Journal of Macroeconomics 75 (2023) 103488

Contents lists available at ScienceDirect

Journal of Macroeconomics

journal homepage: www.elsevier.com/locate/jmacro

Age-specific entrepreneurship and PAYG: Public pensions in Germany☆

Burkhard Heer[a,b,c], Mark Trede[d,*]

[a] University of Augsburg, Department of Economics, Universitatsstr. 16, 86159, Augsburg, Germany
[b] CESifo, Poschingerstr. 5, 81679, Munich, Germany
[c] Netspar, Tilburg University Campus, Koopmans Building, K515, Warandelaan 2, 5037 AB, Tilburg, The Netherlands
[d] University of Muenster, Institute of Econometrics, Am Stadtgraben 9, 48143, Muenster, Germany

ARTICLE INFO

JEL classification:
H55
D31
J11
L26
D15
C68

Keywords:
Entrepreneurship
Aging
Income distribution
Overlapping generations
Social security
Fiscal sustainability

ABSTRACT

We present new empirical evidence on the distribution of earnings, income and wealth among entrepreneurs in Germany. We document that both earnings and income are more concentrated among entrepreneurs than among workers and describe a large-scale overlapping-generations model that replicates the age-earnings profiles of these two household types. As an application, we compute the equilibrium effects of a reform of the German pay-as-you-go pension system in which entrepreneurs must also contribute and receive a pension. We show that in the presence of mobility between workers and entrepreneurs, the expected lifetime utility of all newborn households unanimously declines due to the general equilibrium effects of lower aggregate savings, and welfare losses amount to approximately 0.7% of total consumption. In addition, the integration of self-employed workers into the social security system in Germany does not help to improve its fiscal sustainability, and only an increase in the retirement age to 70 years will help to finance pensions at the present level beyond the year 2050.

1. Introduction

Worldwide, pension systems are being continually reformed. The main cause of stress on pension systems is the global demographic shift towards higher longevity and lower fertility. To counter the effects of the rising dependency ratio, several policies are available. In the recent past, many countries in the OECD have decided to raise the retirement age — on average, the normal retirement age will increase by almost two years by approximately 2060 (OECD, 2019, p. 17). Other policy options include changes in contribution rates during working life or replacement rates of pension. Another major policy measure regards extending the groups who are subject to mandatory contributions, such as the self-employed or entrepreneurs. Across the OECD, voluntary or mandatory access to pension plans varies widely for entrepreneurs and non-standard categories of workers.

Including entrepreneurs in the public pension system is attractive, as they would contribute immediately to the pension system, thus alleviating the current pension burden. Even though their share is relatively small in total population – using the Survey of

☆ We would like to thank Ping Wang and an anonymous referee for helpful comments. Part of this article was written during Burkhard's stay at Fordham University and the Federal Reserve Bank at St. Louis and we would like to thank Jim Bullard, Paul McNelis, Christopher Waller and Christian Zimmermann for their hospitality. The views expressed in this article are ours and do not necessarily reflect the official positions of the Federal Reserve Bank of St. Louis, the Federal Reserve System, or the Board of Governors. All remaining errors are ours.

* Corresponding author.
E-mail addresses: burkhard.heer@wiwi.uni-augsburg.de (B. Heer), mark.trede@uni-muenster.de (M. Trede).

https://doi.org/10.1016/j.jmacro.2022.103488
Received 20 April 2022; Received in revised form 2 December 2022; Accepted 6 December 2022
Available online 9 December 2022

Consumer Finances, Brüggemann (2021) estimates the share of entrepreneurs to amount to 7.4% in the US economy in 2010, while we find a similar share of approximately 8% in Germany using the Social–Economic Panel (SOEP) data during 2002–2017 – they contribute for a significantly larger portion of aggregate economic activity. According to Brüggemann (2021), entrepreneurs generate 17% of total income and own 31% of total net worth in the US in 2010. Considering the population of full-time earners in Germany, the corresponding shares are broadly comparable. Total earnings accruing to entrepreneurs amount to 12%, and their share of net wealth is 32%. In the long run, however, the transitional gain from mandatory social security contributions must be traded off against the future entitlements of these entrepreneurs. There are also arguments in favor of exempting entrepreneurs from social security contributions: compared to regular employees, they have a higher degree of discretion in calculating their contribution base, which might be considered unfair. Furthermore, not being required to pay into the public pension system might induce more individuals to start their own businesses. In this way, economic policy can promote entrepreneurship, and more businesses will mean higher total employment.

In our model, we explicitly account for this economic mechanism described above: the imposition of social security contribution reduces the investment incentives of the entrepreneur. The self-employed need to finance their business by means of their own savings and credit in the financial market. If the social security authority forces the entrepreneurs to form a large part of their precautionary savings for old age in the form of public pension entitlements, they will reduce their old-age savings in the form of entrepreneurial capital. In our calibrated model of the Germany economy, we find that mandatory social security for the entrepreneurs reduce aggregate savings by 3.4%–7.5% depending on the progressivity of the pensions even though entrepreneurs only constitute 8% of the households.

Our model builds upon the work of Cagetti and De Nardi (2006, 2009).[1] While these authors only consider two stages in the life of a household as a worker and a retiree in their model of perpetual youth based upon Blanchard (1985) and Gertler (1999), we extend their model to an overlapping-generations (OLG) framework with annual periods that allows for a more realistic life-cycle description of age-dependent shares of entrepreneurs and savings. In particular, we are able to model the hump-shaped profile of the share of entrepreneurs in each cohort using a large-scale OLG model in the tradition of Auerbach and Kotlikoff (1987) that is calibrated to the characteristics of the Germany economy. Our data sources are the Cross National Equivalent Files and the wealth samples of the Socio-economic Panel (SOEP). We document the different age-earnings profiles of entrepreneurs and workers, the hump-shaped share of entrepreneurs as a function of age, the level of inequality and the degree of mobility between entrepreneurs and workers.[2] The life-cycle model is then used to run counterfactual experiments to investigate the consequences of extending the German pay-as-you-go (PAYG) pension system to entrepreneurs.

We add three results to the literature. (1) We show that both under the present demographics and the population parameters prevailing in the year 2050, the total welfare (as measured by expected lifetime utility) of newborn households decreases if entrepreneurs also contribute to the public pension system in Germany. This qualitative result does not depend on whether pension payments are proportional to contributions or lump-sum. (2) We demonstrate that Germany will be unable to finance the PAYG pension system in the year 2050 through taxes on labor income alone, even if entrepreneurs have to contribute to the public pension system. (3) An increase in the (effective) retirement age to 70 helps to establish the sustainability of the public pension system.

Our policy analysis is related to the extensive literature on the implications of aging for the sustainability of social security systems based on multi-period overlapping-generations models. As one of the earliest and most prominent studies on the consequences of the effects of demographic transition on public pensions, De Nardi et al. (1999) evaluate various policy instruments in a dynamic general equilibrium model with overlapping generations. They advocate a switch to a purely defined contributions system. Nishiyama and Smetters (2007) analyze a 50% privatization of social security and find the welfare effects to be sensitive to the assumptions of a closed economy, missing annuities markets, and the progressivity of pensions. In a more recent study, Kitao (2014) finds that reducing pension benefits is the most efficient policy in the long run. Heer and Irmen (2014) also consider the effect of pension reform on the endogenous growth rate. If labor supply becomes scarcer due to aging, firms have a higher incentive to invest in labor-augmenting technological change. The growth rate effect is shown to be largest for the case of a frozen contribution rate so that the level of pensions falls in comparison to policies with (i) a constant pension level or (ii) a higher retirement rate. Heer et al. (2020) study the sustainability of PAYG public pension systems in the U.S. and 14 European countries. For the present pension systems in these countries, as characterized by the replacement rate of pensions with respect to wage income and the effective retirement age, they find that the majority of continental European countries, including France, Italy, Spain, and Germany, cannot finance pensions beyond the year 2040 through a social security tax on labor income alone. In contrast, the English-speaking countries – the U.S., the UK and Ireland – have sufficient fiscal space to finance future increases in public pension expenditures because their present tax rates on labor income are still a large distance away from the top of the Laffer curve.[3]

This paper is organized as follows. In Section 2, we describe the data from Germany and present age-specific statistics for both workers and entrepreneurs together with inequality measures. Section 3 presents our model with two types of households, workers

[1] The model of de Nardi and Cagetti has also recently been used to study the question of optimal income tax progressivity. In addition to these authors, İmrohoroğlu et al. (2018) introduce superstar entrepreneurs in their model, while Brüggemann (2021) allows for elastic labor supply of the workers and abstracts from intergenerational correlation of earnings or abilities. They both find optimal marginal tax rate of the top income earners in the vicinity of 60%.

[2] Different from Cagetti and De Nardi (2006), however, we do not model an endogenous choice of occupation.

[3] Our discussion of the literature on the sustainability of public pensions in light of the demographic transition is by no means exhaustive. Other articles that have considered demographic transitions and their effects on the pension system include Braun and Joines (2015), Conesa and Garriga (2016), İmrohoroğlu and Kitao (2009), İmrohoroğlu et al. (2016), Krueger and Ludwig (2007), among others.

and entrepreneurs. We allow for both mobility between the two household types and income uncertainty. In addition, income taxes are progressive and pensions in the PAYG system depend on the individual's contributions. The model is calibrated with respect to characteristics of the German economy in Section 4. In Section 5, we describe the distribution of income and wealth both among workers and entrepreneurs in our model and compare them with the empirical observations in Germany. Section 6 considers policy experiments in which entrepreneurs also contribute to the German pay-as-you-go pension system. We find that welfare declines unanimously for all types of newborn households for both proportional and lump-sum pensions. In Section 7, we consider the effects of aging by studying the model economy using forecasts of United Nations (2015) for Germany's demographics. We find that the present pension system cannot be financed by social security taxes on wage income. Including entrepreneurs in the PAYG pension system does not help to establish financial sustainability of social security in Germany; only a postponement of the (effective) retirement age to age 70 helps to finance pensions from contributions on non-capital income. Section 8 concludes. The Online-Appendix[4] describes the stationary equilibrium of the model in more detail.

2. Earnings, income and wealth distribution among workers and entrepreneurs

This section describes the main features of the earnings, income and wealth distribution of entrepreneurs and workers in Germany.

2.1. Data description

Our data source is the German Social–Economic Panel dataset[5] (SOEP) provided by the German Institute for Economic Research/DIW Berlin (Goebel et al., 2019). We make use of the Cross National Equivalent File (CNEF) and the wealth samples, which are part of the SOEP data distribution (Grabka, 2020). The CNEF earnings variable consists of wages and salaries, including training, secondary jobs, bonuses, over-time compensation and profit-sharing, as well as earnings from self-employment, defined as profits before taxes. Gross earnings from self-employment are also reported as a separate variable. Annual hours worked is calculated from individuals' activity calendar data about the average number of hours worked per week and the number of months worked (see Grabka, 2020, for details).

The total population consists of all individuals regardless of whether they are economically active. We restrict our analysis to the economically active population consisting of individuals (i) aged between 20 and 80, (ii) working between 950 and 4000 h per year ("full-time"), and (iii) having strictly positive earnings either from dependent work – workers – or from self-employment – entrepreneurs. In our analysis, entrepreneurs are defined as individuals whose earnings from self-employment constitute at least 75% of their total earnings. For entrepreneurs, earnings from sources other than self-employment are disregarded.[6] Years before 1995 have been dropped so that we can consider both West and East Germany and avoid distortions due to drastic changes in the first few years after reunification. All observations are weighted using either cross-sectional weights or, for issues regarding two periods, longitudinal weights.

Information on gross and net wealth is available only for the years 2002, 2007, 2012 and 2017. Total gross wealth includes housing (ownership of a home and other property), financial assets, building-loan contracts, private insurance, business assets and tangibles. Net wealth is computed as gross wealth minus total debts (mortgages, consumer credits and other debts). The SOEP provides multiple imputations for the wealth variables with item non-response (Grabka and Westermeier, 2015). As we are not primarily concerned with the variation in the estimates, we take into account only one of the imputations. Analyzing the SOEP data, Grabka and Westermeier (2014) describe the overall wealth distribution in Germany. Even though the definition of wealth is rather comprehensive, public pension entitlements originating from the social security system are neglected. Another source of measurement problems is the absence of individuals with extremely high wealth levels in the survey.

The income and wealth variables have been inflation adjusted using annual CPI values published by the Federal Statistical Office (Statistisches Bundesamt, 2020). Table 1 reports descriptive statistics when observations are pooled over all years (1995–2018). Fig. 1 shows the proportion of entrepreneurs per year in the sample. The share increased in the first half of the observation period and slightly decreased since then. Its long-run average is approximately 8%.

2.2. Earnings over time

For both groups (workers and entrepreneurs), we identify individuals as highly productive in year t if their hourly earnings are above the median hourly earnings of their group in year t. Both workers and entrepreneurs can jump between productivity levels from year to year. The mean real hourly earnings of workers and entrepreneurs of both productivity levels are shown by the dashed lines in Fig. 2. The solid lines are the corresponding smoothing splines. The smoothed overall means (for workers and entrepreneurs of both productivity levels) are shown by the dotted lines in the middle. In any year, low-productivity workers earn more per hour than low-productivity entrepreneurs. For highly productive individuals, the relation reverses: the overall mean earnings per hour are larger for entrepreneurs. The spread between high-earnings workers and entrepreneurs increases over time.

4 https://www.wiwi.uni-muenster.de/oeew/sites/oeew/files/downloads/heer-trede-jmacroecon-online-appendix.pdf.

5 Socio-Economic Panel (SOEP), version 35, waves 1984–2018, 2020, doi: 10.5684/soep.v35.

6 Averaged over all observations for entrepreneurs, earnings from other sources contribute less than 0.5% to total earnings.

Table 1
Descriptive statistics: Mean and standard deviation of pooled observations. Hours worked, earnings, pre- and post-government incomes are annual quantities; earnings, incomes and wealth are inflation adjusted (in 2015 Euros).

	Mean	Standard deviation
Age	43.0	11.3
Hours worked	2 104.7	528.3
Earnings	36 973.5	28 907.8
Hourly earnings	17.4	12.0
Pre-government income	60 874.1	47 281.2
Post-government income	44 041.4	28 272.3
Gross wealth	133 186.7	474 993.4
Net wealth	105 776.0	436 893.4

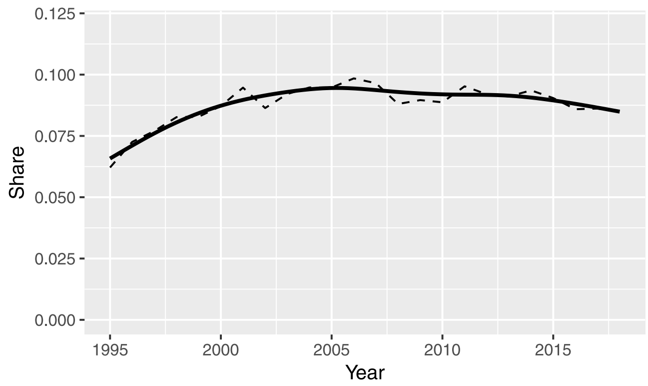

Fig. 1. Proportion of entrepreneurs (dashed line); the solid line is the smoothed time series.

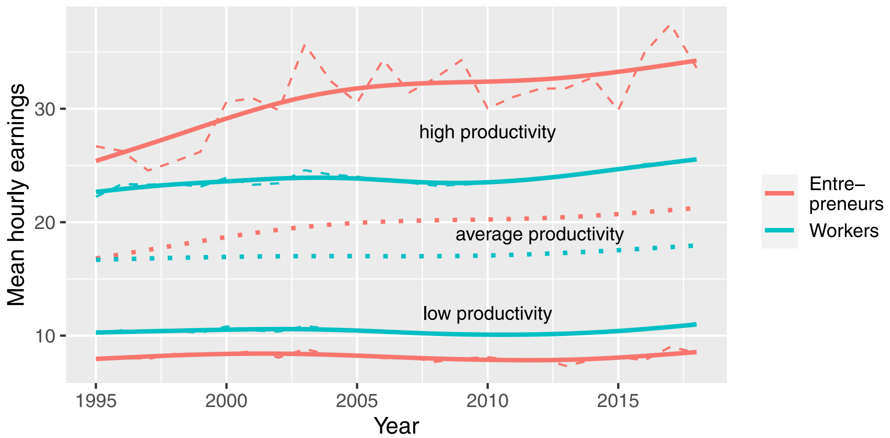

Fig. 2. Mean hourly earnings for entrepreneurs and workers with low or high productivity levels (dashed lines) and the smoothed time series (solid lines); the dotted lines are the overall productivity mean hourly earnings of both groups. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2.3. Age profiles

The age profile of the share of entrepreneurs is depicted in Fig. 3. Individuals are binned into five-year age groups (20–24, 25–29, . . . , 75–80) to obtain more stable estimates. The colored lines show the age profiles for the selected years. Evidently, the lines shift to the right over time, at least for the older age groups. The rising share, especially for individuals aged 45+, is likely to have been caused by labor market policies starting in the 1990s that encouraged the unemployed to move into self-employment (Caliendo et al., 2016). Averaging over time, the share of entrepreneurs is roughly 1% for 21-year-olds.

Mean hourly earnings also depend on age. Fig. 4 shows the age profiles of mean hourly earnings for workers and entrepreneurs of both productivity levels, where the observations have been pooled over all years. With the exception of low-productivity workers, the profiles decline for very young ages. For highly productive individuals (workers or entrepreneurs), the earnings profile then rises until

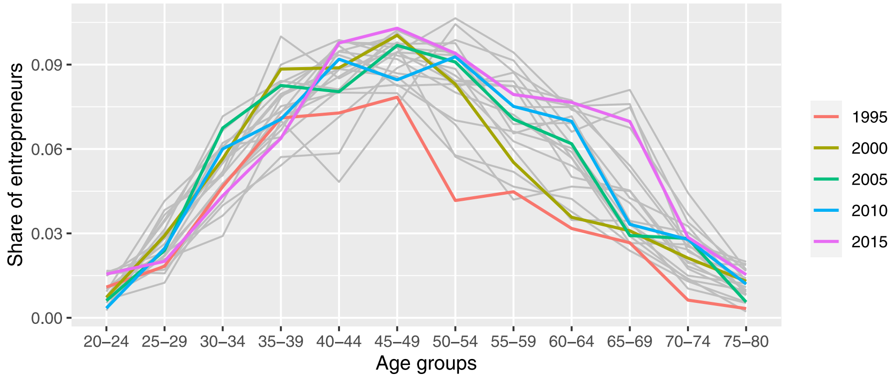

Fig. 3. Age profile of the proportion of entrepreneurs in the total population. Observations are binned into five-year age groups. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

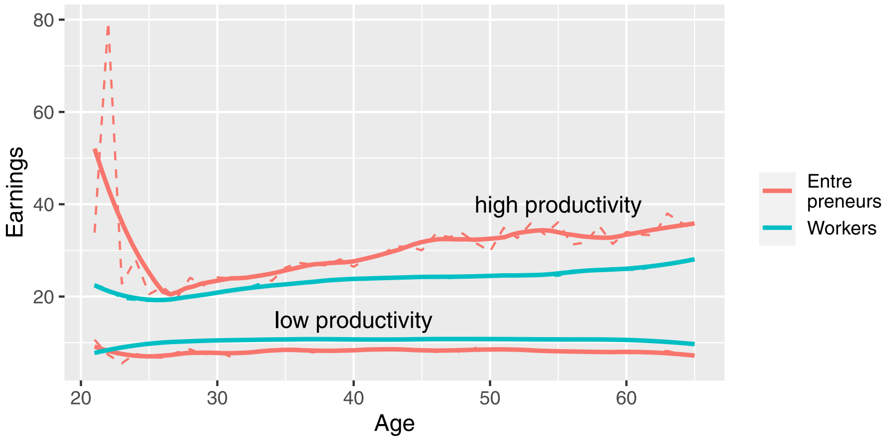

Fig. 4. Age profile of the mean hourly earnings of entrepreneurs and workers of both productivity levels. The solid lines are the smoothed profiles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

retirement. In contrast, workers and entrepreneurs of the low-productivity type experience falling wages after approximately 50 years of age. High-productivity entrepreneurs earn higher hourly wages than high-productivity workers. In contrast, low-productivity entrepreneurs earn less than low-productivity workers.

2.4. Inequality of income and wealth among workers and entrepreneurs

We proceed to describe the evolution of economic inequality. Fig. 5 shows the Gini coefficients of earnings, gross income and net income[7] for workers and entrepreneurs. The level of inequality is considerably higher for entrepreneurs, while the change over time is roughly similar in both groups. Apart from random fluctuations, inequality did not change much before 2000 and then started to increase markedly for approximately 10 years. Since 2010, inequality has been relatively stable again. For both groups, earnings are more unequally distributed than pre-government income. Due to the progressive tax and benefit system, post-government income is the most equally distributed variable. Fuchs-Schündeln et al. (2010) decompose earnings inequality but find that a large share cannot be explained by the observable variables.

Regarding wealth inequality, Table 2 reports the Gini coefficients of gross and net wealth for the years 2002, 2007, 2012, and 2017. Consistent with findings in other studies (Grabka and Westermeier, 2014), wealth inequality is extremely high. Net wealth inequality is higher than gross wealth inequality, indicating that individuals with low gross wealth tend to have higher debt than richer individuals.

The correlation between earnings and wealth is relatively small (see Table 3). This finding is in line with other studies, e.g., Budría Rodríguez et al. (2002). The correlation estimates are rather volatile over time, and there is no clear pattern for the relation between workers and entrepreneurs. The estimates become more stable when we compute rank correlations (not reported), but again, there is no clear distinction between workers and entrepreneurs.

[7] Gross income is identified by the CNEF variable for pre-government income, and net income is defined by the CNEF variable for post-government income (Grabka, 2020).

Fig. 5. Gini coefficients of the earnings, gross income and net income of entrepreneurs and workers. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
Gini coefficients of gross and net wealth.

Year	All		Workers		Entrepreneurs	
	Gross wealth	Net wealth	Gross wealth	Net wealth	Gross wealth	Net wealth
2002	0.7152	0.7738	0.6983	0.7589	0.6447	0.6923
2007	0.7233	0.7932	0.6835	0.7617	0.7322	0.7825
2012	0.7106	0.7782	0.6780	0.7565	0.7232	0.7491
2017	0.7214	0.7765	0.6826	0.7478	0.7332	0.7521

Table 3
Correlation coefficients of gross wealth with earnings and of net wealth with earnings.

Year	All		Workers		Entrepreneurs	
	Gross wealth	Net wealth	Gross wealth	Net wealth	Gross wealth	Net wealth
2002	0.2409	0.2266	0.2181	0.2127	0.2318	0.2042
2007	0.2870	0.2556	0.3014	0.2612	0.2803	0.2500
2012	0.3550	0.3381	0.3670	0.3440	0.4973	0.4671
2017	0.2776	0.2574	0.3635	0.3462	0.2771	0.2575

2.5. Mobility

Individuals can experience two kinds of transitions: they can jump between productivity levels, and they can change from being a worker to being an entrepreneur or vice versa. Assuming time-invariant transition probabilities, the 4 × 4 transition matrix for workers and entrepreneurs of both productivity levels is

$$\Pi = \begin{bmatrix} & \text{worker/low} & \text{worker/high} & \text{entre/low} & \text{entre/high} \\ \text{w/low} & 0.8479 & 0.1427 & 0.0074 & 0.0020 \\ \text{w/high} & 0.1429 & 0.8520 & 0.0014 & 0.0036 \\ \text{e/low} & 0.0932 & 0.0223 & 0.6945 & 0.1901 \\ \text{e/high} & 0.0256 & 0.0488 & 0.1797 & 0.7459 \end{bmatrix}. \tag{1}$$

For example, the probability that a low-productivity worker becomes a high-productivity entrepreneur in the next period is extremely small (0.0020). In each row, the highest probability is on the diagonal element, implying that remaining in the current state is the most likely outcome in the next period. Workers are considerably more persistent in this regard than entrepreneurs.

2.6. Literature review

The majority of empirical studies on income or earnings dynamics revolve around the U.S. and do not differentiate between entrepreneurs and workers. Moreover, most studies consider exclusively earnings or income of male workers. For instance, Guvenen (2009) investigates the risk structure of the labor incomes of men without distinguishing between workers and entrepreneurs. He concludes that heterogeneity in the deterministic component of income paths should be taken into account. Guvenen et al. (2021) offers a detailed statistical analysis of earnings changes using data on male employees. The study shows that earnings shocks are nonnormally distributed and have substantial skewness and kurtosis. A comprehensive overview of the distribution of earnings, income and wealth in the U.S. is given by Budría Rodríguez et al. (2002). They confirm the finding of earlier studies that wealth is by far the most unequally distributed variable, and earnings is less equally distributed than income for the main part of the distribution. Looking at the correlation between earnings and wealth, Budría Rodríguez et al. (2002) report relatively small values (0.47); the correlation between income and wealth (0.60) is also lower than the correlation between earnings and income (0.72). Furthermore, they find that employment status, i.e., whether an individual is a worker, self-employed, retired or a non-worker, is a crucial variable: workers tend to be wealth-poor, whereas retirees have higher wealth on average. The self-employed have a much higher share of income and wealth than their population share.

Quadrini (2000) provides a comparative analysis of the differences in income and wealth between workers and entrepreneurs in the U.S. He finds that there is a high share of entrepreneurs in the top wealth group. Considering the distinction between entrepreneurs and workers is therefore critically important to understanding this strong wealth concentration. Quadrini (2000) further shows that entrepreneurs have a higher wealth-to-income ratio than workers. Cagetti and De Nardi (2006) contrast the wealth share of workers on the one hand, and entrepreneurs and self-employed on the other hand. For the U.S., they report that entrepreneurs and the self-employed own about half of total net worth even though they constitute just 17 percent of the population. They also document a high level of wealth inequality with the top 1 percent households holding roughly 30 percent of total net wealth, the proportion of workers in the top 1 percent is small (under 20 percent). Brüggemann (2021) defines entrepreneurs in a narrower sense as self-employed owners of businesses who actively manage their own business. According to this definition the share of entrepreneurs was 7.4 percent in 2010 in the U.S. Their share of total income amounts to 17 percent, while their wealth share is 31 percent. The Gini coefficient of income (0.65) and wealth (0.77) are both large for entrepreneurs; for workers, the Gini coefficient of income is 0.52, while it is 0.83 for wealth. Brüggemann (2021) reports a transition rate of 0.023 for entry into entrepreneurship and 0.22 as the exit rate. Income and wealth data might be subject to data quality issues. Bhandari et al. (2020) examine the reliability of several U.S. survey datasets on business income and wealth. Their analysis contains all types of business, i.e., it also includes corporations. According to Bhandari et al. (2020), the evidence suggests that survey data "should be treated with great caution", as they create considerable measurement problems, particularly concerning business valuations.

A comparison of the U.S. findings to Germany reveals both similarities and differences. Many features of the data are qualitatively similar but their levels differ quantitatively. For example, both in the U.S. and in Germany, income and earnings inequality is substantially higher for entrepreneurs than it is for workers. However, the Gini coefficients of both groups are at a lower level in Germany. The same is true for wealth inequality but the differences are less pronounced. As to mobility, the transition rate from worker to entrepreneur is substantially less than the transition rate in the reverse direction both in the U.S. and in Germany. However, the U.S. transition rates reported by Brüggemann (2021) are higher than the German rates. While the qualitative empirical regularities of the entrepreneurial sectors are equal to each other in the US and German economies, we appreciate that the quantitative differences will affect the quantitative results in our model presented in the next section. To assess how far the quantitative results derived in the next section carry over from the situation in Germany to the U.S. – or other advanced countries – it is pivotal to consider the differences in inequality and mobility of entrepreneurial income. The lower inequality is among entrepreneurs, the lower are aggregate savings in the entrepreneurial sector.[8] The lower mobility (or, equally, the higher persistence) of high productivity is among the top-income entrepreneurs, the larger is investment by the entrepreneurs. For instance, the two effects on entrepreneurial investment in their firm capital in a country with both lower inequality and lower mobility than in Germany tend to set each other off to some extent. The quantitative results will be reinforced in countries with higher inequality and lower levels of mobility. As we argued in the Introduction and will, again, in Section 4, the behavior of (entrepreneurial) savings is the main economic mechanism driving our results. Hence, our results can at least be carried over to other countries qualitatively. Depending on the levels of inequality and mobility, the quantitative results need to be adjusted up or down.

3. The model

In this section, we introduce our large-scale OLG model with mobility, progressive income taxes and contributions-related pensions. In the benchmark model, entrepreneurs do not contribute to the PAYG pension system.

3.1. Demographics

Households live a maximum of 80 periods. Periods are equal to one year. Households are born at age 1 (corresponding to a real-life age of 20). All agents of age s survive until age $s+1$ with probability ϕ_s, with $\phi_{80} = 0$. Let N_t denote the number of households at period t. We assume that the population grows at a constant rate, $\frac{N_{t+1}}{N_t} = 1+n$.

The first 45 periods, agents are working, while in the last 35 periods, they are retired and receive pensions if they have paid social security contributions while young.

[8] Notice that the savings rate increases with income and, therefore, aggregate savings also rise with income inequality. Huggett and Ventura (2000) explain this empirical observation of the households' savings rates in a calibrated life-cycle model with the help of permanent income and age differences.

3.2. Preferences

The total time endowment is equal to one and allocated between leisure $1-l$ and work l.[9] Instantaneous utility is assumed to be Cobb–Douglas in consumption c and leisure $1-l$:

$$u(c,l)=\frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\sigma}-1}{1-\sigma}, \tag{2}$$

where $1/\sigma$ denotes the intertemporal elasticity of substitution and γ is the weight of consumption in utility.

3.3. Household types and mobility

Households differ with regard to their assets a, accumulated pension contributions x, productivity type $j\in\{1,2\}$, employment type $\epsilon\in\{w,e\}$, and age s. Let $z=(a,x,j,\epsilon,s)$ denote the individual state vector. Households are born as either workers (indexed by "w") or entrepreneurs (indexed by "e") with productivity type j. We assume that the transition between the two employment types, $\epsilon\in\{w,e\}$, and productivity types $j=1,2$, is governed by a homogeneous Markov process. The transition probabilities are described by a matrix $\Pi(\epsilon',j'|\epsilon,j)$, where ϵ' and j' denote the next-period employment and productivity type, respectively. The share of entrepreneurs, ψ, among the newborn generation at age $s=1$ is set so that it is equal to that of the German population, while the share of productivity type $j=1$ ($j=2$) at age $s=1$ is set equal to one half for each employment type $\epsilon\in\{w,e\}$.

3.4. Household taxation

Following Holter et al. (2019), we model the progressive taxation of income from labor. Workers pay social security contributions at a proportional rate τ^p on gross labor income up to a threshold y^{max}, while the capital income tax τ^r and the consumption tax τ^c are imposed as flat-rate taxes. Let $\hat{y}$ denote the pretax non-capital income of the individual worker, entrepreneur or retiree. Accordingly, average income amounts to $\bar{y}$ among the income-tax payers. After-tax (non-capital) income y is given by

$$y=\zeta_0\left(\frac{\hat{y}}{\bar{y}}\right)^{1-\zeta_1} \tag{3}$$

where ζ_0 and ζ_1 denote the tax level parameter and the tax progressivity parameter, respectively. Taxes on non-capital income are given by

$$T(\hat{y})=\hat{y}-y. \tag{4}$$

This functional form of the tax function has the advantage that the level ζ_0 can be changed without affecting tax progressivity ζ_1.

3.5. Contribution-based pensions

The pension system is a pay-as-you-go system. We model the relevant main features of the German pension system. The social security authority collects contributions on gross labor income $\hat{y}$ from workers (up to a threshold y^{max}), but not from entrepreneurs. The pension depends on accumulated contributions x during the working periods. In our model, we use the average rather than the total accumulated contributions (the so-called sum of earnings points in the German pension system) to simplify the computation, while the result, of course, is unchanged.[10] Therefore, the average accumulated earnings of the $(s+1)$-year-old household at the beginning of the next period, x', are a function of its accumulated earnings x at age s in the present period:

$$x'=\begin{cases}\dfrac{(s-1)x+\tau^p\min\{\hat{y},y^{max}\}}{s} & \text{if } \epsilon=w,\quad s=1,\dots,45\\ \dfrac{(s-1)x}{s} & \text{if } \epsilon=e,\quad s=1,\dots,45\\ x & s=46,\dots,80,\end{cases} \tag{5}$$

with initial cumulated contributions equal to zero at the beginning of the life, $x=0$ for $s=1$. Note that workers do not accrue interest on their social security payments in Germany. In the model, we neglect aspects of the German public pension system that are not relevant for the question under consideration such as early (or postponed) retirement schemes, periods of unemployment, sickness or child care etc.

Pensions $p(x)$ are paid proportional to accumulated average contributions irrespective of the household type at the end of the working period[11] and are subject to taxes $T(.)$ on non-capital income.

---

9 For notational convenience, we drop the time period index t from individual and aggregate variables in the following whenever it does not cause any misunderstanding.

10 To convert the average accumulated to the total accumulated contributions, just multiply the former by the number of working periods.

11 This reflects the schedule of the German pay-as-you-go pension system, which is basically proportional to contributions and characterized by a low degree of progressivity. According to OECD (2007), the German pension system is characterized by a progressivity index of 26.7, while the more redistributive public pension systems in the UK and Canada display progressivity indices equal to 81.2 and 86.6.

3.6. Household budget constraints

Worker. The s-year-old worker with productivity j receives total gross labor income, $\hat{y} = \theta^{w,s,j} l w$, in period t, which is equal to the product of his productivity $\theta^{w,s,j}$, his (endogenous) working time l, and the wage per efficiency unit w in period t. He also earns interest r on his wealth a. In addition, all households receive transfers tr from the government. Workers pay wage income taxes $T(\hat{y})$ and social security contributions on their wage income as well as flat-rate taxes τ^r and τ^c on their capital income and consumption. In old age, the worker does not work, $l = 0$, but receives pensions, $\hat{y} = p(x)$. The budget constraint is presented by

$$a' = \begin{cases} \hat{y} - T(\hat{y}) - \tau^p \min\{\hat{y}, y^{max}\} + [1 + (1 - \tau^r)r]\, a + tr - (1 + \tau^c)c & s \leq 45, \\ \\ \hat{y} - T(\hat{y}) + [1 + (1 - \tau^r)r]\, a + tr - (1 + \tau^c)c & s > 45, \end{cases} \tag{6}$$

where a' denotes wealth in period $t + 1$ and

$$\hat{y} = \begin{cases} \theta^{w,s,j} l w & s \leq 45, \\ p(x) & s > 45. \end{cases}$$

Entrepreneur. At the beginning of each period t, the entrepreneur at age s with productivity $j \in \{1, 2\}$ decides how much she invests in working capital k. Her production net of depreciation is presented by

$$f(k, \bar{l}) = \theta^{e,s,j} k^{\alpha} \left(A\bar{l}\right)^{1-\alpha} - \delta k, \tag{7}$$

where $\bar{l} \equiv 0.3$ denotes her constant labor supply and aggregate labor productivity A grows at the exogenous growth rate g:

$$A' = (1 + g)A. \tag{8}$$

Her borrowing costs amount to rk so that her first-order condition with respect to the capital stock k is presented by

$$r = \alpha\theta^{e,s,j} k^{\alpha-1} \left(A\bar{l}\right)^{1-\alpha} - \delta. \tag{9}$$

The taxable entrepreneurial income amounts to[12]

$$\hat{y} = \theta^{e,s,j} k^{\alpha} \left(A\bar{l}\right)^{1-\alpha} - rk - \delta k. \tag{10}$$

In old age, the entrepreneur receives pensions $p(x)$ and does not work or invest in working capital any more, $\bar{l} = 0$ and $k = 0$. The budget constraint of the s-year-old entrepreneur with productivity j and wealth a in period t is given by

$$a' = \hat{y} - T(\hat{y}) + \left[1 + (1 - \tau^r)r\right] a + tr - (1 + \tau^c)c, \tag{11}$$

where

$$\hat{y} = \begin{cases} \theta^{e,s,j} k^{\alpha} \left(A\bar{l}\right)^{1-\alpha} - rk - \delta k & s \leq 45, \\ p(x) & s > 45. \end{cases}$$

Note that households only accumulate contributions to the pension system during periods as a worker.

3.7. Credit market

Entrepreneurs borrow or lend an amount $(k - a)$ from a financial intermediary at rate r, workers lend an amount a, and the corporate sector and the government borrow amounts K_c and D, respectively. Let Ω and K_e denote aggregate wealth and total capital stock in the entrepreneurial sector. In the capital market equilibrium, aggregate wealth is allocated to aggregate capital and government debt:

$$\Omega = K_c + K_e + D. \tag{12}$$

3.8. The household's recursive problem

In the following, we will describe the optimization problem for the working household, $s \leq 45$ and the retired household, $s > 45$, in turn. Let $z = (a, x, j, \epsilon, s)$ denote the individual state vector. $V(z)$ denotes the value function of the household.

[12] In our calibration, we identify $\hat{y}$ with the earnings reported by entrepreneurs in the SOEP.

The young's problem. If a young household at age s with wealth a is a worker, $\epsilon = w$, with productivity type j, he maximizes his value function

$$V(a,x,j,w,s) = \max_{c,l,a'} \left\{ u(c,l) + \beta\phi_s \sum_{\epsilon',j'} \Pi(\epsilon',j'|w,j)\, V(a',x',j',\epsilon',s+1) \right\}, \tag{13}$$

subject to its budget constraint (6), his contributions (5) and the non-negativity constraints on wealth and labor supply, $a' \geq 0$ and $l \geq 0$.

For the entrepreneur, the Bellman equation is presented by

$$V(a,x,j,e,s) = \max_{c,a',k} \left\{ u(c,\bar{l}) + \beta\phi_s \sum_{\epsilon',j'} \Pi(\epsilon',j'|e,j)\, V(a',x',j',\epsilon',s+1) \right\}, \tag{14}$$

subject to her budget constraint (11), her contributions (5) and the non-negativity constraints on wealth, $a' \geq 0$.

The old's problem. During retirement, the household neither works nor becomes an entrepreneur. His working type ϵ and productivity type j remain constant but do not have any effect on his income or utility. Instead, the household receives a pension $p(x)$ depending on his contributions x during working life. For $s \geq 46$, his value function is given by

$$V(a,x,j,\epsilon,s) = \max_{c,a'} \left\{ u(c,0) + \beta\phi_s\, V(a',x,j,\epsilon,s+1) \right\}, \tag{15}$$

subject to

$$a' = p(x) + [1 + (1-\tau_r)r]a + tr - T(p(x)) - (1+\tau^c)c, \tag{16a}$$

$$a' \geq 0. \tag{16b}$$

Note that the state variable x (average contributions to the pension system) remains constant. In addition, the retired worker pays non-capital income taxes $T(.)$ on his pensions.

3.9. Technology

In the non-entrepreneurial (corporate) sector, output is produced with the help of capital and effective labor, according to the standard Cobb–Douglas function:

$$Y_c = AK_c^{\alpha} L_c^{1-\alpha}$$

where L_c denotes (effective) labor input in the non-entrepreneurial sector. Capital K_c also depreciates at rate δ. Firms are competitive and maximize profits, $Y_c - rK_c - wL_c - \delta K_c$, such that factor prices are given by

$$w = (1-\alpha)AK_c^{\alpha} L_c^{-\alpha}, \tag{17a}$$

$$r = \alpha A K_c^{\alpha-1} L_{cs}^{1-\alpha} - \delta. \tag{17b}$$

In equilibrium, corporate profits are equal to zero.

3.10. Government and social security

The government levies total taxes Tax on households consisting of non-capital income and capital income taxes as well as consumption taxes. In addition, the government confiscates all accidental bequests Beq. It pays transfers Tr, provides a certain level G of total public expenditures, and pays interest to the accumulated debt D. In each period, the government budget is financed by issuing government debt:

$$Tr + G + rD - Tax - Beq = D' - D. \tag{18}$$

The social security authority collects contributions at rate τ^p on labor income $\hat{y}$ up to a threshold y^{max} from workers, which it uses to finance pensions. The budget of the social security is balanced such that total pensions P are equal to total contributions.

The stationary equilibrium where the population growth rate, factor prices, and the distribution of the individual state variables are constant is described in the Online-Appendix.[13]

4. Calibration of the benchmark model

We calibrate the model with respect to the characteristics of the German economy in 2020 (see Table 4). For the demographic variables, we use our estimates from the German population. The population growth rate is set equal to −0.07% in 2020, as projected by United Nations (2015). The share of the entrepreneurs in the labor force is calibrated with the help of the SOEP data at 8% (see

[13] https://www.wiwi.uni-muenster.de/oeew/sites/oeew/files/downloads/heer-trede-jmacroecon-online-appendix.pdf.

Table 4
Calibration of benchmark parameters.

Parameter	Value	Description
n	−0.07 %	Population growth rate
β	1.011	Subjective discount factor
γ	0.357	Weight of utility from leisure
$1/\sigma$	1/2	Intertemporal elasticity of substitution
ψ	8.0%	Share of entrepreneurs in labor force
$\bar{l}$	0.3	Steady state labor supply of entrepreneurs
α	0.37	Share of capital income
δ	6.7%	Rate of capital depreciation
g	2.0%	Growth rate of aggregate labor productivity
G/Y	21%	Share of government spending in steady state production
D/Y	62.0%	Public debt–GDP ratio
τ^r	23.0%	Capital income tax
τ^c	15.0%	Consumption tax
ζ_0	0.1138	Income tax level
ζ_1	0.272051	Income tax progressivity
$repl$	38.7%	Replacement rate of pensions relative to gross labor income
$y^{max}/\bar{y}$	2.05	Contributions ceiling relative to average income

Section 2.1). We use the Markov transition matrix (1) to model the dynamics of working-age households both between productivity classes and employment statuses (worker/entrepreneur). We choose our preference parameters as follows. The discount factor $\beta = 1.011$ is taken from Hurd (1989), who, in his estimation procedure, explicitly accounts for mortality risk. Our choice of the discount factor implies a real interest rate equal to 3.0%. The intertemporal elasticity of substitution, $1/\sigma$, is set to 1/2. The parameter $\gamma = 0.357$ is calibrated so that the average working time of the households is equal to that of the entrepreneurs (30% of available time).

The calibration of the production and fiscal parameters follows Trabandt and Uhlig (2011). In particular, we set the production elasticity of capital α, the depreciation rate δ, and the annual growth rate of output g equal to $\alpha = 0.37$, $\delta = 6.7\%$, and $g = 2.0\%$. The government share G/Y amounts to 21%, while the debt-to-GDP ratio D/Y is set to 62%. The tax rates on capital income and consumption are set to $\tau^r = 23\%$ and $\tau^c = 15\%$, respectively. We choose income tax parameters $\zeta_1 = 0.272051$ in our tax function (3). The estimate is taken from Holter et al. (2019) for the case of Germany (married with two children). The parameter $\zeta_0 = 0.1138$ is set to match the income tax rate of the average income $\bar{y}$ implying an average labor income and social security tax equal to 41% as in Trabandt and Uhlig (2011). Our pension contribution rate τ^p is endogenous and amounts to 12.2% and falls short of the empirical value of 18.6% prevailing in Germany in 2020.[14] The annual income threshold with respect to pension contributions (the so-called 'Beitragsbemessungsgrenze') amounted to 78,000 Euro in Germany West in 2018 which equals 205% of the average earnings of workers in the SOEP. Pensions are calibrated so that the replacement rate of pensions relative to average gross labor earnings amounts to 38.7%. Our estimate of the replacement rate is taken from OECD (2019).

The productivity profiles of workers and entrepreneurs, $\theta^{w,s,j}$ and $\theta^{e,s,j}$, $s = 1,\ldots,45$ for the two productivity types $j = 1,2$, are set so that they replicate the empirical mean real hourly earnings profiles presented in Fig. 4. We normalize the age-efficiency profile so that the average efficiency is equal to one. Since the mean hourly earnings of entrepreneurs is an endogenous function of their capital investment, the productivity-age profile $\theta^{e,s,j}$ is not a trivial copy of the profile in Fig. 4 but needs to be calibrated endogenously with the help of the equilibrium conditions of the model. Since this computational task is quite demanding, we proceed as follows. We choose a simplified model. In particular, we assume that there is no mobility between workers and entrepreneurs as well as productivity types, linear income taxation and lump-sum pensions. These assumptions allow us to compute the solution to the model within less than a minute using the Newton–Raphson algorithm. We iterate over the productivity profiles of both workers and entrepreneurs for both productivity types $j \in \{1,2\}$ until we match the empirical profiles of mean real hourly earnings displayed in Fig. 4.[15] Our calibrated efficiency-age profiles of workers, $\{\theta^{w,s,j}\}_{s=1}^{45}$, and entrepreneurs, $\{\theta^{e,s,j}\}_{s=1}^{45}$, are illustrated for the productivity types $j = 1,2$ in Fig. 6. Note that the productivity levels of workers and entrepreneurs of each type almost coincide, except during the first years of working life.

5. Inequality and correlations

The Gini coefficients of earnings, income and wealth for the two subgroups – workers and entrepreneurs – are presented in Table 5. Gross income is defined as all income, including interest income but excluding transfers; net income is defined as gross income plus transfers minus labor and capital income taxes. For this reason, gross income is less concentrated than earnings for

[14] As one explanation of this shortfall, social security expenditures in Germany also cover part of public health expenditures on retired and administrative costs which we do not model.

[15] The simplified model together with its computation and calibration of the age-productivity profiles is described in more detail in the working paper version of this paper in Heer and Trede (2020). In addition, the properties of the model with respect to the distribution and life-cycle profiles of the individual variables are studied.

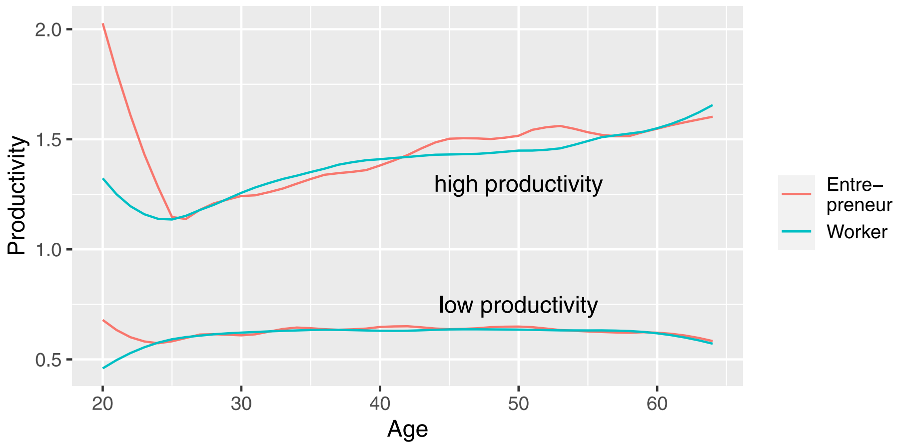

Fig. 6. Age-productivity profiles. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 5
Gini coefficients and correlations in the model with mobility and progressive taxation.

	Workers	Entrepreneurs
Gini coefficients		
Earnings	0.421	0.336
Wealth	0.348	0.361
Gross income	0.352	0.306
Net income	0.337	0.300
Correlation between earnings and wealth		
	0.004	0.135

both workers and entrepreneurs because higher wealth reduces the labor supply and, hence, labor income ceteris paribus. Due to the progressive tax schedule, net income is less concentrated than gross income, and the Gini coefficients fall from 0.352 (0.306) to 0.337 (0.300) for workers (entrepreneurs).[16]

Our values differ slightly from the empirical values for the German economy presented in Fig. 5. The inequality of income among the workers is slightly higher in our model than in the empirical results (e.g., the Gini coefficients of net income amount to 0.337 in the model and 0.28 in Germany), while the model underpredicts the concentration of income among entrepreneurs (e.g., the Gini coefficients of net income amount to 0.300 and 0.35, respectively). The concentration of wealth as measured by the Gini coefficients amounts to 0.348 and 0.361 for workers and entrepreneurs, respectively and is significantly less than empirically. As one possible explanation, we do not model bequests in our model. With respect to the correlation of earnings and wealth, we also find a small value. While empirically, the correlations amount to approximately 0.30 for workers and entrepreneurs during 2002–2017, we obtain somewhat lower correlations of 0.00 and 0.14, respectively.[17]

6. Policy experiment: PAYG pensions for entrepreneurs

In this section, we conduct a policy experiment in which entrepreneurs also pay contributions to the social security system, and compare it to the benchmark case. We make the following assumptions:

1. Entrepreneurs have to pay the same social security tax τ^p on their non-capital income $\hat{y}$ as the workers.
2. Social security taxes τ^p adjust to balance the budget of the social security authority.
3. We assume that the levels of government consumption, G, and public debt, D, (relative to aggregate productivity and population) remain constant under the different scenarios that we consider.
4. As a consequence of the pension reform, we observe various general equilibrium effects. For example, the wealth of entrepreneurs drops because they have to save less for old age. Therefore, tax revenues also fall. The government balances the budget by adjusting its transfers tr.

[16] For the aggregate economy including retirees, we find Gini coefficients of wealth, earnings, gross and net income equal to 0.376, 0.417, 0.396 and 0.382, respectively.

[17] A possible explanation for the empirically low but statistically significant positive correlation of earnings and net wealth among workers might be the endogeneity of individual productivity. If individual productivity levels, for example, depend on the individual's investment in human capital, a positive correlation naturally arises. Similarly, if human capital is not perfectly mobile between generations but the individual's innate abilities depend on those of his or her parents, wealth-rich parents might transfer both physical and human capital to their children.

Table 6
Allocation and welfare effects of pension reform proposals.

	No PAYG	PAYG entrepreneurs	
		Contributions-based pension	Lump-sum pensions
$\tilde{\Omega}$	2.514	2.482	2.325
$\tilde{K}_c$	1.982	1.950	1.825
$\tilde{K}_e$	0.164	0.161	0.162
$\tilde{L}_c$	0.235	0.235	0.221
$\tilde{Y}_c$	0.517	0.514	0.483
$\widetilde{GDP}$	0.560	0.557	0.526
τ^p	12.15%	12.15%	12.08%
$\widetilde{Tr}$	0.102	0.101	0.0890
Lifetime utilities			
Worker, $j = 1$	0%	−0.52 %	−3.29%
Worker, $j = 2$	0%	−0.35 %	−3.36%
Entrepreneur, $j = 1$	0%	−2.17 %	−4.86%
Entrepreneur, $j = 2$	0%	−3.05 %	−5.98%
Expected (newborn)	0%	−0.69 %	−3.56%

We consider the effects of two pension reform proposals in comparison to the benchmark model. In both scenarios, entrepreneurs also contribute to the PAYG pension system. In the first case, pensions are proportional to contributions and the pension schedule $\tilde{p}(\tilde{x})$ is kept constant in comparison with the benchmark model. In the second scenario, we abandon the strict proportionality in the pension system with respect to the contributions and consider lump-sum pensions instead. In this case, the replacement rate of pensions with respect to average income is kept constant.[18]

Table 6 presents the results for selected aggregate variables and welfare effects.[19] Welfare is measured by the consumption equivalent change of expected lifetime utility for the 1-year old with employment and productivity types $\epsilon \in \{e, w\}$ and $j \in \{1, 2\}$ and the newborn who does not know his types yet. In the first entry-column, the benchmark is displayed, while the second entry-column presents the case where also entrepreneurs contribute to the pension system at the same rate as the workers. The third column displays our second scenario of a pension reform with lump-sum pensions.

Comparing the first entry column with the second one, we notice that aggregate savings $\tilde{\Omega}$ drop by 1.3%, from 2.514 to 2.482. If entrepreneurs also contribute to the PAYG pension system in Germany and receive pensions on their contributions during retirement, they reduce savings for old age. As a consequence, the capital stock in the corporate sector, $\tilde{K}^c$, decreases by 1.6%, from 1.982 to 1.950, while capital stock in the entrepreneurial sector $\tilde{K}_e$ even falls by 1.8%, from 0.164 to 0.161. Since the aggregate labor supply is almost unaffected, output in the corporate sector, Y_c, and GDP fall by 0.5% and 0.6%, respectively. We also notice that the contribution rate τ^p remains constant at 12.15% which is a consequence of the proportional pension schedule. Since GDP and, hence, tax revenues fall, government transfers adjust and decline by 1.1%, from 0.102 to 0.101 or, equivalently, from 18.3% to 18.2% of GDP.

Welfare declines significantly for all types of agents. Expected lifetime utilities of all household types, including workers and entrepreneurs, decline. For newborn agents who do not know their type yet, expected lifetime utility falls by 0.69% of total consumption. As an obvious explanation, the general equilibrium effects of lower aggregate savings reduce income and, hence, utility. Note that the welfare effects among the entrepreneurs are stronger than those among the workers. For example, for the productivity type $j = 1$, welfare is decreased by 0.52% (2.17%) of consumption for the worker (entrepreneur). The disincentive effects of income taxation on capital investment by entrepreneurs adds to the detrimental general equilibrium effects, while the insurance effect of the pension system against the risk of longevity is only a subordinated effect for the entrepreneurs' welfare.

In the case of lump-sum pensions (third entry column), we analyze the case where all agents receive the same pension irrespective of their contributions. We set the pension equal to the average pension of all retirees in the benchmark case. As a consequence, aggregate labor declines substantially compared to the benchmark because longer working hours of the workers and higher capital investment of the entrepreneurs do not result in higher pension entitlements. As a consequence, labor in the corporate sector $\tilde{L}_c$ falls by 6.1%, from 0.2354 to 0.221. Associated with this drop in income, aggregate savings decline as well. While the low-productivity workers decrease their savings rate, the high-productivity workers increase theirs. As aggregate income drops, the overall effect is a decline of savings $\tilde{\Omega}$ by 7.5%, from 2.514 to 2.325. Consequently, aggregate capital in the corporate sector, $\tilde{K}^c$, decreases from 1.982 to 1.825. As a result, GDP falls by 6.12%, from 0.5599 to 0.5256.[20]

[18] Fehr et al. (2013) find that the optimal mix between the flat-rate pension share and the earnings-related part should be equal to 30% and 70% of total pensions in Germany, reflecting the trade-off between the increased labor supply distortion on the one hand and the benefit from increased insurance provision against labor market risk on the other hand.

[19] To derive stationary variables (denoted by a tilde), we divide individual variables (except l_t) by aggregate labor productivity, e.g. $\tilde{x}_t \equiv x_t / A_t$, and aggregate variables (with the exception of aggregate labor, $\tilde{L}_t^c \equiv L_t^c / N_t$) by the product of aggregate productivity and population, e.g. $\tilde{\Omega}_t = \Omega_t / (A_t N_t)$.

[20] In order to finance lump-sum pensions to all retirees from lower wage income of the workers but additional contributions from the entrepreneurs, the social security tax rate τ^p almost remains constant and only decreases slightly from 12.15% to 12.08%.

Table 7
Aging and the sustainability of pensions.

	2020	2050		
Entrepreneurs:	No PAYG	No PAYG	PAYG	
			$\tilde{p}(\tilde{x})$	Lump-sum
Retirement age:	65	70	70	70
$\tilde{\Omega}$	2.514	2.693	2.650	2.508
$\tilde{K}_c$	1.982	2.144	2.110	1.997
$\tilde{K}_e$	0.164	0.177	0.174	0.175
$\tilde{L}_c$	0.235	0.236	0.236	0.223
$\tilde{Y}_c$	0.517	0.534	0.531	0.501
$\widetilde{GDP}$	0.560	0.577	0.574	0.546
τ^p	12.15%	11.83%	11.84%	11.53%
$\tilde{T}r$	0.102	0.1064	0.105	0.0095
Lifetime utilities				
Worker, $j = 1$		0%	−0.60%	−2.44%
Worker, $j = 2$		0%	−0.42%	−2.44%
Entrepreneur, $j = 1$		0%	−2.15%	−3.98%
Entrepreneur, $j = 2$		0%	−2.92%	−4.97%
Expected (newborn)		0%	−0.74%	−2.71%

When we investigate the welfare effects of a pension reform that introduces lump-sum benefits irrespective of the contributions, we find again that welfare decreases for all newborns in comparison with the benchmark case of contributions-based pensions (compare the first and the third entry columns). This is remarkable since the social security system redistributes from the agents with high contributions to those with low contributions. However, in the presence of income mobility, workers change their type so that this redistributive effect at the beginning of life levels out over the lifetime.[21] The expected lifetime utility of a newborn decreases by 3.56% of consumption. Not surprisingly, welfare effects are more pronounced for the workers and entrepreneurs who are born with high productivity, $j = 2$. For example, the workers with low and high productivity at birth lose welfare at the amounts of 3.29% and 3.36% of consumption. In addition, welfare losses are higher among entrepreneurs than among workers. While the high-productivity workers incur losses equal to 3.36% of consumption, the high-productivity entrepreneur who receives a lump-sum pension experiences a decline in lifetime utility equal to 5.98% of consumption. In sum, our welfare results provide support against the introduction of pay-as-you-go pensions for entrepreneurs, neither in the form of contributions-related nor lump-sum pensions.

7. Entrepreneurship and aging

In Germany, the old-age dependency ratio, defined as the ratio of those aged 65+ to those aged 20–64, is projected to increase from 38% in 2020 to 64% in 2050 (see United Nations, 2015). In the following, we consider the effects of this imminent aging on equilibrium values of capital, labor and income and the sustainability of public PAYG pensions. We focus on the demographics projected for the German economy in the year 2050. Therefore, we apply the population growth rate and survival probabilities projected by United Nations (2015) but keep the other parameters of the extended model unchanged. In particular, we keep the tax and pension parameters at their respective values in 2020.

We find that the present pension system in Germany is unsustainable. In particular, the maximum possible revenue from the social security tax τ^p which corresponds to the top of the Laffer curve on labor income taxation is insufficient to pay for the pension expenditures.[22] Accordingly, we increased the retirement age until the contributions are able to finance pensions which necessitates an effective retirement age of 70 years.[23] In addition, we studied the sustainability of social security if entrepreneurs also contribute to the PAYG pension system. We find that (1) (mandatory) integration of the entrepreneurs in the social security system does not help to establish financial sustainability and that (2) in this case, the retirement age needs to be raised to 70 years as well.

Therefore, we consider the effects of aging under the assumption that the effective retirement age can be increased to 70 years in Germany by 2050. Table 7 summarizes our results. In the first entry column, we replicate the benchmark case in 2020, keeping in mind that in this case, the retirement age is lower and amounts only to 65. The second entry-column presents the results for the model in the year 2050 if entrepreneurs are not subject to mandatory pension contributions. The results for the two cases where the entrepreneurs contribute to the social security system in 2050 with either proportional or lump-sum pensions are summarized in the third and fourth entry columns of Table 7.

Strikingly, the increase in retirement to the age 70 balances the negative effect of aging, and the aggregate labor supply $\tilde{L}_c$ remains rather constant, increasing from 0.235 to 0.236 (compare the first and second entry columns). The pension contribution

[21] In the working paper version of this paper, we also analyze the case without income mobility and where pensions are only paid to the workers. In this case, expected lifetime utility of the workers with low productivity, $j = 1$, increases after the introduction of lump-sum pensions.

[22] Even in 2018, the federal budget already contributed approximately 68 billion Euros to the financing of social security expenditures, which amounted to 2.04% of GDP.

[23] According to the OECD (2019), the average effective retirement age in Germany amounted to 64.0 years during 2013–2018.

rate even decreases from 12.15% in 2020 to 11.83% in 2050.[24] The workers who receive an income over a longer period of 50 instead of 45 years increase their savings for old age for two reasons. First, their income over their whole working life increases. Second, the survival probabilities are higher in old age so that the weight on instantaneous utility (the survival probabilities times the discount factor) increases. These two savings-increasing factors outweigh the negative effect that stems from a shorter maximum retirement life, which falls from 35 to 30 years. As a consequence, aggregate savings $\tilde{\Omega}$ increase from 2.514 to 2.693. We therefore conclude – in accordance with Heer et al. (2020) – that the German pension system will be sustainable in 2050 if the retirement age is increased to 70.

In the third and fourth columns, we present the cases in which entrepreneurs contribute to the social security system as well. In the case of the proportional pension schedule $\tilde{p}(\tilde{x})$, the main effect on the aggregate variables is again the reduction in aggregate savings and, hence, the capital stock in the corporate sector, $\tilde{K}^c$ which falls from 2.144 to 2.110. The aggregate labor supply in the corporate sector $\tilde{L}^c$ is basically unaffected in the case of a proportional pension schedule $\tilde{p}(\tilde{x})$ so that GDP falls by only 0.5%, from 0.534 to 0.531. The quantitative effects of an integration of the entrepreneurs in the social security system are more pronounced in the case of lump-sum pensions. In this case, GDP even falls by 5.3%. With respect to welfare, our results in the previous section are shown to be robust to demographic change. The integration of entrepreneurs into the German pension system reduces the welfare of all productivity types and the average newborn. Expected lifetime utility of the newborn falls by 0.74% (2.71%) of consumption in the case of proportional (lump-sum) pensions.[25] Again, the two groups of high-productivity households $(j = 2)$ and entrepreneurs suffer disproportionately in comparison with the low-productivity agents $(j = 1)$ and workers. For example, in the case of proportional pensions, low-productivity workers' welfare declines by 0.60% of consumption, while high-productivity entrepreneurs suffer a loss equal to 2.92% in the case of comprehensive social security.

Our result that an increase in the retirement age helps the social security authority to deal with the demographic transition is in accordance with other findings in the literature. For the US economy, De Nardi et al. (1999) presents an early study of the efficiency and welfare effects of a postponement of the retirement age from 65 to 69 years. If the extra burden of aging baby boomers is financed by labor income taxes (their experiment 3), the necessary increase in the tax rate is reduced by 7 percentage points. Over the transition period considered (1980–2120), average welfare gains amount to 49% of GDP. While this early work, due to the computational limits at the time, was unable to consider heterogeneity among workers, Kitao (2014) introduces a realistic wage distribution to confirm the results by De Nardi et al. (1999). He also finds that the increase in retirement age from 66 to 73 years helps to increase the fiscal space in the US economy and has significant welfare effects in the long run. In particular, the new-born household (for the demographics projected for the year 2100) would benefit by as much as $ 41,000 (in 2010 dollars). In a cross-country study, Heer et al. (2020) study the welfare effects of an increase of the retirement age by 5 years in the US and 14 EU-countries. They find that the increase in retirement age by an additional 5 years helps to re-establish sustainability of public pensions in the countries Germany, Ireland and Portugal which, otherwise, would not be able to finance pensions beyond the years 2035–2045. These authors also find similar significant long-run welfare improvements by an increase in the retirement age; however, in comparison with other fiscal measures such as an increase in consumption taxes or a reduction of pension benefits, the relative ranking of these policies depends upon individual country characteristics, i.e. the country's population, fiscal, utility and production parameters.

8. Conclusion

The public pension system in Germany is not sustainable as it is. Increasing the contribution rate on gross wages to the rate associated with maximum revenue will not restore balance in the pension system in the coming decades. The questions of whether broadening the contribution base to include entrepreneurs will help to establish fiscal sustainability – and what are the economic consequences and welfare effects of such a policy – arise.

We present a model that replicates German age-specific entrepreneurship and heterogeneity among both entrepreneurs and workers and between cohorts. We show that the inclusion of entrepreneurs neither helps to establish fiscal sustainability of the pension system nor increases welfare. Only if the retirement age is raised to 70 by the year 2050, contributions will be sufficient to finance pensions. The contribution rate that will balance the social security authority's budget is rather insensitive to the inclusion of entrepreneurs in the social security system.

As a consequence of social security contributions, entrepreneurs will have reduced net income and, hence, lower incentives to invest in their business; therefore, capital in the entrepreneurial sector will decline substantially. In general equilibrium, the accompanying decline in GDP will amount to 0.6% and average expected welfare of the newborn will be reduced by 0.7% of total consumption.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

[24] We find that, due to aging, the maximum of the Laffer curve of labor income taxes is attained at a lower value of the labor income tax rate. Since the labor income tax rate in the year 2020 is already close to those associated with the top of the Laffer curve, the social security contributions rate τ^p cannot increase above the present value.

[25] We refrain from comparing the expected lifetime utility in the years 2020 and 2050. In our model with a calibrated value of the risk aversion coefficient $\sigma > 1$, an increase in longevity results in a decline in expected lifetime utility, even if the consumption-age and labor-supply age profiles remain constant.

Funding sources

We acknowledge support from the Open Access Publication Fund of the University of Muenster.

References

Auerbach, A.J., Kotlikoff, L.J., 1987. Dynamic Fiscal Policy. Cambridge University Press, New York, http://dx.doi.org/10.2307/2233935.

Bhandari, A., Birinci, S., McGrattan, E., See, K., 2020. What do survey data tell us about US businesses? Am. Econ. Rev.: Insights 2 (4), 443–458. http://dx.doi.org/10.1257/aeri.20190304.

Blanchard, O.J., 1985. Debt, deficits and finite horizons. J. Polit. Econ. 93, 223–247. http://dx.doi.org/10.1086/261297.

Braun, R.A., Joines, D.H., 2015. The implications of a graying Japan for government policy. J. Econ. Dyn. Control 57, 1–23. http://dx.doi.org/10.1016/j.jedc.2015.05.005.

Brüggemann, B., 2021. Higher taxes at the top: the role of entrepreneurs. Am. Econ. J.: Macroecon. 13 (3), 1–36. http://dx.doi.org/10.1257/mac.20170441.

Budría Rodríguez, S., Díaz-Giménez, J., Quadrini, V., Ríos-Rull, J.-V., 2002. Updated facts on the U.S. distributions of earnings, income, and wealth. Fed. Reserv. Bank Minneapolis – Q. Rev. 2–35. http://dx.doi.org/10.21034/qr.2631.

Cagetti, M., De Nardi, M., 2006. Entrepreneurship, frictions, and wealth. J. Polit. Econ. 114 (5), 835–870. http://dx.doi.org/10.1086/508032.

Cagetti, M., De Nardi, M., 2009. Estate taxation, entrepreneurship, and wealth. Am. Econ. Rev. 99 (1), 85–111. http://dx.doi.org/10.1257/aer.99.1.85.

Caliendo, M., Künn, S., Weißenberger, M., 2016. Personality traits and the evaluation of start-up subsidies. Eur. Econ. Rev. 86, 87–108. http://dx.doi.org/10.1016/j.euroecorev.2015.11.008.

Conesa, J.C., Garriga, C., 2016. Intergenerational policy and the measurement of tax incidence. Eur. Econ. Rev. 83, 1–18. http://dx.doi.org/10.1016/j.euroecorev.2015.10.009.

De Nardi, M., Imrohoroğlu, S., Sargent, T.J., 1999. Projected U.S. demographics and social security. Rev. Econ. Dyn. 2 (3), 575–615. http://dx.doi.org/10.1006/redy.1999.0067.

Fehr, H., Kallweit, M., Kindermann, F., 2013. Should pensions be progressive? Eur. Econ. Rev. 63, 94–116. http://dx.doi.org/10.1016/j.euroecorev.2013.07.004.

Fuchs-Schündeln, N., Krueger, D., Sommer, M., 2010. Inequality trends for Germany in the last two decades: A tale of two countries. Rev. Econ. Dyn. 13 (1), 103–132. http://dx.doi.org/10.1016/j.red.2009.09.004.

Gertler, M., 1999. Government debt and social security in a life-cycle economy. Carnegie-Rochester Conf. Ser. Public Policy 50, 61–110. http://dx.doi.org/10.1016/S0167-2231(99)00022-6.

Goebel, J., Grabka, M.M., Liebig, S., Kroh, M., Richter, D., Schröder, C., Schupp, J., 2019. The German socio-economic panel (SOEP). J. Econ. Stat. 239 (2), 345–360. http://dx.doi.org/10.1515/jbnst-2018-0022.

Grabka, M.M., 2020. SOEP-Core v35 - Codebook for the $PEQUIV File 1984–2018: CNEF Variables with Extended Income Information for the SOEP. SOEP Survey Papers 772.

Grabka, M.M., Westermeier, C., 2014. Persistently high wealth inequality in Germany. DIW Econ. Bull. 6, 3–15.

Grabka, M.M., Westermeier, C., 2015. Editing and Multiple Imputation of Item Non-response in the Wealth Module of the German Socio-Economic Panel. SOEP Survey Papers 272.

Guvenen, F., 2009. An empirical investigation of labor income processes. Rev. Econ. Dyn. 12, 58–79. http://dx.doi.org/10.1016/j.red.2008.06.004.

Guvenen, F., Karahan, F., Ozkan, S., Song, J., 2021. What do data on millions of U.S. workers reveal about lifecycle earnings dynamics? Econometrica 89 (5), 2303–2339. http://dx.doi.org/10.3982/ECTA14603.

Heer, B., Irmen, A., 2014. Population, pensions, and endogenous economic growth. J. Econ. Dyn. Control 46, 50–72. http://dx.doi.org/10.1016/j.jedc.2014.06.012.

Heer, B., Polito, V., Wickens, M.R., 2020. Population aging, social security and fiscal limits. J. Econ. Dyn. Control 116, 103913. http://dx.doi.org/10.1016/j.jedc.2020.103913.

Heer, B., Trede, M., 2020. Age-Specific Entrepreneurship and PAYG Public Pensions in Germany. CQE Working Paper 91/2020.

Holter, H.A., Krueger, D., Stepanchuk, S., 2019. How do tax progressivity and household heterogeneity affect Laffer curves? Quant. Econ. 10 (4), 1317–1356. http://dx.doi.org/10.3982/QE653.

Huggett, M., Ventura, G., 2000. Understanding why high income households save more than low income households. J. Monetary Econ. 45 (2), 361–397. http://dx.doi.org/10.1016/S0304-3932(99)00058-6.

Hurd, M.D., 1989. Mortality risk and bequests. Econometrica 57 (4), 799–813. http://dx.doi.org/10.2307/1913772.

İmrohoroğlu, S., Kitao, S., 2009. Labor supply elasticity and social security reform. J. Public Econ. 93 (7–8), 867–878. http://dx.doi.org/10.1016/j.jpubeco.2009.05.002.

İmrohoroğlu, S., Kitao, S., Yamada, T., 2016. Achieving fiscal balance in Japan. Intern. Econ. Rev. 57 (1), 117–154. http://dx.doi.org/10.1111/iere.12150.

İmrohoroğlu, A., Kumru, C., Nakornthab, A., 2018. Revisiting tax on top income. Working Paper in Economics and Econometrics 660, Australian National University (ANU).

Kitao, S., 2014. Sustainable social security: Four options. Rev. Econ. Dyn. 17 (4), 756–779. http://dx.doi.org/10.1016/j.red.2013.11.004.

Krueger, D., Ludwig, A., 2007. On the consequences of demographic change for rates of returns to capital, and the distribution of wealth and welfare. J. Monet. Econ. 54 (1), 49–87. http://dx.doi.org/10.1016/j.jmoneco.2006.12.016.

Nishiyama, S., Smetters, K., 2007. Does social security privatization produce efficiency gains? Q. J. Econ. 122 (4), 1677–1719. http://dx.doi.org/10.1162/qjec.2007.122.4.1677.

OECD, 2007. Pensions at a Glance 2007. OECD Publishing, Paris, http://dx.doi.org/10.1787/pension_glance-2007-en.

OECD, 2019. Pensions at a Glance 2019: OECD and G20 Indicators. OECD Publishing, Paris, http://dx.doi.org/10.1787/b6d3dcfc-en.

Quadrini, V., 2000. Entrepreneurship, saving, and social mobility. Rev. Econ. Dyn. 3, 1–40. http://dx.doi.org/10.1006/redy.1999.0077.

Statistisches Bundesamt, 2020. Verbraucherpreisindizes für Deutschland: Lange Reihen ab 1948.

Trabandt, M., Uhlig, H., 2011. The Laffer curve revisited. J. Monet. Econ. 58 (4), 305–327. http://dx.doi.org/10.1016/j.jmoneco.2011.07.003.

United Nations, 2015. World population prospects: The 2015 revision, methodology of the United Nations population estimates and projections. ESA/P/WP.241.