Explaining the decline in the US labor share: taxation and automation

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Abstract

This study provides evidence for the USA that the secular decline in the labor share is not only explained by technical change or globalization, but also by the dynamics of factor taxation, automation capital (robots), and population growth. First, we empirically find indications of co-integration for the period from the last quarter of the 20th to the first decade of the twenty-first century. Permanent effects on factor shares emanate from relative factor taxation. The latter also have a lasting effect on the use of robots. Variance decompositions reveal that taxing contributes to changes in the two income shares and in automation capital. Second, we analyze and calibrate a neoclassical growth model extended to include factor taxation, automation capital, and capital adjustment costs. Labor and automation capital are perfect substitutes, whereas labor and traditional capital are complements. The model replicates the dynamics of the observed functional income distribution in the USA during the 1965–2015 period. Counterfactual experiments suggest that the fall in the labor share would have been significantly smaller if labor and capital income tax rates had remained at their respective level of the 1960s.

Keywords Functional income distribution · Labor income share · Income taxes · Automation capital · Demography · Growth

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1 Introduction

The functional income distribution in most OECD countries has changed significantly over recent decades. Figure 1 displays the labor share of the USA, Japan, and the Euro Area (EA-12) from 1960 to 2018. During this period, these labor shares declined by roughly 5–15 percentage points.

The existing literature emphasizes the role of several factors that contributed to this decline. They include skill-biased technological change (Goldin & Katz, 2008), declining investment good prices (Karabarbounis & Neiman, 2014), a sufficiently high elasticity of substitution between capital and labor (Piketty, 2014), globalization (Elsby et al., 2013; Helpman, 2018), rising profit shares of monopolistic firms and "superstar firm" dynamics (Eggertsson et al., 2019; Autor et al., 2020), or population aging (Irmen, 2021). In contrast with these studies, the present paper emphasizes the effect of taxation and population growth on the implementation of automation capital and the labor share.

The focus of our analysis is on the USA where two related evolutions accompany the decline in the labor share. First, as shown in the left panel of Fig. 2, the difference in the effective tax rates on income from capital and labor shrinks from 1947 to 2010. Over this time span, capital income is taxed more heavily with an average effective tax rate of 41% compared to 23% for labor income. However, compared to their respective levels in 1947 the effective tax rate on income from capital is much lower in 2010, whereas the one for labor income is much higher. Moreover, in the recent past, labor is even taxed more heavily than capital. Second, as shown in the right panel of Fig. 2, in parallel with the declining labor share the population growth rate has fallen, in annualized terms, from 1.7% in 1950 to 0.7% in recent years.
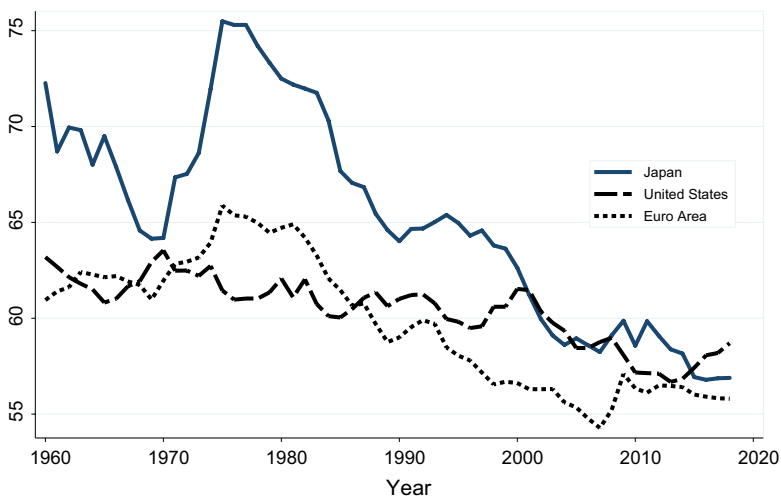
Fig. 1 Labor share, 1960–2018: USA, Japan, and Euro Area (EA-12). *Note:* Labor share is wage income in GDP at current prices; *Source*: AMECO database, OECD
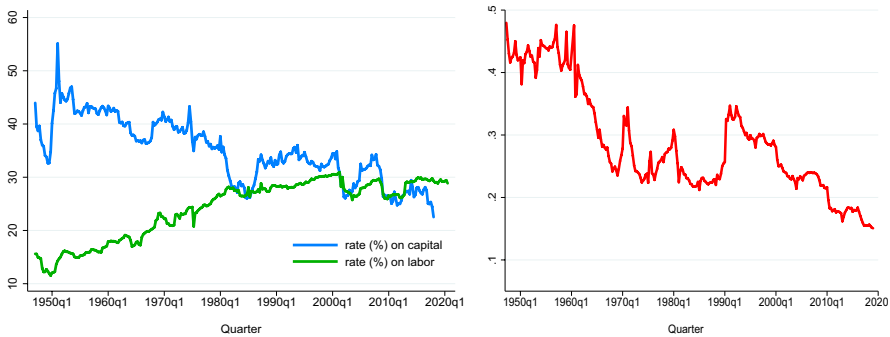
Fig. 2 Left panel: Quarterly US tax rates on income from labor and capital (%); right panel: US population growth rates (%) *Sources:* Left panel: https://paulgomme.github.io; right panel: BEA for population growth, seasonal adjustment: X12-ARIMA

Intuitively, the shrinking difference in the taxation of capital and labor and the decline in population growth induce firms to choose production processes that replace labor more and more with automation capital. On the one hand, the downward trend in the capital income tax rate increases the incentive to build up capital. As long as different types of capital are tied by a no-arbitrage condition, some of the additional capital will come in the form of automation capital.

On the other hand, the labor supply declines at the intensive margin if the labor income tax rate increases. In addition, a declining population growth rate reduces the labor supply at the extensive margin. As argued, e. g., by Heer and Irmen (2014), the relative decline in the labor force at both margins pushes wages up and, therefore, boosts the incentive to engage in labor-saving automation investments.

Hence, these tendencies strengthen the comparative advantage of automation capital in production. This leads to the prediction that the amount of automation capital per worker should increase. Figure 3 confirms this prediction: the time series of automation capital proxied by the (nowcasted) stock of robots per 1000 (full-time) employees in the USA from 1975 to 2010 is clearly increasing. Finally, as automation capital replaces labor, the labor share is expected to decline.

Our analysis derives two main sets of results. First, we provide empirical evidence for the USA supporting the explanation for a declining labor share set out above. We find indications for cointegrating relationships over the period from the first quarter of 1974 to the fourth quarter of 2008 (henceforth, 1974:q1–2008:q4). Permanent effects on factor shares emanate from shocks in relative factor taxation. Changes in relative factor taxation also permanently and sizably affect the use of automation capital. The forecast error variance decomposition (FEVD) analysis of fitted vector error correction (VEC) models reveals that taxing policies account for up to roughly 22% of observed changes in the two income shares and for up to about 35% of the dynamics in automation capital.

The second set of results emanates from the quantitative analysis of a dynamic general equilibrium model that replicates the downward trend in the US labor share from 1965 to 2015. We derive these findings in a new variant of the neoclassical
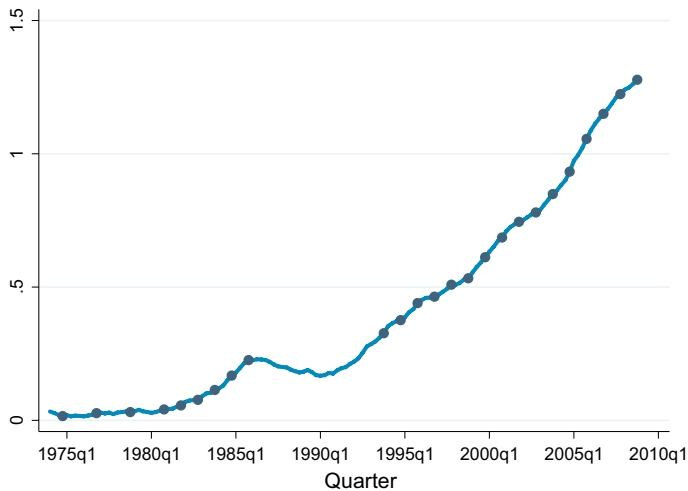
Fig. 3 Robots per 1K workers in the USA: annual and nowcasted quarterly data. *Note*: *Dotted* data points up to 1985:q4 are own calculations based on Tani (1989) and CPS; from 1994:q4 onwards they are obtained from Acemoglu and Restrepo (2019), original source: International Federation of Robotics (IFR); *Solid* blue line: nowcasted (pseudo-)quarterly data; detail on nowcasting is given in Appendix A.1. *Sources*: Tani (1989), Acemoglu and Restrepo (2019), IFR, CPS (Color figure online)

growth model where taxes on capital and labor income as well as population growth play a key role for the dynamics of the functional income distribution. We follow Steigum (2011) and distinguish two types of capital, traditional capital and automation capital (robots). While traditional capital is complementary to labor and automation capital, the latter two factors of production are perfect substitutes. Hence, automation capital substitutes linearly for labor. We extend Steigum's framework in three directions. First, both types of capital investments are subject to adjustment costs. Second, we allow for an endogenous labor supply and, finally, include an active government that charges taxes to finance its consumption.

We use this model to compute the dynamics of the labor share, the stocks of traditional and automation capital, and the evolution of the (endogenous) labor supply over the period 1965–2029. We maintain that automation capital was not introduced into production processes until 1965.[1] Our calibration incorporates the time series of effective capital and labor income tax rates as well as of the population growth rates that are displayed in Fig. 2. The use of quadratic adjustment costs for both types of capital investments is in the tradition of Hayashi (1982). As a result, our model is able to closely replicate the downward trend in the labor share. In particular, in line

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[1] Historical accounts suggest that the use of robots in the USA started slowly in the early 1960s. They were systematically introduced into US manufacturing only in the mid-to-late 1960s (see, e.g., Encyclopedia Britannica, https://www.britannica.com/technology/robot-technology, accessed April 25, 2022), Steigum (2011), p. 543, or Jeremy Gottlieb and David Leech Anderson (https://mind.ilstu.edu/curriculum/medical_robotics/robots_in_beginning.html, accessed April 25, 2022). Figure 3 suggests that robots basically amounted to zero prior to 1980 averaging at less than 1 robot per 5000 workers.

with the data, we match the actual drop in the labor share from 62 to 57% between 1965 and 2015.

The present paper is related to several strands of the literature. First, our empirical analysis contributes to a young and growing empirical literature that studies the interplay between automation and institutional as well as macroeconomic variables in the context of economic growth. To some extent, this literature has revived an old debate about the secular stagnation hypothesis (Hansen, 1939). Acemoglu and Restrepo (2017) find evidence that supports the view that population ageing induces higher GDP per capita growth through automation. This contrasts with the conclusions drawn in Aksoy et al. (2019). So far, the main focus of this literature has been on longitudinal data. The seminal study by Graetz and Michaels (2018) covers 14 years and 14 sectors in 17 countries. It is longitudinal with two cross-sectional dimensions, economies and industries. Acemoglu and Restrepo (2019) are also longitudinal in nature using data on robots and exploiting the geographic variation within the USA. In the context of the rise of robots in China, Cheng et al. (2019) rely on longitudinal survey-based manufacturing firm-level data, too. These studies reveal insightful and, in parts, even causal statistical associations between automation, institutional policies, and macroeconomic variables. However, the empirical establishment of long run—in the sense of cointegrating—relationships between these variables is missing.[2] The present paper seeks to fill this gap.

Second, our theoretical analysis shows that an appropriately augmented variant of Steigum (2011)'s model provides a tractable framework of analysis for macroeconomic phenomena related to the substitution of labor with automation capital that may serve as an alternative to existing models used in the literature including, among others, Acemoglu and Restrepo (2018), Berg et al. (2018), Eden and Gaggl (2018), Irmen (2021), or Hémous and Olsen (2022). Our quantitative analysis suggests that the inclusion of adjustment costs in the spirit of Hayashi (1982) for both types of capital into Steigum's model proves particularly useful for a realistic description of the introduction and the buildup of robots.

Finally, and relatedly, our calibration analysis highlights an important role of factor taxation for the incentives to automate and the evolution of the US functional income distribution. This adds a new positive explanation for the observed decline in the US labor share to the literature mentioned above. In particular, our counterfactual experiments suggest that the labor share in 2015 could have been substantially higher had the tax rates on capital and labor income remained at their 1965 level. These findings support the normative assessment of Acemoglu et al. (2020) who argue that the US tax system generates excessive automation incentives and implies a suboptimally low labor share. In a similar vein, we conclude from a simple welfare experiment that an optimal tax on capital income could be quite high.

The paper is organized as follows. In Sect. 2, we present and interpret the time series evidence for the US economy relying on cointegration analysis and VEC

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[2] Bergholt et al. (2019) make an attempt in this direction. These authors estimate sign-restricted structural vector autoregressive (SVAR) models with permanent shocks to study "medium-run" trend relationships in this context.

models. Section 3 studies a variant of the neoclassical growth model that distinguishes between traditional and automation capital and allows for factor taxation and capital adjustment costs. Sections 3.1–3.4 introduce the model and define the competitive equilibrium with dynamic taxation. Section 3.5 provides an analysis of the steady state without automation capital and analyzes the properties of the asymptotic balanced growth path with automation capital. Section 3.6 explains how we calibrate the model. Section 3.7 features our results on the transition dynamics. Here, we first devise a calibration that replicates the actual decline of the US labor share during the period 1965–2015. Second, we conduct counterfactual experiments suggesting that the effective tax rates on capital and labor income had a significant effect on the labor share over the considered period. Section 3.8 discusses the role of tax policies from a normative point of view. Section 4 concludes. The Appendix includes details on and discussions of our empirical (Sects. A.1–A.4) and our theoretical analysis (Sect. A.5).

2 Empirical analysis

This section provides a test for and a quantification of long-run equilibrium relationships between the US income shares and factor taxation, automation capital, and population growth. Here, 'equilibrium' does not refer to market clearing. Rather, it defines a state of rest. Long-run equilibrium relationships entail systematic movements of macroeconomic variables that an empirical model—compatible with nonstationary, long-run dynamics of time series—seeks to test for. Cointegration suggests the presence of some long-run equilibrium relations that tie the constituents of the modeled system even though some developments may cause permanent changes. Correspondingly, a neoclassical (stochastic) growth model implies several steady-state relations among the natural logarithms of its central variables. A *conditio sine qua non* to fledge such a model with dynamic taxes and automation capital to study functional distribution outcomes is to establish such long-run relationships empirically. A VEC model framework allows for the determination of the dimension of the cointegrating space and for tests of such structures. Thus, the theory of cointegrated time series can provide a test-based justification for the specification of a neoclassical growth model.

Our empirical analysis finds permanent effects of factor taxation and population growth on factor shares and the use of robots, respectively. Pairwise error correcting, i. e., cointegrating, relationships are established for the following pairs of variables: (1) relative factor taxation and the capital share, (2) use of robots and the capital share, and (3) population growth and the labor share. The relationships (1) and (3) are statistically significant at conventional levels. Relationship (2) holds only at a 68% level of confidence which, however, is not unusual in a time series context (see, e. g., Baumeister and Hamilton 2018).

Establishing equilibrium restoring mechanisms in the short and medium-run requires historical series of adequate length and frequency. We document issues related to the compilation and the construction of such time series in the next section. Then, relevant details concerning our methodological framework and the

identification strategy are provided. The section ends with an illustration of the responses of factor shares and automation capital to factor taxation and population growth. Detail on the contribution to variance of relative factor taxation to income shares and to the use of robots is provided.

2.1 Time series

We work with time series in quarterly frequency for the US economy ranging from the first quarter of 1974 to the fourth quarter of 2008. The limiting factor with regard to the span of our sample period is the automation capital series. Tani (1989) provides the numerator of the first part of actual datapoints for the nowcast of the automation capital series as shown in Fig. 3. Underlying is a series of biannual frequency for the 1970s and, from 1980 to 1985, of annual frequency. This series states the industrial robot population in the USA, where an industrial robot is defined by the Industrial Organization for Standardization (ISO).[3] The denominator in the construction of datapoints prior to 1986 shown in Fig. 3 is the corresponding annual average of the seasonally adjusted (SA) number of full-time employees in the USA as collected in the Current Population Survey (CPS). The ISO normed definition of robots and the expression in units of "per thousand workers" allows us to combine it with the corresponding annual data from the International Federation of Robotics (IFR) as provided by Acemoglu and Restrepo (2019) and to nowcast a quarterly series of automation capital. Our nowcast is based on the procedure proposed by Shumway and Stoffer (2008). It relies on the Kalman filter in combination with the expectation maximization (EM) algorithm. A detailed outline of this technique is contained in Appendix A.1. The quarterly series for US effective tax rates on income from capital and labor are provided by Gomme et al. (2011). The construction of the tax rates series is based on the National Income and Product Accounts (NIPA). However, taxes levied on capital and paid by households have to be imputed. The imputation follows a methodology that is generally accepted in the empirical macroeconomic literature (Gomme et al. 2011, p. 266). For further details on the setup of the two series, the interested reader is referred to Appendix A.1.

Fernald (2014) provides the capital share series in quarterly frequency. Quarterly series for the US population (in thousand) stem from the BEA and are provided in the FRED database. We seasonally adjust this series by means of an X12-ARIMA and consider its log first differences transform. Also from FRED we retrieve the BEA series of the US quarterly gross compensation of employees in the form of paid wages and salaries as well as the corresponding GDP series.

These procedures prepare the data set for a multivariate cointegration analysis. It comprises series of quarterly frequency for the US income shares, factor taxation,

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[3] Accordingly, an "industrial robot is an automatic position-controlled reprogrammable multifunctional manipulator having several degrees of freedom capable of handling materials, parts, tools, or specialized devices through variable programmed motions for the performance of a variety of tasks" (Tani 1989, p. 192).
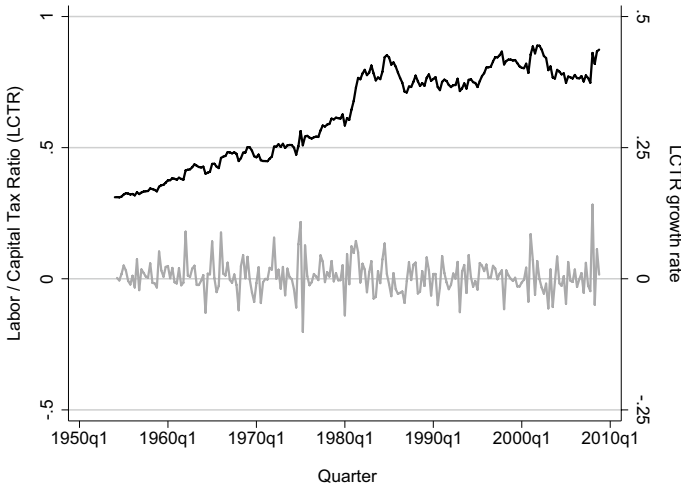
Fig. 4 Labor-tax-to-capital-tax ratio (LCTR), 1954:q1 to 2008:q4. *Note*: black—levels (left ordinate); gray—log first differences (right ordinate); *Source*: Gomme et al. (2011)

population growth, and automation capital. The sample period covers all quarters from 1974 to 2008.

2.2 VEC model analysis

We rely on the maximum likelihood (ML) based framework for estimation and inference in cointegrating systems often referred to as the "Johansen approach" (Johansen, 1995). The variables of interest include the two effective tax rates as well as the two factor shares. Due to the natural log transformation, it is technically feasible to jointly consider the shares of labor and capital income. However, a (near perfect) collinearity prevents the joint integration of both income tax rates into a particular VECM specification. An alternative to including both tax rates is the construction and use of the labor-tax-to-capital-tax ratio (LCTR, Fig. 4) which serves as a workaround in our VECM specifications. It organically takes care of the historical tax policy mix.

Our reduced form (RF) VECM space consists of three dimensions: a relatively exogenous variable X_t (population growth), a policy variable Y_t (the factor tax policy mix, i. e., the LCTR), and a multivariate group of response variables $\mathbf{W}_t$ (our automation capital proxy and the two factor shares), making it

$$\mathbf{Z}_t = \begin{bmatrix} X_t, Y_t, \mathbf{W}_t \end{bmatrix} \Rightarrow \mathbf{Z}_t = \mathbf{A}_1 \mathbf{Z}_{t-1} + \mathbf{A}_2 \mathbf{Z}_{t-2} + \cdots + \mathbf{A}_p \mathbf{Z}_{t-p} + \mathbf{u}_t. \qquad (2.1)$$

In standard VEC notation this becomes

$$\Delta \mathbf{Z}_t = \Gamma_1 \Delta \mathbf{Z}_{t-1} + \Gamma_2 \Delta \mathbf{Z}_{t-2} + \cdots + \Gamma_{p-1} \Delta \mathbf{Z}_{t-p-1} + \Pi \mathbf{Z}_{t-1} + \mathbf{u}_t, \qquad (2.2)$$

where $\Gamma_i = \left(\mathbf{I} - \mathbf{A}_1 - \mathbf{A}_2 - \cdots - \mathbf{A}_p \right)$ for all $i = 1, \ldots, p$. Π may be thought of as consisting of an adjustment speed matrix $\mathbf{a}$ and a long-run coefficient matrix $\mathbf{b}$.

Then, $\Pi = \mathbf{ab}'$, where $\mathbf{b}'\mathbf{Z}_{t-1}$ is the vectorial analogue of the error correction term in the Engel-Granger approach.

2.3 Identification strategy

Our identification scheme for the $\Delta\mathbf{Z}$ vector autoregression (VAR) part of the VECM specification—modeling the short to medium term cyclical dynamics—resembles what has become known as "Slow-r-Fast" scheme in the literature (Stock and Watson 2016, p. 455, pp. 477–478).[4] It delivers a block recursive scheme with an ordering of the respective elements of the partitioned $\mathbf{Z}_t$ vector that is not decisive for the system rotation matrix. In our application, population growth is supposed to be "the least" endogenous and does not respond to the policy variable, i. e., to the LCTR, and to any of the response variables within a quarter. The response variables are comprised of our automation capital measure and the two factor shares. As we focus on the responses to shocks in the tax policy variable, the ordering within $\mathbf{W}_t$ is uncritical. For the $\Delta\mathbf{Z}$/VAR-part of the respective VECM, the following block recursive scheme to identify responses to shocks ε, with orthogonalized analogues η, is implied

$$\begin{pmatrix} \eta_t^X \\ \eta_t^Y \\ \eta_t^{\mathbf{W}} \end{pmatrix} = \begin{pmatrix} H_{XX} & 0 & 0 \\ H_{YX} & H_{YY} & 0 \\ H_{\mathbf{W}X} & H_{\mathbf{W}Y} & H_{\mathbf{WW}} \end{pmatrix} \begin{pmatrix} \varepsilon_t^X \\ \varepsilon_t^Y \\ \varepsilon_t^{\mathbf{W}} \end{pmatrix} \text{ for } \Phi(L)\Delta\mathbf{Z}_t = \begin{pmatrix} \eta_t^X \\ \eta_t^Y \\ \eta_t^{\mathbf{W}} \end{pmatrix}, \qquad (2.3)$$

where $\mathbf{Z}_t$ is partitioned $\mathbf{Z}_t = \left(X_t\ Y_t\ \mathbf{W}_t \right)'$, and $H_{\mathbf{WW}}$ is squared. Our lag order choice $p = 13$ for (2.1), (2.2) is supported by the adequate Likelihood ratio (LR) test.

2.4 Results

The Johansen testing procedure, detailed in "Appendix A.2," fails to reject the null of at most three cointegrating equations of our VECM representation. The second cointegration equation in the Johansen normalization identification clearly indicates a statistically significant equilibrium relationship between the LCTR series and the two factor shares. Additionally, our specification is stable adhering to the implied eigenvalue stability condition.

Impulse response (IR) functions of factor shares in response to a relative increase in taxing labor vis-à-vis capital, i. e., to a positive LCTR shock, are given in Fig. 5. A positive one percent LCTR shock implies a permanent increase in the capital share of production of about 0.64%. In contrast, the labor share response is negative (see the

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[4] The name stems from the order of partitioning of the dependent variables' vector by nature of its, partially sub-vectorial, elements. It has been used, in particular, to identify monetary policy shocks (Bernanke et al., 2005). Under this scheme, so-called slow-moving variables such as output and prices do not respond to monetary policy rate dynamics or to movements in fast-moving variables, such as expectational variables, within the period.
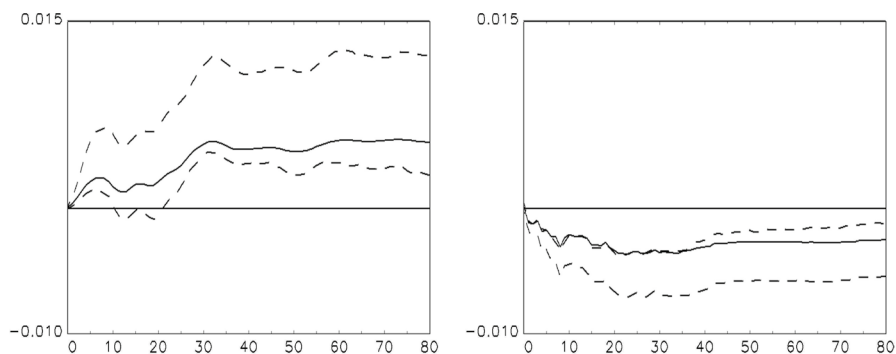
Fig. 5 Orthogonalized IR functions of factor shares to LCTR shock. *Note*: left (right) panel: impulse variable is LCTR; response variable is capital (labor) share of production. Solid lines represent IR functions obtained from a VEC model specification (2.1), (2.2) with a 'Slow-r-Fast' identification scheme for its VAR-part, a cointegration rank of two, and a lag order of 13. The dashed lines depict 68% Hall's percentile method-based bootstrap confidence intervals (see, Benkwitz et al. 2001 for details)

	Step	Capital share	Labor share	Automation capital
Table 1 FEVD values for an LCTR shock by response variable	10	0.010460	0.126208	0.296361
	20	0.050732	0.118803	0.354056
	30	0.080220	0.125973	0.347809
	40	0.085780	0.127680	0.332902
	50	0.086906	0.129162	0.325100
	60	0.090602	0.128036	0.313584
	70	0.091387	0.123696	0.299503
	80	0.091690	0.117447	0.289272

right schedule of Fig. 5). It is also permanent in nature and of similar size. However, the point estimate shows a reaction to a one percent LCTR impulse that lies slightly above a permanent decrease in the labor share of −0.64%. It amounts to approximately −0.59%. Nevertheless, in terms of its size, the decline in response to a one percent LCTR shock covers also −0.64% within the range of reasonable confidence. The latter is—in the time series context—frequently given by a 68% band (Fig. 5).

Table 1 shows FEVD statistics for the three response variables, the capital share, the labor share, and automation capital, given a positive LCTR shock. After about 30–40 quarters, relative factor taxation accounts for up to 9% (13%) of observed changes in the capital (labor) share and for up to 35% of the dynamics in the automation capital series. While the effect starts to peter out after 20 years (falling below 30%) for the use of robots, it only slightly falls for the two factor shares. Bear in mind, in contrast with the following model simulations of Sect. 3, these contributions rely on RF empirical models and empirical series. Thus, they do not truly isolate the effects of factor taxation. Nevertheless, the VECM analysis, following the generic Johansen procedure and resting on a "Slow-*r*-Fast"-style identification

strategy for its VAR-part, has uncovered three significant long-run equilibrium correcting, i. e., cointegrating, relationships (see Appendix A.2 for details). These are given for the following three pairwise (cointegration) relationships: population growth and the labor share, relative factor taxation and the capital share of income, and the use of robots and the capital share of income.[5] This implies that the system is driven by two to three independent stochastic trends. Each of them is shared by the respective pair of cointegrated macroeconomic variables. It allows us to state that there exists, at least, an equilibrium relationship between population growth and the labor share and between relative factor taxation and the capital share, respectively. Each of the pairs of variables tends to develop—in the absence of shocks—in a lockstep that is proportional in nature. This is in the spirit of a long-run equilibrium relationship that entails a systematic co-movement among macroeconomic variables. For example, population growth tends to translate into a labor share dynamics that is proportional to it. This proportionality may only temporarily be perturbed by shocks and overlain by business cycle dynamics. Furthermore, we have illustrated that shocks to relative factor taxation have permanent effects on factor shares. Corresponding variance decompositions reveal that shock in the factor-tax policy mix contribute at some decade-long forecasting horizons to both changes in the two income shares and in automation capital.

3 A neoclassical growth model with dynamic taxes, automation capital, and adjustment costs

We conduct our analysis in a framework that extends the neoclassical growth model in three ways. First, we follow Steigum (2011) and distinguish two types of capital in the aggregate production function, i. e., *traditional capital* (structures, machines) and *automation capital* (robots). Traditional capital and labor are imperfect substitutes, whereas automation capital and labor are perfect substitutes. Second, we introduce adjustment costs associated with the installation of either type of capital. Finally and most importantly, we integrate dynamic taxes on capital and labor income to study their role for the installation of automation capital and the functional income distribution.

The economy comprises a household, a production, and a government sector in an infinite sequence of periods, $t = 0, 1, 2, \ldots, \infty$. At all t, there is a single manufactured good that serves as numéraire. This good may be directly consumed by households, invested, or collected by the government in the form of taxes. If invested, it serves in the next period either as traditional capital or as automation capital. If collected by the government, it is a means to provide contemporaneous government services to each member of the population.

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[5] Note, however, the last of these cointegration relationships is on the fence of being statistically significant (see Appendix A.2). It is significant only for a level of confidence of 68%. In the computation of IR functions shown in Fig. 5, we abstracted from the last of these cointegration relationships and based it on a rank 2 VECM.

3.1 Household sector

There is a single representative household with N_t members. The household size grows at rate n_t, i. e.,

$$N_{t+1} = (1 + n_t)N_t, \quad N_0 > 0. \tag{3.1}$$

The household's inter-temporal utility is

$$U_0 = \sum_{t=0}^{\infty} \beta^t \big[u(c_t, 1 - l_t) + v(g_t) \big], \quad 0 < \beta < 1, \tag{3.2}$$

where c_t, l_t, and g_t denote, respectively, per-capita consumption of the manufactured good, the individual supply of working hours, and the per-capita consumption of the services provided by the government at t. In each period, the household's time budget is normalized to unity. Hence, $1 - l_t$ is leisure at t. We refer to g_t as individual government consumption at t.

The periodic utility of a household member is additively separable in the utility enjoyed from consumption and leisure, $u(., .)$, and the utility derived from government consumption, $v(g_t)$. Hence, government consumption, g_t, does not affect the household's optimization with respect to consumption and labor.

The utility function $u : \mathbb{R}_+ \times [0, 1] \to \mathbb{R}$ is given by

$$u(c, 1 - l) = \frac{\left(c^\theta (1 - l)^{1-\theta} \right)^{1-\eta} - 1}{1 - \eta}, \quad 0 < \theta < 1, \quad \eta > 1, \tag{3.3}$$

where θ and $1 - \theta$ denote the weights attached to consumption and leisure.

Households own two kinds of assets, traditional capital, k_t, and automation capital, p_t (both in per capita terms). These stocks depreciate at the same rate, $\delta \in (0, 1)$, so that their respective accumulation is given by

$$(1 + n_t)k_{t+1} = (1 - \delta)k_t + i_t^k, \tag{3.4a}$$

$$(1 + n_t)p_{t+1} = (1 - \delta)p_t + i_t^p. \tag{3.4b}$$

Here, i_t^k and i_t^p denote per-capita investments in traditional and in automation capital, respectively. Following Hayashi (1982), we allow for symmetrical and quadratic adjustment costs for either type of capital. More precisely, for $x \in \{k, p\}$ an investment i_t^x requires

$$\phi(i_t^x, x_t) = i_t^x + \frac{a_1 \left(i_t^x \right)^2}{2(a_2 + x_t)}, \tag{3.5}$$

units of produced output at t where $a_1 > 0$ and $a_2 > 0$. The (small) constant a_2 allows for the transition from a regime with $p_t = 0$ into one with $p_t > 0$.[6]

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[6] The usual specification of adjustment costs has $a_2 = 0$ (see, e. g., Heer and Scharrer 2018).

Let w_t denote the real wage, r_t^k the rental rate of traditional capital, and r_t^p the rental rate of automation capital at t. Then, the household receives income from labor, $w_t l_t$, taxed at rate $\tau_t^w \in (0,1)$, and interest income on traditional capital, $r_t^k k_t$, and automation capital, $r_t^p p_t$, both taxed at rate $\tau_t^r \in (0,1)$. In addition, the household receives lump-sum government transfers, tr_t. Household income is spent on consumption, c_t, taxed at the constant rate $\tau^c > 0$, and on investment in both types of capital. Accordingly, the household's periodic budget constraint is

$$
\begin{aligned}
(1 + \tau^c)c_t + \phi(i_t^k, k_t) + \phi(i_t^p, p_t) \\
= (1 - \tau_t^w)w_t l_t + (1 - \tau_t^r)r_t^k k_t + (1 - \tau_t^r)r_t^p p_t + tr_t.
\end{aligned}
\tag{3.6}
$$

For given values $k_0 > 0$ and $p_0 \geq 0$, the representative household's optimal plan is a sequence $\{c_t, l_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}\}_{t=0}^\infty$ of per-capita variables that maximizes U_0 subject to (3.4a), (3.4b), (3.6), $l_t \in [0,1]$, and non-negativity constraints on c_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}. The first-order conditions of the household problem are (a detailed derivation of these conditions is found in Appendix A.5.1)

$$
\lambda_t(1 + \tau^c) = \theta c_t^{\theta(1-\eta)-1}(1 - l_t)^{(1-\theta)(1-\eta)},
\tag{3.7a}
$$

$$
\lambda_t(1 - \tau_t^w)w_t \leq (1 - \theta)c_t^{\theta(1-\eta)}(1 - l_t)^{(1-\theta)(1-\eta)-1} \text{ with "} < \text{" only if } l_t = 0,
\tag{3.7b}
$$

$$
q_t^k \leq \lambda_t\left(1 + \frac{a_1 i_t^k}{a_2 + k_t}\right) \text{ with "} < \text{" only if } i_t^k = 0,
\tag{3.7c}
$$

$$
q_t^p \leq \lambda_t\left(1 + \frac{a_1 i_t^p}{a_2 + p_t}\right) \text{ with "} < \text{" only if } i_t^p = 0,
\tag{3.7d}
$$

$$
q_t^k \geq \frac{\beta}{1 + n_t}\left\{\lambda_{t+1}\left[(1 - \tau_{t+1}^r)r_{t+1}^k + \frac{a_1\left(i_{t+1}^k\right)^2}{2(a_2 + k_{t+1})^2}\right] + q_{t+1}^k(1 - \delta)\right\}
\tag{3.7e}
$$

with "$>$" only if $k_{t+1} = 0$,

$$
q_t^p \geq \frac{\beta}{1 + n_t}\left\{\lambda_{t+1}\left[(1 - \tau_{t+1}^r)r_{t+1}^p + \frac{a_1\left(i_{t+1}^p\right)^2}{2(a_2 + p_{t+1})^2}\right] + q_{t+1}^p(1 - \delta)\right\}
\tag{3.7f}
$$

with "$>$" only if $p_{t+1} = 0$,

$$
0 = \lim_{t \to \infty} \beta^t \lambda_t k_{t+1},
\tag{3.7g}
$$

$$0 = \lim_{t \to \infty} \beta^t \lambda_t p_{t+1}. \tag{3.7h}$$

Here, λ_t, q_t^k, and q_t^p are the respective Lagrange multipliers on the periodic budget constraint (3.6) and on the capital accumulation equations (3.4a) and (3.4b).

Conditions (3.7a) and (3.7b) characterize the contemporaneous trade-off between the consumption demand and the labor supply of a household member. Since $u(c, 1 - l)$ is strictly concave on its domain and satisfies the Inada conditions

$$\lim_{c \to 0} \frac{\partial u(c, 1 - l)}{\partial c} = \infty \quad \text{and} \quad \lim_{l \to 1} \frac{\partial u(c, 1 - l)}{\partial l} = -\infty,$$

the optimal plan involves $c_t > 0$ and $l_t < 1$. However, (3.7b) allows for a corner solution $l_t = 0$ that obtains if

$$c_t > \frac{\theta}{1 - \theta} \frac{1 - \tau_t^w}{1 + \tau^c} w_t. \tag{3.8}$$

Hence, the individual labor supply vanishes if c_t, τ_t^w, or τ^c are sufficiently high relative to the wage.

Condition (3.7c) says that $i_t^k > 0$ if the marginal cost of producing one unit of traditional capital, $1 + a_1 i_t^k/(a_2 + k_t)$, expressed in period-$t$ utility as $\lambda_t \left(1 + a_1 i_t^k/(a_2 + k_t)\right)$ is equal to the marginal revenue from selling one unit of traditional capital expressed in period-t utility, $q_t^k \times 1$. Moreover, $i_t^k = 0$ if the costs exceed the benefit for the first marginal unit of investment in traditional capital. Mutatis mutandis, the interpretation of condition (3.7d) for automation capital is the same.

Condition (3.7e) says that $k_{t+1} > 0$ if the return associated with one additional unit of traditional capital per-capita in $t + 1$ shared by a population that has grown by a factor $1 + n_t$ and expressed in period-t utility is equal to the marginal revenue from selling one unit of traditional capital expressed in period-t utility, $q_t^k \times 1$.[7] Mutatis mutandis, the interpretation of condition (3.7f) for automation capital is the same.

Finally, conditions (3.7g) and (3.7h) state the transversality conditions on both types of capital.

3.2 Production sector

The production sector has a single competitive representative firm. Following Steigum (2011), this firm has access to the production function

$$Y_t = A \left[L_t + \kappa P_t\right]^{1-\alpha} K_t^\alpha, \quad A > 0, \tag{3.9}$$

---

[7] The return associated with one additional unit of traditional capital per-capita in $t + 1$ has three components. First, there is the after-tax rate of return $(1 - \tau_{t+1}^r)r_{t+1}^k$. Second, given i_{t+1}^k adjustment costs decline by $a_1 \left(i_{t+1}^k\right)^2/\left(2(a_2 + k_{t+1})^2\right)$. These returns can be consumed and add $\lambda_{t+1}\left[(1 - \tau_{t+1}^r)r_{t+1}^k + a_1\left(i_{t+1}^k\right)^2/\left(2(a_2 + k_{t+1})^2\right)\right]$ to period-$t + 1$ utility. Third, the remaining $1 - \delta$ units can be sold which generates a period-$t + 1$ utility return equal to $q_{t+1}^k(1 - \delta)$.

where L_t is employed hours worked, P_t the amount of hired automation capital, and K_t the amount of hired traditional capital. Hours worked and automation capital are perfect substitutes with a marginal rate of substitution equal to κ.[8]

The representative firm's optimal plan is a sequence $\{K_t, L_t, P_t, Y_t\}_{t=0}^{\infty}$ of factor demands and output supplies that maximizes the net present value of all current and future profits. This comes down to the maximization of periodic profits, Π_t, given by

$$\Pi_t = Y_t - w_t L_t - r_t^k K_t - r_t^p P_t.$$

The corresponding first-order conditions for all t are

$$(1 - \alpha)A\left[\frac{K_t}{L_t + \kappa P_t}\right]^{\alpha} \leq w_t, \text{ with “ < ” only if } L_t = 0, \tag{3.10a}$$

$$(1 - \alpha)\kappa A\left[\frac{K_t}{L_t + \kappa P_t}\right]^{\alpha} \leq r_t^p, \text{ with “ < ” only if } P_t = 0, \tag{3.10b}$$

$$\alpha A\left[\frac{L_t + \kappa P_t}{K_t}\right]^{1-\alpha} = r_t^k. \tag{3.10c}$$

These conditions reflect that K_t is essential but neither L_t nor P_t is.

Let LS_t denote the labor share at t. Using (3.10a)–(3.10c), the latter can be expressed as

$$LS_t = \begin{cases} 1 - \alpha & \text{if } L_t > 0, \ P_t = 0, \\ \frac{1-\alpha}{1+\kappa\frac{P_t}{L_t}} & \text{if } L_t > 0, \ P_t > 0, \\ 0 & \text{if } L_t = 0, \ P_t > 0. \end{cases} \tag{3.11}$$

Hence, in the presence of automation capital the labor share is smaller than $1 - \alpha$. Moreover, it declines in the automation capital intensity, P_t/L_t. This is the result of two reinforcing effects that become visible if the labor share is expressed as the ratio of the marginal to the average product of hours worked, i. e., $LS_t = w_t/(Y_t/L_t)$. A higher automation capital intensity reduces the marginal product of hours worked and boosts its average product.

---

[8] Below, we choose κ to be smaller than unity to obtain reasonable results in our calibrations. In contrast with the actual evolution of the labor share, a specification involving $\kappa = 1$ (and a corresponding value $A = 0.83$ that generates an asymptotic growth rate of 2%) predicts that the labor share drops to 46% after the first year during which automation capital is used, i. e., when $P_t > 0$.

3.3 Government sector

At all t, the government collects taxes and spends its receipts in the form of government consumption and lump-sum transfers while keeping its budget balanced. Let tax_t denote per-capita tax receipts. Then,

$$tax_t = \tau^c c_t + \tau_t^w w_t l_t + \tau_t^r r_t^p p_t + \tau_t^r r_t^k k_t. \tag{3.12}$$

Moreover, the government budget constraint reads

$$N_t g_t + N_t tr_t = N_t tax_t. \tag{3.13}$$

3.4 Dynamic competitive equilibrium with dynamic taxes

Given $K_0 > 0$, $P_0 \geq 0$, $N_0 > 0$, $N_t = (1 + n)^t N_0$, and a sequence of tax rates $\left\{ \tau^c, \tau_t^w, \tau_t^k \right\}_{t=0}^{\infty}$ the dynamic competitive equilibrium with dynamic taxes determines sequences of prices $\left\{ w_t, r_t^k, r_t^p \right\}_{t=0}^{\infty}$, allocations $\left\{ c_t, l_t, i_t^k, i_t^p, k_{t+1}, p_{t+1}, K_t, L_t, P_t, Y_t \right\}_{t=0}^{\infty}$, and government activities $\left\{ g_t, tax_t, tr_t \right\}_{t=0}^{\infty}$. The equilibrium sequences are determined by the following conditions for all $t = 0, 1, 2, \dots, \infty$:

(E1) The plan of the representative household satisfies conditions (3.7a)–(3.7h).
(E2) The plan of the representative firm satisfies (3.10a)–(3.10c).
(E3) The activities of the government satisfy (3.12) and (3.13).
(E4) The labor market clears, i. e.,

$$w_t \geq 0, \quad L_t \leq N_t l_t, \quad \text{and} \quad w_t(L_t - N_t l_t) = 0.$$

(E5) The market for both types of capital clear, i. e.,

$$r_t^k \geq 0, \quad K_t \leq N_t k_t, \quad \text{and} \quad r_t^k(K_t - N_t k_t) = 0,$$
$$r_t^p \geq 0, \quad P_t \leq N_t p_t, \quad \text{and} \quad r_t^p(P_t - N_t p_t) = 0.$$

(E6) The market of the final good clears, i. e.,

$$c_t + g_t + \phi\left(i_t^k, k_t\right) + \phi(i_t^p, p_t) = y_t. \tag{3.14}$$

Since the production function (3.9) exhibits constant returns to scale and factor markets are competitive, (E2) implies $\Pi_t = 0$. (E4) and (E5) describe the usual factor market clearing conditions. The price of labor cannot be negative, demand must not exceed supply, and an excess supply requires $w_t = 0$. Similarly, for the two stocks of capital.

Below it becomes clear that we have to deal with the cases where either $l_t = 0$, i. e., the supply of labor vanishes, or $p_t = 0$, i. e., the supply of automation capital vanishes. In the former case, the demand for hours worked must vanish, too. This is the case for all wages above $w_t = (1 - \alpha)A(K_t/(\kappa P_t))^\alpha$ which is then the equilibrium wage consistent with $L_t = 0$. Combining the latter with Eq. (3.8), we obtain the equilibrium condition under which the individual labor supply vanishes as

$$c_t > \frac{\theta}{1-\theta} \frac{1-\tau^w}{1+\tau^c} (1-\alpha) A \left(\frac{K_t}{\kappa P_t} \right)^\alpha. \tag{3.15}$$

Similarly, if the supply of automation capital is zero, then its demand must vanish. This is the case for all rental rates above $r_t^p = (1-\alpha)\kappa A (K_t/L_t)^\alpha$ which is then the equilibrium rental rate for automation capital consistent with $P_t = 0$.

Finally, (E6) states the goods market equilibrium in per-capita terms, i. e., the supply of produced output is equal to private and government consumption demand plus investment outlays for both types of capital.

3.5 Initial steady state and asymptotic balanced growth path

Let period $t = 0$ correspond to the year 1965. Prior to this period, we assume that the economy is in steady state without automation capital. More precisely, the population growth rate n_0, and the tax rates τ_0^c, τ_0^w, and τ_0^r are constant and equal to the respective values prevailing in 1965. Moreover, the growth rate of the economy is set equal to zero, and labor share is $LS = 1 - \alpha$ (see Appendix A.5.2 for details).

An asymptotic balanced growth path (ABGP) is defined as an equilibrium path to which the economy tends as $t \to \infty$ that satisfies

$$\lim_{t\to\infty} \frac{y_{t+1}}{y_t} = \lim_{t\to\infty} \frac{k_{t+1}}{k_t} = \lim_{t\to\infty} \frac{P_{t+1}}{P_t} = \lim_{t\to\infty} \frac{i_{t+1}^k}{i_t^k}$$
$$= \lim_{t\to\infty} \frac{i_{t+1}^p}{i_t^p} = \lim_{t\to\infty} \frac{c_{t+1}}{c_t} = 1 + \gamma \tag{3.16}$$

for a constant population growth rate, $n_t = n$, constant tax rates, $\tau_t^r = \tau^r$ and $\tau_t^w = \tau^w$, as well as a constant ratio $g_t/y_t < 1$. Here, $\gamma > 0$ is the asymptotic growth rate of per-capita variables.

We explain the derivation and the properties of the unique ABGP in Appendix A.5.2. Here, we simply note that in the limit $t \to \infty$ the production function (3.9) becomes $Y_t = A(\kappa P_t)^{1-\alpha} K_t^\alpha$. As P_t and K_t grow at the same rate along the ABGP, the asymptotic behavior of the model mimics the one of the AK-models of, e. g., Frankel (1962) or Romer (1986), and $\gamma > 0$ will be endogenously determined.

3.6 Calibration

We calibrate the neoclassical growth model with dynamic taxes, automation capital, and adjustment costs to match empirical characteristics of the US economy for the time span from 1965 to 2010. Periods correspond to years. Our calibration strategy comprises four steps. First, a subset of preference and production parameters, $\{\beta, \eta, \theta, \alpha, \delta\}$, is chosen in accordance with the initial steady state corresponding to the year 1965. Second, the adjustment costs parameters a_1 and a_2 are set in accordance with prior empirical estimates and the simulated series of automation capital during the initial phase of the transition. Third, the production parameters A and

κ are jointly chosen to imply an asymptotic endogenous growth rate of per-capita variables, γ, equal to the average annual growth rate during 1965–2010 and a labor share equal to 57% in 2010. Finally, the fiscal and population parameters are set in accordance with the time series evidence for the USA during 1965–2010. Table 2 summarizes the calibration.

Preferences We set $\beta = 0.96$ and $\eta = 2$. These values are commonly applied in dynamic general equilibrium models (see, e. g., in Heer and Maußner 2009). This calibration implies a steady-state real interest rate of 3.75% and an intertemporal elasticity of substitution equal to 1/2. The latter is approximately the midpoint of empirical estimates for $1/\eta$. The weights of consumption and leisure in utility, $\theta = 0.3938$ and $1 - \theta = 0.6062$, are chosen to generate an average labor supply equal to 0.30 in the initial steady state.

Production The production function parameters A and κ are chosen jointly to imply an asymptotic growth rate $\gamma = 2.0\%$ and a labor share equal to 57% in 2010.[9] The production elasticity of traditional capital, $\alpha = 0.38$, is set to imply a labor share equal to 62% in 1965. Following Trabandt and Uhlig (2011), the depreciation rate of capital is $\delta = 7.0\%$. The parameter of the adjustment cost function, $a_1 = 12.0$, which governs the speed of adjustment in the two capital stocks, is taken from Heer and Schubert (2012). The adjustment cost parameter a_2 is a small constant added mainly for numerical purposes. If $a_2 = 0$, then the marginal adjustment costs in the initial steady state without automation capital, $P_t = 0.0$, would be infinite and the accumulation of automation capital would never take off. The long-run effect of the parameter a_2 on aggregate variables is asymptotically zero. Moreover, its magnitude affects the growth rates of automation capital only in the initial phase of the transition. We, therefore, choose $a_2 = 0.10$ so that the simulated growth rates of automation capital P_t (=10.9%) is approximately equal to the annual growth rate of the robots during the period 1974–1980 (=10.7%) as depicted in Fig. 3.

Government The time series of the two income tax rates, τ_t^r and τ_t^w, are the estimates taken from Gomme et al. (2011) for the US economy during 1954–2010 that Fig. 2 displays. In 1965, τ_t^r and τ_t^w amount to 47.3% and 17.9%, in 2010 to 37.1% and 28.9% , respectively. The consumption tax rate $\tau^c = 5.0\%$ and the government share G/Y are chosen as in Trabandt and Uhlig (2011).[10]

3.7 Transition analysis

This section has two parts. First, we study the transition from 1965–2015 of our benchmark economy with automation capital, a strictly positive labor supply, and

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[9] The computation details are described in Appendix A.5.3.

[10] Note that the government share was not constant during the period 1965–2020 but fell from 23% to 18%. As a robustness check, we analyzed whether the decline in government expenditures relative to GDP had a significant quantitative effect on the decline of the labor share and simulated the model accordingly. However, we find that the fall in the government share contributes only 0.04 percentage points to the decline in the labor share in 2010. Therefore, we excluded government consumption from the set of explanatory factors of the labor share decline.

the dynamic tax rates.[11] The main result is that the model explains the actual drop in the labor share from 62% to 57% that took place between 1965 and 2015. Second, we run counterfactual experiments to highlight the role of the tax rates on capital and labor income for the decline in the labor share. We demonstrate that the labor share would have been several percentage points higher if either the tax rate on labor income alone or both tax rates had remained at their 1965 level.

3.7.1 Benchmark

Figures 6 and 7 illustrate the transition dynamics of our model economy for the time span 1965–2030. The transition is computed under the following assumptions:

1. The economy is in steady state prior to 1965 with a labor share equal to 62%. The tax rates on labor and capital income and the population growth rate are set equal to their value in 1965, i. e., 17.9%, 47.3%, and 1.7%, respectively.
2. The transition starts in 1965. The initial capital stock is given by the steady-state stock of traditional capital. Households build up savings and supply labor according to their Euler equations for both types of capital and their first-order conditions with respect to consumption and labor.
3. For the period 1965–2010, the tax rates and the population growth rates are equal to their empirical counterparts.
4. In 1965, the household also starts to invest in automation capital.
5. After 2010, the tax rates on labor and capital income as well as the population growth rate remain constant at 28.3%, 37.1%, and 0.9%, respectively.

The first row of Fig. 6 reveals that the household initially adjusts its asset portfolio and shifts wealth from traditional capital (upper left panel), k_t, to automation capital (upper right panel), p_t. After approximately 30 years, both types of capital start to grow over time. As we argue above, the asymptotic behavior of our model is similar to an AK-model, i. e., the economy converges to a balanced growth path with a growth rate of per-capita variables equal to 2.0%.

In the first year of the transition, the labor supply increases slightly from 0.3 in 1964 to 0.303 in 1965. As initially labor income taxes are low the household substitutes labor intertemporally so that l_t remains higher than 0.3 until 1973. Between 1965 and 2015, the labor supply drops from 0.303 to 0.262 (lower right panel).[12] This is the effect of two channels which operate simultaneously. First, automation capital replaces labor in production. Second, the incentive to supply labor falls with the increase in the tax rate on labor income. Experiment 1 below suggests that the

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[11] In our numerical analysis, we compute the transition dynamics over the period 1965–2175. By 2175, the deviation of the final values from their asymptotic steady state values is less than 0.001%. For reasons of exposition, we only display the first part of the transition. Some additional details on the computation of the transitional dynamics are found in Appendix A.5.4.

[12] Eventually, the labor supply vanishes completely. In our benchmark, $l_t = 0$ is reached in the year 2132.

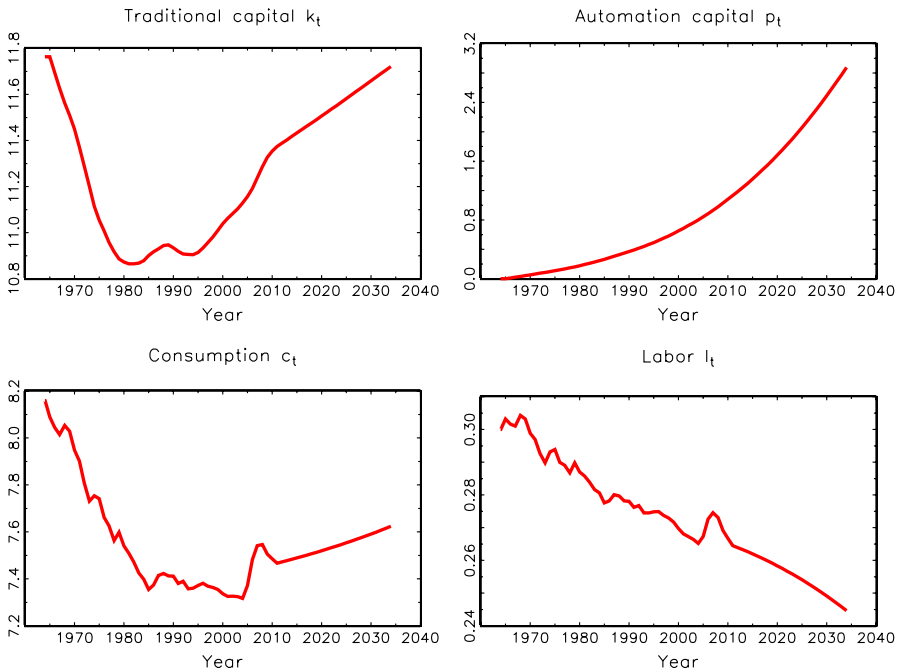**Table 2** Calibration of parameters in the benchmark growth model

Parameter	Value	Description
Preferences		
β	0.96	Discount factor
$1/\eta$	1/2	Intertemporal elasticity of substitution
θ	0.3938	Weight of consumption in utility
Production		
α	0.38	Production elasticity of capital
A	9.60	Total factor productivity
κ	0.019	Marginal rate of substitution between labor and automation capital
δ	0.07	Depreciation rate of capital
(a_1, a_2)	(12.0,0.10)	Adjustment cost parameters
γ	2.0%	Asymptotic growth rate of per-capita variables
Fiscal parameters		
$(\tau^r_{1965}, \tau^r_{2010})$	(47.3%, 37.1%)	Capital income tax rates in 1965 and 2010
$(\tau^w_{1965}, \tau^w_{2010})$	(17.9%, 28.9%)	Labor income tax rates in 1965 and 2010
τ^c	5.0%	Consumption tax rate
G/Y	18%	Share of government spending in production

increase in τ^l_t from 17.9 to 28.3% during 1965–2010 is the driving force behind the decline in l_t.

Per-capita consumption (lower left panel in Fig. 6) falls between 1970 and 1985 for two reasons. First, the net wage income declines with a falling labor supply and increasing tax rates on labor income. During 2004–2010, the labor supply displays a hump-shaped increase which reflects the temporary fall in the labor income tax rate, τ^w_t, during these years (see Fig. 2). Second, the household increases savings as, on average, the tax rates on capital income fall during 1965–2010 (see Fig. 2). Therefore, consumption declines during the first 30 years of the transition. Eventually, the growth effect sets in and consumption starts to increase after 2004. In the long run, consumption grows at the endogenous asymptotic growth rate. Notice the hump-shaped dynamics of consumption during the years 2004–2011 that mirrors the evolution of the labor supply and, hence, of (net) wage income.

The evolution of the factor prices w_t and r^k_t over 1965–2030 are displayed in the upper row of Fig. 7. Their evolution reflects the first-order conditions (3.10a)–(3.10c) that define a factor-price frontier linking r^k_t, r^p_t, and w_t according to

$$r^k_t = \alpha(1-\alpha)^{\frac{1-\alpha}{\alpha}} \kappa^{\frac{1-\alpha}{\alpha}} A^{\frac{1}{\alpha}} \left(r^p_t\right)^{-\frac{1-\alpha}{\alpha}} \quad \text{and} \quad w_t = \frac{r^p_t}{\kappa}.$$

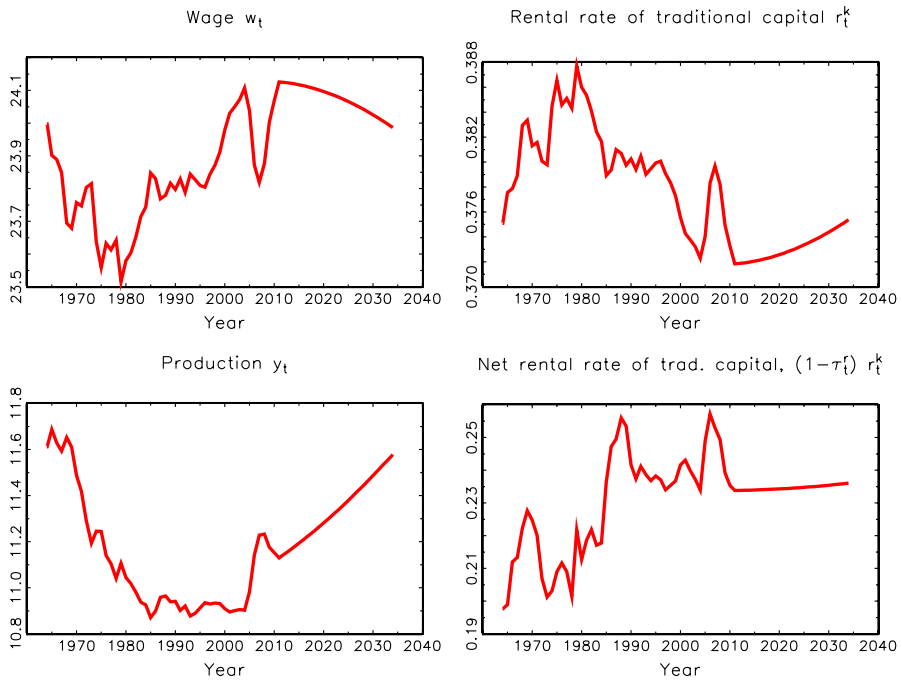**Fig. 6** Transition dynamics in the model with automation capital and adjustment costs, part I

Since traditional capital, k_t, decreases and automation capital increases during 1965–1980, the wage, w_t, falls at the beginning of the transition. Consequently, the rental rate on traditional capital, r_t^k, rises during this period.[13]

The evolution of the labor share, LS_t, is illustrated in Fig. 8. The model (broken green line) is able to replicate the downward trend of the labor share (solid red line) during 1965–2015. In fact, as observed empirically the model generates a drop in the labor share by five percentage points between 1965 and 2015. This effect is explained by the substitution of labor with automation capital. The income share accruing to these two production factors is constant and equal to $1 - \alpha = 62\%$. However, the relative income share of automation capital increases over time at the expense of the residual share for labor.

3.7.2 Experiments

We conduct two counterfactual experiments. In Experiment 1 the tax rate on labor income remains constant at its (low) 1965 level of $\tau_t^w = \tau^w = 17.9\%$, whereas the tax on capital income varies in line with the empirical evidence shown in Fig. 2.

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[13] Asymptotically, the rental rates of traditional and automation capital approach the value given in (A.5.9).

Fig. 7 Transition dynamics in the model with automation capital and adjustment costs, part II

Experiment 2 leaves both tax rates constant at their 1965 level, i. e., in addition the tax rate on capital income remains constant at its (high) 1965 level of $\tau_t^r = \tau^r = 37.1\%$. The counterfactual transition of the labor share is shown in Fig. 9. The actual US labor share and the labor share of our benchmark model appear again as the solid red and the broken green line. In Experiment 1 the decline of the labor share (broken dotted blue line) is less pronounced than in reality and the benchmark. In particular, in 2015 it is 1.3 percentage points higher at 57.9%. Experiment 2 generates an even shallower decline in the labor share to 60.9% (dashed black line).[14] We conclude that the observed increase in the tax on labor income and the decline in the tax on capital income from 1965 to 2015 played an important role for the evolution of the labor share over this period.

To understand the dynamics of the labor share in these experiments relative to the benchmark, it is instructive to compare the co-evolutions of the remaining endogenous variables. For this purpose, Fig. 10 illustrates the dynamics of traditional capital, k_t, automation capital, p_t, consumption, c_t, the labor supply, l_t, the wage, w_t, and the rental rate of traditional capital, r_t^k, (from the upper left to the lower right panel) in these experiments and the benchmark.

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[14] Section A.5.5 has a sensitivity analysis for different values of the Frisch labor supply elasticity. When this elasticity drops, e. g., from 1.64 [as in the benchmark case with utility function (3.3)] to a lower value of 0.64, then in the two experiments the labor shares decline to 57.6% and 60.9%, respectively.

In Experiment 1, the lower labor income tax rate increases the labor supply in the year 2015 from 0.2605 in the benchmark to 0.292. As a consequence, net labor income increases which results in a strong rise of consumption and a moderate rise of savings (the sum of investments in traditional and automation capital, not presented). As is evident from the bottom-left panel of Fig. 10, the increase in the labor supply also implies a rise of the rental rate of capital, r_t^k. Therefore, individual household members adjust their portfolio composition and shift wealth from automation to traditional capital over time. The top row of Fig. 10 reveals that k_t is larger and p_t lower in Experiment 1 than in the benchmark. Accordingly, with a higher labor supply, l_t, and less automation capital, p_t, the labor share increases.

In Experiment 2, the capital income tax rate is permanently higher than in Experiment 1. Therefore, the after-tax rate of return from traditional capital, k_t, falls by approximately 25% in 2015 from 12.2% in Experiment 1 to 9.1% in Experiment 2. Since the growth rate of per-capita variables is endogenous and depends on the capital income tax rate, τ^k, growth slows down and approaches asymptotically 0.85%. For this reason, output, consumption and investment are significantly smaller in Experiment 2 than in Experiment 1. In particular, in 2015, automation capital, p_t, in Experiment 2 (1) only amounts to 0.29 (1.08) units of contemporaneous output which—after noticing that the labor supply does not react strongly to the higher capital income tax rate, τ_t^r—explains the much higher labor share of 60.9%.

## 3.8 Welfare and tax policies

What are the optimal tax rates on labor, automation and traditional capital that a benevolent planner would choose? In the context of our model this question is of interest since the accumulation of robots crowds out labor. In fact, from the year 2132 onwards firms completely dispense with labor so that government revenues can no longer rely on the taxation of wages. Therefore, the optimal capital income tax rate levied on robots and traditional capital in the long run cannot be zero as suggested by Chamley (1986) and Judd (1985). The government would no longer be in a position to finance its exogenous expenditure comprising its consumption and transfers (for a constant tax rate on consumption).[15]

To derive welfare implications and optimal tax policies, (τ_t^r, τ_t^w), we conduct a simple fiscal policy experiment following Grüner and Heer (2000). The government announces an unexpected once-and-for-all change in the capital income tax rate, τ^r, in the year 2020. The labor income tax rate, τ_t^w, adjusts so that the fiscal budget is balanced. Moreover, we assume that - irrespective of the considered tax policy - the time paths of government consumption and transfers, G_t and Tr_t, in absolute terms remain as in the benchmark equilibrium (see Grüner and Heer (2000) for a detailed discussion of this procedure). In addition, we stipulate that the transition from the year 2020 to the new ABGP is completed by 2080. Hence, from this period onward,

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[15] See, e. g., Slavík and Yazici (2014), Costinot and Werning (2018), Thuemmel (2018), or Guerreiro et al. (2022) for alternative justifications of optimal tax or subsidy rates on robots.
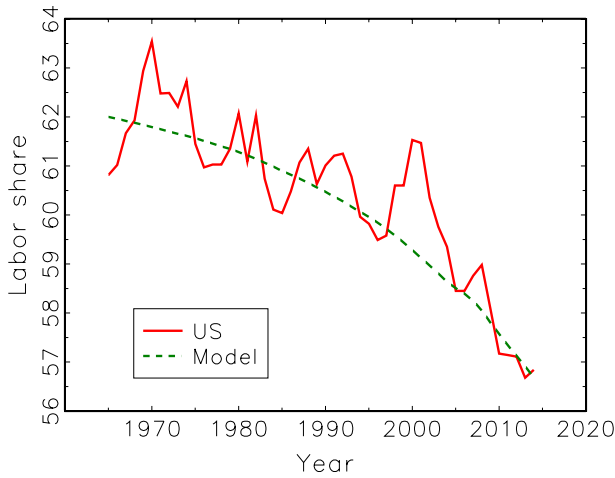
Fig. 8 Labor share dynamics
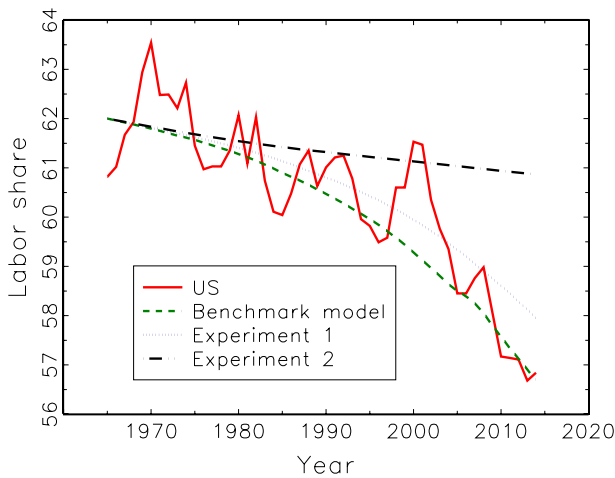
Fig. 9 Labor share and income taxes τ^w and τ^r

the per-capita variables y_t, k_t, p_t, and c_t grow at the endogenous growth rate γ which depends (negatively) on the capital income tax rate τ^r.

In our simulation, we find that the optimal capital income tax rate is 38.2%, about one percentage point higher than in 2020. For the household with intertemporal utility (3.2) this implies a welfare gain of 0.26% of total consumption. Since the optimal capital income tax rate is close to its benchmark value the welfare gain is rather small.

To develop some intuition for this result observe that capital income tax rates smaller than those prevailing in 2020, i. e., values of τ^r below 37%, are insufficient to finance government consumption and transfers until 2069 when labor has
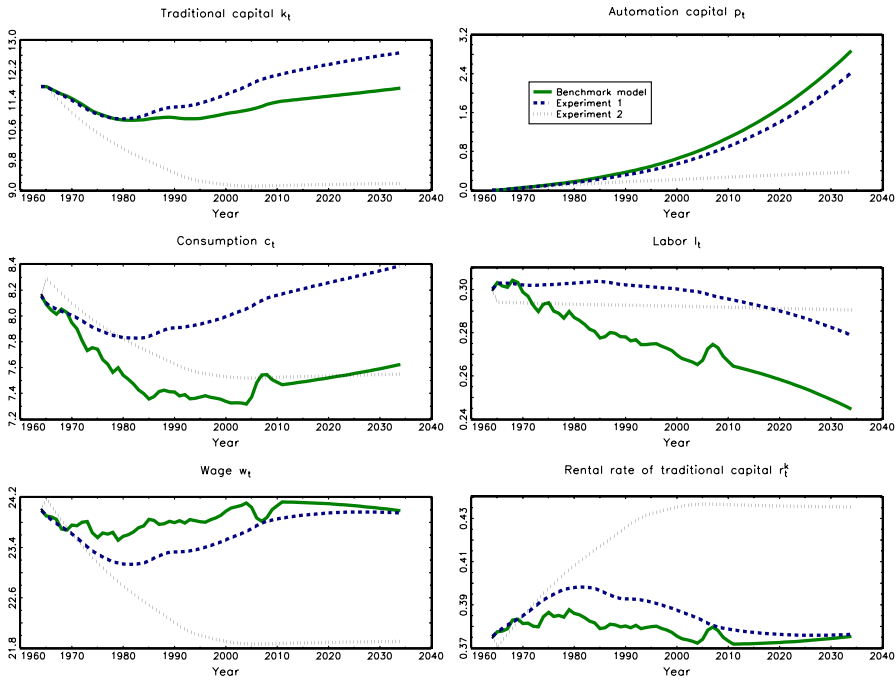
Fig. 10 Transition dynamics in experiments 1 and 2

converged to zero. This reflects two effects of opposite sign that a lower capital income tax rate has on government revenues. First, as the growth rate of per-capita variables is higher than in the benchmark equilibrium, the tax base and government revenues increase. Second, government revenues decline since τ^r is lower. In our model the latter effect dominates. For $\tau^r < 37\%$ this implies that the government can no longer finance its expenditure with higher levels of τ^w_t once the convergence of the labor supply becomes sufficiently close to zero. For capital income tax rates higher than those prevailing in 2020, i. e., values of τ^r exceeding 37%, the growth rate of per-capita variables falls below 2% and, beyond a certain threshold tax level slightly above 40%, total income taxes become again insufficient to finance government expenditures until the year 2080. Hence, the admissible range of capital income tax rates is [37%, 40%]. Over this range welfare is a concave function and peaks at $\tau^r = 38.2\%$.

4 Conclusion

In the empirical part, we find three significant long-run equilibrium correcting, i. e., cointegrating, relationships for the 1974–2008 period. They are given for our factor-tax policy mix variable and the capital share, our use of robots variable and the capital share, and population growth and the labor share. Permanent effects on factor shares emanate from shocks in relative factor taxation. The latter also permanently affect the

use of robots. Variance decompositions reveal that taxing factors contributes long-lastingly to the variation both in the two income shares and in automation capital. Overall, our findings give grounds for setting up and simulating a neoclassical growth model augmented by automation capital, capital adjustment costs, and factor taxation.

In our simulations, we find that tax rates on both labor and capital income have a significant effect on the functional income distribution and, in particular, on the labor share of income. For the US economy, the motivating empirical effect amounts to approximately 4 percentage points over the period 1965–2015. We demonstrate that this stylized fact can be reproduced by a neoclassical growth model with automation capital. Our growth model also predicts a continuing fall in the labor share over the coming decades. However, we would like to interpret this latter finding in a cautious way because we neglect other aggravating factors like artificial intelligence (AI). In our model, the productivity of the automation capital does not increase over time. In future research, we plan to endogenize the investment in AI and its effects on the functional income distribution.

Appendix

The Appendix includes methodological and mathematical details that complement our empirical (Sects. A.1–A.4) and theoretical (Sect. A.5) analysis.

A.1 Construction of central time series

A.1.1 Nowcasting (bi-)annual automation capital to obtain a quarterly series

A.1.1.1 Underlying time series

To-be-nowcasted time series The series we seek to nowcast—or to generate (pseudo-)quarterly data for—is the use of industrial robots per 1000 workers in the USA as provided in Acemoglu and Restrepo (2019). The original source is the IFR. This series is of annual frequency and starts in 1993. It ends in 2014. However, due to data limitations with regard to other series relevant for our analysis we end it in 2008. We merge these data with observations that we construct in the following way. Tani (1989) in Table 1 (col. 3, p. 193) provides data for the industrial robot population in the USA for the years 1974, 1976, 1978, and in annual frequency from 1980 to 1985. As Tani (1989) in Tab. 3 standardizes these data to workers in the manufacturing sector only, we refrain from using his standardized series, but divide the non-standardized robots figures through the respective annual averages of the US full time employees, SA (in 1K), data that we obtain from the CPS. The result is a series of mixed bi-annual, annual frequency with missing values for 1986–1992. Hence, covered years are 1974, 1976, 1978, 1980–1985, 1993–2008. Graphically this series is made of the dots shown in Fig. 3. As this series represents a stock variable (robots per 1000 employees) for particular years, we might interpret it as end-of-year or q4-values.

General strategy and information set series Given the (quasi-)q4-data of robots per 1000 workers, the procedure runs in two main steps. In the first step, the missing

(quasi-)q4-, or annual, values are generated using information from other use of automation capital related variables, for which we have data over the entire period and, at best, at a quarterly observation frequency. In a second step, for the obtained complete annual frequency series, running from 1974 to 2008, an analogue now-casting approach is followed to generate a (pseudo-)quarterly series. Our baseline information set essentially uses variables from the Fernald (2014) database in contemporaneous and first lag expression that recently have been shown by Graetz and Michaels (2018) to be profoundly and significantly associated with robots input: hours worked, labor productivity, different estimates of labor quality (i.e., labor composition), total factor productivity (TFP), and utilization-adjusted TFP. Additionally, we also consider the US tax rate on labor income provided by Gomme et al. (2011) as firms adopt robots mainly for saving on labor costs (besides ensuring uniform quality). Generally, labor costs depend on labor productivity and taxation.

A.1.1.2 Method

Starting point of the procedure is the notion of a general state space model for an n-dimensional time series $\boldsymbol{y}_t$ consisting of a measurement equation that relates the observed data to an m-dimensional state vector $\boldsymbol{\alpha}_t$. The generation of the state vector $\boldsymbol{\alpha}_t$ from the past state $\boldsymbol{\alpha}_{t-1}$, for $t = 1, \dots, T$, is determined by the state equation. The measurement equation has the form

$$\boldsymbol{y}_t = \boldsymbol{Z}_t \boldsymbol{\alpha}_t + \boldsymbol{d}_t + \boldsymbol{u}_t, \quad t = 1, \dots, T. \tag{A.1.1}$$

In (A.1.1), $\boldsymbol{Z}_t$ is an $n \times m$ matrix called measurement or observation matrix, $\boldsymbol{d}_t$ is an $n \times 1$ vector and $\boldsymbol{u}_t \sim$ iid $N(\boldsymbol{0}, \boldsymbol{H}_t)$ is an error vector. The state equation is given by

$$\boldsymbol{\alpha}_t = \boldsymbol{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{c}_t + \boldsymbol{R}_t \boldsymbol{v}_t, \quad t = 1, \dots, T. \tag{A.1.2}$$

In (A.1.2), $\boldsymbol{T}_t$ is an $m \times m$ matrix called transition matrix, $\boldsymbol{c}_t$ is an $m \times 1$ vector, $\boldsymbol{R}_t$ is an $m \times g$ matrix and $\boldsymbol{v}_t \sim$ iid $N(\boldsymbol{0}, \boldsymbol{Q}_t)$ is a $g \times 1$ error vector. The matrices $\boldsymbol{Z}_t$, $\boldsymbol{d}_t$, $\boldsymbol{H}_t$, $\boldsymbol{T}_t$, $\boldsymbol{c}_t$, $\boldsymbol{R}_t$ and $\boldsymbol{Q}_t$ are referred to as system matrices. Usually, it is assumed that the errors of the measurement and the transition equation are uncorrelated, i.e.

$$E[\boldsymbol{u}_t \boldsymbol{v}_t'] = \boldsymbol{0} \quad \forall s, t = 1, \dots, T.$$

Furthermore, it is assumed that the initial state is given by a normal vector

$$\boldsymbol{\alpha}_0 \sim N(\boldsymbol{a}_0, \boldsymbol{P}_0); \quad E[\boldsymbol{u}_t \boldsymbol{a}_0'] = \boldsymbol{0}, \ E[\boldsymbol{v}_t \boldsymbol{a}_0'] = 0, \quad t = 1, \dots, T.$$

In our application of a state-space model, as defined by (A.1.1) and (A.1.2), we seek to generate estimators for the underlying unobserved signal $\boldsymbol{\alpha}_t$ given the data $\boldsymbol{y}_s$, for $s = 1, \dots, S$. Whenever $s = t$ this problem is called filtering, while we speak of smoothing if $s > t$ and forecasting in case $s < t$. The problem of finding such estimators is solved by the Kalman Filter (KF), Kalman Smoother (KS) and forecasting recursions, respectively. The KF is a set of recursion equations (prediction equations and updating equations) that determine the optimal estimates for the state vector $\boldsymbol{\alpha}_t$ given the information available at t (henceforth, I_t). The following definitions are used

$$a_t := E[\alpha_t | I_t] \tag{A.1.3}$$

and

$$P_t := E[(\alpha_t - a_t)(\alpha_t - a_t)' | I_t]. \tag{A.1.4}$$

That is, a_t is the optimal estimator of α_t based on I_t and P_t is the mean square error (MSE) matrix of a_t.

Prediction equations Given a_{t-1} and P_{t-1},

$$\begin{aligned} a_{t|t-1} &= E[\alpha_t | I_{t-1}] \\ &= T_t a_{t-1} + c_t \end{aligned} \tag{A.1.5}$$

$$\begin{aligned} P_{t|t-1} &= E[(\alpha_t - a_{t-1})(\alpha_t - a_{t-1})' | I_{t-1}] \\ &= T_t P_{t-1} T_t' + R_t Q_t R_t. \end{aligned} \tag{A.1.6}$$

And the optimal predictor of y_t is obtained from

$$\begin{aligned} y_{t|t-1} &= Z_t a_{t|t-1} + d_t \\ &= Z_t T_t a_{t-1} + Z_t c_t + d_t \\ &= Z_t (T_t a_{t-1} + c_t) + d_t. \end{aligned} \tag{A.1.7}$$

The corresponding prediction error and its MSE matrix are

$$\begin{aligned} e_t &= y_t - y_{t|t-1} \\ &= y_t - Z_t a_{t|t-1} - d_t \\ &= Z_t \alpha_t + d_t + u_t - Z_t a_{t|t-1} - d_t \\ &= Z_t (\alpha_t - a_{t|t-1}) + u_t \end{aligned} \tag{A.1.8}$$

and

$$E[e_t e_t'] := F_t = Z_t P_{t|t-1} Z_t' + H_t. \tag{A.1.9}$$

Updating equations The moment y_t is the optimal predictor observed and its MSE matrix are updated according to

$$\begin{aligned} a_t &= a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (y_t - y_{t|t-1}) \\ &= a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} (y_t - Z_t \alpha_{t|t-1} - d_t) \\ &= a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} e_t \end{aligned} \tag{A.1.10}$$

$$P_t = P_{t|t-1} - \underbrace{P_{t|t-1} Z_t F_t^{-1}}_{\text{Kalman Gain}} Z_t P_{t|t-1}. \tag{A.1.11}$$

Filter derivation The KF-derivation makes use of the following properties of a bivariate normal distribution. Given y, the distribution of x is normal with

$$E[x|y] = \boldsymbol{\mu}_{x|y} = \boldsymbol{\mu}_x + \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}(\boldsymbol{y} - \boldsymbol{\mu}_y) \qquad (A.1.12)$$

$$Var(x|y) = \boldsymbol{\Sigma}_{xx} - \boldsymbol{\Sigma}_{xy}\boldsymbol{\Sigma}_{yy}^{-1}\boldsymbol{\Sigma}_{yx}. \qquad (A.1.13)$$

For the state vector at $t = 1$,

$$\boldsymbol{\alpha}_1 = \boldsymbol{T}_1\boldsymbol{\alpha}_0 + \boldsymbol{c}_1 + \boldsymbol{R}_1\boldsymbol{v}_1,$$

with $\boldsymbol{\alpha}_0 \sim N(\boldsymbol{a}_0, \boldsymbol{P}_0)$, $\boldsymbol{v}_1 \sim N(\boldsymbol{0}, \boldsymbol{Q}_1)$ and $E[\boldsymbol{\alpha}_0\boldsymbol{v}_1'] = \boldsymbol{0}$. In a linear Gaussian state-space model the initial state vector is normally distributed with

$$\begin{aligned} \boldsymbol{a}_{1|0} &:= E[\boldsymbol{\alpha}_1] = \boldsymbol{T}_1\boldsymbol{a}_0 + \boldsymbol{c}_1 \\ \boldsymbol{P}_{1|0} &:= Var(\boldsymbol{\alpha}_1) = \boldsymbol{T}_1\boldsymbol{P}_{1|0}\boldsymbol{T}_1' + \boldsymbol{R}_1\boldsymbol{Q}_1\boldsymbol{R}_1'. \end{aligned} \qquad (A.1.14)$$

From the measurement equation, it follows that

$$\boldsymbol{y}_1 = \boldsymbol{Z}_1\boldsymbol{\alpha}_1 + \boldsymbol{d}_1 + \boldsymbol{u}_1,$$

with $\boldsymbol{u}_1 \sim N(\boldsymbol{0}, \boldsymbol{H}_1)$ s.t.

$$\begin{aligned} \boldsymbol{y}_{1|0} &:= E[\boldsymbol{y}_1] = \boldsymbol{Z}_1\boldsymbol{a}_{1|0} + \boldsymbol{d}_1 \\ Var(\boldsymbol{y}_1) &= E[(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})'] \\ &= E[(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)'] \\ &= \boldsymbol{Z}_1\boldsymbol{P}_{1|0}\boldsymbol{Z}_1' + \boldsymbol{H}_1. \end{aligned} \qquad (A.1.15)$$

Equations (A.1.14) and (A.1.15) are the prediction equations for $\boldsymbol{\alpha}_1$ and $\boldsymbol{y}_1$ at $t = 0$.

In a next crucial step, one has to find the distribution of $\boldsymbol{\alpha}_1$ conditional on $\boldsymbol{y}_1$ being observed (updating). For this purpose, the joint normal distribution of $(\boldsymbol{\alpha}_1', \boldsymbol{y}_1')$ must be determined. In finding the joint normal distribution, we use

$$\begin{aligned} \boldsymbol{\alpha}_1 &= \boldsymbol{a}_{1|0} + (\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}) \\ \boldsymbol{y}_1 &= \boldsymbol{y}_{1|0} + \boldsymbol{y}_1 - \boldsymbol{y}_{1|0} \\ &= \boldsymbol{Z}_1\boldsymbol{a}_{1|0} + \boldsymbol{d}_1 + \boldsymbol{Z}_1(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}) + \boldsymbol{u}_1. \end{aligned}$$

Note that since

$$\begin{aligned} Cov(\boldsymbol{\alpha}_1, \boldsymbol{y}_1) &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{y}_1 - \boldsymbol{y}_{1|0})'] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{Z}_1\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\} + \boldsymbol{u}_1)'] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\{\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0}\}\boldsymbol{Z}_1' + \boldsymbol{u}_1')] \\ &= E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})\boldsymbol{Z}_1'] + E[(\boldsymbol{\alpha}_1 - \boldsymbol{a}_{1|0})\boldsymbol{u}_1'] \\ &= \boldsymbol{P}_{1|0}\boldsymbol{Z}_1', \end{aligned}$$

$$\begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{y}_1 \end{pmatrix} \sim N\left(\begin{pmatrix} \boldsymbol{a}_{1|0} \\ \boldsymbol{Z}_1\boldsymbol{a}_{1|0} + \boldsymbol{d}_1 \end{pmatrix}, \begin{pmatrix} \boldsymbol{P}_{1|0} & \boldsymbol{P}_{1|0}\boldsymbol{Z}_1' \\ \boldsymbol{Z}_1\boldsymbol{P}_{1|0} & \boldsymbol{Z}_1\boldsymbol{P}_{1|0}\boldsymbol{Z}_1' + \boldsymbol{H}_1 \end{pmatrix} \right).$$

In combination with (A.1.12) and (A.1.13), $(\boldsymbol{\alpha}_1|\boldsymbol{y}_1) \sim N(\boldsymbol{a}_1, \boldsymbol{P}_1)$ follows with

$$
\begin{aligned}
a_1 &= a_{1|0} + P_{1|0} Z_1' (Z_1 P_{1|0} Z_1' + H_1)^{-1} (y_1 - Z_1 a_{1|0} - d_1) \\
&= a_{1|0} + P_{1|0} Z_1' F_1^{-1} e_1
\end{aligned} \tag{A.1.16}
$$

$$
\begin{aligned}
P_1 &= P_{1|0} - P_{1|0} Z_1' (Z_1 P_{1|0} Z_1' + H_1)^{-1} Z_1 P_{1|0} \\
&= P_{1|0} - P_{1|0} Z_1' F_1^{-1} Z_1 P_{1|0}.
\end{aligned} \tag{A.1.17}
$$

Note that (A.1.16) and (A.1.17) are the Kalman Filter updating equations for $t = 1$.

ML-estimation and EM algorithm Let θ denote the parameters of the state-space model. These parameters are embodied in the system matrices. The likelihood of the state-space model is calculated based on the prediction errors e_t with $t = 1, \ldots, T$. The prediction error decomposition of the (negative) log-likelihood is

$$
-2 \ln L(\theta | y) = nT \ln(2\pi) + \sum_{t=1}^{T} \ln |F_t(\theta)| + \sum_{t=1}^{T} e_t'(\theta) F_t^{-1}(\theta) e_t(\theta). \tag{A.1.18}
$$

Shumway and Stoffer (2008) proposed a procedure based on the EM algorithm that is conceptually simpler and more efficient than alternative procedures such as the Newton–Raphson algorithm. The basic idea is that if all states $\alpha_T = \{\alpha_t\}_{t=0}^{T}$ together with the observations $y_T = \{y_t\}_{t=1}^{T}$ were observed, one could consider the entire data space $\{\alpha_T, y_T\}$. The complete data likelihood might, thus, be written as

$$
\begin{aligned}
-2 \ln L(\theta | \alpha, y) = \ &\ln |F_0| + (\alpha_0 - a_0)' F_0^{-1} (\alpha_0 - a_0) \\
&+ n \ln |Q_t| + \sum_{t=1}^{T} (\alpha_t - T_t \alpha_{t-1})' Q_t^{-1} (\alpha_t - T_t \alpha_{t-1}) \\
&+ n \ln |H_t| + \sum_{t=1}^{T} (y_t - Z_t \alpha_t)' H_t^{-1} (y_t - Z_t \alpha_t).
\end{aligned} \tag{A.1.19}
$$

Given the *complete* data without any missing values and mostly in the desired (quarterly) frequency, one could easily obtain the ML-estimates of θ. However, as this is not the case, we may find the ML-estimates based on the *incomplete* data with shortfall by successively maximizing the conditional expectation of the complete data likelihood. This is done in the following steps:

1. Find some initial values for parameters $\theta^{(0)}$,
2. Calculate the incomplete data likelihood $-\ln L(\theta^{(j-1)} | y)$; see Eq. (A.1.18),
3. At iteration $j = 1, 2, \ldots$ use the KF and KS to obtain smoothed values for $\alpha_t^{(S)}$, $P_t^{(S)}$ and $P_{t|t-1}^{(S)}$ for $t = 1, \ldots, T$ based on the parameters $\theta^{(j-1)}$. Use the smoothed values to calculate the conditional expectation of the complete data likelihood

$$Q(\theta|\theta^{(j-1)}) = E\{-2 \ln L(\theta|\alpha, y)|y_n, \theta^{(j-1)}\}$$

$$= \ln |F_0| + \operatorname{tr}\left\{ F_0^{-1} \left[P_0^{(S)} + \left(\alpha_0^{(S)} - a_0 \right)\left(\alpha_0^{(S)} - a_0 \right)' \right] \right\}$$

$$+ n \ln |Q_t| + \operatorname{tr}\left\{ Q^{-1}\left[S_{11} - S_{10}Z_t' - Z_t S_{10} + Z_t S_{00} Z_t' \right] \right\}$$

$$+ n \ln H$$

$$+ \operatorname{tr}\left\{ H^{-1} \sum_{t=1}^{T} \left[\left(y_t - Z_t \alpha_t^{(S)} \right)\left(y_t - Z_t \alpha_t^{(S)} \right)' + Z_t P_t^{(S)} Z_t' \right] \right\},$$

where

$$S_{11} = \sum_{t=1}^{T} \left(\alpha_t^{(S)} \alpha_t^{(S)'} + P_t^{(S)} \right),$$

$$S_{10} = \sum_{t=1}^{T} \left(\alpha_t^{(S)} \alpha_{t|t-1}^{(S)'} + P_{t|t-1}^{(S)} \right) \text{ and}$$

$$S_{00} = \sum_{t=1}^{T} \left(\alpha_{t|t-1}^{(S)} \alpha_{t|t-1}^{(S)'} + P_{t|t-1}^{(S)} \right).$$

4. Update θ_0 according to

$$T_t^{(j)} = S_{10} S_{00}^{-1},$$

$$Q_t^{(j)} = n^{-1}\left(S_{11} - S_{10} S_{00}^{-1} S_{10}' \right) \text{ and}$$

$$H_t^{(j)} = n^{-1} \sum_{t=1}^{T} \left[\left(y_t - Z_t \alpha_t^{(S)} \right)\left(y_t - Z_t \alpha_t^{(S)} \right)' + Z_t P_t^{(S)} Z_t' \right]$$

to obtain $\theta^{(j)}$.

5. Repeat steps 2 to 4 until convergence is achieved (i.e., until parameters or likelihood values stabilize in the sense of differing from their predecessor values by some predetermined small amount κ only).

A.1.1.3 Application

Annual series nowcast Having sketched the nowcasting procedure and indicators set, referred to as information set I_t above, we run the annual series (or q4-value) nowcast for four differently sized set of indicators: I_1 (index t dropped for notational ease) considers 16 series, i.e., in contemporaneous and first lag expression: hours worked, labor productivity, actually used labor composition/quality, labor composition/quality as reported by the Bureau of Labor Statistics (BLS), TFP, utilitization-adjusted TFP, and the tax rate on labor income. The next two considered sets, I_2 and I_3, are similar in size and nowcasting performance: Set I_3 comprises ten series by dropping the two labor composition/quality indicator series and the utilization-adjusted TFP series. Using either of the two TFP series does not alter the nowcasted values. It merely changes fourth or higher decimal places. The same applies to the two different labor
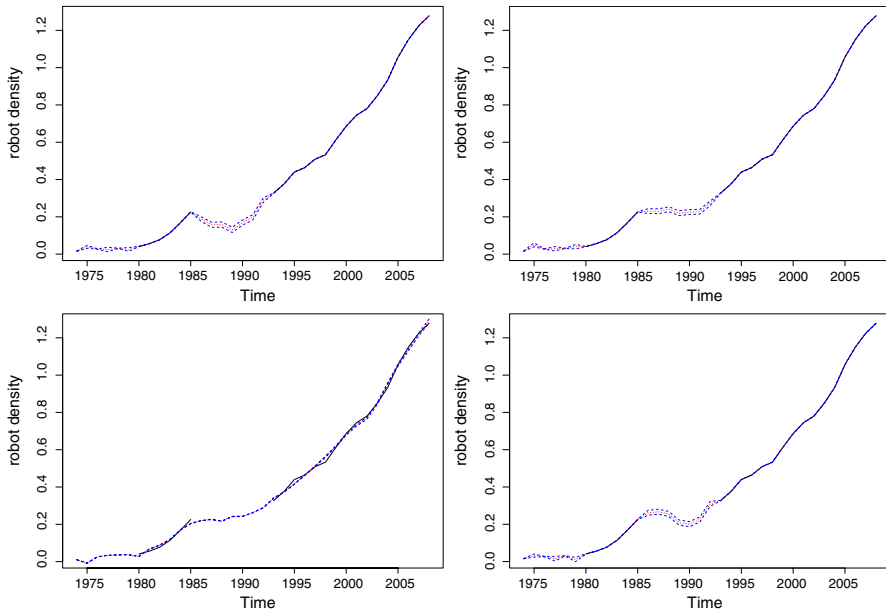
Fig. 11 Annual nowcast of robots per 1K workers for different information sets. *Note*: Underlying indicator sets, from left to right, first row: I_1, I_2, second row: I_3, I_4 (from left to right); black line and dots: empirical values, red dots: smoother values, blue dashed lines: 95% C.I. of prediction errors (Color figure online)

quality indicators. I_2 is of the same size as I_3 but takes into account the labor composition/quality indicator and leaves out the labor tax series. Information set I_4 compared to I_3 includes the labor tax indicator and drops labor productivity.

In line with intuition that—both and primarily—productivity and costs matter with regard to automization, information set I_3 (see the lower left schedule in Fig. 11) generates the most accurate and reasonable annual series nowcast. Note, however, that the annual series is nowcasted only quite imprecisely, i. e., with relatively high prediction errors, for the information sets I_1, I_2, and I_4 (see the upper left, upper right and the lower right schedules in Fig. 11). This circumstance does not allow us to directly and thoroughly assess the robustness of our cointegration analysis with regard to these alternatives. Such an assessment would require Monte Carlo method based simulations that are beyond the scope of this paper. Nonetheless, our choice of the information set I_3 may be justified on efficiency grounds.

Quarterly series nowcast For our quarterly series nowcast, we fill up the I_3-based annual projections and empirical values but now rely on an I_3 analogue using all quarterly instead of just q4 information. The result is shown for different observation and state equation variance values, $\boldsymbol{H}_t$ and $\boldsymbol{Q}_t$, in Fig. 13. The right schedule circle and red dot values correspond to our nowcasted quarterly series of choice and corresponds to the time series (solid blue line) displayed in Fig. 3.

As can be seen from Fig. 12 our nowcasts are slightly sensitive with regard to observation equation variances and produce more accurate predictions of the
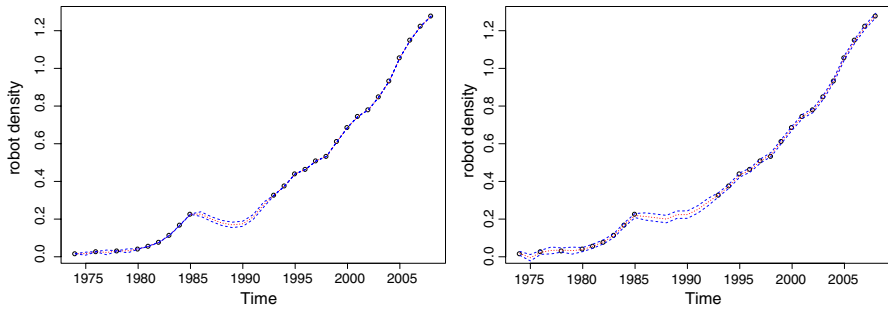
Fig. 12 Observation–equation–variance sensitivity of I_3-based nowcast. *Note*: Variances in observation equation matrix (H_t) doubled from 1 (left) to 2 (right schedule); black circles: empirical values, red dots: smoother values, blue dashed lines: 95% C.I. of prediction errors (Color figure online)
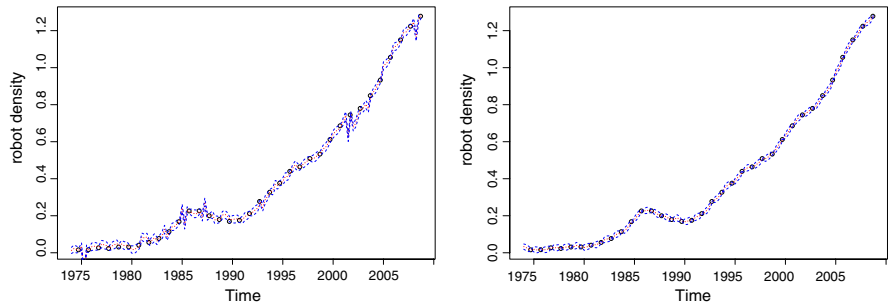
Fig. 13 Observation/state–equation–variance sensitivity of quarterly nowcast. *Note*: Quarterly I_3-based (filling up I_3-based annual nowcast); variances in observation (state) equation matrix H_t (Q_t) are set to 0.01 (1) for left schedule and 0.01 (0.01) for right schedule, respectively; black circles: empirical and nowcasted (q4-/fourth quarter) annual values; remaining legend as for Fig. 12

empirical observations for lower values. Thus, we proceed with the I_3-based annual nowcast with the lower observation equation variance values (Fig. 13).

A.1.2 Construction of the capital tax rate and labor tax rate series

As mentioned above, the capital tax collected from households must be imputed. The corresponding tax rate, τ_h, on general household income—including capital income—is an intermediate input into subsequent calculations. In line with the established approach of the macroeconomic literature, Gomme et al. (2011, p. 266) derive it from the following ratio:

$$\tau_h = \frac{\text{PERSONAL CURRENT TAXES}}{\text{NET INTEREST} + \text{PROPRIETORS' INCOME} + \text{RENTAL INCOME} + \text{WAGES}}.$$

Labor income taxes are obtained via the following straightforward calculation:

$$\text{LABOR INCOME TAXES} = \tau_h [\text{WAGES} + (1 - \alpha)\text{PROPRIETORS' INCOME}] + \text{GSI},$$

where WAGES denotes wages and salaries and GSI contributions for government social insurance, respectively. Total labor income is given by

$$\text{LABOR INCOME} = \text{WAGES} + (1 - \alpha)\,\text{PROPRIETORS' INCOME} + \text{EGSI},$$

where EGSI denotes employer contributions for government social insurance. Finally, the tax rate on labor income can be computed as

$$\tau_L = \frac{\text{LABOR INCOME TAXES}}{\text{LABOR INCOME}}.$$

Gomme et al. (2011) calculate capital income taxes as

$$\begin{aligned}
\text{CAPITAL INCOME TAXES} = \tau_h[&\text{NET INTEREST} + \alpha\text{PROPRIETORS' INCOME} \\
&+ \text{RENTAL INCOME} - (\,\text{HOUSING NET INTEREST} \\
&+ \alpha\ \text{HOUSING PROPRIETORS' INCOME} \\
&+ \text{HOUSING RENTAL INCOME})] + \text{CIT} \\
&+ \text{REAL ESTATE PROPERTY TAXES} \\
&+ \text{OTHER TAXES},
\end{aligned}$$

where CIT denotes corporate income taxes. Capital income is given by

$$\begin{aligned}
\text{CAPITAL INCOME} = &\text{NET OPERATING SURPLUS} \\
&+ \text{CONSUMPTION OF PRIVATE FIXED CAPITAL} \\
&- \text{HOUSING NET OPERATING SURPLUS} \\
&+ (1 - \alpha)(\text{PROPRIETORS' INCOME} \\
&- \text{HOUSING PROPRIETORS' INCOME}).
\end{aligned}$$

In analogy to the labor tax rate, the capital tax rate is obtained from

$$\tau_K = \frac{\text{CAPITAL INCOME TAXES}}{\text{CAPITAL INCOME}}.$$

Gomme et al. (2011) draw these constituent series from the US NIPA. However, several of these series are not available quarterly. See Gomme et al. (2011, pp. 268–269) for details on how the corresponding annual series are converted into quarterly frequency.

A.2 VEC model analysis

As noted above, our RF model space consists of three dimensions: a relatively exogenous variable X_t (population growth), a policy variable Y_t (the factor tax policy mix, i. e., the LCTR), and a multivariate group of response variables $\mathbf{W}_t$ (our automation capital proxy and the two factor shares), making it

$$\mathbf{Z}_t = [X_t, Y_t, \mathbf{W}_t] \Rightarrow \mathbf{Z}_t = \mathbf{A}_1\mathbf{Z}_{t-1} + \mathbf{A}_2\mathbf{Z}_{t-2} + \cdots + \mathbf{A}_p\mathbf{Z}_{t-p} + \mathbf{u}_t. \quad \text{(A.2.1)}$$

In standard VEC notation, this becomes

$$\Delta \mathbf{Z}_t = \Gamma_1 \Delta \mathbf{Z}_{t-1} + \Gamma_2 \Delta \mathbf{Z}_{t-2} + \cdots + \Gamma_{p-1} \Delta \mathbf{Z}_{t-p-1} + \Pi \mathbf{Z}_{t-1} + \mathbf{u}_t, \quad \text{(A.2.2)}$$

where $\Gamma_i = (\mathbf{I} - \mathbf{A}_1 - \mathbf{A}_2 - \cdots - \mathbf{A}_p)$ for all $i = 1, \ldots, p$. Π can be thought of as consisting of an adjustment speed matrix $\mathbf{a}$, and a long-run coefficient matrix $\mathbf{b}$, such that $\Pi = \mathbf{a}\mathbf{b}'$, where $\mathbf{b}'\mathbf{Z}_{t-1}$ is the vectorial analogue of the error correction term in the Engel–Granger approach. For an exemplary unity lag order

$$\Delta \mathbf{Z}_t = \begin{pmatrix} \Delta Y_t \\ \Delta X_t \\ \Delta \mathbf{W}_t \end{pmatrix} = \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta \mathbf{W}_{t-1} \end{pmatrix} + \Pi \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ \mathbf{W}_{t-1} \end{pmatrix} + \mathbf{e}_t$$

$$= \Gamma_1 \begin{pmatrix} \Delta Y_{t-1} \\ \Delta X_{t-1} \\ \Delta W_{1\,t-1} \\ \Delta W_{2\,t-1} \\ \Delta W_{3\,t-1} \end{pmatrix} \quad \text{(A.2.3)}$$

$$+ \begin{pmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{22} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ a_{41} & a_{42} & a_{43} & a_{44} \\ a_{51} & a_{52} & a_{53} & a_{54} \end{pmatrix} \begin{pmatrix} b_{11} & b_{21} & b_{31} & b_{41} & b_{51} \\ b_{12} & b_{22} & b_{32} & b_{42} & b_{52} \\ b_{13} & b_{23} & b_{33} & b_{43} & b_{53} \\ b_{14} & b_{24} & b_{34} & b_{44} & b_{54} \end{pmatrix} \begin{pmatrix} Y_{t-1} \\ X_{t-1} \\ W_{1\,t-1} \\ W_{2\,t-1} \\ W_{3\,t-1} \end{pmatrix} + \mathbf{e}_t.$$

The central requirement of cointegration (*CI*) is a reduced rank of $\Pi = \underset{n \times r}{\mathbf{a}} \left(\underset{n \times r}{\mathbf{b}} \right)'$, i. e., $\mathbf{Z}_t \sim I(1) \Rightarrow \Delta \mathbf{Z}_{t-1} \sim I(0) \Rightarrow \Pi \mathbf{Z}_{t-1} \overset{!}{\sim} I(0)$ for $\mathbf{u}_t \sim I(0)$. This allows for up to $n - 1 = 5 - 1 = 4$ *CI*—relationships of the form $\mathbf{b}'\mathbf{Z}_{t-1} \sim I(0)$, $r \leq n - 1$ cointegrating vectors $\in \Pi$, i. e., r columns of $\mathbf{b}$ form r linearly independent stationary combinations of variables $\in \mathbf{Z}_t$.

As a first step, we perform augmented Dickey-Fuller (ADF) tests for all considered series. Throughout the unit root hypothesis cannot be rejected at a one percent level of significance according to the MacKinnon approximate *p*-values. For all log first differences transforms, the null of a unit root is rejected at every conventional level of significance. The order of integration of variables is $I(1)$ (see "Appendix" for details). In a second step, the appropriate lag length choice is made resorting to likelihood ratio (LR) testing. Here, we follow Schwert (1989) and set $p_{\max} = \left[12 \cdot (T/100)^{\frac{1}{4}} \right]$ quarters where $T = 140$ and brackets denote the nearest integer part of the argument. Hence, $p_{\max} = 13$.

As $\Pi = -(\mathbf{I} - \mathbf{A}_1 - \mathbf{A}_2 - \cdots - \mathbf{A}_p)$, or equivalently $\Pi = \sum_{i=1}^p \mathbf{A}_i - \mathbf{I}$, the Johansen procedure makes use of Granger's Representation Theorem which states that if matrix Π has reduced rank $r < n$ with n denoting the number of non-stationary variables considered, then there exist $n \times r$ matrices $\mathbf{a}$ and $\mathbf{b}$ each with rank r such that $\Pi = \mathbf{a}\mathbf{b}'$ and $\mathbf{b}'\mathbf{Z}_t \sim I(0)$; then, r is the number of cointegration relations (cointegrating rank) and each column of $\mathbf{b}$ is a cointegrating vector. However, before estimating Π, deterministic components of the general system

$$\Delta \mathbf{Z}_t = \Gamma_1 \Delta \mathbf{Z}_{t-1} + \cdots + \Gamma_{p-1} \Delta \mathbf{Z}_{t-p-1}$$

$$+ \mathbf{a} \begin{pmatrix} \mathbf{b} \\ \mathbf{m}_1 \\ \mathbf{d}_1 \end{pmatrix}' \left(\mathbf{Z}_{t-1} \ 1 \ t \right) + \mathbf{m}_2 + \mathbf{d}_2 t + \mathbf{u}_t \qquad (\text{A.2.4})$$

have to be chosen. The above system can discriminate four central versions: (v1) no intercept or trend in the cointegrating equation (CE) or VAR part ($\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{m}_1 = \mathbf{m}_2 = 0$); (v2) intercept and no trend in the CE part and neither intercept nor trend in the VAR part ($\mathbf{d}_1 = \mathbf{d}_2 = \mathbf{m}_2 = 0$), i. e., the no linear trend in data case (first differences have zero mean); (v3) intercept in the CE part and the VAR part, but no trends ($\mathbf{d}_1 = \mathbf{d}_2 = 0$), i. e., no linear trends in levels of data case; (v4) intercept in the CE part and the VAR part paralleled by a linear trend in the CE or in the VAR part, i. e., the linear trend in the CE case, sometimes referred to as the exogenous growth case. Following the Pantula Principle, we start with the most restrictive model, i. e., $r = 0$ in combination with v1, and move gradually to the least restrictive one where $r = n - 1$ in combination with v4. For each gradual step, the trace-test statistics is compared with the critical value and the iteration stops when for the first time the null of no cointegration is not rejected. This determines the order of Π, i. e., the number of cointegration vectors. Besides trace-based rank testing, we cross-check and validate our findings with maximum eigenvalue and information criteria-based cointegration rank tests.

As can be seen from Tables 3 and 4, all series in log levels used in the fitted VECM, as described in the empirical part of the paper, are $I(1)$ and the result of the Johansen test procedure is that there are, at least, three cointegrating relationships.

Table 3 Unit root (UR) and stationarity test statistics

	ADF I	ADF II	PP	KPSS
Log levels				
Population growth	−1.732	−1.679	−1.840	0.399***
Factor tax policy mix (LCTR)	−1.717	−2.003	−5.379	0.988***
Robots per 1K workers	−2.841*	−2.792	−0.481	0.504***
Wage share	−1.253	−2.861	−2.338	0.290***
Capital share	−1.377	−1.937	−1.876	0.361***
Log first differences				
Population growth	−11.36***	−11.39***	−11.36***	0.0631
Factor tax policy mix (LCTR)	−7.887***	−17.13***	−14.08***	0.0359
Robots per 1K workers	−5.086***	−5.468***	−10.61***	0.1030
Wage share	−15.81***	−9.181***	−12.52***	0.0415
Capital share	−6.547***	−6.618***	−4.111***	0.0381

ADF—Augmented Dickey–Fuller (UR under null; I/II = without/with linear trend component), PP—Phillips–Perron (UR under null), KPSS—Kwiatkowski–Phillips–Schmidt–Shin (stationarity under null; with automatic bandwidth selection and autocovariances weighted by quadratic spectral kernel); $^*p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$

The latter concern population growth and the wage share, the factor tax policy mix (LCTR) and the capital share, and robots density and capital share, respectively.

However, the last of these CE relationships is significant at a 68% level of significance only according to the z-statistics of the Johansen normalized restriction test.

A post-estimation stability check confirms three (two) imposed unit moduli of eigenvalues of the companion matrix of our fitted VECM with two exact unit eigenvalues and one very close to one, i.e., with a value of 0.97. All remaining moduli of eigenvalues of the companion matrix are strictly less than one indicating stability. Serial correlation of residuals is clearly rejected by appropriate Lagrange Multiplier tests (LM tests). Against the backdrop of the performed tests (with further

Table 4 Johansen test procedure and test statistics for cointegrating equations

Model	Max rank	Trace stats	5% c.v.	Max EV stats	5% c.v.	Info criterion HQIC
(v1)	0	101.04	59.46	43.72	30.04	−35.52362
	1	57.31	39.89	30.31	23.8	−35.64426
	2	27.00	24.31	17.77	17.89	−35.70900
	3	9.23*	12.53	9.21	11.44	−35.72472*
	4	0.01	3.84	0.01	3.84	−35.72275
(v2)	0	145.46	76.07	68.08	34.4	−35.52362
	1	77.38	53.12	30.36	28.14	−35.81121
	2	47.02	34.91	23.44	22.00	−35.85150
	3	23.58	19.96	17.56	15.67	−35.88699
	4	6.01*	9.42	6.01	9.24	−35.92591*
(v3)	0	133.92	68.52	68.07	33.46	−35.49021
	1	65.85	47.21	30.23	27.07	−35.80259
	2	35.62	29.68	21.39	20.97	−35.86672
	3	14.22*	15.41	12.56	14.07	−35.91094
	4	1.66	3.76	1.66	3.76	−35.93535*
(v4)	0	207.72	87.31	116.61	37.52	−35.49021
	1	91.10	62.99	36.70	31.46	−36.15999
	2	54.39	42.44	25.86	25.54	−36.25025
	3	28.53	25.32	17.42	18.96	−36.30482
	4	11.11*	12.25	11.11	12.52	−36.3426*
(v5)	0	188.16	77.74	111.36	36.41	−35.51998
	1	76.79	54.64	36.56	30.33	−36.17327
	2	40.23	34.55	25.80	23.78	−36.28723
	3	14.43*	18.17	13.46	16.87	−36.36616
	4	0.97	3.74	0.97	3.74	−36.39762*

Trace—trace test; Max EV—maximum eigenvalue test; Info Criterion—information criterion (IC) with IC of choice: HQIC—Hannan–Quinn information criterion; procedure starts with test for zero cointegrating equations (CE), i.e., a maximum rank of zero, and then accepts the first null that is not rejected (indicated by '*'); the Pantula Principle sequence is: (v1) no intercept or trend in CE or in VAR part; (v2) intercept and no trend in CE part and neither intercept nor trend in VAR part; (v3) intercept in CE part and VAR part, but no trends; (v4) intercept in CE part and in VAR part paralleled by linear trend in CE part or in VAR part
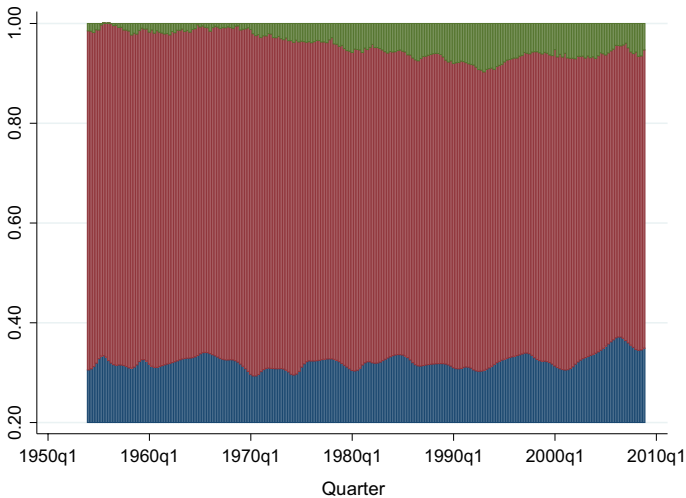
Fig. 14 Capital income share, labor income share, and the elasticity of substitution. *Note*: Blue area: capital income share, red area: labor income share; green area: rest to unity

detail available on request) we assess our VECM specification as being, all in all, acceptable.

A.3 Construction of the adjusted labor share

Our adjusted labor income share is based on BEA time series for GDP and GDI, that is, compensation of employees, paid wages and salaries. The share has been adjusted assuming that a one third add-on to GDI is attributable to the self-employed. Just like any adjustment of the empirical labor share, it represents a crude approximation. Contrary to alternative approaches, it precludes double counting.

A mean difference test for the two factor shares, capital income share and labor income share, summing to one (with mean rounded based on the digit at fourth decimal place) fails to reject the null of unity at any conventional level of significance.

An implied aggregate elasticity of substitution between capital and labor σ that is close to but slightly less than unity, $\sigma \leq 1$, is in line with recent evidence based on longitudinal data and consistent estimates by Glover and Short (2020). A $\sigma \lesssim 1$ (see the sum of the red and blue area in Fig. 14), as used in the empirical part of our paper, excludes the simple capital deepening explanation of the global decline in the labor share. For $\sigma \lesssim 1$, the fall in the labor's share cannot be rationalized by "rising effective capital ratios through physical investment in response to the fall in investment prices" (Glover and Short 2020, p. 35).

A.4 Summary statistics of series used in VECM

See Table 5.

Table 5 Summary statistics of series used (as log levels) in VECM

Series	Mean	SD	Min	Max	Skewness	Kurtosis
Population growth	0.0026	0.0004	0.0021	0.0035	0.7574	2.0393
Factor tax policy mix (LCTR)	0.7417	0.0985	0.4733	0.8891	−1.0011	3.0182
Robots per 1K workers	0.3936	0.3695	0.0149	1.0278	0.9128	2.0670
Wage share	0.6341	0.0299	0.5827	0.6914	0.0355	2.0890
Capital share	0.3247	0.0164	0.2953	0.3717	0.9221	3.0684

Summarized series were transformed to log levels before using them in the VECM as outlined in the empirical part of the paper; robots per 1K workers is the pseudo-quarterly series nowcasted as described above; throughout the observation period ranges from 1974:q1 to 2008:q4 (N obs = 140). All sources of data are given in the text (empirical part of the paper)

A.5 A neoclassical growth model with dynamic taxes, automation capital, and adjustment costs: analytical details

A.5.1 Household optimization problem

The Lagrangian of the household optimization problem is

$$
\begin{aligned}
\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \Bigg\{ & \frac{\left(c_t^\theta(1-l_t)^{1-\theta}\right)^{1-\eta} - 1}{1-\eta} \\
& + \lambda_t \Big[(1-\tau_t^w)w_t l_t + (1-\tau_t^r)r_t^k k_t + (1-\tau_t^r)r_t^p p_t + tr_t - (1+\tau^c)c_t \\
& - \phi(i_t^k, k_t) - \phi(i_t^p, p_t)\Big] \\
& + q_t^k \Big[i_t^k + (1-\delta)k_t - (1+n_t)k_{t+1}\Big] \\
& + q_t^p \Big[i_t^p + (1-\delta)p_t - (1+n_t)p_{t+1}\Big] \\
& + \mu_t^l l_t + \mu_t^{i^k} i_t^k + \mu_t^{i^p} i_t^p + \mu_{t+1}^k k_{t+1} + \mu_{t+1}^p p_{t+1} \Bigg\},
\end{aligned}
$$

where μ_t^x, $x \in \{l, i^k, i^p, k, p\}$, denotes the Lagrange multiplier on the respective constraint $l_t \geq 0, i_t^k \geq 0, i_t^p \geq 0, k_t \geq 0$, and $p_t \geq 0$.

Since the utility function $u(c, 1-l)$ of (3.3) is strictly concave on its domain and satisfies the Inada conditions mentioned in the main text the first-order conditions for the household's optimization problem are

$$\lambda_t(1+\tau^c) = \theta c_t^{\theta(1-\eta)-1}(1-l_t)^{(1-\theta)(1-\eta)}, \tag{A.5.1a}$$

$$\lambda_t(1-\tau_t^w)w_t + \mu_t^l = (1-\theta)c_t^{\theta(1-\eta)}(1-l_t)^{(1-\theta)(1-\eta)-1}, \tag{A.5.1b}$$

$$q_t^k + \mu_t^{i^k} = \lambda_t \phi_{i^k}(i_t^k, k_t), \tag{A.5.1c}$$

$$q_t^p + \mu_t^{i^p} = \lambda_t \phi_{i^p}(i_t^p, p_t), \tag{A.5.1d}$$

$$q_t^k - \frac{\mu_{t+1}^k}{1+n_t} = \frac{\beta}{1+n_t}\left\{\lambda_{t+1}\left[(1-\tau_{t+1}^r)r_{t+1}^k - \phi_k(i_{t+1}^k, k_{t+1})\right] + q_{t+1}^k(1-\delta)\right\}, \tag{A.5.1e}$$

$$q_t^p - \frac{\mu_{t+1}^p}{1+n_t} = \frac{\beta}{1+n_t}\left\{\lambda_{t+1}\left[(1-\tau_{t+1}^r)r_{t+1}^p - \phi_p(i_{t+1}^p, p_{t+1})\right] + q_{t+1}^p(1-\delta)\right\}, \tag{A.5.1f}$$

$$\mu_t^l l_t = 0, \quad \mu_t^{i^k} i_t^k = 0, \quad \mu_t^{i^p} i_t^p = 0, \quad \mu_{t+1}^k k_{t+1} = 0, \quad \mu_{t+1}^p p_{t+1} = 0,$$
$$0 = \lim_{t\to\infty} \beta^t \lambda_t k_{t+1}, \tag{A.5.1g}$$

$$0 = \lim_{t\to\infty} \beta^t \lambda_t p_{t+1}. \tag{A.5.1h}$$

Here, $\phi_{i^x}(i^x, x)$, $x \in \{k, p\}$, denotes the first derivative of the adjustment cost function $\phi(i^x, x)$ with respect to investment i^x. Similarly, ϕ_x denotes the first derivative of this function with respect to the second argument $x \in \{k, p\}$. For the specification of the adjustment cost function ϕ in (3.5), the above first-order conditions boil down to those of (3.7). Standard arguments show that the relevant second-order conditions are satisfied.

A.5.2 Details on the initial steady state and the asymptotic balanced growth path

The *initial steady state* is pinned down by the following 8 equations in the 8 endogenous variables k, l, i^k, c, w, r^k, q^k, and λ:

$$(1+\tau^c)\lambda = \theta c^{\theta(1-\eta)-1}(1-l)^{(1-\theta)(1-\eta)}, \tag{A.5.2a}$$

$$\frac{1-\tau^w}{1+\tau^c}w = \frac{1-\theta}{\theta}\frac{c}{1-l}, \tag{A.5.2b}$$

$$q^k = \lambda\left(1 + \frac{a_1 i^k}{a_2 + k}\right), \tag{A.5.2c}$$

$$\frac{q^k}{\lambda} = \frac{\beta}{1+n}(1-\tau^r)r^k + \frac{a_1(i^k)^2}{2(a_2+k)^2} + \frac{q^k}{\lambda}(1-\delta), \tag{A.5.2d}$$

$$i^k = (n+\delta)k, \tag{A.5.2e}$$

$$c + g + \phi(i^k, k) = Al^{1-\alpha}k^\alpha, \tag{A.5.2f}$$

$$w = (1 - \alpha)A\left[\frac{k}{l}\right]^\alpha, \tag{A.5.2g}$$

$$r^k = \alpha A\left[\frac{l}{k}\right]^{1-\alpha}. \tag{A.5.2h}$$

The derivation of the *Asymptotic Balanced Growth Path (ABGP)* involves the following steps.

The accumulation Eqs. (3.4a) and (3.4b) in conjunction with (3.16) deliver the asymptotic investment ratios

$$\frac{i^k}{k} = \frac{i^p}{p} = (1 + n)(1 + \gamma) - 1 + \delta. \tag{A.5.3}$$

Next consider the first-order conditions with respect to i_t^k and i_t^p given by (3.7c) and (3.7d). Equations (3.16) and (A.5.3) imply that the ratio $i_t^k/(a_2 + k_t)$ converges to $i_t^k/k_t = i^k/k$, whereas $i_t^p/(a_2 + p_t)$ converges to $i_t^p/p_t = i^p/p$. Hence, asymptotically we have

$$\frac{q^k}{\lambda} = 1 + a_1\frac{i^k}{k} \quad \text{and} \quad \frac{q^p}{\lambda} = 1 + a_1\frac{i^p}{p}, \tag{A.5.4}$$

so that $q^k = q^p = q$. Moreover, λ_t and $q_t^k = q_t^p = q_t$ grow at the same rate. Using the latter in (3.7e) and (3.7f) reveals that the ABGP requires

$$\frac{q}{\lambda} = \frac{\beta}{1 + n}\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1 - \tau^r)r_{t+1}^k + \frac{a_1}{2}\left(\frac{i^k}{k}\right)^2\right] + \frac{q_{t+1}}{q_t}\left(\frac{q}{\lambda}\right)(1 - \delta)\right\}, \tag{A.5.5a}$$

$$\frac{q}{\lambda} = \frac{\beta}{1 + n}\left\{\frac{\lambda_{t+1}}{\lambda_t}\left[(1 - \tau^r)r_{t+1}^p + \frac{a_1}{2}\left(\frac{i^p}{p}\right)^2\right] + \frac{q_{t+1}}{q_t}\left(\frac{q}{\lambda}\right)(1 - \delta)\right\}. \tag{A.5.5b}$$

Hence, asymptotically we must have $r^k = r^p$. From (3.10b) and (3.10c) expressed in per-capita variables the latter requires that

$$(1 - \alpha)A\kappa \lim_{t\to\infty}\left(\frac{k_t}{l_t + \kappa p_t}\right)^\alpha = \alpha A \lim_{t\to\infty}\left(\frac{l_t + \kappa p_t}{k_t}\right)^{1-\alpha}. \tag{A.5.6}$$

Since $\lim_{t\to\infty} l_t \in [0, 1]$ (as we show below) the ABGP has

$$\lim_{t\to\infty}\left(\frac{k_t}{l_t + \kappa p_t}\right)^\alpha = \left(\frac{k}{\kappa p}\right)^\alpha \quad \text{and} \quad \lim_{t\to\infty}\left(\frac{l_t + \kappa p_t}{k_t}\right)^{1-\alpha} = \left(\frac{\kappa p}{k}\right)^{1-\alpha}. \tag{A.5.7}$$

With the latter in (3.10b) and (3.10c) we obtain

$$\frac{k}{p} = \frac{\alpha}{1-\alpha} \qquad (A.5.8)$$

and

$$r^k = r^p = A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\kappa^{1-\alpha}. \qquad (A.5.9)$$

Equation (A.5.8) implies that the right-hand side of (3.15) converges to the finite constant

$$\frac{\theta}{1-\theta}\frac{1-\tau^w}{1+\tau^c}(1-\alpha)^{1-\alpha}A\left(\frac{\alpha}{\kappa}\right)^{\alpha}.$$

Since c_t grows asymptotically at a positive rate (3.15) will be satisfied in finite time. Hence, the ABGP has $l = 0$. From (3.7a) this implies that asymptotically

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{q_{t+1}}{q_t} = (1+\gamma)^{\theta(1-\eta)-1}.$$

A.5.3 Calibration details

In this section, we describe the calibration of the two production parameters A and κ. We formulate 5 conditions for the variables A, κ, q^k/λ, k, and i^k. They include the four equations (A.5.10)–(A.5.12) and the condition that the endogenous labor share in our simulation equals 57% in 2010.

The first condition stems from the initial steady state in 1965, where both k_t and i_t^k are constant. Hence, equation (A.5.2e), which describes the accumulation of physical capital k_t, implies

$$i^k = (n+\delta)k. \qquad (A.5.10)$$

The second and the third condition stem from the first-order conditions (A.5.2c) and (A.5.2d) for i_t^k and k_{t+1}. In conjunction with (A.5.2h) and evaluated at the steady state these become

$$\frac{q^k}{\lambda} = 1 + a_1\frac{n+\delta}{a_2+k}k, \qquad (A.5.11a)$$

$$\frac{q^k}{\lambda} = \frac{\beta}{1+n}\left\{(1-\tau^r)\alpha Ak^{\alpha-1}l^{1-\alpha} + a_1\frac{(n+\delta)^2k^2}{2(a_2+k)^2} + \frac{q^k}{\lambda}(1-\delta)\right\}. \qquad (A.5.11b)$$

The fourth condition is derived from the asymptotic growth rate of per-capita variables, $\gamma = 2.0\%$. The value of γ derives from

$$\frac{q}{\lambda} = \frac{\beta(1+\gamma)^{\theta(1-\eta)-1}}{1+n}\left\{(1-\tau^r)A\alpha^{\alpha}(1-\alpha)^{1-\alpha}\kappa^{1-\alpha} + \frac{a_1}{2}\left(\frac{i^k}{k}\right)^2 + \frac{q}{\lambda}(1-\delta)\right\}, \qquad (A.5.12)$$

which expresses (A.5.5a) along the ABGP.

The fifth condition imposes in our simulation a value of 57% on the endogenous labor share in 2010.

These conditions constitute a complex nonlinear system of five equations in five endogenous variables. We use an iterative procedure to solve this computational problem. We start with a guess for the parameter A. Next, we solve (A.5.10)–(A.5.12) for the endogenous variables κ, k, i^k and q^k/λ given $\gamma = 2.0\%$. Finally, we simulate the model and compute the labor share in the year 2010. If the labor share is equal to 57%, we stop. Otherwise we update our guess for A. To speed up the convergence in the outer loop over A, we use the secant method described in Sect. 11.5.1 of Heer and Maußner (2009).

A.5.4 Details on the computation of the transition dynamics

To compute the transition dynamics, we need to solve a difference equation system in the state variables $\{k_t, p_t\}_{t=1965}^{2175}$. We choose a time horizon of 210 years (= periods) so that the growth rates of the variables stabilize and are numerically close to their exact asymptotic counterparts.

As endogenous variables of our difference equation system, we use consumption, c_t, investment in both capital stocks, i_t^k and i_t^p, and labor l_t. The difference equations system include the household's first-order conditions (3.7c), (3.7e), (3.7f), and the resource constraint (3.14). With the endogenous variables, it is straightforward to compute the dynamics of the two capital stocks, k_t and p_t, from (3.4) for given initial values $k_{1965} > 0$ and $p_{1965} = 0$ taken from the steady state without automation capital.

Using k_t, p_t, and l_t, we compute the factor prices w_t, r_t^k, and p_t. From the first-order conditions (3.7a), (3.7c) and (3.7d) we calculate λ_t, q_t^k, and q_t^p. Hence, the values of all variables that show up in the equilibrium conditions of our model are pinned down. For the endogenous variables in the year 2176 that are also needed to compute the transition dynamics, we assume that c_t, i_t^k, and i_t^p grow at the asymptotic growth rate, while l_t is equal to zero (which occurs in the year 2132 in our simulation of the benchmark economy).

In essence, we have to solve a nonlinear equations problem in $4 \times 210 = 840$ variables. This is a non-trivial task. The problem is to find a good initial value for the endogenous variables during 1965–2175. We, therefore, proceed as follows:[16]

Algorithm: Computation of the Transition Dynamics in the Benchmark Model with Automation Capital

Step 1: Compute the initial steady state in the year 1964 without automation capital.

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[16] The Gauss computer code is available from the authors upon request.

Step 2: Compute the final steady state of the model with $\kappa = 0$ and, hence, without automation capital, for the tax rates and the population growth rate prevailing in 2015.

Step 3: Project a transition path for the model without automation capital (with $\kappa = 0$) for $\{i_t^k, c_t, l_t\}_{t=1965}^{2015}$ in the form of a linear adjustment.

Step 4: Solve the simple model without automation capital ($\kappa = 0$).

Step 5: Use the transition path from Step 4 with $p_t \equiv 0$ for $t = 1965, \ldots, 2015$ as initial guess for the computation of the transition in the model with automation capital. Assume that the variables $\{i_t^k, i_t^p, c_t\}$ grow at the rate γ after the final period, while l_t falls at the rate γ.

Step 5: Iterate over the time horizon of the transition T by incremental steps of one year. Use the transition path of the previous iteration as an initial guess assuming that in the period $T + 1$, the variables $\{i_t^k, i_t^p, c_t\}$ grow at the rate γ, while l_t falls at rate γ if it is larger than zero or remains equal to zero otherwise.

Step 6: Stop when the dynamics of the model during the period 1965–2015 do not change any more and the endogenous variables have reached their asymptotic values in the period T.

A.5.5 Sensitivity analysis: utility function and Frisch labor supply elasticity

The response of the labor supply and, hence, the labor share with respect to a change in income taxation is mainly driven by the response of the labor supply to a change in the net wage rate. This section provides a sensitivity analysis of our results with respect to the labor supply elasticity.

In our benchmark equilibrium, the instantaneous utility function is given by (3.3). The implied Frisch labor supply elasticity, η^F, is

$$\eta^F = \frac{1 - \theta(1 - \eta)}{\eta} \frac{1 - l}{l}.$$

Thus, in the initial steady state (with $l = 0.30$) we have $\eta^F = 1.63$. This value is in the upper range of empirical estimates. Though, estimates of the Frisch labor supply elasticity based upon microeconometric studies vary considerably. MaCurdy (1981) and Altonij (1986) estimate values of 0.23 and 0.28 using PSID data, while Killingsworth (1983) finds a value of 0.4. Domeij and Floden (2006) argue that these estimates are biased downward due to omitted borrowing constraints. In macroeconomic studies such as Trabandt and Uhlig (2011), a value of unity is often chosen to account for the effects of higher wages on labor along both the intensive and extensive margins. These findings lead us to test the sensitivity of our results for a different utility function that exhibits a Frisch labor supply elasticity equal to 0.64, a value much lower than 1.63.

Following Fehr et al. (2013), consider the following instantaneous utility function

$$u(c, 1 - l) = \frac{1}{1 - \eta} \left[c^{1 - \frac{1}{\rho}} + \theta(1 - l)^{1 - \frac{1}{\rho}} \right]^{\frac{1 - \eta}{1 - \frac{1}{\rho}}}, \tag{A.5.13}$$
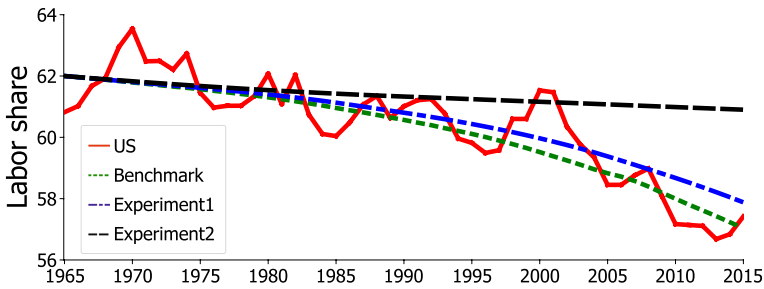
Fig. 15 Sensitivity analysis: labor share and income taxes τ^w and τ^r

where ρ denotes the intra-temporal elasticity between consumption and leisure and $1/\eta$ is again the intertemporal elasticity of substitution. Then, the implied Frisch labor supply elasticity is (see, Fehr et al. 2013, p. 99).

$$\eta^F = \frac{l}{1-l}[\xi\eta + (1-\xi)\rho], \quad \xi = \frac{\theta^\rho w^{1-\rho}}{1 + \theta^\rho w^{(1-\rho)}}.$$

We use the value $\rho = 0.60$ from Fehr et al. (2013) so that the Frisch labor supply elasticity is equal to 0.641 in the initial steady state. The latter is a reasonable intermediate value between those estimated by the microeconometric studies and those applied in macroeconomic simulations using Dynamic General Equilibrium models.[17]

Figure 15 presents our results for the behavior of the labor share in our model under the utility function (A.5.13) and in the two experiments 1 and 2 where we set the labor income tax rate, τ^w, and both income tax rates, τ^w and τ^r, equal to their values prevailing in 1965, respectively.

Comparing Figs. 9 and 15 reveals that the combined contribution of the two tax rates for the explanation of the labor share decline is approximately the same. Under the utility function (3.3) the labor income tax and capital tax rate explain 18% and 60% of the decline in the labor share (Fig. 9), under the utility function A.5.13, the explanatory shares amount to 12% and 66%, respectively (Fig. 15). Unsurprisingly, the explanatory power of the labor income tax rates for the decline in the labor share shrinks with a lower labor supply elasticity, while that of the capital income tax rate even increases.

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[17] We also re-calibrated the parameter set $(\theta, A, \kappa, a_2) = (0.317, 9.5, 0.019, 0.11)$ so that (1) the average labor supply in 1965 is equal to 0.30, (2) the asymptotic growth rate is equal to 2.0%, (3) automation capital grows at 10.9% during the initial years 1975–1980, and (4) the labor share amounts to 57% in 2010.

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