

Floquet Majorana fermions in superconducting quantum dots

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1. Introduction

There are condensed matter systems which can hold collective quasiparticles that are their own antiparticles, therefore satisfying the Majorana condition [1–3]. These quasiparticles are termed Majorana Fermions (MFs) and follow non-abelian statistics. Detection of MFs in solid state systems has been recently experimentally proposed [4–6,53]. Recently, the interest in encoding a qubit in these kinds of excitations has grown due to the possibility to be non-local, a property which has a great potential in quantum computation due to the robustness of the qubit against local perturbations [7]. Furthermore, how to tune MFs in condensed matter systems is one of the main purposes of research in the emergent field of topological quantum computation.

In the last years, different works have shown how the application of ac fields enriches the properties of these quasiparticles and facilitate their tunability. For instance, it is possible to generate Floquet Majorana fermions (FMFs) as steady-states of non-equilibrium systems which present interesting properties for

quantum computation: non-locality and non-abelian statistics [8,9].

In every system with particle–hole symmetry, the quasiparticles come in pairs $\gamma_{-E}^\dagger = \gamma_E$, therefore they can hold MFs as long as the energy can be tuned to zero. One of the simplest and most tunable system with particle hole symmetry is a double quantum dot (QD) connected via an s-wave superconductor [10]. It is well known that the proximity effect induces Cooper pairs correlations across the dots [11,12] generating effectively superconductivity [13]. Interestingly, fractional Josephson effect, a signature of the presence of MFs [5,14,15], in a quadruple quantum dot in the presence of an s-wave superconductor has been predicted by Markus Büttiker and coworkers [16].

The advantage that configurations of a few QDs connected to s-wave superconductors present, in order to generate and detect MFs, in comparison with nano-wires [17–19] or long QD chains [20–22] proposals is their great tunability, while in the latter the MFs have topological protection.

In this paper we analyze two different configurations of QDs in proximity to superconducting leads such that Cooper pair correlations are induced between the neighboring dots as long as the coherence length is larger than the distance between them. We include periodically driven gates and search for the conditions for appearance of FMFs. The paper is organized as follows: in [Section](#)

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2 we present the model, in Section 3 we discuss the generation of FMFs in a double and a triple superconducting QD. Finally, we present our conclusions in Section 4.

2. Undriven system

Systems of QDs coupled to s-wave superconductors have been a subject of study [11,13,16] because the proximity effect induces Cooper pair correlations that can be easily detected due to the low number of degrees of freedom in QDs. In a system where neighboring QDs are coupled through superconducting reservoirs as in Fig. 1, in the limit of large superconducting gap the superconductors can be traced out and an effective Hamiltonian for the dots is obtained [23,24] as

$$H = \sum_{i,\sigma} \mu_{i,\sigma} d_{i,\sigma}^\dagger d_{i,\sigma} + \sum_{i,\sigma} (t_{i,i+1} d_{i,\sigma}^\dagger d_{i+1,\sigma} + \Delta_{i,i+1} d_{i,\sigma} d_{i+1,\bar{\sigma}} + h. c.), \quad (1)$$

which already contains effective superconductivity between neighboring dots. The fermionic operator $d_{i,\sigma}$ represents the annihilation of an electron in the i -QD with spin σ . The symbol $\bar{\sigma}$ means the opposite spin to σ , which can be $\sigma = \uparrow, \downarrow$. μ_i is the onsite energy in i -QD, the parameter $t_{i,i+1}$ is the effective tunneling probability from dot i to dot $i+1$ through the superconductor by virtual occupation of the above gap excitations and $\Delta_{i,i+1}$ is the effective superconducting amplitude due to the superconductor connecting the i and $i+1$ dots. If a large magnetic field is applied to the dots only one spin comes into play. However, the magnetic fields have to be non-collinear in order to have s-wave type Cooper pair correlations (see Fig. 1) [10]. In this configuration, it is more natural to work on the basis of the quantization axes given by the magnetic field in each dot. For that purpose, we have to perform the rotation

$$d_{2,\sigma} \rightarrow \cos \frac{\theta}{2} d_{2,\sigma} + \sigma \sin \frac{\theta}{2} d_{2,\bar{\sigma}} \quad (2)$$

as the magnetic field in the central QD forms an angle θ with the magnetic fields in the left and right QDs (see Fig. 1). The low-frequency Hamiltonian will be given by Eq. (1) by neglecting the contribution from the high-energy spin direction in each dot (keeping $\sigma = \downarrow$):

$$H = \sum_i \mu_i d_i^\dagger d_i + \sum_i \left\{ t'_{i,i+1} d_i^\dagger d_{i+1} + \Delta'_{i,i+1} d_i d_{i+1} + h. c. \right\}, \quad (3)$$

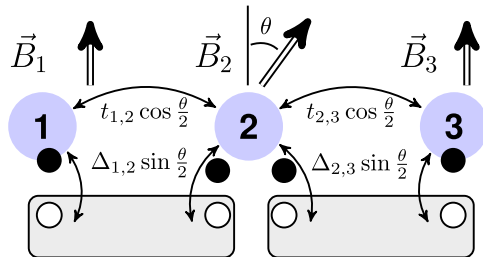


Fig. 1. Scheme of three QDs coupled by tunnel and coupled to a superconductor. The existence of Cooper pairs generates correlations of the type $d_{i,\sigma} d_{i+1,\bar{\sigma}}$ in the effective Hamiltonian for the QDs. The applied magnetic fields and their directions are also shown in the picture. The angle θ controls the ratio $\Delta_{i,i+1}/t_{i,i+1}$ (see text below).

where $d_i \equiv d_{i,1}$, $t'_{i,i+1} \equiv t_{i,i+1} \cos(\theta/2)$ and $\Delta'_{i,i+1} \equiv \Delta_{i,i+1} \sin(\theta/2)$. Therefore the normal and superconducting tunneling amplitudes are renormalized and their renormalization depends on the angle between the magnetic field directions. This dependence introduces a simple way to tune externally the coupling parameters of the system [10].

In order to obtain the excitation spectrum of the system the Hamiltonian is written in the Nambu basis $\Psi = (d_1, d_1^\dagger, \dots, d_N, d_N^\dagger)$ as

$$H = \frac{1}{2} \Psi^\dagger h \Psi + \frac{1}{2} \sum_i \mu_i. \quad (4)$$

For a triple QD h reads

$$h = \begin{pmatrix} \mu_1 & 0 & t'_{1,2} & -\Delta'_{1,2} & 0 & 0 \\ 0 & -\mu_1 & \Delta'_{1,2} & -t'_{1,2} & 0 & 0 \\ t'_{1,2} & \Delta'_{1,2} & \mu_2 & 0 & t'_{2,3} & -\Delta'_{2,3} \\ -\Delta'_{1,2} & -t'_{1,2} & 0 & -\mu_2 & \Delta'_{2,3} & -t'_{2,3} \\ 0 & 0 & t'_{2,3} & \Delta'_{2,3} & \mu_3 & 0 \\ 0 & 0 & -\Delta'_{2,3} & -t'_{2,3} & 0 & -\mu_3 \end{pmatrix}. \quad (5)$$

The eigensystem of h ($h \mathbf{v}_i = \lambda_i \mathbf{v}_i$) determines the quasiparticles, given by $\gamma_i = \mathbf{v}_i^\dagger \Psi$. A zero-energy solution, $\lambda_i = 0$, implies the presence of a pair of Majorana quasiparticles.

In the case of a double QD one can choose an angle such that $\Delta'_{1,2} = \pm t'_{1,2}$ and if $\mu_1 = 0$, there are two MFs given by

$$\gamma_1 = \frac{1}{\sqrt{2}} (d_1 \mp d_1^\dagger), \\ \gamma_2 = \frac{1}{\sqrt{2} \sqrt{1 + \delta^2}} \left\{ (d_2 \pm d_2^\dagger) - \delta (d_1 \pm d_1^\dagger) \right\}, \quad (6)$$

where $\delta = \mu_2/2t'_{1,2}$. Only in the case where $\mu_2 = 0$ the MFs are spatially separated [10]. In the case of a triple QD, assuming $\Delta'_{i,i+1} = \pm t'_{i,i+1}$ and $\mu_1 = 0$, there are two MFs given by

$$\gamma_1 = \frac{1}{\sqrt{2}} (d_1 \mp d_1^\dagger), \\ \gamma_2 = \frac{(d_3 \pm d_3^\dagger) - \alpha (d_2 \pm d_2^\dagger) + \beta (d_1 \pm d_1^\dagger)}{\sqrt{2} \sqrt{1 + \alpha^2 + \beta^2}}, \quad (7)$$

where $\alpha = \mu_3/2t'_{2,3}$ and $\beta = \mu_2\mu_3/4t'_{1,2}t'_{2,3}$. In the case where μ_2 or μ_3 are zero the MFs are spatially separated [25]. Interestingly, the manipulation of the onsite-energies allows us to change the localization of the MFs, which would be relevant for their detection in transport [10].

3. Floquet Majorana fermions

In the following, we will apply external ac fields in order to change periodically the onsite energies of the QDs and in this way obtain FMFs as steady-state solutions of the non-equilibrium problem.

For every system described by a time-periodic Hamiltonian a set of solutions exists, called Floquet states, which have the form $|\psi_n(t)\rangle = e^{-i\epsilon_n t} u_n(t)$, where $|u_n(t)\rangle$ are time periodic functions called Floquet modes and ϵ_n are the so-called quasienergies [26–28]. As the quasienergies are only defined modulo Ω , where $\Omega = 2\pi/T$ and T is the period of the Hamiltonian, a system with particle-hole symmetry (with excitations in pairs $\gamma_{-\epsilon}^\dagger = \gamma_\epsilon$) will hold FMFs if $\epsilon = 0, \pm \Omega/2$. If the frequency is large enough, it is a good approximation to consider the time-averaged Hamiltonian to describe the dynamics. For lower frequencies, where multi-photon processes are relevant, the dynamics becomes more involved but there is also a way to find an effective time-independent

Hamiltonian which includes as many photon processes as necessary [29,30].

The motivation to consider periodically driven quantum systems is the fact that their time-evolution is governed by an effective time-independent Hamiltonian, whose properties can be engineered according to the particular purposes. This method, called Floquet engineering, has been employed to achieve dynamic localization [31–33], photon-assisted tunneling [26,34] or nobel topological band structures [35–41,54,55].

The application of degenerate perturbation theory in the extended Floquet Hilbert space provides a high-frequency expansion (in powers of $1/\Omega$) for this effective Hamiltonian, such as $H_F = \sum_{\nu=0}^{\infty} H_F^{\nu}$ [29]. With the definition of the Fourier components of the time-periodic Hamiltonian

$$H_m = \frac{1}{T} \int_0^T dt e^{-im\Omega t} H(t), \quad (8)$$

the leading orders of the expansion for the effective Hamiltonian are

$$\begin{aligned} H_F^0 &= H_0, \\ H_F^1 &= \sum_{m=1}^{\infty} \frac{[H_m, H_{-m}]}{m\Omega}, \\ H_F^2 &= \sum_{m \neq 0} \left(\frac{[H_{-m}, [H_0, H_m]]}{2(m\Omega)^2} + \sum_{m' \neq 0, m} \frac{[H_{-m'}, [H_{m'-m}, H_m]]}{3mm'(\Omega)^2} \right), \end{aligned} \quad (9)$$

These terms will be considered below in order to obtain FMFs in two different configurations of driven quantum dots: QDs and TQDs.

The time periodic perturbation applied to the i -QD is

$$V(t) = \sum_i A_i \cos(\Omega t + \varphi_i) d_i^{\dagger} d_i. \quad (10)$$

In order to study the effect of an external driving at high frequency, it is convenient to move to the interaction picture which transfers the time-dependence to the tunneling terms by means of the unitary transformation: $U(t) = \exp\{-i \int_0^t V(t') dt'\}$. Only the non-diagonal elements change under the transformation depending on whether they commute or not with the time-periodic term:

$$\begin{aligned} [d_i^{\dagger} d_{i+1}, V(t)] &= d_i^{\dagger} d_{i+1} (A_{i+1} \cos(\Omega t + \varphi_{i+1}) \\ &\quad - A_i \cos(\Omega t + \varphi_i)), \\ [d_i d_{i+1}, V(t)] &= d_i d_{i+1} (A_{i+1} \cos(\Omega t + \varphi_{i+1}) \\ &\quad + A_i \cos(\Omega t + \varphi_i)). \end{aligned} \quad (11)$$

Therefore, the renormalization of the tunneling and the superconducting pairing depends on the symmetry of the driving, i.e., on the intensities applied in the different dots and on the phase difference of the ac gate voltages between the different dots. As an example of this, in the case of two QDs if an ac gate potential is applied to each of them with the same amplitude ($A_1 = A_2$) and frequency, the tunneling term does not change if the phases are equal but it does if the phase difference is π and the opposite happens for the superconducting pairing (see Eq. (11) [42,43]).

3.1. Superconducting double QD:

In the present work, we are interested in a configuration such that both the tunnel and the superconducting amplitudes are equally renormalized by the ac voltages. By inspection of Eq. (11), one can see that this corresponds to driving one of the gates periodically, it means $A_1 = A_0$ and $A_2 = 0$. In this case, the Fourier

components of the time-dependent Hamiltonian are

$$H_m = \begin{pmatrix} \mu_1 \delta_{n,0} & 0 & t_m & -\Delta_m \\ 0 & -\mu_1 \delta_{n,0} & \Delta_{-m} & -t_{-m} \\ t_{-m} & \Delta_m & \mu_2 \delta_{n,0} & 0 \\ -\Delta_{-m} & -t_m & 0 & -\mu_2 \delta_{n,0} \end{pmatrix}, \quad (12)$$

where $t_m \equiv t'_{1,2} \mathcal{J}_m(A_0/\Omega)$, $\Delta_m \equiv \Delta'_{1,2} \mathcal{J}_m(A_0/\Omega)$ and \mathcal{J}_m is the m -Bessel function of first kind. The zeroth order effective Hamiltonian only predicts spatially separated FMFs if $\mu_{1,2} = 0$ and $\Delta'_{1,2} = \pm t'_{1,2}$ (see Eq. (6)). However, the following order corrections allow us to generate new sweet spots for FMFs. The first order correction is zero and the effect of the second one is the renormalization of $t'_{1,2}$ and $\Delta'_{1,2}$ to some effective values given by

$$t_{\text{eff}} = t'_{1,2} \mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{4t'^2_{1,2}}{\Omega^2} f\left(\frac{A_0}{\Omega}\right), \quad (13)$$

$$\Delta_{\text{eff}} = \Delta'_{1,2} \mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{4\Delta'^2_{1,2}}{\Omega^2} f\left(\frac{A_0}{\Omega}\right), \quad (14)$$

where $f(A_0/\Omega)$ is a function of all the Bessel functions. Considering only two sidebands ($\mathcal{J}_{n,-n}(A_0/\Omega) = 0$ for $n > 2$) its analytical expression becomes

$$f\left(\frac{A_0}{\Omega}\right) = \mathcal{J}_1^2\left(\frac{A_0}{\Omega}\right) \left(\mathcal{J}_0\left(\frac{A_0}{\Omega}\right) + \mathcal{J}_2\left(\frac{A_0}{\Omega}\right) \right). \quad (15)$$

As the ratio between the intensity and the frequency of the ac field increases more terms contribute to $f(A_0/\Omega)$.

The key point in the previous discussion is that the renormalization of $t'_{1,2}$ and $\Delta'_{1,2}$ by the ac field makes it possible to choose the driving amplitude such that $\Delta_{\text{eff}} = \pm t_{\text{eff}}$ even when $\Delta'_{1,2} \neq t'_{1,2}$. This is exactly what we observe in the quasienergy spectrum (see Fig. 2). In this calculation, the on-site energies μ_1 and μ_2 are set to zero and the static normal and superconducting tunnelings are different, i.e., $\Delta'_{1,2} \neq t'_{1,2}$. At high frequencies all the quasienergies are zero at the zeros of the function $\mathcal{J}_0(A_0/\Omega)$ (approximately $A_0/\Omega = 2.40, 5.52, 8.65$, etc.) and there are no FMFs.

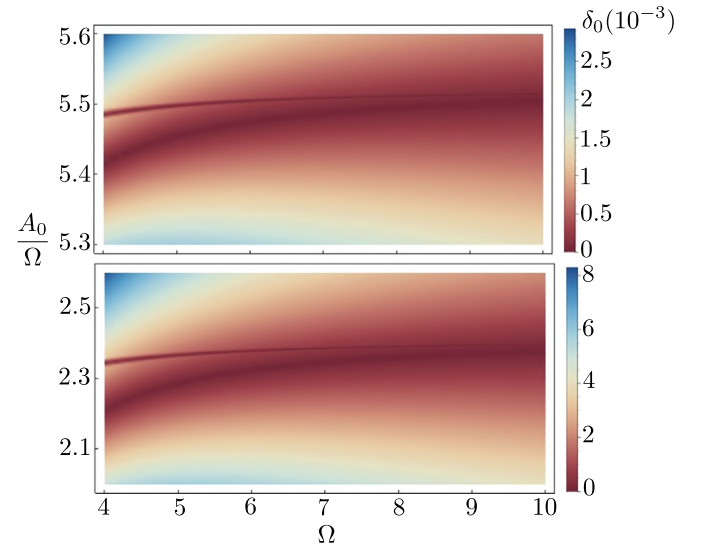


Fig. 2. Quasienergy gap δ_0 for a superconducting double QD as a function of the amplitude and frequency of the driving. The dark regions correspond to closed gap, i.e., zero quasienergy. The plot shows that the 4-fold degeneracy at high frequency at the zeros of the Bessel function $\mathcal{J}_0(A_0/\Omega)$ splits into two different sweet spots with FMFs as the frequency decreases. The bottom plot shows the region around the first zero and the top plot around the second zero. Parameters: $\mu_1 = \mu_2 = 0$, $\Delta'_{1,2} = 1$, $t'_{1,2} = 0.8$, $\varphi_3 - \varphi_1 = 0$. All the energies are in units of $\Delta'_{1,2}$, which is set to 1.

As the second order correction becomes important, i.e., as the frequency decreases, two different driving amplitudes allow for the condition required to the existence of FMFs: the one for which $\Delta_{\text{eff}} = t_{\text{eff}}$ and the one for which $\Delta_{\text{eff}} = -t_{\text{eff}}$. This is why at lower frequencies there are two quasienergy gap closings around each zero of the Bessel function, i.e., two different sweet spots (the bottom panel of Fig. 2 shows the gap around the first zero, ~ 2.40 and the top panel around the second one ~ 5.52). In the following we generalize this method for generation of FMFs to a largest system, i.e., to an array of three QDs.

3.2. Superconducting triple QD:

Analogously to the case of the double QD, we use the driving fields such that all the non-diagonal terms of the Hamiltonian are renormalized in the same way by the ac field at high-frequency. That implies driving the left and right dots with ac gate voltages such that $A_1 = A_3 = A_0$ and $A_2 = 0$. Let us choose for simplicity $t \equiv t'_{1,2} = t'_{2,3}$ and $\Delta \equiv \Delta'_{1,2} = \Delta'_{2,3}$. We are going to analyze the presence of FMF as a function of the different parameters of the present setup, in particular of the phase difference between the ac voltages. With the driving fields in phase $\varphi \equiv \varphi_3 - \varphi_1 = 0$, the Fourier components of the time-dependent Hamiltonian are

$$H_m = \begin{pmatrix} \mu_1 \delta_{n,0} & 0 & t_m & -\Delta_m & 0 & 0 \\ 0 & -\mu_1 \delta_{n,0} & \Delta_m & -t_m & 0 & 0 \\ t_m & \Delta_m & \mu_2 \delta_{n,0} & 0 & t_m & -\Delta_m \\ -\Delta_m & -t_m & 0 & -\mu_2 \delta_{n,0} & \Delta_m & -t_m \\ 0 & 0 & t_m & \Delta_m & \mu_3 \delta_{n,0} & 0 \\ 0 & 0 & -\Delta_m & -t_m & 0 & -\mu_3 \delta_{n,0} \end{pmatrix} \quad (16)$$

where $t_m \equiv t \mathcal{J}_m(A_0/\Omega)$ and $\Delta_m \equiv \Delta \mathcal{J}_m(A_0/\Omega)$. Due to the driving symmetry, if we keep only the zero order term of the expansion for the effective Hamiltonian all the non-diagonal terms vanish at the zeros of $\mathcal{J}_0(A_0/\Omega)$ so there is no effective tunneling or superconducting pairing and the quasienergies are $\pm \mu_i$ for $i = 1, 2, 3$. In the following, we show how the higher order corrections to this high-frequency approximation generate FMFs around these zeros. We will focus on the case $\mu_1 = \mu_2 = 0$, $\mu_3 \neq 0$ and $\Delta_i \neq \pm t_i$ such that there are no MFs in the static case. In Fig. 3, we plot the gap of the quasienergy spectrum as a function of the amplitude and the frequency of the driving. In the limit of high-frequency the effective tunneling and superconducting pairing are zero so there is a four-fold degeneracy at $\epsilon = 0$ and there are no MFs. At lower frequencies, these zero-quasienergy pairs appear at different amplitudes in which the Majorana condition is satisfied, two different sweet spots. This is due to the second order correction to the effective Hamiltonian. The largest effect of this term is a correction of the tunneling amplitudes, which becomes

$$t_{\text{eff}} = t \mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{4t(2t^2 - \Delta^2)}{\Omega^2} f\left(\frac{A_0}{\Omega}\right), \quad (17)$$

$$\Delta_{\text{eff}} = \Delta \mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{4\Delta(2\Delta^2 - t^2)}{\Omega^2} f\left(\frac{A_0}{\Omega}\right). \quad (18)$$

One difference with the double QD system is that in this case a small effective tunneling between dots 1 and 3 appears due to virtual processes. The expression for this long-range tunneling is

$$\tau_{1,3} = \frac{\mu_3(\Delta^2 - t^2)}{\Omega^2} \sum_{m=1}^{\infty} \frac{\mathcal{J}_m\left(\frac{A_0}{\Omega}\right)^2}{m^2}. \quad (19)$$

Moreover, the chemical potentials μ_2 and μ_3 are shifted, such

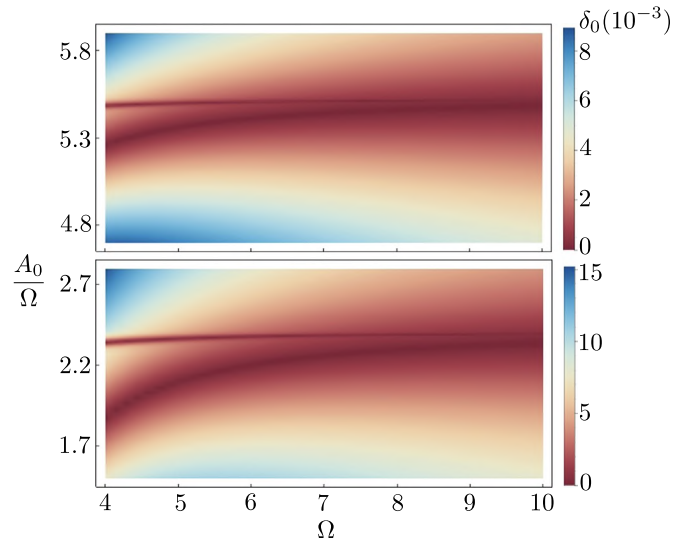


Fig. 3. Quasienergy gap δ_0 for a superconducting triple QD as a function of the amplitude and frequency of the driving. The dark regions correspond to closed gap, it means, zero quasienergy. The plot shows that the 4-fold degeneracy at high frequency at the zero of the Bessel function $\mathcal{J}_0(A_0/\Omega)$ splits into two different sweet spots with FMFs as the frequency decreases. The bottom plot shows the region around the first zero and the top plot around the second zero. Parameters: $\mu_1 = \mu_2 = 0$, $\mu_3 = 1.5$, $\Delta = 1$, $t = 0.8$, $\varphi_3 - \varphi_1 = 0$. All the energies are in units of Δ , which is set to 1.

that

$$\mu_{2\text{eff}} = 2 \frac{\mu_3(\Delta^2 - t^2)}{\Omega^2} \sum_{m=1}^{\infty} \frac{\mathcal{J}_m\left(\frac{A_0}{\Omega}\right)^2}{m^2}, \quad (20)$$

$$\mu_{3\text{eff}} = \mu_3 - 2 \frac{\mu_3(\Delta^2 + t^2)}{\Omega^2} \sum_{m=1}^{\infty} \frac{\mathcal{J}_m\left(\frac{A_0}{\Omega}\right)^2}{m^2}. \quad (21)$$

The shift in the chemical potentials only changes the localization of the states (see Eq. (7)) and the effect of the long-range tunneling is small. In order to probe this, we plot in Fig. 4 the quasienergy spectrum around zero and the functions $\Delta_{\text{eff}} \pm t_{\text{eff}}$ as a function of the driving amplitude. The sweet spots are very close to the zeros

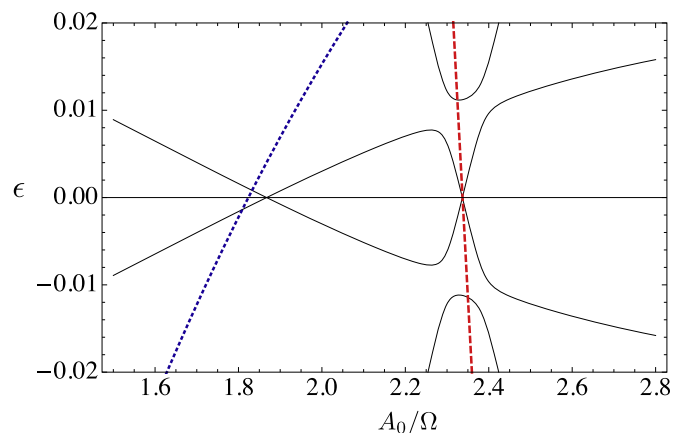


Fig. 4. Lower part of the quasienergy spectrum for a superconducting triple QD as a function of the amplitude of the driving. The dotted (blue) line corresponds to $t_{\text{eff}} - \Delta_{\text{eff}}$ and the dashed (red) to $t_{\text{eff}} + \Delta_{\text{eff}}$. We show that the FMFs appear close to the conditions $\Delta_{\text{eff}} = \pm t_{\text{eff}}$. Parameters: $\mu_1 = \mu_2 = 0$, $\mu_3 = 1.5$, $\Delta = 1$, $t = 0.8$, $\Omega = 4$, $\varphi_3 - \varphi_1 = 0$. All the energies are in units of Δ , which is set to 1. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

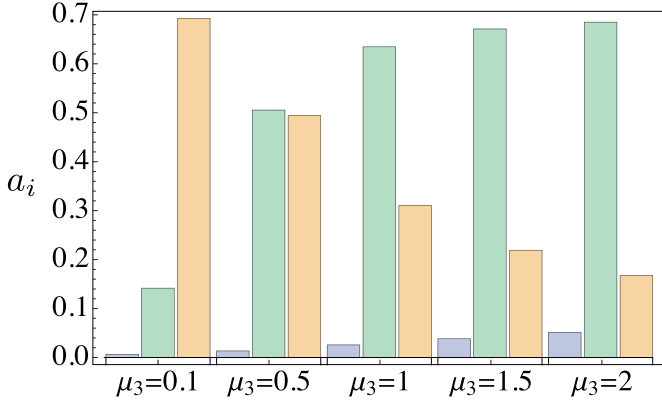


Fig. 5. Spatial location of the second FMF appearing when $t_{\text{eff}} = \Delta_{\text{eff}}$. The blue, green and orange bars are respectively a_1 , a_2 and a_3 (see Eq. (22)). Parameters: $\mu_1 = \mu_2 = 0$, $\Delta = 1$, $t = 0.8$, $\Omega = 4$, $\varphi_3 - \varphi_1 = 0$. All the energies are in units of Δ , which is set to 1. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

of these functions, indicating that the effect of $\tau_{1,3}$ is small. Finally, we calculate the localization of the FMFs found in this configuration. We choose the FMF that appears when $t_{\text{eff}} = \Delta_{\text{eff}}$ (left zero in Fig. 4). The Majorana pairs are given by

$$\begin{aligned} \gamma_1 &= \frac{1}{\sqrt{2}}(d_1 - d_1^\dagger), \\ \gamma_2 &= a_1(d_1 - d_1^\dagger) + a_2(d_2 - d_2^\dagger) + a_3(d_3 - d_3^\dagger), \end{aligned} \quad (22)$$

with normalization $\sum_{i=1}^3 2a_i^2 = 1$. In the bar diagram in Fig. 5 the value of the constants a_i for different values of the chemical potential μ_3 is plotted. Interestingly, Fig. 5 shows that for certain values of the chemical potential μ_3 the two FMFs are spatially separated and that it is possible to tune the position of γ_2 .

Furthermore, as we will see below, the phase difference between the local ac gate voltages within each dot plays an important role. Then, in order to conclude the analysis about the generation of FMFs in a triple QD configuration, we will show that the existence of sweet spots depends on the relative phase between the driving fields. When the two fields have opposite phases, $\varphi_3 - \varphi_1 = \pi$, the zero order term of the expansion (9) does not change with respect to the previous case where $\varphi_3 - \varphi_1 = 0$. However, the following corrections depend on the phase difference. We have calculated the effective tunneling amplitudes in the case $\varphi_3 - \varphi_1 = \pi$ and the result is

$$t_{\text{eff}} = t\mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{2t(t^2 + \Delta^2)}{\Omega^2}f\left(\frac{A_0}{\Omega}\right), \quad (23)$$

$$\Delta_{\text{eff}} = \Delta\mathcal{J}_0\left(\frac{A_0}{\Omega}\right) - \frac{2\Delta(t^2 + \Delta^2)}{\Omega^2}f\left(\frac{A_0}{\Omega}\right). \quad (24)$$

Therefore the functions $\Delta_{\text{eff}} \pm t_{\text{eff}}$ become zero for the same value of A_0 in contrast with the previous case for $\varphi = 0$. In Fig. 6 we show the gap of the quasienergies as a function of the phase difference and the amplitude of the driving field. The measurement of this φ -dependence would be an important signature of the existence of FMFs.

The existence of these exotic dynamical quasiparticles can be detected by connecting two metallic leads and measuring transport [10,25,44,45]. The signatures of FMFs will be present in the differential conductance measurement by the fulfillment of the Floquet sum rule [46]. It is expected that FMFs could be measured by transport by tuning the parameters of the ac driving and therefore the normal and superconducting couplings.

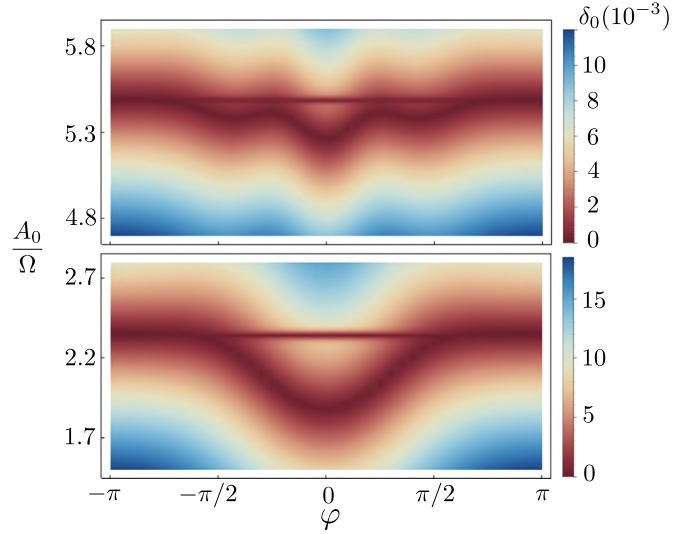


Fig. 6. Quasienergy gap δ_0 for a superconducting triple QD as a function of the amplitude and the relative phase of the driving fields $\varphi = \varphi_3 - \varphi_1$. The dark regions correspond to closed gap, it means, zero quasienergy. The plot shows that one of the sweet spots varies with the phase, while the other does not. The bottom plot shows the region around the first zero of the function $\mathcal{J}_0(A_0/\Omega)$ and the top plot around the second zero. Parameters: $\mu_1 = \mu_2 = 0$, $\mu_3 = 1.5$, $\Delta = 1$, $t = 0.8$, $\Omega = 4$. All the energies are in units of Δ , which is set to 1.

4. Conclusions

To summarize, we have discussed the existence of FMFs in two different configurations of QDs driven by ac gate voltages and coupled through superconductor leads. The simplicity of these systems and their tunability in comparison with other proposed setups which provide MFs deserve to consider them as suitable solid state devices to host MFs. We have shown the existence of FMFs by means of the expansion of an effective Floquet Hamiltonian in power series. By modifying the frequency of the driving field applied to a double QD it is possible to control the existence of a series of sweet spots. Moreover, we analyze as well a driven triple QD and we predict the existence of sweet spots as a function of the relative phase of the local drivings. This method for FMFs generation can be extended to chains of QDs with more than three atoms. One would expect that as the number of QDs increases, the localization of the FMFs changes and Eq. (22) would be generalized. Experimentally, the recent achievements in the fabrication and control of triple [47–50] and even quadruple semiconductor QDs [51], also for driven configurations [52], open the avenue for the experimental realization of hybrid configurations with superconductor contacts where FMFs can be experimentally investigated.

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References

- [1] F. Wilczek, *Nat. Phys.* 5 (2009) 614.
- [2] J. Alicea, *Rep. Prog. Phys.* 75 (2012) 076501.
- [3] C. Beenakker, *Ann. Rev. Condens. Matter Phys.* 4 (2013) 113.
- [4] V. Mourik, et al., *Science* 336 (2012) 1003.
- [5] L.P. Rokhinson, X. Liu, J.K. Furdyna, *Nat. Phys.* 8 (2012) 795.
- [6] S. Nadj-Perge, et al., *Science* 346 (2014) 602.
- [7] C. Nayak, S.H. Simon, A. Stern, M. Freedman, S. Das Sarma, *Rev. Mod. Phys.* 80 (2008) 1083.
- [8] D.E. Liu, A. Levchenko, H.U. Baranger, *Phys. Rev. Lett.* 111 (2013) 047002.
- [9] L. Jiang, et al., *Phys. Rev. Lett.* 106 (2011) 220402.
- [10] M. Leijnse, K. Flensberg, *Phys. Rev. B* 86 (2012) 134528.
- [11] A. Martin-Rodero, A. Levy-Yeyati, *Adv. Phys.* 60 (2011) 899.
- [12] S. de Franceschi, L. Kouwenhoven, C. Schönberger, W. Wernsdorfer, *Nat. Nanotechnol.* 5 (2010) 703.
- [13] B. Sothmann, S. Weiss, M. Governale, J. König, *Phys. Rev. B* 90 (2014) 220501.
- [14] F. Domínguez, F. Hassler, G. Platero, *Phys. Rev. B* 86 (2012) 140503.
- [15] P. Jacquod, M. Büttiker, *Phys. Rev. B* 88 (2013) 241409.
- [16] B. Sothmann, J. Li, M. Büttiker, *N. J. Phys.* 15 (2013) 085018.
- [17] M. Leijnse, K. Flensberg, *Semicond. Sci. Technol.* 27 (2012) 124003.
- [18] R. Lutchyn, J. Sau, S. Das Sarma, *Phys. Rev. Lett.* 105 (2010) 077001.
- [19] Y. Oreg, G. Refael, F. von Oppen, *Phys. Rev. Lett.* 105 (2010) 177002.
- [20] A.Y. Kitaev, *Phys. Usp.* 44 (2001) 131.
- [21] J.D. Sau, S. Das Sarma, *Nat. Commun.* 3 (2012).
- [22] I.C. Fulga, A. Haim, A.R. Akhmerov, Y. Oreg, *N. J. Phys.* 15 (2013) 045020.
- [23] A.R. Wright, M. Veldhorst, *Phys. Rev. Lett.* 111 (2013) 096801.
- [24] J. Eldridge, M.G. Pala, M. Governale, J. König, *Phys. Rev. B* 82 (2010) 184507.
- [25] M.-X. Deng, S.-H. Zheng, M. Yang, L.-B. Hu, R.-Q. Wang, *Chin. Phys. B* 24 (2015) 037302.
- [26] G. Platero, R. Aguado, *Phys. Rep.* 395 (2004).
- [27] H. Sambi, *Phys. Rev. A* 7 (1973) 2203.
- [28] J.H. Shirley, *Phys. Rev. B* 138 (1965) 979.
- [29] A. Eckardt, E. Anisimovas, arXiv:1502.06477.
- [30] N. Goldman, J. Dalibard, *Phys. Rev. X* 4 (2014) 031027.
- [31] F. Grossman, T. Dittrich, P. Jung, P. Hänggi, *Phys. Rev. Lett.* 67 (1991) 516.
- [32] C.E. Creffield, G. Platero, *Phys. Rev. B* 69 (2004) 165312.
- [33] A. Gómez-León, G. Platero, *Phys. Rev. B* 84 (2011) 121310.
- [34] F. Gallego-Marcos, R. Sánchez, G. Platero, *J. Appl. Phys.* 117 (2015) 112808.
- [35] J. Inoue, A. Tanaka, *Phys. Rev. Lett.* 105 (2010) 017401.
- [36] N.H. Lindner, G. Refael, V. Galitski, *Nat. Phys.* 7 (2011) 490.
- [37] A. Gómez-León, G. Platero, *Phys. Rev. Lett.* 110 (2013) 200403.
- [38] P. Delplace, A. Gómez-León, G. Platero, *Phys. Rev. B* 88 (2013) 245422.
- [39] A.G. Grushin, A. Gómez-León, T. Neupert, *Phys. Rev. Lett.* 112 (2014) 156801.
- [40] A. Gómez-León, P. Delplace, G. Platero, *Phys. Rev. B* 89 (2014) 205408.
- [41] M. Benito, A. Gómez-León, V.M. Bastidas, T. Brandes, G. Platero, *Phys. Rev. B* 90 (2014) 205127.
- [42] Y. Li, Y. Wang, F. Zhong, arXiv:1301.3623, 2013.
- [43] Y. Li, A. Kundu, F. Zhong, B. Seradjeh, *Phys. Rev. B* 90 (2014) 121401.
- [44] K. Flensberg, *Phys. Rev. B* 82 (2010) 180516.
- [45] M.X. Deng, et al., *Phys. Lett. A* 378 (2014) 2256.
- [46] A. Kundu, B. Seradjeh, *Phys. Rev. Lett.* 111 (2013) 136402.
- [47] M.C. Rogge, R.J. Haug, *N. J. Phys.* 11 (2009) 113037.
- [48] G. Granger, et al., *Phys. Rev. B* 82 (2010) 075304.
- [49] M. Busl, et al., *Nat. Nanotechnol.* 8 (2013) 261.
- [50] R. Sánchez, et al., *Phys. Rev. Lett.* 112 (2014) 176803.
- [51] T. Takakura, et al., *Appl. Phys. Lett.* 104 (2014) 113109.
- [52] F. Braakman, P. Barthelemy, C. Reichl, W. Wegschneider, L. Vandersypen, *Nat. Nanotechnol.* 8 (2013) 432.
- [53] M.T. Deng, C.L. Yu, G.Y. Huang, M. Larsson, P. Caroff, H.Q. Xu, *Nano Letters* 12 (2012) 6414.
- [54] Q.J. Tong, J.H. An, J. Gong, H.G. Luo, C.H. Oh, *Phys. Rev. B* 87 (2013) 201109(R).
- [55] M. Thakurathi, A.A. Patel, D. Sen, A. Dutta, *Phys. Rev. B* 88 (2013) 155133.