoods enables us to incorporate problem-specific knowledge, such as truck distance as a key cost driver (Ostermeier et al., 2021). An overview of our solution framework is shown in Fig. 5.

We generate an initial truck tour with one of two possible start procedures, depending on the given problem instance (see Section 5.1). This truck tour is then evaluated and complemented to a full solution by finding the optimal robot schedule using an MIP (see Section 5.2). Next, a GVNS is used to improve truck tours with respect to depots visited, drop-off locations and direct truck deliveries (see Section 5.3). It consists of a shaking step and a subsequent Variable Neighborhood Descent (VND). Within the GVNS, tours are again assessed by the robot scheduling MIP from Section 5.2.

5.1. Initial truck tour generation

There are start heuristics for classical VRPs (i.e., truck delivery only) and TnR routing (i.e., robot delivery only) available in current literature. Our approach combines these two modes and thus chooses between truck and robot delivery based on efficiency. We found in our numerical experiments that above a certain number of mandatory truck deliveries, the order of these deliveries is crucial for solution quality. Below a certain number of truck deliveries, the robot deliveries have a greater impact on the solution and total costs. Leveraging these insights, we propose two alternative principles for generating start solutions, depending on the number of truck deliveries required. They differ in terms of which deliveries are considered and in which order. Deliveries which can be completed by both truck and robot are treated as robot deliveries in this step.

Robot deliveries first, truck deliveries second. In the event of less than σ mandatory truck deliveries, we generate a tour that includes both robot and truck deliveries in a two-step approach. First, stops at drop-off and depot locations are sequentially appended to the tour based on the *priority rule (PR)* “go to the location from which most robot deliveries can be started such that they reach customers on time”. Truck delivery customers are ignored in the first step. As soon as robot customers are assigned to a stop, they are not considered for later stops. This rule results in a sequence of depot and drop-off points, which could be non-feasible since robot availability is not yet considered. In the second step, the truck deliveries required are inserted sequentially, each customer at the position of the tour where the smallest deviation is caused. We therefore obtain a complete tour consisting of drop-off locations and stops at truck delivery customers.

Only truck deliveries. In the event of at least σ truck deliveries, we solve a *VRP with time windows* (see model provided in Appendix A) for truck delivery customers, thus ignoring robot deliveries completely. The corresponding VRP can be solved optimally for small problem sizes, while for larger problem sizes the best solution found within a given time limit τ is used. This results in a truck tour that visits all customers requiring truck delivery, starting from the start location. This route then serves as starting solution for the GVNS. Despite lacking the consideration of robot drop-off locations, this enables us to obtain an efficient basis for the truck routing as the direct truck deliveries are decisive for the final tour, including drop-off and depot locations.

5.2. Tour evaluation and robot scheduling

Feasibility of truck tours. All solutions obtained (including the start solution) need to be assessed with respect to robot availability to prevent non-feasible tours. A truck tour is only feasible if the total number of available robots (initial number of robots on the truck δ plus all robots at depots visited on the tour $r_a, a \in R$) is equal to or larger than the number of customers not visited by

Table 4
Additional parameters and variables for robot scheduling.

Truck tour parameters	
U	Index set of stops on the truck tour $u \in \{1, 2, \ldots\}$
Y	Tuple of truck stops, where element $y(u)$ is the uth stop of the truck tour, $y(u) \in L$
$\tilde{C}$	Set of customers not visited by truck (i.e., not in Y)
t_u	Arrival time at truck stop $u, u \in U$
$c^{\mathrm{T}}_{u,k}$	Cost of serving customer $k, k \in \tilde{C}$, from stop $u, u \in U$
Decision and auxiliary variables	
$x_{u,k}$	Binary: 1, if customer $k, k \in \tilde{C}$, is supplied from stop $u, u \in U$; 0 otherwise
q_u	Number of robots aboard the truck at departure from stop $u, u \in U$
$r_{a,u}$	Number of available robots in location $a, a \in L$, after the uth truck stop

truck (i.e., customers that are not on the truck route). We append the closest unvisited depot to the end of the tour as long as the number of available robots is not sufficient.

Robot scheduling for given truck route. Once feasibility is ensured, the corresponding robot movements for the truck route in question must be defined, i.e., all remaining customers must be assigned to a truck stop, from which the corresponding robot will start. This transforms the truck tour into a full solution. We apply an MIP proposed by Boysen et al. (2018) and enhanced by Ostermeier et al. (2021) to assign customers to the truck stops on the route. This is necessary to evaluate the quality of a route that has been found. In contrast to the MIP from Section 4, which included the decision on truck movements, we do not need duplicates of robot drop-off (D) and depot locations (R). This leads to $L := C^{\mathrm{m}} \cup C^{\mathrm{o}} \cup D \cup R$ being the set of all locations potentially reachable by truck. We assume the truck tour to be given as a tuple Y, where $y(u)$ is the location of the uth stop, $y(u) \in L$. Note that customers that are part of the truck route (i.e., served by truck) can be ignored in this step. We denote the set of remaining customers to be served by robot as $\tilde{C}$, with $\tilde{C} \subseteq C^{\mathrm{o}} \cup C^{\mathrm{r}}$. Table 4 summarizes the notation of truck tour parameters and decision variables.

The actual arrival time at each truck stop $t_u, u \in U$ for a given tour Y can be calculated using Eqs. (28)–(30). Equation (28) states that the truck tour starts at time zero. For drop-off and depot locations, only truck travel times determine the arrival time (Eq. (29)). For customer locations, the beginning of the respective time window also has to be considered to prevent premature deliveries (Eq. (30)).

$$t_1 = 0 \tag{28}$$

$$t_u = t_{u-1} + \vartheta^t_{y(u),y(u-1)} \qquad \forall u : y(u) \in D \cup R \tag{29}$$

$$t_u = max(t_{u-1} + \vartheta^t_{y(u),y(u-1)}; d_{y(u)} - \epsilon_k) \qquad \forall u : y(u) \in C \tag{30}$$

Based on arrival times, the total cost $c^{\mathrm{T}}_{u,k}$ of supplying a customer k from stop u is denoted by Eq. (31). It comprises the robot usage cost (at rate c^{r}) for travel time, waiting time at the customer (in the event the robot arrives before the time window) and the time to return to the closest depot ϑ^{b}_k. Finally, delay costs are added.

$$c^{\mathrm{T}}_{u,k} := c^{\mathrm{r}}(\vartheta^r_{y(u),k} + (d_k - \epsilon_k - t_u - \vartheta^r_{y(u),k})^+ + \vartheta^b_k) + c^{\mathrm{l}}(t_u + \vartheta^r_{y(u),k} - d_k)^+ \qquad \forall u \in U, k \in \tilde{C} \tag{31}$$

The variables $x_{u,k}$, $r_{a,u}$ and q_u define where each customer's robot is started, how many robots are available in each location and on the truck after every stop. The robot scheduling MIP can then be formulated as follows.

$$\min \ F(Q, X, R) = \sum_{u \in U} \sum_{k \in \tilde{C}} x_{u,k} \cdot c^{\mathrm{T}}_{u,k} \tag{32}$$

subject to

$$\sum_{u \in U} x_{u,k} = 1 \qquad \forall k \in \tilde{C} \tag{33}$$

$$r_{a,u} = r_{a,u-1} \qquad \forall a \in R, u \in U : a \neq y(u) \tag{34}$$

$$r_{a,u} \leq r_{a,u-1} + q_{u-1} - q_u - \sum_{k \in \tilde{C}} x_{u,k} \qquad \forall a \in L, u \in U : a = y(u) \tag{35}$$

$$q_0 = \delta \tag{36}$$

$$r_{a,0} = r_a \qquad \forall a \in R \tag{37}$$

$$r_{a,u} = 0 \qquad \forall a \in L \setminus R, u \in U \tag{38}$$

$$x_{u,k} \in \{0, 1\} \qquad \forall u \in U, k \in \tilde{C} \tag{39}$$

$$r_{a,u} \geq 0 \qquad \forall a \in R, u \in U \tag{40}$$

$$0 \leq q_u \leq K \qquad \forall u \in U \tag{41}$$

The objective function (32) minimizes total robot and delay costs. Constraint (33) ensures that exactly one robot is sent to each remaining customer. Constraint (34) states that if a depot is not visited, the number of available robots remains the same. Constraint (35) keeps track of the number of robots in locations visited and aboard the truck after every stop. Equations (36) and (37) define the initial number of robots in the depots and on the truck. Constraint (38) ensures that robots cannot be stored at drop-off locations or customers. Constraints (39)–(41) define the variable domains.

5.3. General variable neighborhood search

For improving the truck tour, we apply a GVNS as described by Hansen & Mladenović (2018), which tries to improve the initial routing solution by exploiting problem-specific knowledge. It conducts several cycles of shaking and subsequent local search using a VND procedure. Both the shaking and the VND rely on neighborhoods. These are defined by operators, such that every neighborhood contains all truck tours that can be generated by applying the respective operator to the incumbent truck tour. Algorithm 2 summarizes the GVNS applied. The inner while loop constitutes the VND (with its improvement neighborhood k_i), the outer one conducts the shaking (with shaking neighborhood k_s) and stores

Algorithm 2 Detailed GVNS procedure (adapted from Hansen & Mladenović (2018)).

Require: Starting solution π_s
  $\pi_{best} = \pi_s$; // best solution found
  $k_s = 1$ // shaking neighborhood
  while $k_s \leq$ number of shaking neighborhoods **do**
    improvement = false
    // perform several VND runs with same shaking neighborhood:
    for $j = 1$ to α **do**
      $k_i = 1$
      $\pi_{current}$ = random(shakeneighborhood(π_{best}, k_s)) // shaking neighborhood k_s
      while $k_i \leq$ number of VND neighborhoods **do**
        π_{k_i} = best(improveneighborhood($\pi_{current}, k_i$) // improvement neighborhood k_i
        if Z(π_{k_i}) <Z(π_{best}) **then**
          $\pi_{best} = \pi_{k_i}$
          $k_i = 1$
          improvement = true
        else
          $k_i = k_i + 1$; // next neighborhood
        end if
      end while
      if improvement = true **then**
        $k_s = 0$
        break
      end if
    end for
    $k_s = k_s + 1$
  end while
  return π_{best}

the best known solution. The parameter α in the for loop determines the number of VND iterations for every shaking neighborhood. To evaluate truck tours, the GVNS repeatedly uses the robot scheduling MIP.

Shaking. The shaking phase of the GVNS is used to diversify the search. Neighborhoods are obtained by varying the truck tour of a previously generated solution and reoptimizing the robot movements. The neighborhoods are applied in the given order, one in each shaking phase, and used to generate α new solutions. For each of these solutions we apply a separate VND in the next step. When a shaking step has led to an improvement, the process restarts from the first neighborhood. The search is complete after all shaking neighborhoods have been used without improvements.

- **Depot insertion.** This operator inserts a new robot depot into the tour. Since robot availability is crucial for finding an efficient robot schedule, selecting different depots can enable tour improvements.
- **Detour insertion.** This operator inserts a drop-off point or a customer with optional truck delivery into the truck tour that leads to a detour of half the delivery area's side length or above. It is used to diversify the search by causing a large change in the current truck tour.
- **Swap stop.** This operator swaps two random stops (of which each can be a drop-off point, robot depot or customer) of a truck tour. This may again lead to large detours and thus widens the search space.
- **Stop relocation.** This operator shifts a stop to a later or earlier point on the tour.
- **Customer reshuffling.** This operator instigates the most extensive tour change. The rationale for this operator is that a truck delivery with a high delay can lead to a strong cost increase and therefore should be avoided. An example is presented in Fig. 6. The original tour is reduced to only the required truck deliveries by removing any other stops (step 1 in Fig. 6). For the remaining steps, we consider the original arrival times at the customers. For every customer with a late delivery (customers 3 and 4 in the example), we generate tour candidates by inserting this stop at earlier points of the tour such that customers are reached before their deadline. Customer 3 with a deadline at time 11 can only be inserted before customer 1, as customer 1 was originally reached at time 12. Customer 4 with the deadline at time 30 can be inserted before and after customer 1 (as $12 < 30$), but not after customer 2 (as arrival time at customer 2 is $32 > 30$). In total, this results in one tour candidate for customer 3 and two tour candidates for customer 4 plus the reduced tour (step 1). All tours derived in steps 1 to 3 form the 'customer reshuffling' shaking neighborhood (i.e., four tours in our example). If this neighborhood contains more than n^{shuffle} tours, we reduce it by considering only the shortest tour of each step (resulting in three tours in our example). The following VND will then construct a new solution around the reshuffled truck deliveries by inserting robot depots and drop-off points, which potentially leads to extensive changes compared to the original tour.

VND. The VND is used to improve the truck tour. It relies on multiple neighborhoods of the incumbent solution that are searched sequentially. The VND restarts from the first neighborhood when a better solution is found. This continues until all neighborhoods of the incumbent solution have been searched and no improvement has been found. Each neighborhood contains all tours that can result from applying its operator to the incumbent tour.

- **Remove a non-depot.** Removes a drop-off point or a customer with optional truck delivery from the current truck tour. Since truck distance is a main cost driver, this often leads to improvements. Required truck deliveries cannot be removed in this step.
- **Remove a depot.** Removes a depot from the current truck tour. The removal of a depot may lead to non-feasible solutions. In this case additional depots will be appended within the feasibility check.
- **Add depot.** Adds a new depot to the existing truck tour. Additional depots can increase robot availability on parts of the tour and lead to better robot schedules at reduced costs.
- **Add a non-depot.** Adds a drop-off point to the existing truck tour. This may reduce robot travel times by bringing the truck closer to nearby customers.
- **Swap two stops.** By changing the order of stops, truck distance can be reduced or delays at the later stop can be avoided.
- **Relocate a stop.** This operator primarily aims at improving arrival times at customers. In particular when the truck arrives at a customer too early and is forced to wait for the time window, shifting this customer to a later point of the tour can reduce total time and delays.

The order of improvement neighborhoods ensures that tours are kept short, and that we start with the smallest neighborhoods. This reduces the computational effort by limiting the number and complexity of the robot scheduling MIP (Eqs. (32)–(41)) that has to be solved to evaluate the tours. Since in neighborhoods 'add depot' and 'add non-depot', several hundreds of combinations of inserted location and insertion position of the tour exist, neighborhoods are limited to the $n^{\max}$ shortest tours. This again reduces computational effort based on known problem characteristics.

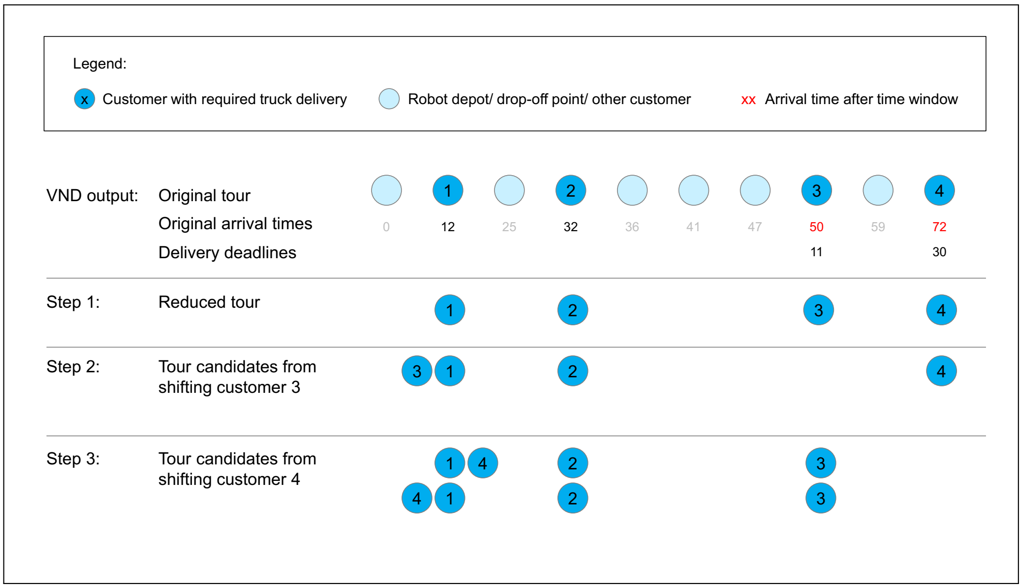

Fig. 6. Customer reshuffling procedure (example).

6. Numerical examples

This section analyzes the performance of our MTR heuristic. First, we describe the instances and parameters used in our experiments (Section 6.1). Next, we compare our approach to a benchmark (Section 6.2) to assess the performance of our algorithm. Further experiments assess the impact of both required and optional truck deliveries. We compare different fulfillment concepts for home delivery depending on the share of truck deliveries required (Section 6.3) and analyze the impact of time windows on the routing (Section 6.4). Further, the influence of handling times is analyzed (Section 6.5). Finally, we discuss the impact of customer distribution (Section 6.6), and cost rates for the truck and delays (Section 6.7). Our approach was implemented in Python (using PyCharm 2018.3.5 Professional Edition) with Gurobi (version 8.0.1) as MIP solver and executed on a 64-bit PC with an Intel Core i7-8650U CPU (4×1.9 GHz), 16 GB RAM, and Windows 10 Enterprise.

6.1. Instance and parameter setting

In our numerical experiments we aim at analyzing the performance of our MTR heuristic in comparison to related approaches. To enable a fair comparison and to evaluate the impact of direct deliveries we leverage the test data provided by Ostermeier et al. (2021) (http://www.vrp-rep.org/datasets/item/2020-0005.html). The data set comprises instances for TnR routing and resembles the general setting of our problem but ignores the possibility of direct truck deliveries. The data setting is as follows. Customer locations are picked randomly from all buildings in a 4 km^2 area in northern Munich (Germany), using OpenStreetMap (OpenStreetMap Foundation, 2019) to create instances with $|C| = 50$ customers. To account for direct truck deliveries, we assume that the first 12% of customers (which the instances list in random order) require truck delivery ($|C^{\mathrm{m}}|/|C|$=0.12). The remaining customers require robot delivery ($C^{\mathrm{r}} = C/C^{\mathrm{m}}$). This means there are no optional truck deliveries in the default case ($C^{\mathrm{o}} = \emptyset$). The impact of optional truck deliveries will be analyzed separately. Note that our assumption for $|C^{\mathrm{m}}|/|C|$ is in line with the estimate reported by Forbes (2019), that technically 75 to 90% of Amazon deliveries could be made by autonomous vehicles, and will be subject to a sensitivity analysis in the following. There are $|R| = 25$ evenly distributed robot depots, and $|D| = 48$ uniform-randomly distributed drop-off points in the area. All delivery time windows have the same length $\epsilon = 10$ min. The end of a customer's time window is generated based on the direct travel time of the truck from its random starting position to the customer. This travel time is multiplied by a uniform-randomly distributed factor from the interval $[\rho_{\min}, \rho_{\max}] = [5, 8]$. This procedure simulates an assignment of customers to vehicles such that reasonable tours are made possible. The initial number of robots is $r_a = 10$ for every depot $a, a \in R$. The capacity of the truck is $K = 8$ robots and it is fully loaded at the start ($\delta = 8$). The average speed of the truck is 30 km/h and the average speed of the robots 5 km/h. A handling time per truck stop of $\mu = 40$ s is assumed in addition to travel times. Note that we assume the same travel distances for the truck and robots. In practice, robots would often be able to take shortcuts, which adds to their advantages. There are 20 instances generated with 25, 50, 75 and 100 customers respectively, resulting in a total of 80 instances used in our experiments. All results presented show the average of the corresponding 20 solutions. We further apply the cost rates empirically quantified by Ostermeier et al. (2021). These are $c^{\mathrm{d}} = 0.20$ €/km and $c^{\mathrm{t}} = 30$ €/h for the truck, $c^{\mathrm{r}} = 0.50$ €/h for robot use and $c^{\mathrm{l}} = 5$ €/h for delivery delays. For a more detailed discussion of costs in last-mile delivery we further refer to Brotcorne, Perboli, Rosano, & Wei (2019).

Lastly, we allow $\alpha = 4$ VND iterations per shaking neighborhood (executed in parallel), a maximum of $n^{\mathrm{shuffle}} = 4$ for the *customer reshuffling* shaking neighborhood and a maximum VND neighborhood size of $n^{\max} = 90$ tours. The threshold for the selection of the start heuristic is set at $\sigma = 2$ and its time limit τ at 3 min.

6.2. Performance comparison

There are no existing solution approaches to MTR and only a couple of publications on TnR (see literature analysis). To the best of our knowledge, Boysen et al. (2018) and Ostermeier et al. (2021) provide the currently most developed approaches in this research area. As we provide a generalization of the problem, we will use a special case of our problem that is equivalent to the problems in the benchmark. We compare our MTR heuristic to the LS approach by Ostermeier et al. (2021), as the authors study the TnR concept with a total cost objective, i.e., without the possibil-

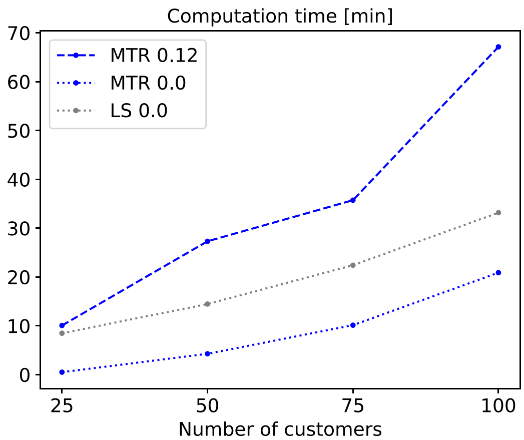

Fig. 7. Comparison of our MTR approach to the benchmark (i.e., LS approach by Ostermeier et al., 2021) for a share of 0% and 12% of truck deliveries.

ity of truck deliveries. Their numerical studies show that their LS approach outperforms the approach by Boysen et al. (2018) in finding cost-optimal tours. However, due to its structure, the LS is not suitable for incorporating truck deliveries and all customers must be visited by robot. We consequently apply our MTR heuristic to solve both instances without truck deliveries and additionally instances with the restriction of 12% truck deliveries required. In the special case of our setting without truck deliveries, the problem is identical to the one solved by Ostermeier et al. (2021). The MTR heuristic reaches a solution quality differing only 0.3 to 2.0% from the LS in these cases. The computation times for different problem sizes are shown in Fig. 7. The scenario with 12% truck deliveries is labeled 'MTR 0.12' and the one without truck deliveries 'MTR 0.0'. We see that the MTR approach outperforms the LS when only robot deliveries are required, reducing the computation time by 37 to 94% (for 100 and 25 customers respectively). This shows that despite the focus of our MTR approach on a mixed delivery structure, it works efficiently and effectively for a related problem without direct truck deliveries. When truck deliveries are required, the computation effort increases, but remains at a level acceptable for an application in practice. The reason for the increase is the strong impact of truck deliveries on costs and the resulting longer search for better alternatives. A premature truck delivery on the route forces the truck to wait until the delivery time window starts. This causes additional costs and potential delays at later stops. A late truck delivery, on the other hand, causes delay costs at the respective customer. Robot deliveries are more flexible as they can be started at different stops of a truck tour such that small changes to a given tour often show a minor impact on robot schedule and delay costs.

Additionally, we found that the MIP for the entire MTR-RP ((1)–(27)) could not be solved exactly within three hours for six customers, even if stops are not duplicated, branching is supported by a relaxed MIP version and a feasible start solution is provided to the solver. An average MIP gap of 52% remained.

6.3. Comparison of delivery concepts

This section compares the delivery concepts given in Table 5 for a varying share of truck deliveries required.

Figure 8 shows the total costs, computation times, average delay and total truck distance for the concepts analyzed. We henceforth highlight the default setting described in Section 6.1 with a bold x-label. Note that TD was solved without consideration of the earliest delivery time, i.e., delivery can occur before the time window to reduce computational complexity. This leads to an advantage for TD and an underestimation of the improvements due to MTR. Despite this simplification, optimality could not be proved within the computation time limit of three hours. We therefore report properties of the best solutions found and the lower bound of the objective value ('TD LB'). Further, MTR and STR are identical for 0% of truck deliveries.

Computation time. Runtime increases significantly for a mixed planning (i.e., MTR and MTR OT) as soon as truck deliveries are required. The actual locations of individual customers are the main driver of computation times. Single customers can significantly increase the problem complexity and the respective runtimes if they require truck delivery and cause a large detour for the truck. For example, for MTR, the standard deviation across the 20 instances relative to the average objective value increases by 25% when the share of truck deliveries increases from 0 to 12%. In line with this, runtimes for MTR OT are higher due to the potential additional truck stops. STR on the other hand reveals a decrease in runtime as more truck deliveries are outsourced to a separate routing problem.

Cost impact of MTR. All robot concepts outperform a solution with truck deliveries only (TD). This is in line with current literature (see Ostermeier et al., 2021). MTR OT is the option with the lowest total costs in all examples. Total costs increase significantly for all concepts involving robots as soon as truck deliveries are required (i.e., comparing 0 and 4% truck deliveries required). In the STR case, this is due to the truck delivery tour needed in addition to the robot delivery tour. In the MTR and MTR OT cases, it can be attributed to reduced flexibility given the stops required for truck delivery.

A further increase in the share of truck deliveries leads to a moderate increase in total costs. Comparing a combined truck and robot delivery to a separate delivery (i.e., MTR vs. STR) of more than 4% truck deliveries results in cost savings of between 20 and 24% in favor of a mixed delivery. This highlights the advantage of our MTR heuristic's ability to combine truck and robot deliveries into one tour. Compared to TD, MTR reduces costs by 43% in the default case with 12% truck deliveries. This highlights the attractiveness of delivery by truck and robots even for situations in which not all deliveries can be made by robot.

MTR OT vs. MTR. MTR OT chooses the delivery mode for all customers that can be supplied by truck or robot. An increasing share of these customers is served by truck as the share of required truck deliveries increases. When no truck deliveries are required, only 1% of the customers are served by truck. When truck deliveries are required, the truck delivery share among the 'optional' customers C^o steadily increases from 18% (in the case with 4% required truck deliveries) to 35% (in the case with 20% required truck deliveries). The reason for this increase is that additional truck deliveries are only efficient when they are close to the tour. As the share of required truck deliveries increases, the tour gets longer and thus more of the 'optional' customers are suitable for truck delivery with a small deviation. However, the cost advantage of additional optional truck deliveries (i.e., MTR OT vs. MTR) is low with up to 2% savings. Due to this small cost advantage of MTR OT compared to MTR, we restrict most of our remaining analyses to the comparison of MTR vs. STR for better readability.

Delay and truck distance. The logistical performance with respect to delays is comparable for all robot concepts. This shows that all deliveries can be made by a single tour without compromising on delivery performance. MTR and MTR OT show a minimum delay when 8% of truck deliveries are considered. The rea-

Table 5
Overview of delivery concepts.

Concept	Description	Rationale	Solution approach
TD: Truck-only delivery	Only deliveries by truck to all customers	Benchmark to assess MTR benefits	MIP from Appendix A
MTR: Mixed truck and robot delivery	Tour with mandatory truck deliveries and all other deliveries by robot ($C^r = C \setminus C^m$)	Approach of this paper	MTR heuristic
MTR OT: MTR with optional truck deliveries	MTR extended by optional truck deliveries, i.e., in addition to mandatory truck deliveries, all other deliveries can be made by truck ($C^o = C \setminus C^m$)	Approach of this paper and assessing optional truck delivery	MTR heuristic
STR: Separate truck and robot tours	Separate planning of one TD tour for truck deliveries and one TnR tour for robot deliveries (i.e., two simultaneous tours)	Serves as benchmark to assess benefits of MTR heuristic vs. existing TnR heuristics	TD tour by MIP from Appendix A; TnR tour by MTR heuristic

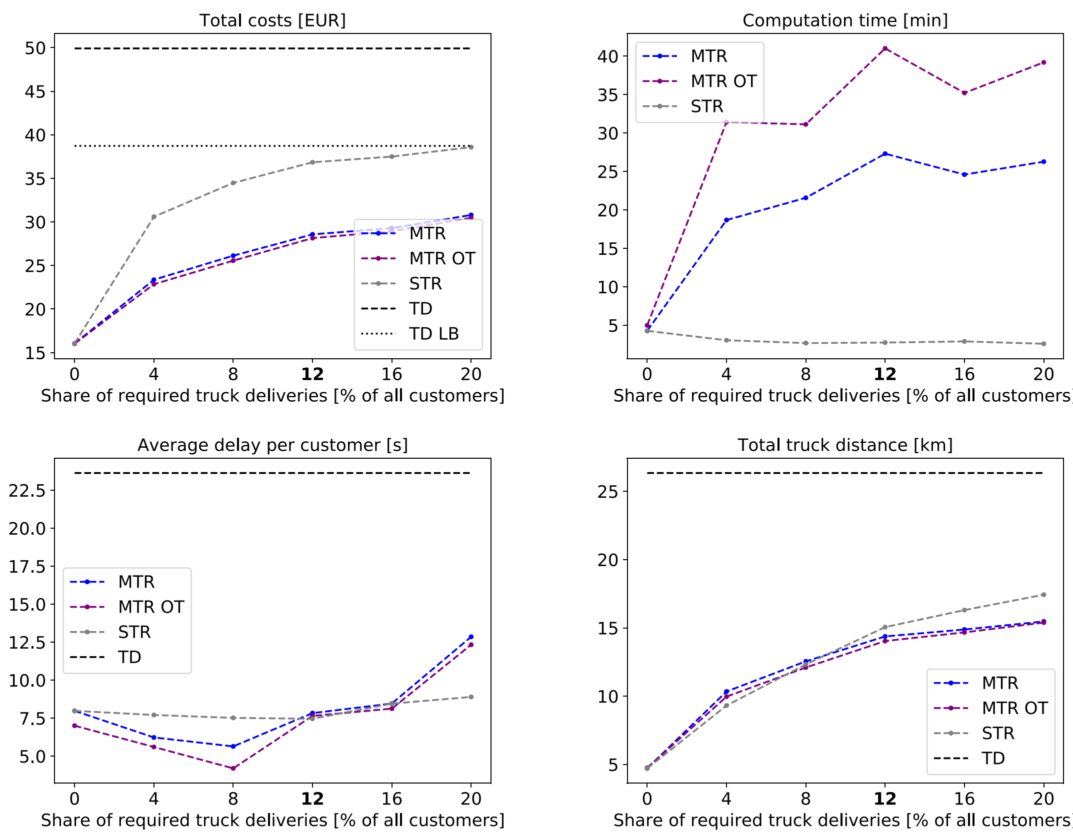

Fig. 8. Comparison of different delivery modes for a varying share of required truck deliveries.

son for this is that including additional stops at truck delivery customers (and thus forcing the truck to make a longer tour) can improve punctuality as less distance needs to be covered by robots. A further increase in truck deliveries then leads to additional delays caused by longer truck tours and a later launch of robots at the last drop-off points. The latter effect also leads to a decreasing advantage of MTR OT when more than 8% of deliveries are required by truck. The truck is already overwhelmed serving the customers who require truck deliveries such that optional truck deliveries are hardly made in addition. The development of covered truck distance is similar across the three concepts using robots. It shows a flattening increase for an increasing number of truck deliveries. In the default case, MTR reduces truck mileage by 45% compared to TD, showing that it is able to reduce pollution and traffic even when truck deliveries are necessary. The steady increase in mileage can be reasoned by the tight time windows considered. The truck in the MTR scenario must go on a criss-cross route to satisfy all the time windows at customer stops. We therefore analyze a changing time window structure in the following.

6.4. Analysis of the time window structure

Time windows limit the degree of freedom for the routing. This section analyzes the impact of the time window length for both truck and robot deliveries. We analyze both customer groups separately since the impact of a customer's time window on the overall solution is higher when the truck needs to visit the customer and meet the time window. This can lead to detours or waiting time affecting all other deliveries as well, while a robot delivery has little effect on other deliveries.

Time window length for truck deliveries. Figure 9 shows the performance of MTR vs. STR depending on the change in time window length for truck deliveries. Every time window change is made symmetrically, i.e., in the case of a 10 min change, start and end of the time windows are shifted by 5 min each. 0 corresponds to the default case.

Cost and computation time are reduced if time windows become wider due to increased flexibility. The cost decrease runs

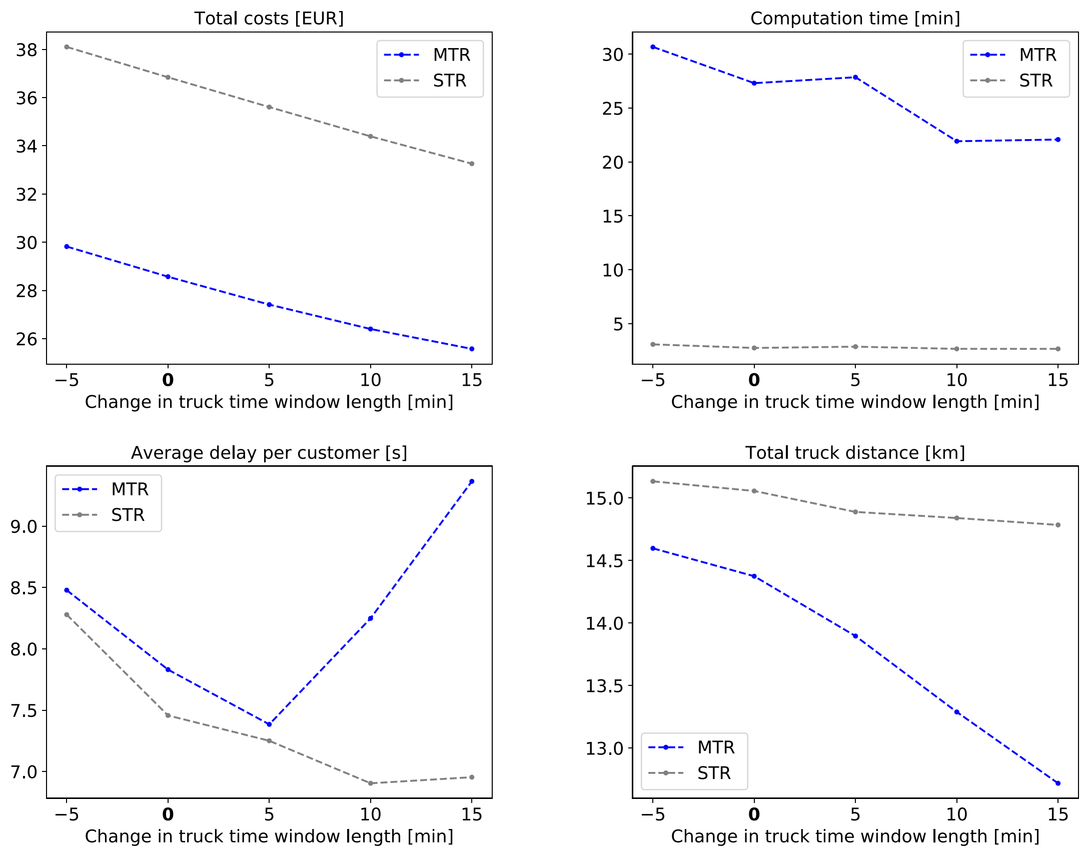

Fig. 9. Comparison of MTR vs. STR for varying length of truck delivery time windows.

in parallel for MTR and STR such that MTR's cost advantage is stable at 21 to 23%. The driver of the cost decrease is reduced truck usage both for MTR and STR. STR achieves only a moderate truck distance reduction, but at the same time reduces delays and keeps robot use stable since the separate robot delivery tour is not affected. MTR achieves a larger distance reduction at the cost of increasing delays and robot use. This means that although the time windows become wider, MTR uses this opportunity to further reduce truck distance and allow longer robot travel, resulting in a very small increase in delays. Additionally, we considered a scenario without time windows for truck deliveries. Even in this scenario, a cost saving of 19% is achieved by MTR compared to STR. This is possible as truck deliveries can be added freely at beneficial points of the route such that deviations are minimized.

Time window length for robot deliveries. We further analyze the impact of robot delivery time windows. Since these time windows could have an impact on the delivery mode chosen by MTR OT, we include it into this analysis. The results for the corresponding changes are shown in Fig. 10.

As could be expected, costs of the MTR are hardly affected by these changes since truck tours are dominated by truck deliveries. The only effect of wider time windows is reduction in delays. For STR, the TnR route changes slightly. The distance becomes longer, while robot cost and delays decrease. This leads to a minor cost reduction as robot deliveries only account for 38% of total costs and truck deliveries are not affected. MTR OT continues to find opportunities for slight improvements by changing the delivery mode from robot to truck delivery for some customers. The number of these customers (around 6 customers) and the cost advantage created (1 to 2% compared to MTR) remain constant with changing time window length. This shows that the customers' location mainly defines whether customers are suitable for truck delivery or not. The results highlight that both MTR and STR can fulfill tight time windows for robot deliveries at little additional cost. The MTR approach outperforms the STR concept with separate planning of truck and robot deliveries across all scenarios.

6.5. Impact of handling times

The handling time for a truck delivery can vary depending on how far the driver walks to the customer's door and how quickly the customer responds. We therefore analyze the impact of considerably longer handling times for deliveries by truck within a sensitivity analysis in Fig. 11. As the handling time increases, so do total costs for both approaches. For MTR this leads to additional delays (on a low absolute level) that further contribute to the cost increase. STR benefits from a second truck and therefore keeps delays low. MTR responds to this by further reducing the truck distance and the number of stops at robot depots and drop-off points. The number of customer stops cannot be reduced, since only customers requiring truck delivery are served by truck. However, these customers are partly shifted to the end of the truck tour, such that their handling times do not cause delays for subsequent robot deliveries. Despite the route adaptions by MTR, its cost advantage compared to STR diminishes to 14% (at 160 s handling time) and 9% (at 280 s handling time). In practice this means that (i) handling times should be kept short, e.g., by allowing the driver to leave a parcel at the door instead of waiting for the customer, (ii) time windows offered to the customers must take handling times into account (as in our example, time windows are too tight for increased handling times), and (iii) even with very high handling times MTR remains cost-competitive.

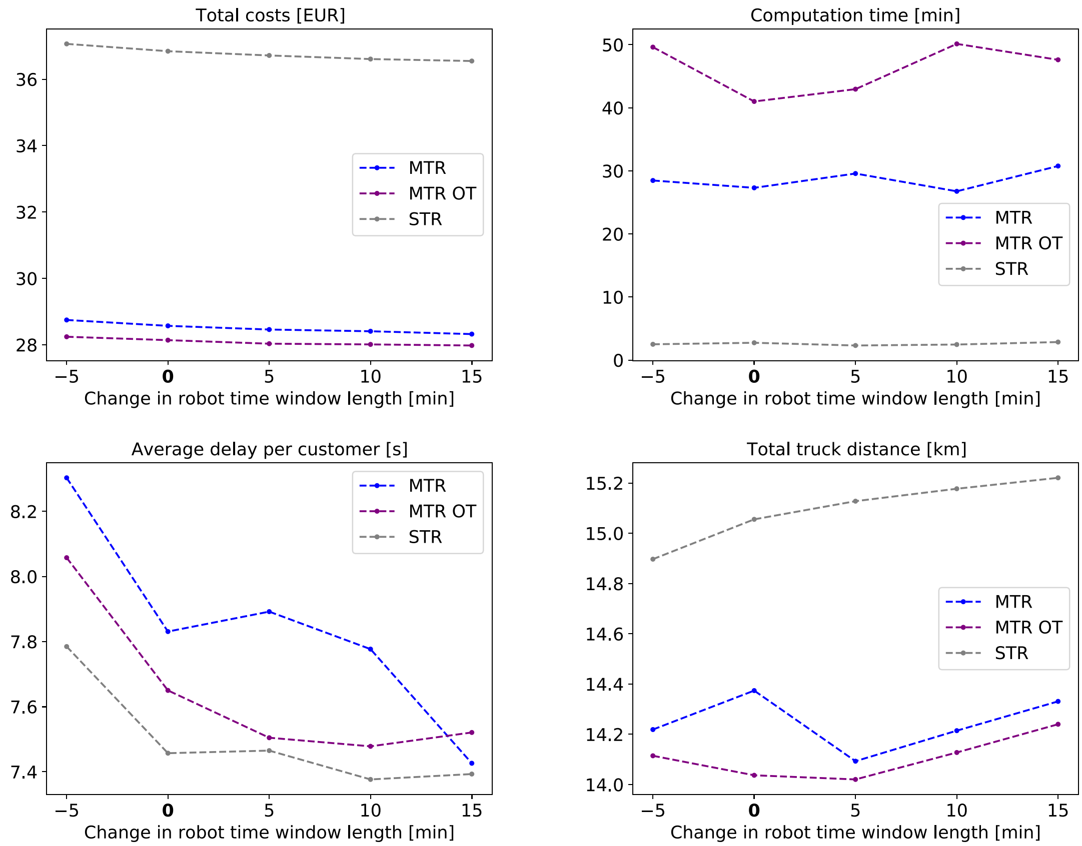

Fig. 10. Comparison of MTR and MTR OT vs. STR for varying length of robot delivery time windows.

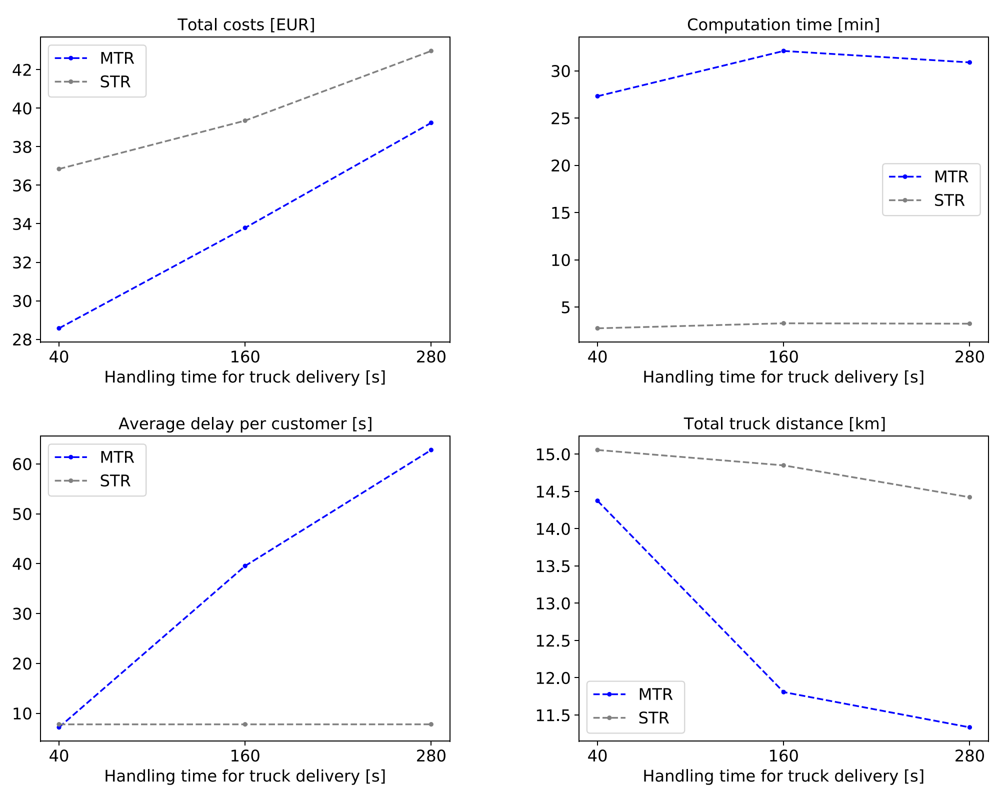

Fig. 11. Comparison of MTR vs. STR for varying handling times of truck delivery.

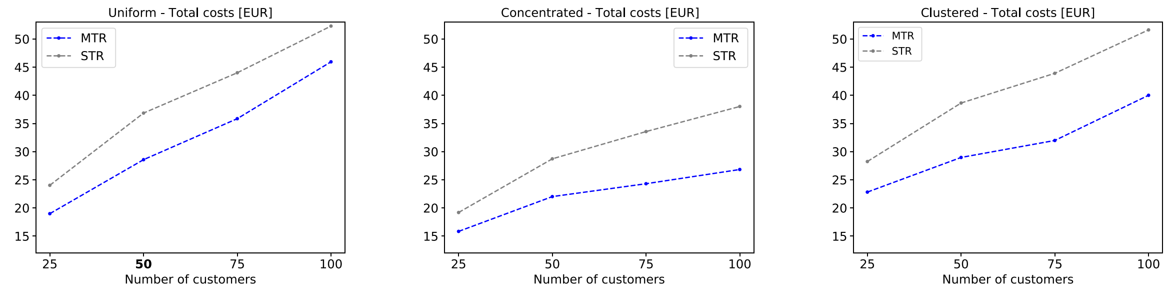

Fig. 12. Cost comparison of MTR vs. STR for varying customer and depot distributions.

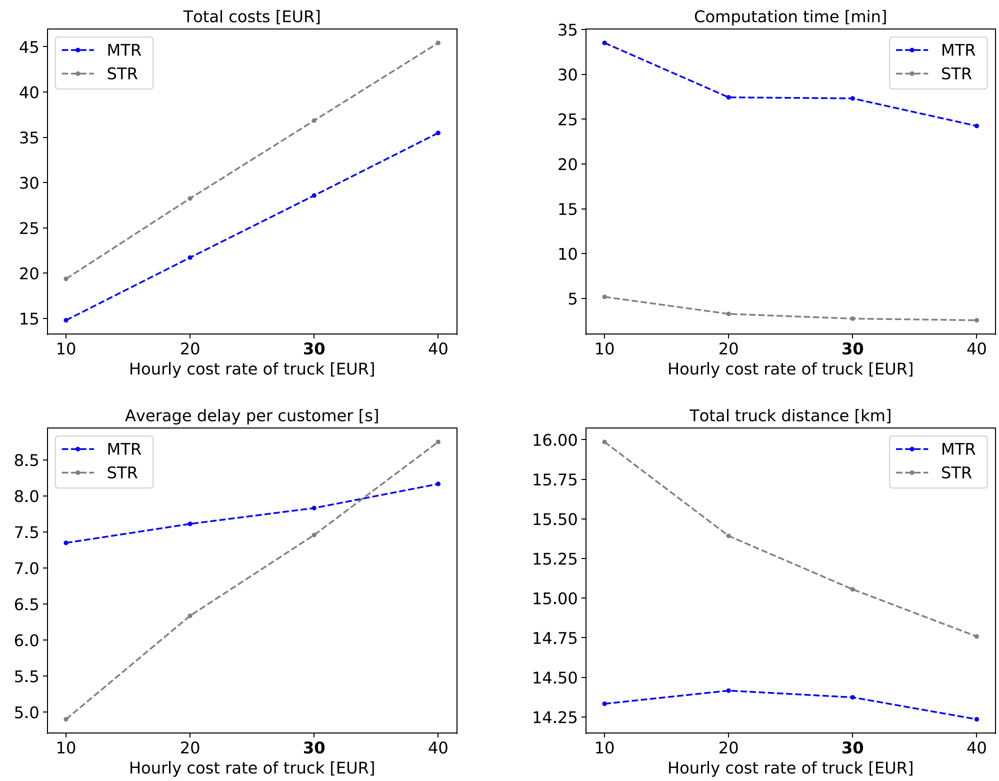

Fig. 13. Comparison of MTR vs. STR for varying hourly truck cost rates.

6.6. *Impact of delivery area setting*

The spatial distribution of customers and depots can have a strong impact on a concept's performance. We therefore analyze total costs of MTR vs. STR for different distribution types. The uniform distribution of our default setting is compared to two alternatives: a concentrated distribution, where customers and the equidistant depots are located centrally in a $2 \times 2\,\text{km}^2$ square area, and a clustered distribution, where two customer clusters are considered, one in the lower left and one in the upper right quadrant of the original $4 \times 4\,\text{km}^2$ square area. The depot distribution in the clustered distribution remains equidistant in the whole square area (as in the "uniform" case). The number of customers is varied from 25 to 100 (where our default case corresponds to the uniform distribution of 50 customers). The MIP used to solve the truck delivery tour part of STR could not be solved to proven optimality within three hours in the 100-customer case. The best-known solutions are reported. The results are summarized in Fig. 12. We further illustrate the results of the different settings using exemplary routing solutions in the appendix (see Appendix B).

Total costs show a near linear increase for both alternative concepts. The MTR approach is able to sustain or even expand its cost advantage for an increasing number of customers. In the concentrated setting, for instance, cost savings increase from 17% (25 customers) to 29% (100 customers). Concentrated customers are beneficial for both MTR and STR, as total travel distances decrease. However, the MTR is able to better exploit the advantages of a concentrated or clustered distribution as all customers are served by a single tour, while travel distances are further reduced and time windows are met. Our MTR approach robustly leads to significant savings in all settings presented.

6.7. *Impact of costs*

Impact of truck costs. The hourly cost rate of the truck is mostly driven by the driver's salary. We therefore provide a sensitivity analysis on the truck cost rate c^t, which corresponds to a Western European salary level in our default case. Figure 13 displays our findings. Total costs increase proportionally for both approaches, leading to stable cost savings of 22 to 24% through MTR. STR is more sensitive to changing costs. The higher the truck costs, the

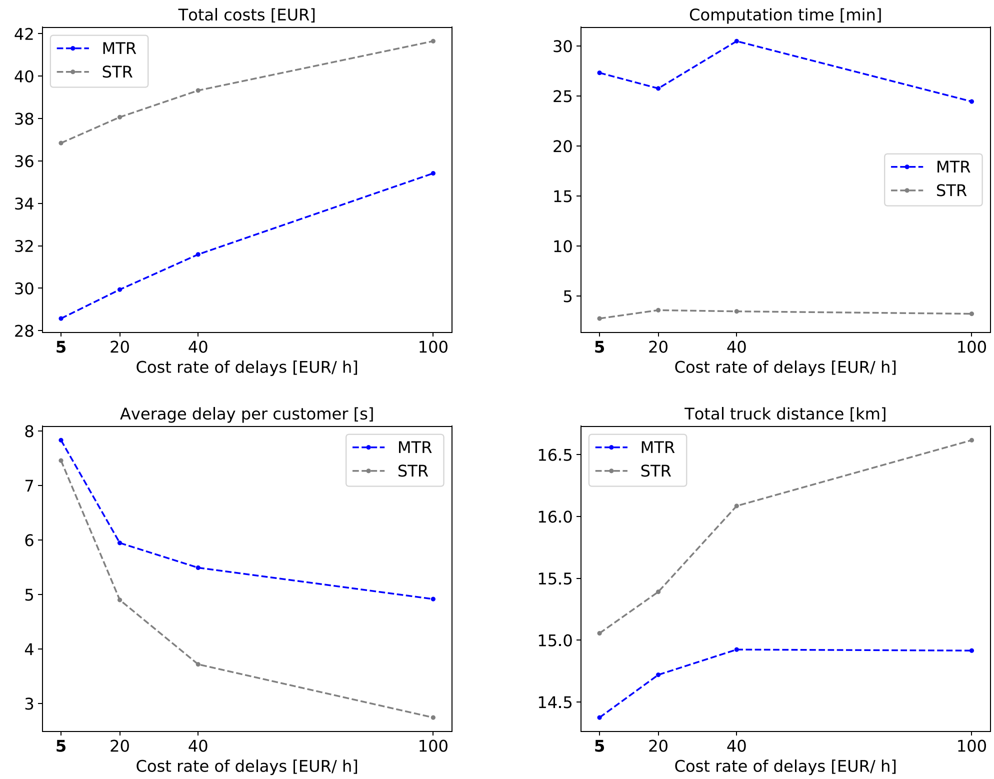

Fig. 14. Comparison of MTR vs. STR for varying delay cost rates.

higher the delays. The increase in delays goes along with a decrease in truck distance. The MTR solution on the other hand is not sensitive to changing costs with respect to delays and truck distance. In the 10 €/h scenario, the MTR approach therefore results in 10% less mileage at a cost of a 50% higher average delay compared to STR.

Impact of delay costs. We have shown that increasing truck costs may lead to increasing delays within the MTR approach. In our final test we therefore assess how MTR performs for varying delay costs c^l. The results are summarized in Fig. 14. The cost curves show that MTR savings slightly decrease as the importance of delays increases. However, MTR achieves cost savings of 15% even for a 100 €/h delay cost rate. Since the applied instances are chosen to be challenging with respect to delivery times, neither of the two approaches can eliminate delays completely. STR is able to reduce delays more as it uses two vehicles instead of one. The price of this is an increasing truck distance, while MTR's truck distance is stable. In summary, the STR concept minimizes delays compared to our MTR approach, but at the cost of longer truck tours. From a total cost perspective, MTR enables significant cost savings even if the costs of delays are high.

7. Conclusion

Our work shows that the MTR concept is a valuable extension of the existing TnR concept to enable further applications in the retail industry. It combines autonomous robot deliveries with classical truck deliveries (e.g., for bulky orders). We present a comprehensive model formulation for this home delivery concept and solve it using a tailored GVNS solution framework. The GVNS is competitive compared to existing TnR routing algorithms as it outperforms the prevailing LS approach in terms of runtime and equals its solution quality for a robot-only delivery. The extension presented enables practitioners to assess and operate an MTR system that can completely replace classical truck tours.

Our analyses show that the MTR concept reduces costs and truck mileage by more than 40% compared to classical truck delivery, even when a share of customers has to be supplied by truck. To give some further detail, the experiments show that (i) direct truck deliveries have a large impact on costs and solution structure (e.g., 46% higher costs and 119% higher mileage due to 4% of truck deliveries with MTR), (ii) by including direct truck deliveries in the tour, our approach leads to savings of up to 24% compared to a separation of truck and robot deliveries, and (iii) adapting the time windows for truck deliveries can help to further reduce costs and travel distance. Additional analyses highlight the benefits of a mixed delivery concept and show that the MTR results are robust across different settings. Applied to settings without truck deliveries, our approach is 37 to 94% faster than an existing state-of-the art approach.

While we address an important extension for TnR delivery, there are several other aspects that can be assessed in future research. Our model could test technical additions and infrastructure specifics such as faster robot travel on bike lanes. Robot movements between depots may further help to increase robot availability in depots visited by truck. The exchange of robots between depots might therefore be a next step. In line with this, our model could be extended to include the pickup of robots at drop-off points on the tour. This means that robots could be sent to locations other than robot depots. Stochastic travel times and pickups from customers could be considered to generalize the problem. Other innovative last-mile delivery concepts could be compared to MTR to derive guidance on which concept and fleet mix to implement in which setting. To date, the TnR and MTR routing approaches have focused on a single truck tour. The use of multiple tours and the corresponding allocation of customers to different tours is required in settings with higher order volumes. Ultimately, the problem presented demonstrates situations of high complexity and unique structure for which alternative solution approaches can be tested. Those could assist in accelerating computation, dealing with larger problem sizes or evidencing optimality. We show that the MTR can significantly reduce total mileage and transportation costs. A detailed analysis of the impact on noise and CO_2 emissions of the different transportation modes constitutes a future area of research.

Acknowledgment

We would like to thank the editor and three reviewers for their valuable advice on improving this work.

Appendix A. A MIP model for the VRP with time windows

For solving the VRP with time windows, we introduce the following MIP model, which we adapted from Ostermeier et al. (2021) to incorporate time windows instead of only deadlines. It minimizes the cost of traditional truck delivery assuming the same cost factors as in the MTR case. We further assume the same processing time of 40 sec. for every customer k (included in ϑ^t_{ik}). We introduce the set of available vehicles F, which contains only one vehicle in our case. The binary decision variable s_{fij} is 1 if vehicle f travels from location i to location j and 0 otherwise. Finally, auxiliary decision variable t_k denotes the arrival time at customer k and t^T_f the total tour time of vehicle f. This leads to the objective function (A.1), which incorporates the cost of truck distance, truck time and delays. Constraint (A.2) ensures every customer is visited exactly once. (A.3) and (A.4) keep track of the earliest possible arrival times at customers. Constraint (A.5) ensures no customer is served before his/ her time window and Constraint (A.6) derives the delays from the arrival times. (A.7) defines the total operating time of each truck. (A.8) and (A.9) establish flow constraints for the trucks at every stop. Constraints (A.10) to (A.12) define the solution space.

$$\min \sum_{f\in F}\sum_{i\in C\cup\{\gamma\}}\sum_{j\in C\cup\{\gamma\}} c^{\mathrm{d}}\lambda_{ij}s_{fij} + \sum_{f\in F} c^{\mathrm{t}}t^T_f + \sum_{k\in C} c^{\mathrm{l}}\upsilon_k \tag{A.1}$$

subject to

$$\sum_{i\in C\cup\{\gamma\}}\sum_{f\in F} s_{fik} = 1 \qquad \forall k \in C \tag{A.2}$$

$$t_k \geq \vartheta^t_{\gamma k} - M\cdot(1 - s_{f\gamma k}) \qquad \forall k\in C, f\in F \tag{A.3}$$

$$t_j \geq t_i + \vartheta^t_{ij} - M\cdot(1 - s_{fij}) \qquad \forall i,j\in C, f\in F \tag{A.4}$$

$$t_k \geq d_k - \epsilon \qquad \forall k\in C \tag{A.5}$$

$$\upsilon_k \geq t_k - d_k \qquad \forall k\in C \tag{A.6}$$

$$t^T_f \geq t_k + \vartheta^t_{k\gamma} - M\cdot(1 - s_{fk\gamma}) \qquad \forall k\in C, f\in F \tag{A.7}$$

$$\sum_{i\in C\cup\{\gamma\}} s_{fik} = \sum_{i\in C\cup\{\gamma\}} s_{fki} \qquad \forall k\in C, f\in F \tag{A.8}$$

$$\sum_{k\in C} s_{f\gamma k} \leq 1 \qquad \forall f\in F \tag{A.9}$$

$$s_{fij} \in \{0,1\} \qquad \forall i,j\in C, f\in F \tag{A.10}$$

$$t_k \geq 0;\ \ \upsilon_k \geq 0 \qquad \forall k\in C \tag{A.11}$$

$$t^T_f \geq 0 \qquad \forall f\in F \tag{A.12}$$

Appendix B. Exemplary tours for the different delivery area settings

Figures B.15, B.16 and B.17 show exemplary routing solutions for the uniform, concentrated and clustered scenarios defined in Section 6.6. Please note the different scales of the figures for the different settings. The solid blue arrows represent the truck legs, the dotted blue lines the robot legs. In the concentrated setting, the truck distance can be reduced compared to the uniform setting as the customers are closer to each other (2km vs. 4km delivery area). The distance is also reduced in the clustered setting, even though the time windows are still assigned randomly (not separated by cluster, see Section 6.1). Since the robots can arrive early and wait for the desired time window, it is often most efficient for the truck to serve the clusters sequentially and therefore reduce the distance traveled. The longest distance that needs to be covered in the clustered setting is the way between the two clusters.

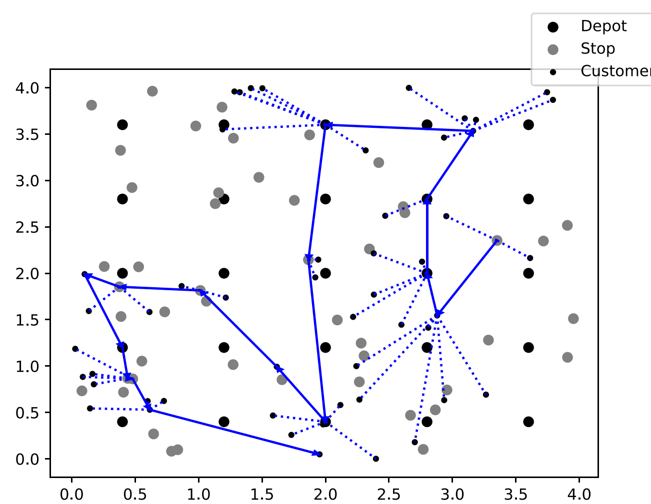

Fig. B.15. MTR routing result for the uniform customer distribution.

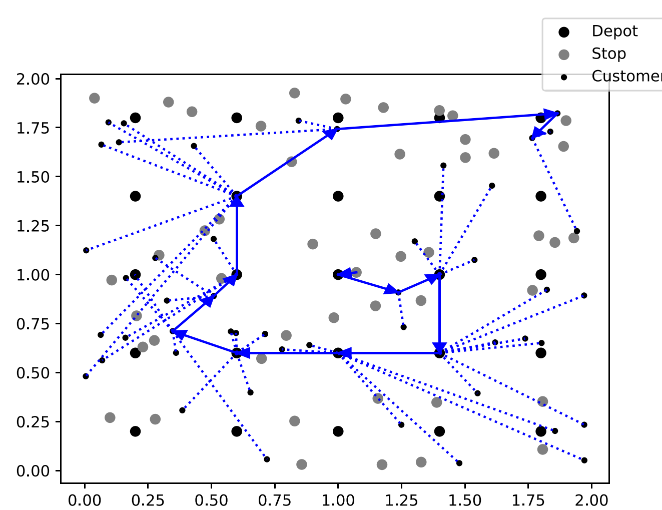

Fig. B.16. MTR routing result for the concentrated customer distribution.

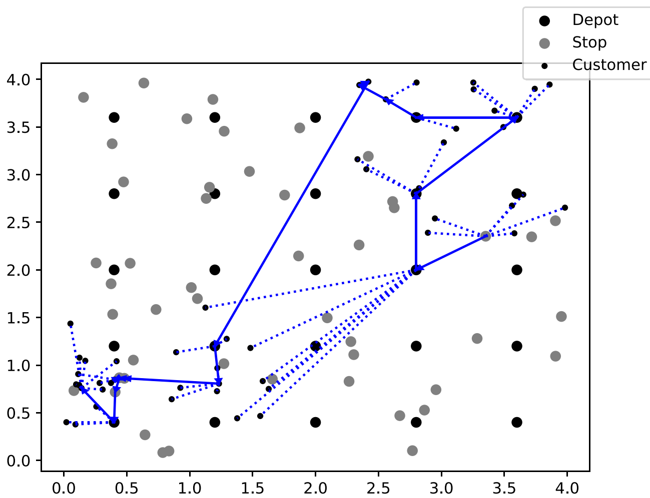

Fig. B.17. MTR routing result for the clustered customer distribution.

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