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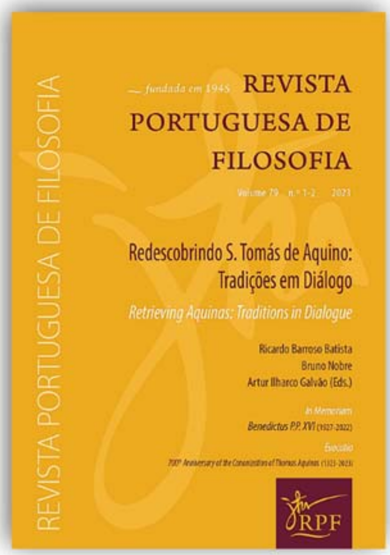

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An Axiomatization of the Thomasic Ontology of Composition

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Abstract

This treatise delves into the construction and formalization of a precise theory to encapsulate a pivotal facet of Thomas Aquinas's ontology. In particular, it focuses on the development of a formal language that is adequate for formulating Aquinas's understanding of the simultaneous nature of divinity as both an object of subsistence and a universal, individual, and actuating form—a concept evocative of the original Platonic sense of form. The presented axiomatized theory as a medium for expounding the ontological principles enunciated by Thomas Aquinas. A comprehensive set of axioms is systematically justified, exhibiting their congruence with Aquinas's ontological doctrines. To elucidate this harmony, an extensive series of theorems is deduced, closely aligned with the verbal tenets of Aquinas's teachings. The inherent consistency of the axiom system is proven by the provision of a geometrico-topological model tailored to support it.

Keywords: adequacy, Aquinas, axiomatic formulation, Divinity, formal language, ontology.

I. Introduction

In this treatise a formal language will be constructed in which an essential part of Thomas Aquinas's ontology can be precisely formulated. In the formal language, an axiomatization of this part of his ontology will be presented, and its exegetic adequacy shown by deducing a long series of theorems that are all in verbal accordance with the ontological teachings of Thomas. It will be made plausible that no theorem contradicting his ontology can be derived. The consistency of the axiom-system will be demonstrated by providing a geometrico-topological model for it. Finally, the axiom-system will be significantly extended and given an ontological interpretation that is very plausibly in agreement with the intentions of Thomas Aquinas. The texts referred to are the *Summa theologiae*, *Summa contra gentiles*, *De ente et essentia*, and *In Aristotelis librum de anima commentarium*.

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II. Methodological preliminaries

Before I begin, a remark concerning method is in order. This treatise has been written in the conviction that the logical reconstruction of philosophical doctrines from the history of philosophy can be of value for our understanding of them (if they can be at all subjected to such a treatment). This conviction is not uncontroversial. It is in the nature of a logical reconstruction that it contains certain deviations from the original that is being reconstructed. In a logical reconstruction, inconsistencies are avoided – that is, *inessential* inconsistencies, which are due to carelessness; essentially inconsistent theories are not amenable to logical reconstruction. (Sometimes, however, an attempt at logical reconstruction is necessary in order to show that a philosophical theory is essentially inconsistent.) In a logical reconstruction, instances of ambiguity and vagueness may be clarified into alternative non-ambiguous and non-vague logical sub-reconstructions. The theoretical horizon of a logical reconstruction is normally wider than that of the original text: it normally points out conclusions that the author of the original did not think of, or at least did not mention; these 'new insights', however, must not be contrary to the spirit of the original; if they were, the logical reconstruction would be inadequate as an interpretation of the original. A logical reconstruction employs logical resources of which the author had no, or only an inadequate, idea. A logical reconstruction proves conclusions that the author of the original merely stated on the strength of his intuitions or arrived at by entirely inconclusive arguments. A logical reconstruction is more, sometimes much more, systematic than the original, connecting results that are not connected in the original. However, it may also disconnect results that are connected in the original: in case no justifiable logical bond can be found between them.

If the original text is open to logical reconstruction, then the mentioned deviations of *its* logical reconstruction from it – if they remain within limits – will contribute not to its distortion but rather to its clarification, revealing, as it were, what the author would have said if he had had the modem logical techniques at his disposal.

III. The scope of the reconstruction

The present logical reconstruction refers to the ontological doctrines of Thomas Aquinas concerning the composition (*compositio*) of existent objects[1] – that is, of *res* in the appropriate sense: of substances (*substantiae primae*), and also of quasi-substances, i.e., human souls and God[2] – *by their (ontologically) fundamental aspects*. Aquinas recognizes five fundamental aspects of an object: its *matter,* its *pure substantial form,* its *being,* its *essence,* and its *actuating substantial form.* It should be noted that a substantial form is *not* ipso facto – qua substantial form – a form which is a substance; it is, qua substantial form, merely a *fundamental* form *of* a substance or quasi-substance.

Accordingly, five functional terms are introduced: m(t), f(t), s(t), w(t), a(t), where "t" can be replaced by any object-variable or object-name. They are to be read as 'the matter of t', 'the pure form of t' (short for 'the pure substantial form of t'), 'the being of t', 'the essence of t', 'the actuating form of t' (short for 'the actuating substantial form of t').

Normally, object-aspects are not objects (there are, however, exceptions); thus, it is not generally meaningful to speak, for example, of the essence of the being of an object, or of the being of the being of an object. The formal language will consequently be constructed in such a manner that functional terms embedded in functional terms – like w(f(t)), s(w(f(t))), f(s(m(t))), etc. – are not well-formed. To allow functional terms to be well-

1. In what follows, the word "object" is always to be taken in the sense of '*existent* object'. Moreover, it should always be born in mind that the rather nondescript term "object" is here being used in a special, technical sense: to refer to the *bearers* (or *possessors*) of the fundamental compositional aspects that Thomas's doctrines are about. The use of the term "object" in this treatise must not be taken to imply a depreciation of the dignity of what it is applied to (as when human beings and God are said to be "objects").
2. Note that the Thomasic reason for God's quasi-substantiality is quite different from the reason for the quasi-substantiality of human souls. Human souls, according to Thomas, are not substances *in the full* (or: *strong*) sense for the reason that – although they can and for some time do exist on their own (according to Thomas): without a body – their disembodied form of existence is not their normal or natural state; in their normal or natural state, they are parts of human beings and thus have an imperfection which prevents them from being substances in the full (strong) sense (see *Summa theologiae*, 1, 29, 1, and 1, 75, 2). God, according to Thomas, is not a substance in the proper sense because God is not in any category (*genus*) and because 'substance' in the proper sense is a category (see Summa theologiae, 1, 3, 5). It seems that Thomas's main motivation for excluding God from the genus of substance is to stress the incomparability of God in relation to the created beings. – Thomas nevertheless applies 'substantia' to the human soul and to God on several occasions; for example, such a use of 'substantia' is evidently implied in quotations 40 and 46 in this treatise. The best way to deal with this situation is to say that Thomas is using 'substantia' on such occasions in an analogically extended sense; that, in literal parlance, he is saying that the human soul and God are *subsistences* (i.e., objects: substances or *quasi-substances*); cf. the *Sed contra* of *Summa theologiae*, 1, 75, 2, where 'substantia' is defined – not in the full and not in the proper sense, but in the analogically extended sense – merely as *aliquid subsistens*).

formed which are formed like the mentioned examples is warranted by nothing in the writings of Aquinas.

According to Aquinas, certain objects have no matter; for such objects, the function *the matter of* is initially not defined. However, a complete definition (that is, a definition for all objects) is secured for this function by assuming an empty aspect of every object, and by stipulating that if an object has no matter, then its matter is its empty aspect. Accordingly, the functional term c(t) is introduced, which is to be read as 'the empty aspect of t'.

Aspects of the same object, according to Aquinas, combine to form an aspect of the object or the object itself, and Aquinas – as has been said – recognizes five (fundamental) aspects of an object. Accordingly, we have a dyadic functor '+' such that only functional expressions with '+' that have the forms (φ(t) + φ'(t)), ((φ(t) + φ'(t)) + φ''(t)), (φ(t) + (φ'(t) + φ''(t))) are well-formed, where φ, φ', φ'' may each be replaced by "m", "f", "s" – *and*, in addition, by "c" (though this is non-Thomasic, it is warranted by the benefits it has for the formulation of Thomasic doctrines, as will be seen). But why is one not also allowed to replace φ, φ', and φ'' each by "w" and "a"? One is not allowed to do so because the forms of expression just presented refer to the formal language *without* defined expressions (i.e., to the language in basic notation), and with the help of the composition functor '+' w(t) and a(t) can, in fact, be *defined*, in keeping with the writings of Aquinas. The first definition is this:

w(t) := (f(t) + m(t)) – *the essence of an object is its pure form combined with its matter*.

If one followed Aquinas to the letter, this definition would *not* be adequate for all objects; it would only be adequate for material objects. But the introduction of c(t) makes it possible to regard it as the general definition of essence, without getting into conflict with Thomasic views. Let "*g*" designate some immaterial object; then 'w(*g*) = (f(*g*) + m(*g*))' is equivalent to 'w(*g*) = f(*g*)', where the latter statement corresponds to the Thomasic definition of essence for immaterial objects: *the essence of an immaterial object is its pure form*. The equivalence is easily proven: Since *g* is immaterial we have 'm(*g*) = c(*g*)', hence we also have '(f(*g*) + m(*g*)) = (f(*g*) + c(*g*))', and therefore '(f(*g*) + m(*g*)) = f(*g*)' – for the empty aspect of *g* adds nothing to the pure form of *g*. Thomas says:

1. *In hoc* ergo *differt* essentia substantiae compositae [sive materialis] et substantiae simplicis [sive immaterialis], quod essentia substantiae compositae non est tantum forma, sed complectitur formam et materiam; essentia autem substantiae

simplicis est forma tantum (*De ente et essentia*, caput 4, 25 [of the continuously enumerated sections]).

Whereas Aquinas, when speaking of composition, always means *proper composition,* that is, the composition of different, non-empty aspects of the same object, there is, in this reconstruction of his doctrines, also a place for *improper composition,* that is, for the composition of an aspect of an object with itself or with the empty aspect of the object. (Note that by being different, object-aspects – at least the essential object-aspects considered by Aquinas – are *distinct*, since they cannot be proper parts of each other or overlap.)

The second definition is this:

a(t) := (f(t) + s(t)) – *the actuating form of an object is its pure form combined with its being* (or: *the actuating form of an object is the composition of its pure form and its being*).

Aquinas does not verbally distinguish between the actuating form of an object and its pure form, and on the whole he seems to be unaware of their being distinct (in most objects). However, his doctrines can be consistently interpreted *only* by considering the pure form of an object to be normally distinct from its actuating form. In the following quotations Aquinas is referring to the actuating form of an object:

2. ex forma et materia relinquitur esse substantiale quando componuntur (*De ente et essentia*, 6, 34).

3. Per formam enim, quae est actus materiae, materia efficitur ens actu et hoc aliquid (*De ente et essentia*, 2, 6).

And Aquinas adds:

4. Unde illud quod superadvenit non dat esse actu simpliciter materiae, sed esse actu tale [...] Unde, quando talis forma acquiritur, non dicitur generari simpliciter, sed secundum quid (*De ente et essentia*, 2, 6).

In the quotation below, however, Aquinas is referring to the pure form of an object:

5. esse substantiae compositae non est tantum formae, nec tantum materiae, sed ipsius compositi; essentia autem est secundum quam res esse dicitur. Unde oportet ut essentia, qua res denominatur ens, non tantum sit forma nec tantum materia,

sed utrumque, quamvis huiusmodi esse suo modo sola forma sit causa (*De ente et essentia*, 2, 6bis).

Here Aquinas names, beside form and matter, a third ultimate component in the composition of a material substance: its being (*esse*), whereas in the previous quotations he only mentions form and matter, obviously intending that they by themselves suffice to constitute the object. This apparent discrepancy can be resolved by supposing that in the last quotation Aquinas means by 'forma' the pure form of the object, which together with the matter of the object composes its essence, which in turn enters into composition with the being of the object to constitute the object itself; *whereas* in the previous quotations Aquinas means by 'forma' the pure form of the object in *composition with its* being, that is: the actuating form of the object, which together with the matter of the object composes the object itself. It amounts to the same whether the pure form and the matter are first composed to constitute the essence, and then the essence and the being are composed to constitute the object; or whether the pure form and the being are first composed to constitute the actuating form, and then the actuating form and the matter are composed to constitute the object.

In the next quotation the first instance of the word 'forma' means the actuating form of the object, the second instance, however, its pure form:

6. In substantiis autem compositis ex materia et forma est duplex compositio actus et potentiae: prima quidem ipsius substantiae, quae componitur ex materia et forma; secunda vero, ex ipsa substantia iam composita et esse; quae etiam potest dici ex 'quod est' et 'esse'; vel ex 'quod est' et 'quo est' (*Summa contra gentiles*, liber 2, capitulum 54).

In this passage, we also have an example of the equivocal use of the word 'substantia' in the writings of Aquinas; the first instance of this word signifies the same as 'res per se subsistens' (the same as '[individual] substance' in the analogically extended sense: cf. footnote 2), the second and third, however, the same as 'essentia seu natura' ('essence'). Aquinas is quite aware of this equivocation; in *Summa theologiae*, pars 1, quaestio 29, articulus 2, he explicitly distinguishes the two meanings of 'substantia' (following Aristotle):

7. substantia dicitur dupliciter. Uno modo dicitur substantia *quidditas rei,* quam significat definitio, secundum quod dicimus quod *definitio significat substantiam rei:* quam quidem substantiam Graeci *usiam* vocant, quod nos *essentiam* dicere

possumus. – Alio modo dicitur substantia *subiectum vel suppositum quod subsistit in genere substantiae*.

In contrast, Aquinas seems not to be aware of the equivocation in his use of the word ‘forma’; he apparently does not differentiate between what we have here been calling ‘the pure form’ and what we have here been calling ‘the actuating form’ of an object. The identification of what, on the strength of his own theory, is non-identical is bound to lead to some confusion, as we shall see (in section VIII, subsequent to quotation 11). (Thomas’s use of ‘forma’ for stating what is valid in his ontology concerning the actuating form of an object is, it seems, predominant over his use of ‘forma’ for stating what is valid in his ontology concerning the pure form of an object.)

The matter of a material object cannot enter into composition with the being of that object (while the pure form of *any* object enters into composition with the being of that object to constitute its actuating form); there is no ‘actuating matter’ of a material object. Matter is actuated by the actuating form (compare quotations 2 and 3); the complementary view of pure form being actuated by the actuating matter is absurd for Aquinas; not matter but pure form is the ‘vehicle’ (and actuating form is the effective ‘bringer’) of being:[3]

8. Forma tamen potest dici ‘quo est’, secundum quod est essendi principium (*Summa contra gentiles*, 2, 54).

9. quamvis huiusmodi esse suo modo sola forma sit causa (the last phrase of quotation 5).

10. materia vero non habet esse nisi per formam (*De ente et essentia*, 6, 36).

In consequence, the composition function is initially not defined if the arguments of the function are the matter of a material object and the being of that object. However, we can stipulate that, for any material object, the composition of its matter with its being is its empty aspect.

IV. Preliminaries concerning the syntax of the language of reconstruction

On the basis of Thomasic doctrine, the possibilities of expression by means of the composition functor ‘+’ are drastically limited in the intended

3. The pure form and the being it carries are, in combination, the actuating form.

formal language. The limitations can be summed up as follows (leaving aside the limitation that does not directly concern compositional expressions, namely, that there is to be no embedding of functional terms in functional terms):

(a) A well-formed compositional expression in basic notation contains at most two instances of '+'.

(b) Only expressions having the form f(t), m(t), s(t), c(t) may occur in a well-formed compositional expression in basic notation (i.e., without defined expressions) as argument-expressions that are not themselves compositional expressions.

(c) Exactly one object-variable or exactly one object-name occurs in a well-formed compositional expression.

These restrictions can be justified as follows:

(a') Thomas Aquinas does not consider more complex compositions than can be expressed by compositional expressions in basic notation that contain at most two instances of '+'.

(b') Aquinas does not in general consider the composition of an object with an object, or of an object with an object-aspect; he only considers the composition of an object-aspect with an object-aspect, where the non-composed object-aspects are, for Aquinas, *pure form*, *matter*, and *being* – with *the empty aspect* added as a technical tool for theory building. (Occasionally, however, an object-aspect is identical with an object: with the object it is an aspect of.)

(c') Aquinas does not in general consider the composition of aspects of different objects; he only considers the composition of aspects of the same object. (Occasionally, however, an aspect of one object is identical with an aspect of another object.)

In spite of the restrictions just described, it is still possible to generate infinitely many well-formed compositional expressions. But this is possible only if there are infinitely many object-designators, because for each object-designator (object-variable or object-name) the number of well-formed compositional expressions 'around it' is finite.

The intended axiom-system will be constructed in such a manner as to make it provable that all well-formed compositional expressions around the object-designator t are reducible to t, c(t), m(t), f(t), s(t), w(t) [:= (f(t) + m(t))], or a(t) [:= (f(t) + s(t))]. As has been said, Aquinas recognizes only five aspects of an object; in addition to those five aspects, we have, for reasons of formal simplification, the empty aspect of an object. By composition of aspects of an object there issues an aspect of this object (one of the mentioned six) or the object itself. In special cases, the reduction of compositional expressions can be carried further than this. For example, if t designates an immaterial object, we have m(t) = c(t) and w(t) = f(t).

Which predicates should belong to the envisaged formal language? As basic (undefined) predicate, only the identity-predicate '=' will be a constituent of it. With respect to the sentences and open sentences that are generable with the help of '=', no further restrictions are imposed; such restrictions – as, for example, requiring that one and the same object-designator (on its own, in a functional term, or in a compositional expression) has to occur on the left side and the right side of '=' – would not be justifiable by the writings of Aquinas. As will become apparent, a great many other predicates for Thomasic ontological distinctions can be defined with the help of the identity-predicate, the aspect-expressions, and the logical expressions. The rendering of 'est' by 'is identical with' in the present context of considering the Thomasic doctrine of the composition of objects by their aspects is, of course, a matter of interpretation; this rendering can be said to be overwhelmingly suggested by the relevant passages in the writings of Aquinas.

V. The syntax of the formal language T

The reflections in sections III and IV are summed up and made precise by the following definition of the formal language T:

1. Object-variables (OVs) of T
(a) '*x*' is an OV of T;
(b) if t is an OV of T, then t' is an OV of T;
(c) OVs of T are only expressions that are generable by (a) and (b).

2. Object-names (ONs) of T
(a) '*g*' is an ON of T;
(b) if t is an ON of T, then t' is an ON of T;
(c) ONs of T are only expressions that are generable by (a) and (b).

3. Object-designators (ODs) of T
t is an OD of T *iff* t is an OV of T or an ON of T.

4. Primary aspect-expressions (PAEs) of T
(a) If t is an OD of T, then m(t), f(t), s(t) and c(t) are PAEs of T;
(b) PAEs of T are only expressions that are generable by (a).

5. Secondary aspect-expressions (SAEs) of T
(a) If t is an OD of T and φ(t) and φ'(t) are PAEs of T, then (φ(t) + φ'(t)) is a SAE of T;
(b) SAEs of T are only expressions that are generable by (a).

6. Aspect-expressions (AEs) of T

(a) PAEs and SAEs of T are AEs of T;
(b) if t is an OD of T and φ(t) is a PAE of T and (φ'(t) + φ''(t)) is a SAE of T, then (φ(t) + (φ'(t) + φ''(t))) and ((φ'(t) + φ''(t)) + φ(t)) are AEs of T.
(c) AEs of T are only expressions that are generable by (a) and (b).

7. Compositional expressions (CEs) of T
β is a CE of T *iff* β is an AE of T but not a PAE of T.

8. Tertiary aspect-expressions (TAEs) of T
γ is a TAE of T *iff* γ is an AE of T but neither a PAE nor a SAE of T.

9. Entity-designators (EDs) of T
δ is an ED of T *iff* δ is an OD or an AE of T.

10. Primary sententials (PSLs) of T
(a) If δ and δ' are EDs of T, then (δ = δ') is a PSL of T;
(b) PSLs of T are only expressions that are generable by (a).

11. Sententials (SLs) of T
(a) PSLs of T are SLs of T;
(b) if σ and σ' are SLs of T, then ¬σ, (σ ∧ σ'), (σ ∨ σ'), (σ ⊃ σ'), (σ ≡ σ') are SLs of T;
(c) if σ is a SL of T in which in certain places X [perhaps only one place] there occurs the ON ν of T, and if υ is an OV of T that does not occur in σ and replaces ν in all places X in σ, with σ[υ] resulting from σ, then ∀υσ[υ] and ∃υσ[υ] are SLs of T;
(d) SLs of T are only expressions that are generable by (a), (b), and (c).

12. Primary sentences (PSs) of T
σ is a PS of T *iff* is a PSL of T in which no OV of T occurs.

13. Sentences (Ss) of T
σ is a S of T *iff* σ is a SL of T in which no OV of T occurs free.

1. – 13. determines the syntax of T. The intended interpretation of T has been outlined, but of course there remains much to be said about it. The logical operators ¬, ∧, ∨, ⊃, ≡, ∀, ∃ are to be read as 'not', 'and', 'or' (in the sense of '*not* neither_, nor_'), 'if_, then_' (truth-functionally understood), '_if and only if_' (truth-functionally understood), 'for all objects', 'for some object'. Finally, parentheses can be omitted in accordance with the following rules:

(i) External parentheses can be omitted.
(ii) In the sequence +, =, ¬, ∧, ∨, ⊃, ≡, syntactic binding-power is decreasing from left to right.[4]

4. Note that this rule allows one to write ¬δ = δ' instead of ¬(δ = δ'), which certainly requires some getting used to. If one writes (δ ≠ δ') as a definitional variant of ¬(δ = δ'), then '≠' (difference) binds as strongly as '=' (identity). Thus one can write ¬δ ≠ δ' instead of ¬(δ ≠ δ') [and instead of ¬¬(δ = δ'), and instead of ¬¬δ = δ'].

(iii) In a conjunction / disjunction the parentheses around the first or second member of the conjunction / disjunction can be omitted if that member is itself a conjunction / disjunction.

VI. The logic of T

Before proceeding to the formulation of an axiom-system in T that is adequate, in the intended interpretation of T, for capturing a part of Thomas Aquinas's ontology, the logic has to be described by the use of which theorems are to be deduced from the axioms. That logic is classical first-order predicate-logic with identity and functions. However, there is one restriction to this logic in this particular case of its application: Only ODs of T are quantifiable, which means that the deduction rules ∀υσ[υ] → σ[δ] and σ[δ] → ∃υσ[υ] ('→' is short for 'logically implies') may only be applied if δ is an OD of T; the appropriate form of those deduction rules for the present purposes can, therefore, be represented like this: ∀υσ[υ] → σ[t] and σ[t] → ∃υσ[υ].[5] This restriction is in keeping with the intended interpretation of ∀ and ∃: 'for all objects', 'for some object', because, under the intended interpretation of AEs of T, an AE of T, for example 'f(*g*')', will normally not refer to an object but only to an aspect of it. Aspects of objects which are not objects are, therefore, not quantified over. It is, moreover, impossible to refer to them directly, that is, to refer to them without referring at the same time to some object. This is a consequence of there being no simple names in T for aspects of objects that are not objects. Under the intended interpretation, these reference-semantic features mirror the ontologically dependent status of object-aspects which are not objects, in contrast to the ontologically independent status of objects. Thomas Aquinas would have said that object-aspects which are not objects are less real (have less being) than objects; the former entities have their being only in the latter.

Note that the second-order deduction rule for ∀: *If* Π, σ' → σ[δ], *then* Π, σ' → ∀υσ[υ], *provided* δ *does not occur free in* Π, σ' → ∀υσ[υ], and the second-order deduction rule for ∃: *If* Π, σ[δ] → σ', *then* Π, ∃υσ[υ] → σ', *provided* δ *does not occur free in* Π, ∃υσ[υ] → σ', can *in any case* (that is, whether one has the representation of Thomas Aquinas's views in mind or not) only be truthfully formulated *with the restriction that* δ *is to be an* OD

5. It goes without saying that if σ[t] is deduced from ∀υσ[υ] and t is a variable of T, that then t is to be a variable that is free in σ[t] (not bound by a quantifier in σ[t]) at least in the intended places of substituting t for υ.

of T.[6] *Without that restriction*, counterexamples can be found; for example, the following counterexample to the second-order deduction rule for ∃: It is true that $\mathrm{f}(g') = g' \rightarrow \exists x(\mathrm{f}(x) = x)$, and $\mathrm{f}(g')$ does not occur at all in $\exists x'(x' = g') \rightarrow \exists x(\mathrm{f}(x) = x)$; but, obviously, $\exists x'(x' = g') \rightarrow \exists x(\mathrm{f}(x) = x)$ is not true.

Since only ODs of T are quantifiable and we nevertheless want to make unrestricted use of the deduction-rules for identity, the basic deduction-rules for identity cannot be formulated in the following manner: $\rightarrow \forall x(x = x)$, $\rightarrow \forall x\forall x'(x = x' \supset (\sigma[x] \supset \sigma[x']))$. And they also cannot be formulated in the following manner: $\rightarrow \mathrm{t} = \mathrm{t}$, $\rightarrow \mathrm{t} = \mathrm{t}' \supset (\sigma[\mathrm{t}] \supset \sigma[\mathrm{t}'])$. Rather, they must be put in the following way: $\rightarrow \delta = \delta$, $\rightarrow \delta = \delta' \supset (\sigma[\delta] \supset \sigma[\delta'])$ (where δ is any ED of T).

VII. The axiomatic system TO

The axiom-system TO ('Thomasic ontology') consists of the following axioms (arranged into groups; the first three groups are presented via axiom-schemata). [In what follows $\varphi[\upsilon]$ and $\varphi'[\upsilon]$ is a PAE or a SAE of T having υ as its OV. But note that in $(\varphi[\upsilon] + \varphi'[\upsilon])$ it cannot be the case that both $\varphi[\upsilon]$ and $\varphi'[\upsilon]$ are SAEs of T and the expression is still well-formed – according to the syntax of T.]

A1 Every S of T having the form
$\forall\upsilon(\varphi[\upsilon] + \varphi'[\upsilon] = \varphi'[\upsilon] + \varphi[\upsilon])$
is an axiom of TO.

A2 (a) Every S of T having the form
$\forall\upsilon(\varphi[\upsilon] + \varphi[\upsilon] = \varphi[\upsilon])$
is an axiom of TO.
(b) Every S of T having the form
$\forall\upsilon(\varphi[\upsilon] + \mathrm{c}(\upsilon) = \varphi[\upsilon])$
is an axiom of TO.

A3 Every S of T having the form
$\forall\upsilon(\varphi[\upsilon] + \varphi'[\upsilon] = \varphi[\upsilon] \supset \varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi[\upsilon] = \mathrm{c}(\upsilon))$
is an axiom of TO.

B1 $\forall x(x = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x))$ [or making use of the definition of *essence*: $\forall x(x = \mathrm{w}(x) + \mathrm{s}(x))$]

B2 $\forall x((\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x) = (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x))$ [or making use of the definition of *essence* and *actuating form*: $\forall x(\mathrm{w}(x) + \mathrm{s}(x) = \mathrm{a}(x) + \mathrm{m}(x))$]

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6. The restriction is automatically fulfilled if one replaces "δ" by "t" in the second-order deduction rules (it being understood that "t" stands only for an OD of T). Note that "Π" stands for the background assumptions made (if there are any), e. g., for axiomatic principles.

B3 (a) $\forall x \neg f(x) = m(x)$ [or: $\forall x(f(x) \neq m(x))$]
(b) $\forall x \neg s(x) = m(x)$
(c) $\forall x \neg f(x) + s(x) = m(x)$[7] [or: $\forall x(f(x) + s(x) \neq m(x))$
B4 (a) $\forall x \neg x = c(x)$
(b) $\forall x \neg f(x) = c(x)$
(c) $\forall x \neg s(x) = c(x)$
(d) $\forall x \neg f(x) + s(x) = c(x)$
(e) $\forall x \neg f(x) + m(x) = c(x)$
B5 $\forall x(x = f(x) \supset m(x) = c(x))$
B6 $\forall x(\neg m(x) = c(x) \supset m(x) + s(x) = c(x))$
B7 $\forall x(\neg m(x) = c(x) \supset (f(x) + m(x)) + f(x) = c(x) \wedge (f(x) + m(x)) + m(x) = c(x))$
B8 $\forall x(\neg f(x) = s(x) \supset (f(x) + s(x)) + f(x) = c(x) \wedge (f(x) + s(x)) + s(x) = c(x))$

Concerning A1: The composition-function is commutative. The composition of aspect α of an object and aspect β of the same object is identical with the composition of aspect β of that object with aspect α of that object. Aquinas would surely have agreed.

Concerning A2 *and* A3: The conjunction of A2(a) and A2(b) is logically equivalent with the converse of A3, that is, with $\forall\upsilon(\varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi'[\upsilon] = c(\upsilon) \supset \varphi[\upsilon] + \varphi'[\upsilon] = \varphi[\upsilon])$, from which we obtain A2*: $\forall\upsilon(\varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi'[\upsilon] = c(\upsilon) \vee \varphi[\upsilon] = c(\upsilon) \supset \varphi[\upsilon] + \varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi'[\upsilon] + \varphi[\upsilon] = \varphi'[\upsilon])$. From A3, on the other hand, we get A3*: $\forall\upsilon(\varphi[\upsilon] + \varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi'[\upsilon] + \varphi[\upsilon] = \varphi'[\upsilon] \supset \varphi'[\upsilon] = \varphi[\upsilon] \vee \varphi'[\upsilon] = c(\upsilon) \vee \varphi[\upsilon] = c(\upsilon))$. This means that from A2 and A3 there follows a pair of theorem-schemata (i.e., A2* and A3*) that state the sufficient and necessary condition for improper composition, that is, composition that is not properly speaking *composition*. We already had occasion to mention that object-aspects cannot be proper parts of each other; otherwise, A2 and A3 could not be formulated in the manner presented above, but would have to take care of the possibility that '$\varphi''[\upsilon]$ is a proper part of $\varphi[\upsilon]$' is true. (Of course one may, in a sense, truthfully say that '$m(g)$ is a proper part of $w(g)$ [$:= f(g) + m(g)$]'; but this is analogous to saying 'object *a* is a proper part of the proposition *that F(a)*', *not* analogous to saying '$\lambda x(x = a)$[8] is a proper part of $\lambda x(x = a \vee x = b)$ [presupposing $a \neq b$]'. If it *were* analogous to the latter, then, indeed, we would have for a material object *g*: $w(g) + m(g) = w(g)$, and at the same time: $\neg m(g) = w(g)$ and $\neg m(g) = c(g)$, in other words: we would have a counterexample to A3.)

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7. For reading this, remember that '+' binds more strongly than '=', and '=' more strongly than '¬'.

8. Let 'λ' be the operator of class-abstraction. Thus, $\lambda x A[x]$ is the class of all x such that $A[x]$, which class can also be designated (a tiny bit less concisely) like this: $\{x: A[x]\}$.

Concerning B1: An object is composed of its essence and its being, and its essence is, in turn, composed of its pure (substantial) form and its matter. Aquinas states this explicitly for material objects (see quotation 6). In view of the possibility that the matter of an object is its empty aspect, we can make his statement apply to all objects – *without* incurring any consequences that contradict his doctrine (as will be amply seen below).
Concerning B2: We have already given a justification of this axiom above, in the middle of section III: The composition of essence and being is identical with the composition of actuating form and matter. For this reason, Aquinas sometimes says that a material object is composed of its form (that is, its *actuating form*) and its matter (see for instance quotation 2), and sometimes that there is a double composition in a material object: its essence is composed of its form (that is, its *pure form*) and its matter, and the material object itself is composed of its essence and its being (see quotation 6). Again, the possibility that the matter of an object is its empty aspect allows us to make an insight primarily reached for material objects apply to all objects (without in any way contradicting Thomas).

It must be emphasized that B2 is far from stating the *associativity of* '+'. In fact, the assumption of the associativity of '+' would make TO almost inconsistent: Assume $\neg \mathrm{m}(x) = \mathrm{c}(x)$; hence by B5: $\underline{\neg x = \mathrm{f}(x)}$; and by B6: $\mathrm{m}(x) + \mathrm{s}(x) = \mathrm{c}(x)$; by B1, B2, and the *associativity of* '+': $x = \mathrm{f}(x) + (\mathrm{s}(x) + \mathrm{m}(x))$; hence by $\mathrm{m}(x) + \mathrm{s}(x) = \mathrm{c}(x)$ [in view of A1]: $x = \mathrm{f}(x) + \mathrm{c}(x)$; hence by A2(b): $\underline{x = \mathrm{f}(x)}$ – contradiction. Thus, if the associativity of '+' were assumed, TO would be saved from being inconsistent only by the non-assumption of $\exists x \neg \mathrm{m}(x) = \mathrm{c}(x)$, though assuming this is utterly plausible, considering that it is just about undeniable that some objects are material objects.
Concerning B3: B3 is undeniable under the assumption of the ontology of Thomas Aquinas. Given that ontology, neither the pure form nor the actuating form nor the being of an object is its matter. It will be proved below that no object is its matter and that the essence of no object is the matter of the object.
Concerning B4: This axiom characterizes *the empty aspect of* – an aspect-function Aquinas does not consider – in relation to the other aspect-functions and in relation to objects, in the following manner: The empty aspect of an object and the object itself, or the empty aspect of the object and an aspect of it that is not its matter, are in no case identical. However, B4 does not exclude that the empty aspect of some object is its matter.
Concerning B5: Under the intended interpretation, B5 says that if an object is its pure form, that then it is an immaterial object – which completely agrees with what Aquinas says about objects that are forms.

Concerning B6, B7, B8: Axiom B6 has already been justified above (at the end of section III); it expresses the stipulation there proposed. The axioms B7 and B8 have the same role as B6: the role of completing the definition of the composition-function for cases in which it is *initially* not defined. We have no information as to what Aquinas considered to result by the composition of the essence and the pure form, or the essence and the matter, of a material object; and we have no information as to what Aquinas considered to result by the composition of the actuating form and the pure form, or the actuating form and the being, of an object whose pure form and being are different; hence we must consider the composition-function to be *initially* not defined for these cases. (Concerning the composition of the matter and the being of a material object, we have *positive* evidence that Aquinas regarded it *as impossible*: being can come to matter only via form; see quotation 10.) B6, B7, and B8 may be called "the reduction-axioms", from the important role they play in the reduction of all AEs around a given OD to basic AEs around it or to the OD itself. This reduction, programmatically described in section IV, will be carried out in section X. The uses of B7 and B8 in the logical reconstruction of Thomasic ontology are, however, not exhaustively described by these remarks. The impression of an *ad hoc* character of B7 and B8 will be dispelled as we move on to the proving of theorems.

In view of the intended interpretation of TO relative to Thomasic doctrine, I repeat two definitions (but *now* as part of the formal exposition; concerning the Thomas-interpretational justification of these definitions, see section III):

D1 w(t) := (f(t) + m(t)) (for all ODs t of T)

D2 a(t) := (f(t) + s(t)) (for all ODs t of T)

VIII. The deductive exposition of TO

T1 $\forall x(\mathrm{m}(x) = \mathrm{c}(x) \supset x = \mathrm{a}(x))$
(Every immaterial object is its actuating form)
Proof: Assume $\mathrm{m}(x) = \mathrm{c}(x)$; by B1, $x = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)$; hence $x = (\mathrm{f}(x) + \mathrm{c}(x)) + \mathrm{s}(x)$; by A2(b), $\mathrm{f}(x) + \mathrm{c}(x) = \mathrm{f}(x)$; hence $x = \mathrm{f}(x) + \mathrm{s}(x)$, hence by D2: $x = \mathrm{a}(x)$.

T2 $\forall x(x = \mathrm{a}(x) \supset \mathrm{m}(x) = \mathrm{c}(x))$
(Every object that is its actuating form is immaterial)
Proof: Assume $x = \mathrm{a}(x)$; hence by D2: $x = \mathrm{f}(x) + \mathrm{s}(x)$; by B1, $x = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)$; hence $\mathrm{f}(x) + \mathrm{s}(x) = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)$; by B2, $(\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x) = (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)$; hence $\mathrm{f}(x) + \mathrm{s}(x) = (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)$, and hence $(\mathrm{f}(x) + \mathrm{s}(x))$

+ $m(x) = f(x) + s(x)$; hence by A3: $m(x) = f(x) + s(x) \vee m(x) = c(x)$; by B3(c), $\neg m(x) = f(x) + s(x)$; hence $m(x) = c(x)$.

T3 $\forall x(m(x) = c(x) \supset w(x) = f(x))$
(The essence of an immaterial object is its [pure] form)
Proof: Assume $m(x) = c(x)$; by D1, $w(x) = f(x) + m(x)$; hence $w(x) = f(x) + c(x)$; by A2(b), $f(x) + c(x) = f(x)$; hence $w(x) = f(x)$.

T4 $\forall x(w(x) = f(x) \supset m(x) = c(x))$
(Every object whose essence is its form is an immaterial object)
Proof: Assume $w(x) = f(x)$, hence by D1: $f(x) + m(x) = f(x)$, hence by A3: $m(x) = f(x) \vee m(x) = c(x)$; by B3(a), $\neg m(x) = f(x)$; hence $m(x) = c(x)$.

Concerning T3 and T4, see quotation 1. Concerning T1, consider the following quotation:

11. In his igitur quae non sunt composita ex materia et forma, in quibus individuatio non est per materiam individualem, id est per hanc materiam, sed ipsae formae per se individuantur, oportet quod ipsae formae sint supposita subsistentia. Unde in eis non differt suppositum et natura (*Summa theologiae*, 1, 3, 3).

This evidence for T1 is somewhat vitiated by the fact that Thomas, in the quoted passage, transfers what is valid of actuating form to pure form – which he shouldn't do (if his ontology is to be coherent). The context makes it clear that, in momentary confusion, he *in fact* intends to assert, 'All immaterial objects are their pure forms'. But, by Thomas Aquinas's own lights, it is of course false that all immaterial objects are their pure forms. A *created* immaterial object (an angel, for example) is not its pure form, and consequently – since the essence of an immaterial object is its pure form – it (the created immaterial object) is not its essence. Aquinas, however, deduces from 'All immaterial objects are their pure forms'[9] – with the correct presupposition that the essence of an immaterial object is its pure form – 'All immaterial objects are their essences' ('Unde in eis non differt suppositum et natura'). In other places, Thomas is perfectly clear about the fact (in his ontology) that a created immaterial object is not its pure form – because, since it is created, its being, *and thereby itself*, is distinct from its essence, that is, from its pure form:

9. In the article from which quotation 11 is taken, Thomas quite generally identifies essence – 'essentia vel natura' – and pure form '[quae] comprehendit in se illa tantum quae cadunt in definitione speciei', which is contradicting what he says in other places; see quotation 5.

12. *Secundo modo* invenitur essentia in substantiis creatis intellectualibus, in quibus est aliud esse quam essentia earum, quamvis essentia sit sine materia (*De ente et essentia*, 5, 31).

13. oportet quod in intelligentiis sit esse praeter formam; et ideo dictum est quod intelligentia est forma et esse (*De ente et essentia*, 4, 26).[10]

These quotations imply that a created immaterial object is *properly* composed of its being and its essence, that is, its pure form; hence it is not identical with its pure form. Thus, Thomas by the equivocation in his use of the word 'forma' is led to imagining a proposition valid *relative to pure form* which is not valid relative thereto, but rather is valid *relative to actuating form:* 'In his igitur quae non sunt composita ex materia et forma [...] oportet quod ipsae formae sint supposita subsistentia' (see quotation 11).

T2 says about objects that are their actuating forms what B5 says about objects that are their pure forms: that they are immaterial. If B5 agrees with Thomasic doctrine (and it does), so certainly does T2.

T5 $\forall x(\mathrm{m}(x) = \mathrm{c}(x) \wedge \neg \mathrm{w}(x) = \mathrm{s}(x) \supset \neg x = \mathrm{f}(x))$
(Every immaterial created object is not its pure form)
Proof: Assume $\mathrm{m}(x) = \mathrm{c}(x) \wedge \neg \mathrm{w}(x) = \mathrm{s}(x)$; hence by T3: $\mathrm{w}(x) = \mathrm{f}(x)$; by B1 and D1: $x = \mathrm{w}(x) + \mathrm{s}(x)$; hence $x = \mathrm{f}(x) + \mathrm{s}(x) \wedge \neg \mathrm{f}(x) = \mathrm{s}(x)$. Assume $x = \mathrm{f}(x)$; hence $\mathrm{f}(x) = \mathrm{f}(x) + \mathrm{s}(x)$, and hence $\mathrm{f}(x) + \mathrm{s}(x) = \mathrm{f}(x)$; hence by A3: $\mathrm{s}(x) = \mathrm{f}(x) \vee \mathrm{s}(x) = \mathrm{c}(x)$; hence by B4(c): $\mathrm{s}(x) = \mathrm{f}(x)$ – contradicting $\neg \mathrm{f}(x) = \mathrm{s}(x)$.

T6 $\forall x(\neg x = \mathrm{f}(x) \wedge \mathrm{m}(x) = \mathrm{c}(x) \supset \neg x = \mathrm{w}(x))$
(Every immaterial object that is not its pure form is not its essence)
Proof: Assume $\neg x = \mathrm{f}(x) \wedge \mathrm{m}(x) = \mathrm{c}(x)$; hence by T3: $\mathrm{w}(x) = \mathrm{f}(x)$; hence $\neg x = \mathrm{w}(x)$.

T5 and T6 formally state as provable theorems the Thomasic principles that I have just now (above, in front of quotation 12) made use of.

T7 $\forall x(x = \mathrm{f}(x) \supset x = \mathrm{a}(x) \wedge x = \mathrm{s}(x) \wedge x = \mathrm{w}(x))$
(Every object that is its pure form is its actuating form, its being, and its essence)
Proof: Assume $x = \mathrm{f}(x)$; hence by B5: $\mathrm{m}(x) = \mathrm{c}(x)$; hence by T1: $\underline{x = \mathrm{a}(x)}$; hence by D2: $x = \mathrm{f}(x) + \mathrm{s}(x)$; hence $\mathrm{f}(x) + \mathrm{s}(x) = \mathrm{f}(x)$; hence by A3: $\mathrm{s}(x) = \mathrm{f}(x) \vee \mathrm{s}(x) = \mathrm{c}(x)$; hence by B4(c): $\mathrm{s}(x) = \mathrm{f}(x)$; hence $\underline{x = \mathrm{s}(x)}$; hence by B1: $\mathrm{s}(x) = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)$; hence by A1: $\mathrm{s}(x) + (\mathrm{f}(x) + \mathrm{m}(x)) = \mathrm{s}(x)$; hence by A3: $\mathrm{f}(x) + \mathrm{m}(x) = \mathrm{s}(x) \vee \mathrm{f}(x) + \mathrm{m}(x) = \mathrm{c}(x)$; hence by B4(e) and D1: $\mathrm{w}(x) = \mathrm{s}(x)$; hence $\underline{x = \mathrm{w}(x)}$.

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10. These assertions do not hinder Aquinas from also asserting, in the very same work (and rather close to the location from where quotation 13 is taken): 'intelligentiae quidditas est ipsamet intelligentia, ideo quidditas vel essentia eius est ipsum quod est ipsa' (*De ente et essentia*, 4, 28).

T7 logically contains a (Thomasically valid) principle – namely, $\forall x(x = f(x) \supset x = w(x))$ – which Aquinas may be implicitly using in quotation 11 when obtaining from the (Thomasically) invalid sentence $\forall x(m(x) = c(x) \supset x = f(x))$ the likewise invalid sentence $\forall x(m(x) = c(x) \supset x = w(x))$. Other principles Aquinas may be using in quotation 11 when validly obtaining the said invalid conclusion from the said invalid premise are the following: $\forall x(m(x) = c(x) \supset w(x) = f(x))$ (i.e., T3; this is the most likely candidate) and $\forall x(x = f(x) \wedge m(x) = c(x) \supset x = w(x))$. The last-mentioned principle is, given B5, deductively equivalent with $\forall x(x = f(x) \supset x = w(x))$. Therefore, since that principle (the one first mentioned in the preceding sentence) is very easy to prove, there is an easier way of arriving at $\forall x(x = f(x) \supset x = w(x))$ than the way via the proof of T7:

T8 $\forall x(x = f(x) \wedge m(x) = c(x) \supset x = w(x))$
(Every immaterial object that is its pure form is its essence)
Proof: Assume $x = f(x) \wedge m(x) = c(x)$; hence by T3: $w(x) = f(x)$; hence $x = w(x)$.

T7 shows that objects that are their pure form are, in a certain sense, *simple objects*; we shall have occasion to come back to this.

T9 $\forall x(x = w(x) \supset w(x) = s(x))$
(Every object that is its essence is uncreated)
Proof: Assume $x = w(x)$; by B1 and D1: $x = w(x) + s(x)$; hence $w(x) + s(x) = w(x)$; hence by A3: $s(x) = w(x) \vee s(x) = c(x)$; hence by B4(c): $w(x) = s(x)$.

T10 $\forall x(w(x) = s(x) \supset x = w(x))$
(Every uncreated object is its essence)
Proof: Assume $w(x) = s(x)$; by B1 and D1: $x = w(x) + s(x)$; hence $x = s(x) + s(x)$; hence by A2(a): $x = s(x)$; hence $x = w(x)$.

The proof of T10 already contains the proof of

T11 $\forall x(w(x) = s(x) \supset x = s(x))$
(Every uncreated object is its being)

And we also have

T12 $\forall x(x = s(x) \supset w(x) = s(x))$
(Every object that is its being is uncreated)
Proof: Assume $x = s(x)$; by B1 and D1: $x = w(x) + s(x)$; hence $w(x) + s(x) = s(x)$; hence by A1: $s(x) + w(x) = s(x)$; hence by A3: $w(x) = s(x) \vee w(x) = c(x)$, hence by B4(e) and D1: $w(x) = s(x)$.

We have all the time been reading ‘m(x) = c(x)’ as ‘x is an immaterial object’, and ‘w(x) = s(x)’ as ‘x is an uncreated object’. According to stipulation, the matter of an object is its empty aspect if the object is immaterial; if, on the other hand, the object is material, then, clearly, its matter is not its empty aspect. This justifies reading ‘m(x) = c(x)’ as ‘x is an immaterial object’.

Moreover, according to Aquinas, the totality of objects is divided into the *one* uncreated object, God, and the many created objects. God is the only object whose essence is its being:

14. Hinc est quod Exodi III proprium nomen Dei ponitur esse “QUI EST”: quia eius solius proprium est quod sua substantia non sit aliud quam suum esse (*Summa contra gentiles*, 2, 52).

Consequently, the essence of every uncreated object is its being, and every object whose essence is its being is uncreated. The second statement in the following quotation is logically equivalent to the second conjunct of the preceding (conjunctive) sentence:

15. cuilibet rei creatae suum esse est ei per aliud: alias non esset creatum. Nullius igitur substantiae creatae suum esse est sua substantia (*Summa contra gentiles*, 2, 52).[11]

These considerations justify the reading of ‘w(x) = s(x)’ as ‘x is an uncreated object’.

To make the two readings which have just been justified ‘official’ (i.e., formally accepted), and in order to have convenient building blocks for further predicate-formations in TO, I introduce the following two definitions:

D3 $\quad$ M(t) := ¬m(t) = c(t) (for all ODs t of T)

D4 $\quad$ C(t) := ¬w(t) = s(t)

With the help of the predicates M(t) and C(t), the four principal Thomasic categories of objects can be defined:

D5 $\quad$ D(t) := ¬M(t) ∧ ¬C(t)

11. Here the word “substantia” is used in two senses in one sentence. But this is harmless (it does not lead to misunderstandings), since the relational character of the second use of “substantia”, and the lack of that character in the first use, is quite explicit. Aquinas, presumably, was aware of the difference in meaning, and, quite rightly, thought it tolerable.

D6 $\mathrm{I(t)} := \neg\mathrm{M(t)} \wedge \mathrm{C(t)}$

D7 $\mathrm{E(t)} := \mathrm{M(t)} \wedge \neg\mathrm{C(t)}$

D8 $\mathrm{B(t)} := \mathrm{M(t)} \wedge \mathrm{C(t)}$

According to the teaching of Thomas Aquinas, the third category (given by D7) is empty. There are no material uncreated objects (for example, *elementa* in the sense of the Pre-Socratics, having a quasi-divine character):

16. Per hoc autem [quod omnia quae sunt a Deo sunt] excluditur antiquorum Naturalium error, qui ponebant corpora quaedam non habere causam essendi. Et etiam quorumdam qui dicunt Deum non esse causam substantiae caeli, sed solum motus (*Summa contra gentiles*, 2, 15).

In TO we can prove

T13 $\neg\exists x \mathrm{E}(x)$

(There are no material uncreated objects)

Proof: By D7, $\neg\exists x\mathrm{E}(x)$ is equivalent to $\neg\exists x(\mathrm{M}(x) \wedge \neg\mathrm{C}(x))$, hence to $\forall x(\mathrm{M}(x) \supset \mathrm{C}(x))$, which is equivalent, by D3 and D4, to $\forall x(\neg\mathrm{m}(x) = \mathrm{c}(x) \supset \neg\mathrm{w}(x) = \mathrm{s}(x))$, hence to $\forall x(\mathrm{w}(x) = \mathrm{s}(x) \supset \mathrm{m}(x) = \mathrm{c}(x))$ [Every uncreated object is immaterial]. The latter can be proved as follows: Assume $\mathrm{w}(x) = \mathrm{s}(x)$; hence $\mathrm{s}(x) + \mathrm{f}(x) = \mathrm{w}(x) + \mathrm{f}(x)$; hence by D1: $\mathrm{s}(x) + \mathrm{f}(x) = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x)$; by B4(d): $\neg\mathrm{f}(x) + \mathrm{s}(x) = \mathrm{c}(x)$; hence by A1: $\neg\mathrm{s}(x) + \mathrm{f}(x) = \mathrm{c}(x)$; hence $\neg(\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x) = \mathrm{c}(x)$; hence by B7: $\mathrm{m}(x) = \mathrm{c}(x)$.

From quotation 14 we may gather: *If the being of x is caused by an object other than x, then the being of x is different from the essence of x.*[12] The converse – *If the being of x is different from the essence of x, then the being of x is caused by an object other than x* – is also valid, according to Aquinas. As evidence for this, one can point to quotation 15, or to the following quotation:

17. oportet quod omnis talis res, cuius [esse] est aliud quam natura sua, habeat esse ab alio (*De ente et essentia*, 4, 27; see also *Summa theologiae*, 1, 3, 4).

Consequently, we can read '$\mathrm{C}(x)$' – that is, '$\neg\mathrm{w}(x) = \mathrm{s}(x)$' – also as 'the being of x is caused by an object other than x', and '$\neg\mathrm{C}(x)$' – which is logically

12. I argue via contraposition: Suppose the being of x is identical with the essence of x; hence – in view of quotation 14 – x is identical with God; hence the being of x is not caused by an object other than x (else, the being of God would be caused by an object other than God – which is absurd). Therefore: If the being of x is caused by an object other than x, then the being of x is different from the essence of x.

equivalent to ‘w(x) = s(x)’ – also as ‘the being of x is not caused by an object other than x’ – which for Thomas is equivalent to ‘the being of x is not caused by *any* object’, since self-causation is, according to him, impossible:

18. nec tamen invenitur, nec est possibile, quod aliquid sit causa efficiens sui ipsius; quia sic esset prius seipso, quod est impossibile (*Summa theologiae*, 1, 2, 3).

19. Non autem potest esse quod ipsum esse sit causatum ab ipsa forma vel quidditate rei, dico sicut a causa efficiente; quia sic aliqua res esset causa sui ipsius, et aliqua res seipsam in esse produceret, quod est impossibile (*De ente et essentia*, 4, 27).

From T13 we can easily deduce:

T14 $\forall x(M(x) \equiv B(x))$
(The material objects are the created bodies)
Proof: From T13 by use of D7: $\forall x(M(x) \supset C(x))$; hence $\forall x(M(x) \equiv M(x) \wedge C(x))$; hence by D8: $\forall x(M(x) \equiv B(x))$.

T15 $\forall x(\neg C(x) \equiv D(x))$
(The uncreated – uncaused – objects are the divine objects)
Proof: From T13 (using D7): $\forall x(\neg C(x) \supset \neg M(x))$; hence $\forall x(\neg C(x) \equiv \neg M(x) \wedge \neg C(x))$; hence by D5: $\forall x(\neg C(x) \equiv D(x))$.

D5 mirrors the Thomasic conception of divinity: a divine object is an uncreated (uncaused) immaterial object. This conception is the Judaeo-Christian conception of divinity with an Aristotelian touch; this touch is emphasized by the fact that ‘uncreated object’ is taken to mean as much as ‘object whose essence is its being’.

It does not follow from Thomas’s theory of object-composition *alone* that there are divine objects, or that there are (created) *intelligences* (‘substantiae creatae intellectuales [immateriales]’, ‘intelligentiae’), or indeed that there are (created) bodies (‘corpora’). Accordingly, neither $\exists xD(x)$ nor $\exists xI(x)$ nor $\exists xB(x)$ (nor their negations) can be proved in TO, although the truth not only of $\exists xB(x)$ but also of $\exists xD(x)$ and $\exists xI(x)$ (under the given interpretation of T) was, of course, indubitable for Aquinas.

In the ontological doctrines here under consideration, Aquinas does not consider so-called *abstract objects,* numbers, for example, or geometrical figures (which entities one might think of subsuming under category I(x)); hence they are not included in the universe of discourse. There is also a more substantial justification for their exclusion:

20. corpus mathematicum non est per se existens, ut Philosophus probat (*Summa contra gentiles*, 1, 20).

Clearly, ‘corpora mathematica’ – and certainly *all* abstract ‘objects’ – are not objects in the full sense (of substance or at least quasi-substance) for Aquinas.

Further on in this treatise, extensions of TO will be presented in which $\exists x\mathrm{I}(x)$ and $\exists x\mathrm{D}(x)$ are provable. But TO *itself* is very weak in its existential assumptions; not even the entirely unproblematic assertion $\exists x\mathrm{M}(x)$ can be deduced in it. However, TO, while entailing no further existential commitments than that there is at least one object, entails (under the intended interpretation of T) that there are no material uncreated (uncaused) objects (as demonstrated). Here TO is following Aquinas perfectly.

The second part of the proof of T13 can be rephrased in the following manner: By B4(d) and A1: $\forall x\neg \mathrm{s}(x) + \mathrm{f}(x) = \mathrm{c}(x)$ [1]; hence $\forall x(\mathrm{w}(x) = \mathrm{s}(x) \supset \neg\mathrm{w}(x) + \mathrm{f}(x) = \mathrm{c}(x))$ [2]; by B7, D1: $\forall x(\neg\mathrm{m}(x) = \mathrm{c}(x) \supset \mathrm{w}(x) + \mathrm{f}(x) = \mathrm{c}(x))$ [3]; hence $\forall x(\neg\mathrm{m}(x) = \mathrm{c}(x) \supset \neg\mathrm{w}(x) = \mathrm{s}(x))$ [4], and hence $\forall x(\mathrm{w}(x) = \mathrm{s}(x) \supset \mathrm{m}(x) = \mathrm{c}(x))$ [5]. In this way, it becomes easier to bring out the intuitive ideas behind the proof: The pure form (just as the essence) of an object enters into composition with the being of the object (that is, their combination is not the object’s empty aspect) [1]; here one may cite

21. esse est actualitas omnis formae vel naturae (*Summa theologiae*, 1, 3, 4).

Hence, if the essence of an object is identical with the being of the object, then its pure form enters into composition with its essence [2]. But the pure form of a material object does not enter into composition with its essence [3]; *there is nothing in a material object that is constituted by combining its essence and its pure form*; hence the essence of a material object is different from its being [4], and therefore an object whose essence is its being is immaterial [5].

IX. Divine objects and their simplicity

By D5, T11, and T10, theorems about divine objects can be deduced that correspond to Thomas’s dicta about God:

T16 (a) $\forall x(\mathrm{D}(x) \supset \neg\mathrm{M}(x))$ – 22. Deus non est corpus (*Summa contra gentiles*, 1, 20).
(b) $\forall x(\mathrm{D}(x) \supset \mathrm{w}(x) = \mathrm{s}(x))$ – 23. in Deo non est aliud essentia vel quidditas quam suum esse (*Summa contra gentiles*, 1, 22).
(c) $\forall x(\mathrm{D}(x) \supset x = \mathrm{w}(x))$ – 24. Deus est sua essentia (*Summa contra gentiles*, 1, 21).
(d) $\forall x(\mathrm{D}(x) \supset x = \mathrm{s}(x))$ – 25. Deus non solum est sua essentia, ut ostensum est, sed etiam suum esse (*Summa theologiae*, 1, 3, 4).

By T13 and T9 we obtain

T17 $\forall x(\mathrm{M}(x) \supset \neg x = \mathrm{w}(x))$.

And Aquinas says accordingly:

26. in rebus compositis ex materia et forma, necesse est quod differant natura vel essentia et suppositum [seu res] (*Summa theologiae*, 1, 3, 3).

Proceeding in the reverse order of formal theorem and exegetical justification, we first find Aquinas saying the following:

27. Si autem sint aliquae formae creatae non receptae in materia, sed per se subsistentes, ut quidam de angelis opinantur, erunt quidem infinitae secundum quid, inquantum huiusmodi formae non terminantur neque contrahuntur per aliquam materiam: sed quia forma creata sic subsistens habet esse, et non est suum esse, necesse est quod ipsum eius esse sit receptum et contractum ad determinatam naturam (*Summa theologiae*, 1, 7, 2).

And in accordance with this quotation we then have:

T18 (a) $\forall x(\mathrm{I}(x) \supset x = \mathrm{a}(x))$ (by D6, D3, and T1)
(b) $\forall x(\mathrm{I}(x) \supset \neg x = \mathrm{s}(x))$ (by D6, D4, and T12)
(c) $\forall x(\mathrm{I}(x) \supset x = \mathrm{s}(x) + \mathrm{f}(x))$ (by T18(a), D2, A1)

All of this amply shows that the theorems and definitions of TO mirror Thomasic doctrine.

In *Summa theologiae*, 1, 3, 3, Aquinas deduces (in his own words, of course) T16(c) from the *invalid* sentence $\forall x(\mathrm{m}(x) = \mathrm{c}(x) \supset x = \mathrm{f}(x))$ – 'Every immaterial object is its pure form' (S) (which I already had occasion to remark upon). He does so with the help of the principles $\forall x(\mathrm{m}(x) = \mathrm{c}(x) \supset \mathrm{w}(x) = \mathrm{f}(x))$ (T3) and $\forall x(\mathrm{D}(x) \supset \mathrm{m}(x) = \mathrm{c}(x))$ (T16(a)). From S and (implicit) T3, he first gets the (*invalid*) sentence $\forall x(\mathrm{m}(x) = \mathrm{c}(x) \supset x = \mathrm{w}(x))$ – see quotation 11; and then, in immediate continuation of quotation 11, he writes:

28. Et sic, cum Deus non sit compositus ex materia et forma, ut ostensum est [T16(a)], oportet quod Deus sit sua deitas [id est, sua essentia], sua vita, et quidquid aliud sic de Deo praedicatur.

Thus, starting from an invalid premise, Thomas obtains a (Thomasically) valid conclusion. The partly invalid premises made use of in *Summa theologiae,* 1, 3, 3, to obtain $\forall x(\mathrm{D}(x) \supset x = \mathrm{w}(x))$ can also be use d to obtain

$\forall x(D(x) \supset x = f(x))$: the latter is an immediate consequence of S and T16(a). That sentence ('Every divine object is its pure form'), too, is valid in spite of the invalid premise from which it is derived; it is just as valid as $\forall x(D(x) \supset x = a(x))$ ('Every divine object is its actuating form'), which one gets from T16(a) by T1. The Thomasic validity of the two sentences emerges from the following quotation:

29. unumquodque agens agit per suam formam: unde secundum quod aliquid se habet ad suam formam, sic se habet ad hoc quod sit agens. Quod igitur primum est et per se agens, oportet quod sit primo et per se forma. Deus autem est primum agens, cum sit prima causa efficiens, ut ostensum est. Est igitur per essentiam suam forma; et non compositus ex materia et forma (*Summa theologiae*, 1, 3, 2).

In this quotation, Aquinas certainly did not intend to refer to pure form rather than to actuating form, or vice versa, since he did not distinguish between them. Indeed, *with respect to divine objects*, Aquinas is quite right in this non-distinction (but *not* with respect to all objects); for the actuating form and the pure form of a divine object are provably identical (see T19 below). Thus, quotation 29 can be taken to provide Thomasic evidence for $\forall x(D(x) \supset x = f(x))$ as well as for $\forall x(D(x) \supset x = a(x))$, since these sentences are provably equivalent (on the basis of T19). (And therefore, since $\forall x(D(x) \supset x = a(x))$ is provable, as has already been shown, $\forall x(D(x) \supset x = f(x))$ is also provable – *without* using any invalid premise.) We have:

T19 $\forall x(D(x) \supset a(x) = f(x))$
Proof: Assume $D(x)$, hence by D5: $\neg M(x) \wedge \neg C(x)$; hence by D3 and D4: $m(x) = c(x) \wedge w(x) = s(x)$; hence by D1: $m(x) = c(x) \wedge f(x) + m(x) = s(x)$; hence $f(x) + c(x) = s(x)$; hence by A2(b): $f(x) = s(x)$; hence $f(x) + s(x) = f(x) + f(x)$; hence by D2 and A2(a): $a(x) = f(x)$.

We can also prove the converse of T19:

T20 $\forall x(a(x) = f(x) \supset D(x))$
(If the actuating form of an object is its pure form, then the object is divine)
Proof: Assume $a(x) = f(x)$; hence by D2: $f(x) + s(x) = f(x)$; hence by A3: $s(x) = f(x) \vee s(x) = c(x)$; hence by B4(c): $s(x) = f(x)$; by B1: $x = (f(x) + m(x)) + s(x)$; hence by B2: $x = (f(x) + s(x)) + m(x)$; hence $x = (f(x) + f(x)) + m(x)$; hence by A2(a): $x = f(x) + m(x)$; hence by D1: $x = w(x)$; hence by T9: $w(x) = s(x)$; hence by D4: $\neg C(x)$; hence by T15: $D(x)$.

T19 and T20 make precise what is meant by 'the pure form of an object is *normally distinct* from its actuating form' (cf. section III, the paragraph before quotation 2). The pure form of an object is distinct from its actuating form if and only if that object is not a divine object – which, certainly, is normally the case.

There are many equivalence statements regarding the predicate D(x) that are provable in TO (beside the equivalence theorem T15 and the trivial definitional equivalences):

T21 (a) $\forall x(D(x) \equiv a(x) = f(x))$ (by T19, T20)
(b) $\forall x(D(x) \equiv x = w(x))$ (by T15, D4, T9, T10)
(c) $\forall x(D(x) \equiv x = s(x))$ (by T15, D4, T11, T12)
(d) $\forall x(D(x) \equiv s(x) = f(x))$
Proof: From $a(x) = f(x)$: $\underline{s(x) = f(x)}$ (see the proof of T20); from $\underline{s(x) = f(x)}$: $a(x) = f(x)$ (see the proof of T19); hence by T21(a): T21(d) – what was to be proven.
(e) $\forall x(D(x) \equiv x = f(x))$
Proof: From D(x): $x = s(x) \wedge s(x) = f(x)$, by T21(c) and T21(d); hence $\underline{x = f(x)}$; from $\underline{x = f(x)}$: $x = s(x)$, by T7; hence by T21(c): D(x).
(f) $\forall x(D(x) \equiv s(x) = a(x))$
Proof: From D(x): $a(x) = f(x) \wedge s(x) = f(x)$, by T21(a) and T21(d); hence $\underline{s(x) = a(x)}$; from $\underline{s(x) = a(x)}$: $s(x) = f(x) + s(x)$, by D2; hence by A1: $s(x) + f(x) = s(x)$; hence by A3: $f(x) = s(x) \vee f(x) = c(x)$; hence by B4(b): $f(x) = s(x)$; hence by T21(d): D(x).
(g) $\forall x(D(x) \equiv w(x) = a(x))$
Proof: From D(x): $x = f(x) \wedge x = w(x) \wedge a(x) = f(x)$, by T21(e), T21(b), and T21(a); hence $\underline{w(x) = a(x)}$; from $\underline{w(x) = a(x)}$: $f(x) + m(x) = f(x) + s(x)$, by D1 and D2; hence by B1: $x = (f(x) + s(x)) + s(x)$; hence by B4(a): $\neg(f(x) + s(x)) + s(x) = c(x)$; hence by B8: $f(x) = s(x)$; hence by T21(d): D(x).

We have a much shorter sequence of in TO provable equivalence statements regarding the predicate ¬M(x):

T22 (a) $\forall x(\neg M(x) \equiv x = a(x))$ (by T1, T2, D3)
(b) $\forall x(\neg M(x) \equiv w(x) = f(x))$ (by T3, T4, D3)

It is interesting to compare T22(a) with T21(e), and T22(b) with T21(g), and the two pairs with each other. In the pair T22(b) and T21(g) the role of ‘f(x)’ and ‘a(x)’ is inverse to the role of ‘f(x)’ and ‘a(x)’ in the pair T22(a), T21(e).

From T21 and T22 together, the radical simplicity of a divine object can be deduced. An object is said to be *radically simple* if and only if every (fundamental) aspect of it that is different from its empty aspect is identical with the object itself, or in other words: if it has no proper (fundamental) components. The decisive theorem is this:

T23 $\forall x(D(x) \supset x = f(x) \land x = w(x) \land x = s(x) \land x = a(x))$
(A divine object is radically simple)
Proof: Assume $D(x)$; hence $\neg M(x)$, by D5; hence $x = f(x) \land x = w(x) \land x = s(x) \land x = a(x)$, by T21(e), (b), (c) from $D(x)$ [concerning the first three conjuncts], and by T22(a) from $\neg M(x)$ [concerning the fourth and last conjunct].

It can easily be seen that the converse of T23 is also provable. *The reading* given to T23 (see the sentence immediately below it) can be justified as follows: Suppose x is a divine object: $D(x)$, and $\varphi[x]$ is an aspect of x that is different from $c(x)$; hence $\varphi[x]$ is different from $m(x)$ (since $m(x) = c(x)$ because of $\neg M(x)$ – which follows from $D(x)$). Then by T28 – proven below – $\varphi[x]$ is identical to x, or $f(x)$, or $w(x)$, or $s(x)$, or $a(x)$. In each of these cases, $\varphi[x]$ is identical to x (making use of T23 for the four cases other than the first case). Therefore: Every aspect of x[13] that is different from $c(x)$ is identical with x, that is: x is radically simple.

T23 corresponds to the Thomasic doctrine of the total simplicity of God:

30. quod Deum omnino esse simplicem, multipliciter potest esse manifestum. Primo quidem per supradicta. Cum enim in Deo non sit compositio, neque quantitativarum partium, quia corpus non est; neque compositio formae et materiae: neque in eo sit aliud natura et suppositum; neque aliud essentia et esse […] manifestum est quod Deus nullo modo compositus est, sed est omnino simplex. [...] Unde, cum Deus sit ipsa forma, vel potius ipsum esse, nullo modo compositus esse potest (*Summa theologiae*, 1, 3, 7).

The degree of composition of an object is the number of its proper components, that is: the number of its (fundamental) aspects that are different from its empty aspect and different from the object itself. The degree of composition of a divine object is, evidently, *zero*.

An object is said to be *radically composite* if its degree of composition is maximal. Material objects (that is, created bodies, according to T14) are radically composite, as we shall see. We first prove the following two theorems:

T24 $\forall x \neg x = m(x)$
(No object is its matter)

13. Note that every aspect of x is designated by some AE of T around "x". There are no aspects of x that are not designated by some AE of T around "x". This follows from the intended interpretation of T.

Proof: Assume $x = \mathrm{m}(x)$; by B1: $x = (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)$; hence by B2: $x = (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)$; hence $\mathrm{m}(x) = (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)$; hence by A1: $\mathrm{m}(x) + (\mathrm{f}(x) + \mathrm{s}(x)) = \mathrm{m}(x)$; hence by A3: $\mathrm{f}(x) + \mathrm{s}(x) = \mathrm{m}(x) \lor \mathrm{f}(x) + \mathrm{s}(x) = \mathrm{c}(x)$ – which contradicts the conjunction of B3(c) and B4(d).

Aquinas says:

31. Esse autem non dicitur de materia, sed de toto; unde materia non potest dici quod est, sed ipsa substantia [tota res] est id quod est (*Summa contra gentiles*, 2, 54).[14]

T25 $\forall x \neg \mathrm{w}(x) = \mathrm{m}(x)$
(The essence of no object is its matter)
Proof: Assume $\mathrm{w}(x) = \mathrm{m}(x)$; hence by D1: $\mathrm{f}(x) + \mathrm{m}(x) = \mathrm{m}(x)$; hence by A1: $\mathrm{m}(x) + \mathrm{f}(x) = \mathrm{m}(x)$; hence by A3: $\mathrm{f}(x) = \mathrm{m}(x) \lor \mathrm{f}(x) = \mathrm{c}(x)$, which contradicts the conjunction of B3(a) and B4(b).

And Aquinas says:

32. materia non est ipsa substantia rei, nam sequeretur omnes formas esse accidentia, sicut antiqui Naturales opinabantur: sed materia est pars substantiae (*Summa contra gentiles*, 2, 54).

33. Quod enim materia sola rei non sit essentia, planum est, quia res per essentiam suam cognoscibilis est, et in specie ordinatur vel in genere; sed materia neque cognitionis principium [est], neque secundum eam aliquid ad genus vel speciem determinatur, sed secundum id quo [quod?] aliquid actu est (*De ente et essentia*, 2, 5).

We then have:

T26 (a) $\forall x(\mathrm{M}(x) \supset \neg x = \mathrm{m}(x) \land \neg x = \mathrm{f}(x) \land \neg x = \mathrm{w}(x) \land \neg x = \mathrm{s}(x) \land \neg x = \mathrm{a}(x))$

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14. The phrase "[id] *quod est*" can be taken to designate the entire object, i.e., the individual substance or quasi-substance; this is how the phrase is to be understood in quotation 31. It can also be taken to designate merely the *essence* (the "substantia", in another sense) of the object; this is how it is to be understood in quotation 6. The phrase "[id] *quo est*", in contrast, can be taken to designate the *being* ("esse") of the object; this is how it is to be understood in quotation 6. It can also be taken to designate the *actuating form* of the object; this is perhaps how "id quo aliquid actu est" should be understood in quotation 33 – if "id quo aliquid actu est" has not been put erroneously for "id *quod* aliquid actu est", i.e., for "essentiam"; that such a slip occurred seems not unlikely, given the context: Thomas is certainly speaking, in the greater part of quotation 33, about the *essences* of objects, justifying that an object's essence is not its matter. In any case, the determination of species that the actuating form of an object provides is not perfect (see T55, T57, and T58 in section XXII), whereas the determination of species that is provided by the essence of an object *is* perfect (see T56).

(b) $\forall x(M(x) \supset \neg m(x) = f(x) \wedge \neg m(x) = w(x) \wedge \neg m(x) = s(x) \wedge \neg m(x) = a(x) \wedge \neg f(x) = w(x) \wedge \neg f(x) = s(x) \wedge \neg f(x) = a(x) \wedge \neg w(x) = s(x) \wedge \neg w(x) = a(x) \wedge \neg s(x) = a(x))$
(c) $\forall x(M(x) \supset \neg c(x) = m(x) \wedge \neg c(x) = f(x) \wedge \neg c(x) = w(x) \wedge \neg c(x) = s(x) \wedge \neg c(x) = a(x))$
Proof:
(a) Assume M(x); by T24: $\neg x$ = m(x); and, because of M(x), by B5, D3: $\neg x$ = f(x); and by T21(b), Tl6(a): $\neg x$ = w(x); and by T21(c), Tl6(a): $\neg x$ = s(x); and by T2, D3: $\neg x$ = a(x).
(b) Assume M(x); by B3(a): ¬m(x) = f(x); by T25: ¬m(x) = w(x); by B3(b): ¬m(x) = s(x); by B3(c), D2: ¬m(x) = a(x); and, because of M(x), by T4, D3: ¬f(x) = w(x); and by T21(d), Tl6(a): ¬f(x) = s(x); and by T21(a), T16(a): ¬f(x) = a(x); and by T15, T16(a), D4: ¬w(x) = s(x); and by T21(g), T16(a): ¬w(x) = a(x); and by T21(f), Tl6(a): ¬s(x) = a(x).
(c) Assume M(x); hence by D3: ¬c(x) = m(x); the other non-identities in the consequent of T26(c) follow by B4 alone.

It is apparent from T26 that a material object has at least five proper components (that is: aspects that are different from the object's empty aspect and from the objects itself). And there cannot be more than five proper components in an object (see T28); hence the degree of composition of a material object is maximal, and it is *radically composite*.

The intelligences, in turn, are neither radically simple nor radically composite. While the degree of composition of divine objects is *zero*, and that of material objects *five*, the degree of composition of created immaterial substances is *two*:

T27 (a) $\forall x(I(x) \supset \neg x = m(x) \wedge \neg x = f(x) \wedge \neg x = w(x) \wedge \neg x = s(x) \wedge x = a(x))$
(b) $\forall x(I(x) \supset f(x) = w(x) \wedge \neg f(x) = s(x))$
(c) $\forall x(I(x) \supset c(x) = m(x) \wedge \neg c(x) = f(x) \wedge \neg c(x) = w(x) \wedge \neg c(x) = s(x) \wedge \neg c(x) = a(x))$
Proof:
(a) Assume I(x); hence by D6: ¬M(x) ∧ C(x); hence by T15, T21(e): $\neg x$ = f(x); and by T15, T21(b): $\neg x$ = w(x); hence by Tl8(b): $\neg x$ = s(x); and by Tl8(a): x = a(x).
(b) Assume I(x); hence by D6: ¬M(x) ∧ C(x); and by T3, D3: f(x) = w(x); and by T15, T21(d): ¬f(x) = s(x).
(c) Assume I(x); hence by D6, D3: c(x) = m(x); the other non-identities in the consequent of T27(c) follow by B4 alone.

In view of T27, f(x), w(x), and s(x) are proper components of x – assuming I(x); and they are the only proper components of x (in view of T28; if φ[x] is m(x), then it is not a proper component of x, since m(x) = c(x); if φ[x] is a(x), then it is not a proper component of x, since a(x) = x). Of the aspects f(x), w(x), and s(x), only two are distinct (by T27(b)). Thus, the degree of composition of x is *two*.

Occasionally Aquinas calls intelligences as well as God 'substantiae simplices' (see quotation 1). However, in doing so, he has in mind an extended sense of 'simplex' (*not* radical simplicity):

34. Non est autem opinandum quod, quamvis substantiae intellectuales non sint corporeae, nec ex materia et forma compositae, nec in materia existentes sicut formae materiales, quod propter hoc divinae simplicitati adaequentur (*Summa contra gentiles*, 2, 52; consider, in this connection, quotations 12 and 13).

X. The Reduction Theorem and its proof

This section contains the formulation and the proof of the *Reduction Theorem* (for TO), and the formulations and the proofs of some corollaries of that theorem.

Definition: An AE α of T is in TO reducible to the EDs $\beta_1, \ldots, \beta_n$ of T if and only if $\alpha = \beta_1 \lor \ldots \lor \alpha = \beta_n$ is provable in TO.

Reduction Theorem: If t is an OD of T and φ[t] is an AE of T, then φ[t] is reducible in TO to t, f(t), m(t), s(t), f(t) + m(t), f(t) + s(t), c(t), or in short: to RS[t] (designating the sequence just described).

Proof:

Let t be an OD of T; there are four PAEs of T around t: f(t), m(t), s(t), and c(t); with these 16 SAEs of T around t can be formed, and 128 TAEs of T around t; there are no other AEs of T around t.

Because of A1, 6 of the 16 SAEs of T around t are reducible in TO to their respective commutation (for example, f(t) + m(t) is the commutation of m(t) + f(t), and vice versa); hence every SAE of T around t is reducible in TO to RS[t] *if* the remaining 10 SAEs of T around t are reducible in TO to RS[t]; let the remaining 10 be these:

(i) c(t) + c(t), m(t) + m(t), f(t) + f(t), s(t) + s(t);
(ii) m(t) + c(t), f(t) + c(t), s(t) + c(t);
(iii) m(t) + s(t);
(iv) f(t) + s(t), f(t) + m(t);

Because of A2(a), each AE in row (i) is reducible in TO to a PAE of T around t – and therefore to RS[t].

Because of A2(b), each AE in row (ii) is reducible in TO to a PAE of T around t – and therefore to RS[t].

Because of B6, A1, A2(b), the AE in row (iii) is reducible in TO to: c(t), s(t) – and therefore to RS[t].

Every AE in row (iv) is trivially reducible in TO to RS[t].

We have now established *Lemma 1: Every* SAE *of* T *around* t *is reducible in* TO *to* RS[t].

Because of A1, 64 of the TAEs of T around t are reducible in TO to their respective commutations – for example, in such a manner that, in each case, the relevant commutation has the form $(\varphi[t] + \varphi'[t]) + \varphi''[t]$; hence every TAE of T around t is reducible in TO to RS[t] *if* the remaining 64 TAEs around t are reducible in TO to RS[t].

For these remaining TAEs, each having the form $(\varphi[t] + \varphi'[t]) + \varphi''[t]$, we obtain:

(*i*) If $\varphi[t] + \varphi'[t]$ is $\alpha(t) + \alpha(t)$, then the TAE is reducible in TO to a SAE of T around t by A2(a) – and therefore to RS[t] by *Lemma 1*.

(*ii*) If $\varphi[t] + \varphi'[t]$ is $\alpha(t) + c(t)$ or $c(t) + \alpha(t)$, then the TAE is reducible in TO to a SAE of T around t by A2(b) and Al – and therefore to RS[t] by *Lemma 1*.

(*iii*) If $\varphi[t] + \varphi'[t]$ is $f(t) + m(t)$ or $m(t) + f(t)$;
then, if $\varphi''[t]$ is $s(t)$, the TAE is reducible in TO to t by B1 and A1 – and therefore to RS[t];
then, if $\varphi''[t]$ is $c(t)$, the TAE is reducible in TO to $f(t) + m(t)$ by A2(b) and Al – and therefore to RS[t];
then, if $\varphi''[t]$ is $f(t)$, the TAE is reducible in TO to: $c(t)$, $f(t)$ – by B7, A1, A2[15] – and therefore to RS[t];
then, if $\varphi''[t]$ is $m(t)$, the TAE is reducible in TO to: $c(t)$, $f(t)$ – by B7, Al, A2(b) – and therefore to RS[t].

(*iv*) If $\varphi[t] + \varphi'[t]$ is $m(t) + s(t)$ or $s(t) + m(t)$;
then, if $\varphi''[t]$ is $s(t)$, the TAE is reducible in TO to $s(t)$ by B6, A1, A2[16] – and therefore to RS[t];
then, if $\varphi''[t]$ is $c(t)$, the TAE is reducible in TO to a SAE of T around t by A2(b) – and therefore to RS[t] by *Lemma 1*;
then, if $\varphi''[t]$ is $f(t)$, the TAE is reducible in TO to: $f(t)$, $f(t) + s(t)$ – by B6, A2(b), A1 – and therefore to RS[t];
then, if $\varphi''[t]$ is $m(t)$, the TAE is reducible in TO to: $m(t)$, $s(t)$ – by B6, A2(b), A1 – and therefore to RS[t].

(*v*) If $\varphi[t] + \varphi'[t]$ is $f(t) + s(t)$ or $s(t) + f(t)$;
then, if $\varphi''[t]$ is $s(t)$, the TAE is reducible in TO to: $c(t)$, $f(t)$ – by B8, A2(a), A1 – and therefore to RS[t];
then, if $\varphi''[t]$ is $c(t)$, the TAE is reducible in TO to $f(t) + s(t)$ by A2(b) and A1 – and therefore to RS[t];

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15. One needs both A2(a) and A2(b).

16. This is perhaps not so easily seen. There are two cases: $m(t) = c(t)$ and $\neg m(t) = c(t)$. In the second case, $m(t) + s(t) = c(t)$ by B6; and therefore: $s(t) + m(t) = c(t)$ by A1; hence $(m(t) + s(t)) + s(t) = s(t)$ and $(s(t) + m(t)) + s(t) = s(t)$ by A1 and A2(b). In the first case, $m(t) + s(t) = s(t)$ by A1 and A2(b); and $s(t) + m(t) = s(t)$ by A2(b); hence $(m(t) + s(t)) + s(t) = s(t)$ and $(s(t) + m(t)) + s(t) = s(t)$ by A2(a).

then, if φ”[t] is f(t), the TAE is reducible in TO to: c(t), f(t) – by B8, A2(a), A1 – and therefore to RS[t];
then, if φ”[t] is m(t), the TAE is reducible in TO to t by B2, B1, A1 – and therefore to RS[t].

We have now established *Lemma 2: Every* TAE *of* T *around* t *is reducible in* TO *to* RS[t].

Since every PAE of T around t is trivially reducible in TO to RS[t], and since every AE of T around t is either a PAE or a SAE or a TAE around t, we obtain by *Lemma 1* and *Lemma 2: Every* AE of T *around* t *is reducible in* TO *to* RS[t]. This result establishes the *Reduction Theorem*.

The *Reduction Theorem* entails (making use of D1 and D2) the following theorem (in which φ[x] is any AE of T around x):

T28 $\forall x(\varphi[x] = x \lor \varphi[x] = \mathrm{f}(x) \lor \varphi[x] = \mathrm{m}(x) \lor \varphi[x] = \mathrm{s}(x) \lor \varphi[x] = \mathrm{w}(x) \lor \varphi[x] = \mathrm{a}(x) \lor \varphi[x] = \mathrm{c}(x))$

T28 is logically equivalent to $\forall x(\neg\varphi[x] = x \land \neg\varphi[x] = \mathrm{c}(x) \supset \varphi[x] = \mathrm{f}(x) \lor \varphi[x]= \mathrm{m}(x) \lor \varphi[x] = \mathrm{s}(x) \lor \varphi[x] = \mathrm{w}(x) \lor \varphi[x] = \mathrm{a}(x))$, which says (straightforwardly) that there are *at most* five proper components in an object, namely, f(x), m(x), s(x), w(x), and a(x).

In the wake of the *Reduction Theorem*, two further observations – with some interest – come to mind:

First

A PSL of T is called ‘undecided in TO’ if and only if neither the PSL itself nor its negation is provable in TO. It can easily be shown that of the PSLs of T which can be formed by using only the material listed in RS[t] (that is: t, f(t), m(t), s(t), f(t) + m(t), f(t) + s(t), c(t); cf. the *Reduction Theorem*) at most (and very probably *exactly*) the following eleven PSLs are undecided in TO (and, of course, PSLs that are equivalent to them by A1 and/or the symmetry of identity): m(t) = c(t), f(t) = t, f(t) = s(t), f(t) = f(t) + m(t), f(t) = f(t) + s(t), s(t) = t, s(t) = f(t) + m(t), s(t) = f(t) + s(t), f(t) + s(t) = t, f(t) + m(t) = t, f(t) + s(t) = f(t) + m(t). Based on the proven theorems of TO, these PSLs can be grouped in two equivalence-lists:

Divinity	*Immateriality*
f(t) = t	
s(t) = t	
f(t) = s(t)	m(t) = c(t)
f(t) + m(t) = t	f(t) + s(t) = t
f(t) = f(t) + s(t)	f(t) = f(t) + m(t)
s(t) = f(t) + m(t)	
s(t) = f(t) + s(t)	
f(t) + s(t) = f(t) + m(t)	

Every SL in the left list implies every SL in the right list.

Second

Let t be an OD of T and φ[t] = φ'[t] a PSL of T (and note that φ[t] and/or φ'[t] can here be the same expression as t). It can be shown:

T29 φ[t] = φ'[t] ≡
φ[t] = r[t] ∧ φ'[t] = r'[t] ∧ r[t] = r'[t] ∨
φ[t] = r[t] ∧ φ'[t] = k'[t] ∧ r[t] = k'[t] ∨
φ[t] = k[t] ∧ φ'[t] = r'[t] ∧ k[t] = r'[t] ∨
φ[t] = k[t] ∧ φ'[t] = k'[t] ∧ k[t] = k'[t],
where r[t], k[t], r'[t], k'[t] belong to RS[t]; r[t], k[t] are the ultimate reducts of φ[t], and r'[t], k'[t] are the ultimate reducts of φ'[t]; possibly, some or all expressions out of r[t], k[t], r'[t], k'[t] are identical.[17]
Proof: The part of the proof that concerns the direction from the right side of the equivalence to its left side is trivial. Assume φ[t] = φ'[t]; in consideration of the proof of the *Reduction Theorem* (see footnote 17) we have: (φ[t] = r[t] ∨ φ[t] = k[t]) ∧ (φ'[t] = r'[t] ∨ φ'[t] = k'[t]), and hence φ[t] = r[t] ∧ φ'[t] = r'[t] ∨ φ[t] = r[t] ∧ φ'[t] = k'[t] ∨ φ[t] = k[t] ∧ φ'[t] = r'[t] ∨ φ[t] = k[t] ∧ φ'[t] = k'[t]; hence the right side of the equivalence follows because of φ[t] = φ'[t].

For obvious reasons T29 may be called the *Normal Form Theorem* (for TO). Here is an example of its application:

m(*x*) + s(*x*) = m(*x*) ≡ m(*x*) + s(*x*) = c(*x*) ∧ m(*x*) = m(*x*) ∧ c(*x*) = m(*x*) ∨
m(*x*) + s(*x*) = c(*x*) ∧ m(*x*) = m(*x*) ∧ c(*x*) = m(*x*) ∨

17. From the proof of the *Reduction Theorem* it is apparent that an AE φ[t] of T around the OD t has at most two alternative ultimate reducts (expressions in RS[t]): r[t] and k[t]. In case φ[t] has just one ultimate reduct, the expressions r[t] and k[t] are identical. In case φ[t] belongs to RS[t], the ultimate reduct of φ[t] is φ[t]. In case φ[t] is the OD t itself, the ultimate reduct of φ[t] is t.

$m(x) + s(x) = s(x) \wedge m(x) = m(x) \wedge s(x) = m(x) \vee$
$m(x) + s(x) = s(x) \wedge m(x) = m(x) \wedge s(x) = m(x)$,[18] hence
$m(x) + s(x) = m(x) \equiv m(x) + s(x) = c(x) \wedge c(x) = m(x) \vee m(x) + s(x) = s(x) \wedge s(x) = m(x)$, i.e., $m(x) + s(x) = m(x) \equiv m(x) + s(x) = c(x) \wedge c(x) = m(x)$ (by B3(b)), hence by A1, A2(b), B4(c): $\neg m(x) + s(x) = m(x)$.

XI. The proof of the consistency of TO

I now proceed to the proof of the consistency of TO. The consistency of TO will be proved by providing a verifying model for it in an interpreted semiformal language T' that contains T as a sublanguage.

The ODs of T (OVs and ONs of T) are the *second-order* ODs of T'; they *speak about* (that is, are used for *quantifying over* or for *referring to*) the circles in a Euclidean plane (normal circles, with finite positive radius), which are identified with certain sets of points in that plane. (The points inside a circle belong to the circle.) The *first-order* ODs of T', in contrast, speak about the points in the plane. In addition, there are designators for miscellaneous sets of points in the plane (as needed), and designators for real numbers (variables, names, functional expressions). While the second-order OVs of T' are x, x', x'', etc., the first-order OVs of T' are y, y', y'', etc.

In a circle x, conceived as a set of points, there can be distinguished certain proper subsets, for example, the set to which belongs only the centre of x, the set of all points in the periphery of x, the set of all points which lie (properly) between the centre and the periphery of x. I define:

Firstly, for all second-order ODs t of T':
(a) m(t) := λ[19]y(y = the centre of t);[20]
(b) f(t) := λy(y lies [properly] between the centre of t and the periphery of t);[21]
(c) s(t) : = λy(y is in the periphery of t);[22]

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18. According to the proof of the *Reduction Theorem*, '$m(x) + s(x)$' has two alternative ultimate reducts: '$c(x)$' and '$s(x)$'.
19. 'λ' is here employed as the operator of class-abstraction.
20. The centre of t is the point of t whose distance from any two points of t that have distance d(t) from each other is d(t)/2, where d(t) is the furthest distance between points of t (in other words, the length of the diameter of t).
21. y lies between the centre of t and the periphery of t *iff* ∃y'(y' is in the periphery of t, and y is on the straight line between the centre of t and y', but neither identical to y' nor identical to the centre of t).
22. y is in the periphery of t *iff* y∈t and ∃y'(y'∈t and the distance between y and y' is d(t)). (Concerning d(t), see footnote 20.)

(d) c(t) := the empty set $[\lambda y'(y' \neq y')]$.

Secondly, more generally for all designators α and β of T' that refer to sets of points in the (intended) plane:
(e) α is connected with β := $\neg\alpha = \beta \wedge \exists y \exists y'(y \in \alpha$ and $y' \in \beta$ and y' can be reached from y without touching a point that belongs neither to α nor to β) $\vee \neg\alpha = \beta \wedge (\alpha = \lambda y'(y' \neq y') \vee \beta = \lambda y'(y' \neq y'))$;
(f) $\alpha + \beta := \lambda y(($α is connected with β $\wedge$ α and β have no element in common $\vee$ $\alpha = \beta) \wedge (y \in \alpha \vee y \in \beta))$.

With the help of these definitions, the axioms of TO can be proven on the basis of certain elementary geometrical facts about circles, employing elementary set theory in a two-sorted predicate-logical framework (it should be noted that the only sets referred to are sets of *individuals*, i.e., sets of geometrical points):

Proof of A1:

"$\varphi'[\upsilon]$ is connected with $\varphi'[\upsilon]$" is provably equivalent (on the basis of the geometrical background theory and definition (*e*)) to "$\varphi'[\upsilon]$ is connected with $\varphi[\upsilon]$". Therefore, obviously, "($\varphi[\upsilon]$ is connected with $\varphi'[\upsilon] \wedge \varphi[\upsilon]$ and $\varphi'[\upsilon]$ have no element in common $\vee\ \varphi[\upsilon] = \varphi'[\upsilon]) \wedge (y \in \varphi[\upsilon] \vee y \in \varphi'[\upsilon])$" is provably equivalent to "($\varphi'[\upsilon]$ is connected with $\varphi[\upsilon] \wedge \varphi'[\upsilon]$and $\varphi[\upsilon]$ have no element in common $\vee\ \varphi'[\upsilon] = \varphi[\upsilon]) \wedge (y \in \varphi'[\upsilon] \vee y \in \varphi[\upsilon])$". Hence by the principles of elementary set theory and definition *(f)*: $\varphi[\upsilon] + \varphi'[\upsilon] = \varphi'[\upsilon] + \varphi[\upsilon]$.

Proof of A2(a):

"($\varphi[\upsilon]$ is connected with $\varphi[\upsilon] \wedge \varphi[\upsilon]$ and $\varphi[\upsilon]$ have no element in common $\vee$ $\varphi[\upsilon] = \varphi[\upsilon]) \wedge (y \in \varphi[\upsilon] \vee y \in \varphi[\upsilon])$" is provably equivalent to "$y \in \varphi[\upsilon]$" (on purely logical grounds). Hence by the principles of set theory and definition *(f)*: $\varphi[\upsilon] + \varphi[\upsilon] = \varphi[\upsilon]$.

Proof of A2(b):

(i) Assume: ($\varphi[\upsilon]$ is connected with $c(\upsilon) \wedge \varphi[\upsilon]$and $c(\upsilon)$ have no element in common $\vee\ \varphi[\upsilon] = c(\upsilon)) \wedge (y \in \varphi[\upsilon] \vee y \in c(\upsilon))$; hence by definition *(d)*: $y \in \varphi[\upsilon] \vee y \in \lambda y'(y' \neq y')$, and hence because of $\neg y \in \lambda y'(y' \neq y')$: $y \in \varphi[\upsilon]$.

(ii) Assume $y \in \varphi[\upsilon]$; hence $\underline{y \in \varphi[\upsilon] \vee y \in c(\upsilon)}$; $c(\upsilon) = y \in \lambda y'(y' \neq y')$ by definition *(d)*; hence $\neg\varphi[\upsilon] = c(\upsilon) \wedge (\varphi[\upsilon] = \lambda y'(y' \neq y') \vee c(\upsilon) = \lambda y'(y' \neq y'))$; hence by definition *(e)*: $\underline{\varphi[\upsilon] \text{ is connected with } c(\upsilon)}$. Moreover, $\underline{\varphi[\upsilon] \text{ and } c(\upsilon) \text{ have no element in common}}$, because of $c(\upsilon) = y \in \lambda y'(y' \neq y')$. Consequently: ($\varphi[\upsilon]$ is connected with $c(\upsilon) \wedge \varphi[\upsilon]$ and $c(\upsilon)$ have no element in common $\vee\ \varphi[\upsilon] = c(\upsilon)) \wedge (y \in \varphi[\upsilon] \vee y \in c(\upsilon))$.

From (i) and (ii) (that together demonstrate the co-extensionality of the relevant set-defining predicates) one obtains by the principles of set theory and definition *(f)*: $\varphi[\upsilon] + c(\upsilon) = \varphi[\upsilon]$.

Proof of A3:

Assume φ[υ] + φ'[υ] = φ[υ] (the *first assumption*); assume ¬φ'[υ] = c(υ) (the *second assumption*); what must be deduced from these two assumptions is φ'[υ] = φ[υ] (in order to obtain a proof of A3).

I first demonstrate (i): ∃*y*(*y*∈φ[υ]), and I then demonstrate (ii): ∃*y*(*y*∈φ[υ]) ⊃ φ'[υ] = φ[υ]; from (i) and (ii), φ'[υ] = φ[υ] follows by *modus ponens*.

(i) Assume ¬∃*y*(*y*∈φ[υ]); hence φ[υ] = λ*y*'(*y*' ≠ *y*'); because of ¬φ'[υ] = c(υ) (the *second assumption*), we have by definition *(d)*: ∃*y*(*y*∈φ'[υ]), and therefore: *y*∈φ[υ] + φ'[υ] by definition *(f)*, since (*y*∈φ[υ] ∨ *y*∈φ'[υ]) ∧ φ[υ] is connected with φ'[υ] ∧ φ[υ] and φ'[υ] have no element in common [by *y*∈φ'[υ], φ[υ] = λ*y*'(≠ *y*'), φ'[υ] ≠ λ*y*'(≠ *y*'), and definition *(e)*]; hence ¬φ[υ] + φ'[υ] = φ[υ] – contradicting the *first assumption*.

(ii) Assume *y*∈φ[υ], hence *y*∈φ[υ] + φ'[υ] (since φ[υ] + φ'[υ] = φ[υ], according to the *first assumption*), hence by definition *(f)* (and set theory): <u>φ[υ] is connected with φ'[υ] ∧ φ[υ] and φ'[υ] have no element in common ∨ φ[υ] = φ'[υ]</u>; of this *underlined disjunction*, assume the *first disjunct*: φ[υ] is connected with φ'[υ] ∧ φ[υ] and φ'[υ] have no element in common; because of ¬φ'[υ] = c(υ) (the *second assumption*), we have by definition *(d)*: ∃*y*''(*y*''∈φ'[υ]), and therefore: *y*''∈φ[υ] + φ'[υ] by definition *(f)*, since (*y*''∈φ[υ] ∨ *y*''∈φ'[υ]) ∧ φ[υ] is connected with φ'[υ] ∧ φ[υ] and φ'[υ] have no element in common (cf. the *first disjunct*); but ¬*y*''∈φ[υ], since φ[υ] and φ'[υ] have no element in common and *y*''∈φ'[υ]; hence: ¬φ[υ] + φ'[υ] = φ[υ] – contradicting the *first assumption*. Consequently (the *first disjunct* having been excluded), we have the *second disjunct* of the *underlined disjunction*: φ[υ] = φ'[υ], i.e., φ'[υ] = φ[υ]. It has now been shown: ∃*y*(*y*∈φ[υ]) ⊃ φ'[υ] = φ[υ]. [If it could happen that "*y*" occurs free in φ'[υ] = φ[υ], then of course another variable must be – and can be – used instead of "*y*" in the deduction in (ii); but in fact it cannot happen: due to the syntax of the expressions schematically represented by "φ'[υ]" and "φ[υ]". Note that "*y*" does not occur free in the assumptions Π – definitions included – made by adopting the model under consideration, nor in the two assumptions the proof of A3 starts out with.]

Proof of B1:

B1 is established on the basis of the principles of set theory if the relevant predicates (those underlined below) are shown to be co-extensional. This is done in the following way:

(i) Assume <u>*y*∈*x*</u>; hence [since *x* is a circle and using definitions *(a)*, *(b)*, *(c)*]: *y*∈m(*x*) ∨ *y*∈f(*x*) ∨ *y*∈s(*x*).

In the first and second case [contained in the disjunction just deduced], *y*∈f(*x*) + m(*x*) by definition *(f)* [and the principles of set theory, of course], since in those two cases [in consideration of *(a)*, *(b)*, and *(e)*]: (*y*∈f(*x*) ∨ *y*∈m(*x*)) ∧ f(*x*) is connected with m(*x*) ∧ f(*x*) and m(*x*) have no element in common; hence: <u>*y*∈(f(*x*) + m(*x*)) + s(*x*)</u> by *(f)*, since (*y*∈f(*x*) + m(*x*) ∨ *y*∈s(*x*)) ∧ f(*x*) + m(*x*) is connected with s(*x*) ∧ f(*x*) + m(*x*) and s(*x*) have no element in common [in consideration of *(f)*, as applied to f(*x*) + m(*x*), *(c)*, and *(e)*, always remembering that *x* is a circle].

In the third case [contained in the above disjunction, the one first deduced], $\underline{y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)}$ by *(f)*, since $(y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ is connected with $\mathrm{s}(x) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ and $\mathrm{s}(x)$ have no element in common [the justification has already been noted; the only difference is that the part $y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)$ is now deduced from $y \in \mathrm{s}(x)$, and not from $y \in \mathrm{f}(x) + \mathrm{m}(x)$].

(ii) Assume $\underline{y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)}$; hence by *(f)*: $y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)$, hence by *(f)*: $y \in \mathrm{f}(x) \vee y \in \mathrm{m}(x) \vee y \in \mathrm{s}(x)$; hence, since x is a circle, by *(a)*, *(b)*, *(c)*: $\underline{y \in x}$.

Proof of B2:

(i) Assume $\underline{y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)}$; hence by definition *(f)*: $y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)$; hence by *(f)*: $y \in \mathrm{f}(x) \vee y \in \mathrm{m}(x) \vee y \in \mathrm{s}(x)$.

In the first and third case [contained in the disjunction just deduced], $y \in \mathrm{f}(x) + \mathrm{s}(x)$ by *(f)*, since $(y \in \mathrm{f}(x) \vee y \in \mathrm{s}(x)) \wedge \mathrm{f}(x)$ is connected with $\mathrm{s}(x) \wedge \mathrm{f}(x)$ and $\mathrm{s}(x)$ have no element in common [in consideration of *(b)*, *(c)*, and *(e)*, remembering that x is a circle]; hence $\underline{y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)}$ by *(f)*, since $(y \in \mathrm{f}(x) + \mathrm{s}(x) \vee y \in \mathrm{m}(x)) \wedge \mathrm{f}(x) + \mathrm{s}(x)$ is connected with $\mathrm{m}(x) \wedge \mathrm{f}(x) + \mathrm{s}(x)$ and $\mathrm{m}(x)$ have no element in common [in consideration of *(f)*, as applied to $\mathrm{f}(x) + \mathrm{s}(x)$, *(a)*, and *(e)*, always remembering that x is a circle].

In the second case [contained in the above disjunction, the one first deduced], $\underline{y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)}$ by *(f)*, since $(y \in \mathrm{f}(x) + \mathrm{s}(x) \vee y \in \mathrm{m}(x)) \wedge \mathrm{f}(x) + \mathrm{s}(x)$ is connected with $\mathrm{m}(x) \wedge \mathrm{f}(x) + \mathrm{s}(x)$ and $\mathrm{m}(x)$ have no element in common [the justification has already been noted; the only difference is that the part $y \in \mathrm{f}(x) + \mathrm{s}(x) \vee y \in \mathrm{m}(x)$ is now deduced from $y \in \mathrm{m}(x)$, and not from $y \in \mathrm{f}(x) + \mathrm{s}(x)$].

(ii) Assume $\underline{y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{m}(x)}$; hence by *(f)*: $y \in \mathrm{f}(x) \vee y \in \mathrm{s}(x) \vee y \in \mathrm{m}(x)$.

In the first and third case [i.e., $y \in \mathrm{f}(x)$, $y \in \mathrm{m}(x)$], $y \in \mathrm{f}(x) + \mathrm{m}(x)$ by *(f)*, since $(y \in \mathrm{f}(x) \vee y \in \mathrm{m}(x)) \wedge \mathrm{f}(x)$ is connected with $\mathrm{m}(x) \wedge \mathrm{f}(x)$ and $\mathrm{m}(x)$ have no element in common; hence $\underline{y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)}$ by *(f)*, since $(y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ is connected with $\mathrm{s}(x) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ and $\mathrm{s}(x)$ have no element in common.

In the second case [i.e., $y \in \mathrm{s}(x)$], $\underline{y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{s}(x)}$, since $(y \in \mathrm{f}(x) + \mathrm{m}(x) \vee y \in \mathrm{s}(x)) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ is connected with $\mathrm{s}(x) \wedge \mathrm{f}(x) + \mathrm{m}(x)$ and $\mathrm{s}(x)$ have no element in common.

Proof of B3 *and* B4:

B3 and B4 are immediately evident on the basis of the definitions *(a) – (f)* since $\forall x \mathrm{Circle}(x)$ is an axiom of the deductive system used for proving the consistency of TO.

Proof of B5:

B5 results trivially since $\forall x \neg x = \mathrm{f}(x)$ is a theorem of the system used for proving the consistency of TO. ('No circle is its inner content without the centre.']

Proof of B6:

Since $\forall x \neg \mathrm{m}(x) = \mathrm{c}(x)$ is a theorem of the system used for proving the consistency of TO , B6 is equivalent to $\forall x(\mathrm{m}(x) + \mathrm{s}(x) = \mathrm{c}(x))$.

Assume $y \in \mathrm{m}(x) + \mathrm{s}(x)$; hence by *(f)*: m($x$) is connected with s($x$) ∧ m($x$) and s($x$) have no element in common ∨ m(x) = s(x); but according to definitions *(a)*, *(c)*, *(e)*, and the axiom $\forall x$Circle(x), m(x) is not connected with s(x), and ¬m(x) = s(x). Therefore: $\neg\exists y(y \in \mathrm{m}(x) + \mathrm{s}(x))$, and hence: $\mathrm{m}(x) + \mathrm{s}(x) = \lambda y'(y' \neq y')$; hence by *(d)*: $\mathrm{m}(x) + \mathrm{s}(x) = \mathrm{c}(x)$.

Proof of B7:

Since $\forall x \neg \mathrm{m}(x) = \mathrm{c}(x)$ is a theorem, B7 is equivalent to $\forall x((\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x) = \mathrm{c}(x)) \wedge \forall x((\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{m}(x) = \mathrm{c}(x))$.

(i) Assume $y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x)$; hence by *(f)*: f($x$) + m($x$) is connected with f($x$) ∧ f($x$) + m($x$) and f($x$) have no element in common ∨ f(x) + m(x) = f(x). However, f(x) + m(x) and f(x) have an element in common ∧ ¬f(x) + m(x) = f(x). [For this result consider the following truths: $\exists y'(y' \in \mathrm{f}(x))$, $\exists y''(y'' \in \mathrm{m}(x))$, f($x$) is connected with m($x$), f($x$) and m($x$) have no element in common; hence: $\neg y'' \in \mathrm{f}(x)$, but $y'' \in \mathrm{f}(x) + \mathrm{m}(x)$, in virtue of $y'' \in \mathrm{m}(x)$; and $y' \in \mathrm{f}(x)$ and $y' \in \mathrm{f}(x) + \mathrm{m}(x)$, in virtue of $y' \in \mathrm{f}(x)$.] Therefore: $\neg\exists y(y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x))$, and hence: $(\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x) = \lambda y'(y' \neq y')$; hence by *(d)*: $(\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{f}(x) = \mathrm{c}(x)$.

(ii) Assume $y \in (\mathrm{f}(x) + \mathrm{m}(x)) + \mathrm{m}(x)$; continue *mutatis mutandis* as in (i).

Proof of B8:

Because of $\forall x \neg \mathrm{f}(x) = \mathrm{s}(x)$, B8 is equivalent to $\forall x((\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{f}(x) = \mathrm{c}(x)) \wedge \forall x((\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{s}(x) = \mathrm{c}(x))$.

(i) Assume $y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{f}(x)$; hence by *(f)*: f($x$) + s($x$) is connected with f($x$) ∧ f($x$) + s($x$) and f($x$) have no element in common ∨ f(x) + s(x) = f(x). However, f(x) + s(x) and f(x) have an element in common ∧ ¬f(x) + s(x) = f(x). Therefore: $\neg\exists y(y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{f}(x))$, and hence $(\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{f}(x) = \lambda y'(y' \neq y')$; hence by *(d)*: $(\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{f}(x) = \mathrm{c}(x)$.

(ii) Assume $y \in (\mathrm{f}(x) + \mathrm{s}(x)) + \mathrm{s}(x)$; continue *mutatis mutandis* as in (i).

The verifying model for TO I have presented is trivial only with respect to B5. Note also that, considered in Thomasic lights, the presented model is “atheistic”, because $\forall x \neg x = \mathrm{f}(x)$ and $\forall x \neg \mathrm{f}(x) = \mathrm{s}(x)$ are true in it (cf. the proof of B5 and the proof of B8); it is, moreover, “materialistic”, because $\forall x \neg \mathrm{m}(x) = \mathrm{c}(x)$ is true in it (cf. the proof of B6 and the proof of B7). But let the second-order ODs of T’ (OVs x, x', x'', etc., and ONs g, g', g'', etc.) *speak about* (i.e., be used for *quantifying over* or *referring to*) the spheres with positive finite radius in an infinite Euclidean space SP – *plus* the sphere in SP ‘whose centre is everywhere and whose surface nowhere’, that is: SP itself (called ‘the super-sphere’). Those spheres are certain sets of points in SP (SP is, of course, *the* set of points in SP). The first-order ODs of T’ speak about the points in SP (hence the first-order OVs of T’ – y, y', y'', etc. – are used for quantifying over the points in SP). I define:

For all second-order ODs t of T’:

(a') m(t) := λ*y*(*y* is in the surface of t);[23]
(b') s(t) := λ*y*(*y* is a centre of t);[24]
(c') f(t) := λ*y*(∃*y*'∃*y*"(*y*' is a centre of t ∧ *y*" is in the surface of t ∧ *y* is [properly] between *y*' and *y*"[25]) ∨ ¬∃*y*"(*y*" is in the surface of t) ∧ *y* is a centre of t).

The rest, *(d') – (f')*, is identical to *(d) – (f)*. Note that m(t) is now being interpreted with respect to spheres (see *(a')*) *in orientation as* s(t) was previously interpreted with respect to circles (cf. *(c)*), and that s(t) is now being interpreted with respect to spheres (see *(b')*) *in orientation as* m(t) was previously interpreted with respect to circles (cf. *(a)*).

Any sphere in SP is either a *normal* (finite) sphere or *the super-sphere*. For normal spheres *x* in SP, we have: ¬m(*x*) = c(*x*), s(*x*) = λ*y*(*y* = the centre of *x*), λ*y*(*y* = the centre of *x*) ≠ *x*, f(*x*) = λ*y*∃*y*"(*y*" is in the surface of *x* ∧ *y* is [properly] between the centre of *x* and *y*"), ¬s(*x*) = f(*x*), ¬*x* = f(*x*). For the super-sphere *g* in SP (which sphere is SP itself), we have: m(*g*) = c(*g*), s(*g*) = *g*, f(*g*) = s(*g*) [all three equations result because there is no *r* that is a maximal distance between points of *g*; see *(a')*, *(b')*, *(c')*, and footnotes 23 and 24], therefore: *g* = f(*g*). B5 is now true in a non-trivial way. With respect to *g*, B1 is proved as follows: *g* = s(*g*); hence *g* = (s(*g*) + c(*g*)) + s(*g*) by A2[26] (the proof of which is the same as before); hence *g* = (f(*g*) + m(*g*)) + s(*g*) [because of f(*g*) = s(*g*), m(*g*) = c(*g*)]. With respect to normal spheres, B1 is proved as previously, with respect to the model of circles.

XII. Preview of the further procedure

Inconsistency, we have seen, is not a charge that can be raised against Thomas Aquinas's central ontological doctrines about the (substantial) form, the essence, the being and the matter of objects, and the laws of their composition in objects (i.e., in existing substances and quasi-substances),

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23. *y* is in the surface of t *iff* *y*∈t and ∃*r*(*r* is a maximal distance between points of t and ∃*y*'(*y*'∈t and d(*y*, *y*') = *r*)). Here "*r*" and "d(*y*, *y*')" are designators for real numbers (as has been said, T' also contains such designators): "*r*" is a variable that is used for quantifying over the real numbers), and "d(*y*, *y*')" is a functional expression, which is to be read as "the distance between *y* and *y*' " (and refers to a uniquely determined real number for each *y* and each *y*').
24. *y* is a centre of t *iff* *y*∈t and ∀*r*(*r* is a maximal distance between points of t ⊃ ∀*y*'∀*y*"(*y*'∈t and *y*"∈t and d(*y*', *y*") = *r* ⊃ d(*y*', *y*) = *r*/2 and d(*y*", *y*) = *r*/2)).
25. That is: *y* is on the straight line between *y*' and *y*", but is neither identical to *y*' nor identical to *y*" (cf. footnote 21).
26. *g* = s(*g*) = s(*g*) + s(*g*) [by A2(a)] = (s(*g*) + c(*g*)) + s(*g*) [because s(*g*) + c(*g*) = s(*g*) by A2(b)].

whether the objects be material or immaterial, created or uncreated. Note, incidentally, that the Thomasic doctrine of the real distinction between the essence and the being (or *esse*) in created substances is a trivial theorem of TO (see D4); the special form that doctrine assumes if it is applied to created *immaterial* substances is also a theorem of TO, and not an entirely trivial one (see T18(c)).

I am now going to enrich T and TO, which makes it possible to capture – in a formal language and axiomatic system – an even larger portion of Thomas Aquinas's ontology than is captured by TO. As before, all steps of theory-building will be made in close correspondence to the words of Thomas Aquinas himself. The extensions of T and TO I propose will serve to strengthen the implicit definition of Thomasic terms that is provided by the original axiom-system. Among other things, I will provide a formal representation of Thomasic individuation principles (there are several, not just one). However, the formal approach is abandoned in the final part of the paper, where – after having exhaustively treated the formal or structural interrelations of the ontological notions involved in Thomas's theory of the fundamental composition of objects – I analyze the *conceptual content itself* of those notions. I will conclude with a synopsis of Thomas's theory of forms.

XIII. An extension of T and TO: human beings and souls

T enriched by the monadic predicates L(t) and H(t) constitutes the language T*. The syntactical rules of T (see section V) are rewritten for T*. The rewriting simply consists in replacing "T" by "T*", with one exception: the specification of PSLs for T* is this:

*10**. Primary sententials (PSLs) of T*
(a) If δ and δ' are EDs of T*, then (δ = δ') is a PSL of T*;
(b) if t is an OD of T*, then L(t) and H(t) is a PSL of T*;
(c) PSLs of T* are only expressions that are generable by (a) and (b).

The intended interpretation of T* is the same as that of T, with the addition that a sentential of L(t) of T* is to be read as 't is a living object [i.e., living substance or quasi-substance]', and a sentential of H(t) of T* as 't is a human substance [hence: human object]'.

I continue with a definition that can be given in T* (but not already in T):

D9 $A(\delta) := \exists\upsilon(L(\upsilon) \wedge M(\upsilon) \wedge \delta = a(\upsilon))$ [for all EDs δ and OVs υ of T*, provided υ does not occur in δ]

According to the intended interpretation of T* and in view of the Thomasic doctrine which states that a soul is the actuating form of a living body, A(δ), as defined by D9, can be read as 'δ is a soul [anima]'. If t refers to a living body (in other words, if '$L(t) \wedge M(t)$' is true), then 'a(t)' can be read as 'the soul of t'. Aquinas says:

35. anima est primum quo vivimus, cum tamen vivamus anima et corpore: ergo anima est forma corporis viventis. Et haec est definitio superius de anima posita, quod anima est actus primus physici corporis potentia vitam habentis (*In Aristotelis librum de anima commentarium*, liber 2, lectio 4, 271 [of the continuously enumerated sections]).

It is evident that 'forma' in this quotation does not mean *pure form* but *actuating form*; if it were otherwise, then the soul would not be '*actus* primus physici corporis potentia vitam habentis' (the emphasis is mine). Moreover, if 'forma' did not mean *actuating form* but *pure form* in quotation 35, it would be incorrect to call the composite of body and soul 'this something' ('hoc aliquid'), in other words: it would be incorrect to call it an 'object' (more particularly, a 'material object').[27] It would only be correct to call the composite of body, soul, *and* being (*esse*) 'this something'. But Thomas is quite unambiguous in this regard:

36. compositum ex anima et corpore dicitur *hoc aliquid* (*Summa theologiae*, 1, 75, 2).

Consider also

37. ex anima et corpore resultat unum esse in uno composito (*De ente et essentia*, 4, 29).

From this last quotation, it is evident that the *esse* is nothing properly added to 'anima et corpus' but results already by the very union of the two, which can only be because the *esse* is already included in the 'anima',[28] the *actuating* form of the 'compositum' (i.e., of the living body).

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27. An 'object' in the sense of 'substance or quasi-substance' is precisely what I take Thomas to mean by 'hoc aliquid', and Aristotle by '*tode ti*'.
28. It cannot be included in the 'corpore' because for Aquinas not matter but 'forma' is the vehicle/the bringer of being; see quotations 8, 9, and 10 (and the remarks referring to them).

Evidently, ‘corpus vivens’ and ‘corpus potentia vitam habens’ have different meanings. A living body is *obviously* an *object* (a plant, an animal, a human being), while a body that potentially has life is *the matter* (or more correctly speaking in view of section XXIII: *the first representative* of the matter) of a living body, and therefore *not obviously* an *object*.[29] Aquinas uses the word ‘corpus’ (if it does not simply mean *material object*) both in the sense of ‘corpus vivens’ and in the sense of ‘corpus potentia vitam habens’, and it must be determined from the context what exactly is meant by him. When he says ‘compositum ex anima et corpore dicitur *hoc aliquid*’, then he means by ‘corpus’ the same as is meant by ‘corpus potentia vitam habens’; when, however, he says

38. Ex praemissis igitur manifeste ostendi potest animam humanam non corrumpi, corrupto corpore (*Summa contra gentiles*, 2, 79),

he is using ‘corpus’ in the sense of ‘corpus vivens’.

D9 is the first definition that introduces a predicate which forms well-formed expressions not only with ODs of T* but also with other EDs of T*. Another definition that introduces such a predicate is the following:

D10 Sub(δ) := ∃υ(δ = υ) [for all EDs δ and OVs υ of T*, provided υ does not occur in δ]

According to the intended interpretation of T*, Sub(δ), as defined by D10, can be read as ‘δ is a [individual] substance or quasi-substance’, or as ‘δ is an object’, or as ‘δ subsists’. ‘∀*x*Sub(*x*)’ is a trivial logical truth; it means, according to the intended interpretation, that every object is an object. But, from it, we cannot validly infer for every ED δ of T*: Sub(δ), which, if it could be validly inferred for every such δ, would mean (according to the intended interpretation) that all *considered* entities, be they quantified over or merely designated, are *objects* (i.e., a substance or a quasi-substance). This would certainly contradict Thomasic doctrine. Fortunately, that step of inference cannot be validly taken, because the deductive restrictions that were specified for the logic of T (see section VI) also apply to the logic of T*.

TO, rewritten for T* and enriched by the following three axioms:

29. According to T24, no object is its matter. But *perhaps* the matter of an object is sometimes *another* object? However, this option of theory formation will be closed by T48.

C1 $\forall x(H(x) \supset L(x) \land M(x))$
C2 $\forall x(H(x) \supset \exists x'(I(x') \land x' = a(x)))$
C3 $\exists x H(x)$,

constitutes a part of the system TO*.

Concerning C1: According to the intended interpretation, C1 asserts that every human substance is a living material object. But is not *Socrates* a human substance which is *not* a living material object? Here it must be recalled that we are using 'object' in the sense of 'existent object' (see section III). Now, 'existent' may mean the same as 'now existent' or the same as 'at some time existent', and accordingly also two meanings of 'living' have to be distinguished (at least for the purposes of this treatise), since it is certainly true that

39. vivere enim est esse viventis (*Summa contra gentiles*, 2, 57).

If we take 'existent' to mean the same as '*now* existent', and 'living' to mean the same as '*now* living', then Socrates turns out to be *not* a human substance (because he is not an object, because he is not an existent object, because he is not a *now* existent object) – just as he turns out to be not a living (i.e., *now* living) material object. If, however, we take 'existent' to mean the same as '*at some time* existent', and 'living' to mean the same as '*at some time* living', then Socrates can certainly be regarded as a human substance (and hence as an object, an existent object, an *at some time* existent object) – *and* certainly as a living (i.e., *at some time* living) material object. Instead of "human substance", we say more familiarly "human being", and instead of "living material object" "living body". It should be born in mind that these predicates (and all other ordinary language predicates put forward as readings of the formal predicates that form sententials with the ODs of T*) have an *existential* (and substantial or quasi-substantial) import, according to the intended interpretation of T*. – There can be no question but that C1 squares with Thomasic (but not so easily with Cartesian or Platonic) doctrine.

Concerning C2: C2 asserts that the actuating form – that is, in view of C1, *the soul*[30]– of any human being is a created immaterial object (an *intelligence*). This *cannot* be expressed in the following manner: $\forall x(H(x) \supset$

30. C1 requires that human beings are living bodies, and in the case of a living body x, the locution "the soul of x" can be used instead of "a(x)" ["the actuating form of x"], as was determined earlier in this section.

I(a(x))), since this is not a well-formed expression of T*. For according to D6 and D3, 'I(a(x))' would entail 'm(a(x)) = c(a(x))'; but the latter expression (and therefore also the former) is ungrammatical according to the syntax of T (see sections III and V) – and also, of course, according to the syntax of T* (cf. its description above). Concerning the Thomasic doctrinal justification of C2, consider the following:

40. Est ergo distinctio earum [intelligentiarum] ad invicem, secundum gradum potentiae et actus; ita quod intelligentia superior, quae magis propinqua est primo [enti], habet plus de actu et minus de potentia, et sic de aliis. Et hoc completur in anima humana, quae tenet ultimum gradum in substantiis intellectualibus (*De ente et essentia*, 4, 29).[31]

Concerning C3: C3 simply states an empirical fact: there is at least one human being. C3 is the first axiom of the formal theory (emerging in TO and TO*) that states the existence of a specific *kind* of object.

Aquinas would probably have accepted $\forall x(\mathrm{H}(x) \equiv \mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \exists x'(\mathrm{I}(x') \wedge x' = \mathrm{a}(x)))$ as true (given its intended interpretation). Not so a modern Thomist, who knows that the universe is much larger than Aquinas thought it to be. For all we know, the universe may well contain a living body whose actuating form is a created immaterial object, but which body is not a human being (say, because it has an amoeba-like appearance).

C2 is an axiom that many people these days are likely to reject; for, using C3, we can deduce from C2: $\exists x'\mathrm{I}(x')$ – *There is at least one created immaterial object*. C1, in contrast, is not so likely to be rejected; using C3, we can deduce from C1: $\exists x\mathrm{M}(x)$ – *There is at least one material object* (but, in view of T13, it has to be a *created* one).

There is convincing evidence that Thomas would have accepted $\forall x(\mathrm{H}(x) \equiv \mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \mathrm{Sub}(\mathrm{a}(x)))$ as true (given its intended interpretation): We have C1 and

41. Relinquitur igitur animam humanam, quae dicitur intellectus vel mens, esse aliquid incorporeum et subsistens (*Summa theologiae*, 1, 75, 2),

31. Note that Thomas is here saying that the human soul is an intellectual *substance* (and not merely that it is an immaterial *object* [*res*], which, as such, might merely be a quasi-substance). He is certainly not always ready to go thus far. But probably he is here, instead of being inconsistent, merely using the word "substantia" in an extended sense (the sense of 'aliquid subsistens'; see footnote 2). What is clear is that all other created immaterial objects are *substances*, and substances *not* only in an extended sense.

which supports $\forall x(\mathrm{H}(x) \supset \mathrm{Sub}(\mathrm{a}(x)))$.[32] Moreover, Thomas states

42. relinquitur quod, cum animae brutorum animalium per se non operentur, non sint subsistentes: similiter enim unumquodque habet esse et operationem (*Summa theologiae*, 1, 75, 3),

which supports $\forall x(\neg\mathrm{H}(x) \wedge \mathrm{L}(x) \wedge \mathrm{M}(x) \supset \neg\mathrm{Sub}(\mathrm{a}(x)))$, since Thomas would accept (a) that every non-human living material object is a 'brute animal' or a plant, and (b) that the souls of plants (like the souls of 'brute animals' according to quotation 42) do not subsist (i.e., are not objects). From C1, $\forall x(\mathrm{H}(x) \supset \mathrm{Sub}(\mathrm{a}(x)))$, and $\forall x(\neg\mathrm{H}(x) \wedge \mathrm{L}(x) \wedge \mathrm{M}(x) \supset \neg\mathrm{Sub}(\mathrm{a}(x)))$ we get $\forall x(\mathrm{H}(x) \equiv \mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \mathrm{Sub}(\mathrm{a}(x)))$ as an obvious logical consequence.

'$\mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \mathrm{Sub}(\mathrm{a}(x))$' and '$\mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \exists x'(\mathrm{I}(x') \wedge x' = \mathrm{a}(x))$' are, *initially*, not provably equivalent sententials of T*: from '$\exists x'(\mathrm{I}(x') \wedge x' = \mathrm{a}(x))$' we get '$\mathrm{Sub}(\mathrm{a}(x))$' (by D10), but, so far, we do not get '$\exists x'(\mathrm{I}(x') \wedge x' = \mathrm{a}(x))$' from '$\mathrm{Sub}(\mathrm{a}(x))$'. However, the gap between '$\mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \mathrm{Sub}(\mathrm{a}(x))$' and '$\mathrm{L}(x) \wedge \mathrm{M}(x) \wedge \exists x'(\mathrm{I}(x') \wedge x' = \mathrm{a}(x))$' can be closed, in keeping with Thomasic doctrine, by adding two further axioms to TO* (and remembering D6):

C4 $\forall x\forall x'(x' = \mathrm{a}(x) \vee x' = \mathrm{f}(x) \supset \neg\mathrm{M}(x'))$
(If an object is the actuating or pure form of an object, then it is immaterial)

C5 $\forall x\forall x'(\mathrm{M}(x) \wedge x' = \mathrm{a}(x) \supset \mathrm{C}(x'))$
(If an object is the actuating form of a material object, then it is created)

C4, of course, logically contains B5 and T2 as special cases (in view of D3). It is a consequence of C5 (together with C1) that the soul of a human being is not a divine object, or in other words, $\forall x(\mathrm{H}(x) \supset \neg\exists x'(\mathrm{D}(x') \wedge x' = \mathrm{a}(x)))$: Assume $\mathrm{H}(x)$, $\exists x'(\mathrm{D}(x') \wedge x' = \mathrm{a}(x))$ [for *reductio*]; hence by C1: $\mathrm{M}(x)$, and hence $\exists x'(\mathrm{M}(x) \wedge x' = \mathrm{a}(x) \wedge \mathrm{D}(x'))$; hence by C5: $\exists x'(\mathrm{C}(x')\ \&\ \mathrm{D}(x'))$ – which is a contradiction in view of D5.

XIV. The entity-predicates corresponding to the object-aspects

Following the lead of D9 and D10, we can define a whole series of *entity*-predicates (in contradistinction to *object*-predicates – M(t), C(t), I(t),

32. Note that, in contrast to '$\mathrm{I}(\mathrm{a}(x))$', '$\mathrm{Sub}((\mathrm{a}(x))$' is syntactically correct. As is apparent from D10, '$\mathrm{Sub}((\mathrm{a}(x))$' does not involve the substitution of '$\mathrm{a}(x)$' into a PAE of T*. '$\mathrm{I}(\mathrm{a}(x))$', in contrast, *does* involve the substitution of '$\mathrm{a}(x)$' into PAEs of T* (see D6, D3).

D(t), B(t), E(t) – which do not take AEs of T*, or of T, as arguments of predication):

D11 (a_1) FP(δ) := ∃υ(δ = f(υ)), (a_2) FA(δ) := ∃υ(δ = a(υ)), (b) S(δ) := ∃υ(δ = s(υ)), (c) W(δ) := ∃υ(δ = w(υ)), (d) Mat(δ) := ∃υ(M(υ) ∧ δ = m(υ)), (e) N(δ) := ∃υ(δ = c(υ)), (f) F(δ) := FP(δ) ∨ FA(δ) [for all EDs δ and OVs υ of T*, provided υ does not occur in δ]

The *definienda* of the seven definitions comprised by D11 are to be read, in the order of their occurrence, as 'δ is a pure (substantial) form', 'δ is an actuating (substantial) form', 'δ is an *esse*' [instead of 'δ is a being', which is ambiguous: 'δ is a being' could also mean the same as 'δ is an entity'], 'δ is an essence', 'δ is a (parcel of) matter' ['δ is a *materia designata*'; cf. quotation 59 in section XX], 'δ is an empty aspect', 'δ is a (substantial) form'. Since we are still concerned with Thomas Aquinas's doctrine on the *fundamental* composition of objects, *simpliciter substantial* forms are the only forms we are dealing with (with the exception of *the existence* [*esse*] of *x*, for any object *x*, which is a form and, for *most x*, a non-substantial form; see section XXIV). This is the reason why I say 'form' instead of 'substantial form' (that is, I *usually* say so – leaving aside occasional reminders of what is really intended by the shorter expression; in section XXIV, however, I will speak in the fully explicit way). But of course Aquinas also recognizes forms that are not *simpliciter* substantial, and forms that are *simpliciter* non-substantial: forms that are a substantial form only of some objects that they are a form of, and forms that are a substantial form of no object that they are a form of.

By being a substantial form a form is not automatically a *subsistent* form, that is, a form which is an *object*, perhaps even a *substance* in the full and proper sense. Without doubt, however, some substantial forms are for Aquinas subsistent forms (and now I continue the numbering of theorems – all theorems of TO being also theorems of TO*):

T30 ∃*x*'FA(*x*')
(There is an object which is an actuating form)
Proof: By C3: ∃*x*H(*x*); hence by C2: ∃*x*∃*x*'(I(*x*') ∧ *x*' = a(*x*)), and hence ∃*x*'∃*x*(*x*' = a(*x*)); hence by D11(a_2): ∃*x*'FA(*x*').

That is to say, the existence of subsistent actuating forms – namely, of human souls – follows, in view of C2, from the existence of human beings. I will prove later on – on the basis of one more axiom of TO* – that there is also a subsistent pure form.

Being an immaterial object coincides with being an object which is an actuating form (and therefore, *nota bene*, on the basis of T1, T2, and D3 also with being an object which is *its* actuating form):

T31 $\forall x(\neg M(x) \equiv FA(x))$
Proof:
(i) Assume $\neg M(x)$; hence by D3, T1: $x = a(x)$, and hence: $\exists x'(x = a(x'))$, hence by D11(a_2): $FA(x)$.
(ii) Assume $FA(x)$; hence by D11(a_2): $\exists x'(x = a(x'))$, hence by C4: $\neg M(x)$.

Furthermore we have:

T32 $\forall x(D(x) \supset FP(x))$
(Every divine object is a subsistent pure form, i.e., a pure form which is an object)
Proof: Assume $D(x)$; hence by T21(e): $x = f(x)$, and hence $\exists x'(x = f(x'))$; hence by D11(a_1): $FP(x)$.

The converse of T32 is not provable (and I leave it at that).[33] Furthermore we have:

T33 $\forall x(FP(x) \supset FA(x))$
(Every subsistent pure form is a subsistent actuating form)
Proof: Assume FP(x), hence by D11(a_1): $\exists x'(x = f(x'))$; hence by C4: $\neg M(x)$; hence by D3, T1: $x = a(x)$, and hence $\exists x'(x = a(x'))$; hence by D11(a_2): $FA(x)$.

From T33 we easily obtain by D11(f):

T34 $\forall x(F(x) \equiv FA(x))$
(The subsistent forms are the subsistent actuating forms)

And from the general equivalences T34 and T31 together, we obtain:

T35 $\forall x(M(x) \equiv \neg F(x))$
(Every object [which does not mean: *every entity*] *is either a material object or a subsistent form)*

33. Further considerations that are relevant for the question of whether $\forall x(FP(x) \supset D(x))$ should be provable in TO* follow in section XV; see, in particular, the end of section XV.

XV. Subsistent *esse*

Aquinas holds that there is at most one subsistent *esse* (an *esse* which is an object):

43. Esse autem, in quantum est esse, non potest esse diversum: potest autem diversificari per aliquid quod est praeter esse; sicut esse lapidis est aliud ab esse hominis. Illud igitur quod est esse subsistens, non potest esse nisi unum tantum (*Summa contra gentiles*, 2, 52).

In other words, '$\exists x S(x) \supset \exists x(S(x) \wedge \forall x'(S(x') \supset x = x'))$' is true for Thomas (in the intended interpretation), and it is logically equivalent to a further axiom of TO*:

C6 $\forall x \forall x'(S(x) \wedge S(x') \supset x = x')$

Moreover, Thomas certainly believes that there is a divine object:

C7 $\exists x D(x)$

By making use of C6 and C7, the following theorems can be deduced:

T36 $\exists x FP(x)$
(There is a subsistent pure form)
Proof: T32, C7.
T37 $\exists! x S(x)$
(There is precisely one subsistent esse)
Proof: By C7: $\exists x D(x)$; hence by T21(c): $\exists x(x = s(x))$, and hence $\exists x \exists x'(x = s(x'))$; hence by D11(b): $\exists x S(x)$; hence by C6: $\exists! x S(x)$. [The general definition of 'precisely one' is this: $\exists! \upsilon \sigma[\upsilon] := \exists \upsilon \sigma[\upsilon] \wedge \forall \upsilon \forall \upsilon'(\sigma[\upsilon] \wedge \sigma[\upsilon'] \supset \upsilon = \upsilon')$.]
T38 $\forall x(D(x) \supset S(x))$
(Every divine object is a subsistent esse)
Proof: Assume $D(x)$; hence by T21(c): $x = s(x)$, and hence $\exists x'(x = s(x'))$; hence by D11(b): $S(x)$.
T39 $\forall x \forall x'(D(x) \wedge D(x') \supset x = x')$
(There is at most one divine object)
Proof: T38, C6.

Concerning the Thomasic background of C6 and T38 (and its proof with the help of T21(c)), and the Thomasic background of T39 (and its proof with the help of T38, C6), consider the following quotation in addition to quotation 43:

44. Esse proprium uniuscuiusque rei est [in quantum est esse?] unum tantum. Sed ipse Deus est esse suum, ut supra ostensum est. Impossibile est igitur esse nisi unum Deum (*Summa contra gentiles*, 1, 42).

T40 $\exists! x D(x)$
(There is exactly one divine object)
Proof: C7, T39.

T41 $\forall x(D(x) \equiv S(x))$
(The divine objects are the subsistent "esses")
Proof: T38 is one half of T41, and already proven. Assume $S(x)$; by C7: $\exists x' D(x')$, and hence by T38: $\exists x'(D(x') \wedge S(x'))$; hence by C6 [because of $S(x)$, $S(x')$]: $x = x'$, and therefore: $D(x)$ [because of $D(x')$].

I introduce, as part of the logical inventory of T*, the functor of definite description 'ι' (the Greek letter *iota*), which forms *definite object-descriptions* (DODs) of T* from monadic predicates of T* by binding the free OV in such predicates. The DODs of T* are not counted among the ONs of T* although, in one respect, they function like ONs of T*: if the formative predicate of a DOD is true of exactly one object, then the DOD names that object; *otherwise*, it names *the moon* (say). Syntactically, a DOD of T* can stand anywhere in an SL of T* where an ON of T* can stand. However, DODs of T*, in contrast to ONs of T*, are not fit for all-generalization.[34] Employing the (standard) logic of the ι-functor, we can prove:

T42 $\iota x D(x) = \iota x S(x)$
(God is the subsistent esse)
Proof: T41, T40.

And Thomas says:

45. Deus est ipsum esse per se subsistens (*Summa theologiae*, 1, 44, 1).

The reading of '$\iota x D(x)$' – 'the divine object' – as 'God' is made formally official by the following definition:

D12 $\mathbf{d} := \iota x D(x)$

D12 is immediately employed in proving

34. Certain premises that do not contain "$\iota x D(x)$" (see C7 together with T39, or T40 all by itself) logically imply $D(\iota x D(x))$. But they certainly do not logically imply $\forall x' D(x')$ (although "$\iota x D(x)$" does not occur in this latter sentence of T*).

T43 $\mathbf{d} = \mathrm{s}(\mathbf{d}) \wedge \forall x(x = \mathrm{s}(x) \supset x = \mathbf{d})$
(God is the only object that is its being)
Proof: By T40: $\exists! x\mathrm{D}(x)$, and hence $\mathrm{D}(\iota x\mathrm{D}(x))$; hence by D12: $\mathrm{D}(\mathbf{d})$; hence by T21(c): $\mathbf{d} = \mathrm{s}(\mathbf{d})$. And assume $x = \mathrm{s}(x)$; hence $\exists x'(x = \mathrm{s}(x')) \wedge \exists x'(\mathbf{d} = \mathrm{s}(x'))$ [the second conjunct follows because of $\mathbf{d} = \mathrm{s}(\mathbf{d})$]; hence by D11(b) and C6: $x = \mathbf{d}$.

Compare – with T43 and its proof – the following quotation (which is a continuation of quotation 43):

46. Illud […] quod est esse subsistens, non potest esse nisi unum tantum. Ostensum est autem quod Deus est suum esse subsistens. Nihil igitur aliud praeter ipsum potest esse suum esse. Oportet igitur in omni substantia quae est praeter ipsum, esse aliud ipsam substantiam et eius esse (*Summa contra gentiles*, 2, 52).

Furthermore the following theorems can be proved:

T44 $\mathrm{w}(\mathbf{d}) = \mathrm{s}(\mathbf{d}) \wedge \forall x(\mathrm{w}(x) = \mathrm{s}(x) \supset x = \mathbf{d})$
(God is the only object whose essence is its being)
Proof: By T40: $\exists! x\mathrm{D}(x)$, and hence $\mathrm{D}(\iota x\mathrm{D}(x))$; hence by D12: $\mathrm{D}(\mathbf{d})$, hence by D5, D4: $\mathrm{w}(\mathbf{d}) = \mathrm{s}(\mathbf{d})$. And assume $\mathrm{w}(x) = \mathrm{s}(x)$; hence by D4 and T15: $\mathrm{D}(x)$; hence by T39 and $\mathrm{D}(\mathbf{d})$: $x = \mathbf{d}$.

T45 $\mathrm{W}(\mathbf{d}) \wedge \mathrm{FA}(\mathbf{d}) \wedge \mathrm{FP}(\mathbf{d})$
(God is a subsistent essence, a subsistent actuating form, and a subsistent pure form)
Proof: By T40, D12: $\mathrm{D}(\mathbf{d})$; hence by T21(b): $\mathbf{d} = \mathrm{w}(\mathbf{d})$, and hence $\exists x'(\mathbf{d} = \mathrm{w}(x'))$; hence by D11(c): $\underline{\mathrm{W}(\mathbf{d})}$. By $\mathrm{D}(\mathbf{d})$, T16(a), D3, T1: $\mathbf{d} = \mathrm{a}(\mathbf{d})$, and hence $\exists x'(\mathbf{d} = \mathrm{a}(x'))$; hence by D11($\mathrm{a}_2$): $\underline{\mathrm{FA}(\mathbf{d})}$. By $\mathrm{D}(\mathbf{d})$, T21(e): $\mathbf{d} = \mathrm{f}(\mathbf{d})$, and hence $\exists x'(\mathbf{d} = \mathrm{f}(x'))$; hence by D11($\mathrm{a}_1$): $\underline{\mathrm{FP}(\mathbf{d})}$.

While, given the present state of the system TO*, it is possible to prove both that God is the only object that is its essence, and that God is the only object that is its pure form, it is neither possible to prove that *God is the only subsistent essence* (the only object that is an essence), nor that *God is the only subsistent pure form* (the only object that is a pure form). To my knowledge, Thomas nowhere asserts that God is the only subsistent essence. And of course he nowhere asserts that God is the only subsistent pure form, since he does not distinguish between actuating form and pure form (which non-distinction is correct for God – see T21(a) – but incorrect for all other objects). Thus, to Thomas Aquinas's mind, there simply are many subsistent forms: God, human souls, other created intelligences, in other words: all immaterial objects. But the two *italicized* propositions (see above in this paragraph) can certainly be considered to be in the spirit of Thomasic ontology. If $\forall x\forall x'(\mathrm{W}(x) \wedge \mathrm{W}(x') \supset x = x')$ and $\forall x\forall x'(\mathrm{FP}(x) \wedge$

$FP(x') \supset x = x'$) were added as axioms to TO*, those propositions would become provable in TO* (given the intended interpretation of T*), and in consequence also the following would be provable: $\mathbf{d} = \iota x S(x) = \iota x W(x) = \iota x FP(x)$ – *God is the subsistent esse, which is the subsistent essence, which is the subsistent pure form*. I nevertheless refrain from adding the mentioned principles as axioms to TO*: they are in the Thomasic spirit, but they were certainly not evident to Thomas (while C6, their structural companion, *was* evident to him).

XVI. Non-subsistent matter

Whereas Thomas Aquinas acknowledges many subsistent forms (and we may add, in the Thomasic spirit, what Thomas himself did not assert: all of these forms are *actuating*, precisely one of them is also *pure*), and precisely one subsistent *esse*, and at least one subsistent essence (and once more we may add, in the Thomasic spirit, what Thomas himself did not assert: there is *no other* subsistent essence), Thomas does not acknowledge a subsistent matter (see quotation 10):

C8 $\neg\exists x Mat(x)$

And of course we can add to this axiom

C9 $\neg\exists x N(x)$
(There is no subsistent empty aspect)

On the basis of D10 and D11(d), C8 is equivalent to

T46 $\forall x'(M(x') \supset \neg Sub(m(x')))$,

and on the basis of D10 and D11(e), C9 is equivalent to

T47 $\forall x'\neg Sub(c(x'))$.

T47 has B4(a), $\forall x\neg x = c(x)$, as a logical consequence. From T46 and T47, we obtain:

T48 $\forall x'\neg Sub(m(x'))$
(The matter of no object is an object)
Proof: (i) Assume $M(x')$; hence by T46: $\neg Sub(m(x'))$. (ii) Assume $\neg M(x')$; hence by D3: $m(x') = c(x')$; by T47: $\neg Sub(c(x'))$; hence $\neg Sub(m(x'))$.

T24 – $\forall x \neg x = m(x)$ – is a direct logical consequence of T48. As T48 is a generalization of T24, so $\forall x' \neg W(m(x'))$ is a generalization of T25: $\forall x \neg w(x) = m(x)$. But in order to obtain $\forall x' \neg W(m(x'))$ – *The matter of no object is an essence* – as a theorem, we need to add further axioms:

C10 $\forall x'(M(x') \supset \neg W(m(x')))$
C11 $\forall x' \neg W(c(x'))$

From these axioms,

T49 $\forall x' \neg W(m(x'))$

follows in the same manner as T48 follows from T46 and T47. (Because the axioms C4, C9, C11 logically contain, as special cases, axioms of the system TO – namely, B5, B4(a), B4(e) – C4, C9, C11 can replace those axioms of TO.)

XVII. Thomasic individuation principles

I leave TO* in the state attained so far. Whatever TO* will finally amount to, let TO** be TO* enriched by the *Thomasic individuation axioms*. Such individuation axioms have the form $\forall \upsilon \forall \upsilon'(B_{con}[\varphi[\upsilon] = \varphi[\upsilon']] \supset A[\upsilon, \upsilon'])$[35] or the form $\forall \upsilon \forall \upsilon'(A[\upsilon, \upsilon'] \supset \varphi[\upsilon] = \varphi[\upsilon'])$, where φ is an AE of T* having υ or υ' as its OV ([$\varphi[\upsilon]$ is φ with υ, $\varphi[\upsilon']$ is φ with υ').

Is every S of T* having the form $\forall \upsilon \forall \upsilon'(\varphi[\upsilon] = \varphi[\upsilon'] \supset \upsilon = \upsilon')$ a Thomasic individuation principle? Even if it were so, we nevertheless could not consistently add 'Every S of T* having the form $\forall \upsilon \forall \upsilon'(\varphi[\upsilon] = \varphi[\upsilon'] \supset \upsilon = \upsilon')$ is an axiom of TO**' to the specification of the other axioms of TO**, since we can already in TO* prove the following theorem:

T50 $\exists x \exists x'(a(x) = a(x') \wedge \neg x = x')$
(There are objects that are different although their actuating forms are identical)
Proof: By C3: $\exists x H(x)$;hence by C2: $\exists x(H(x) \wedge \exists x'(I(x') \wedge x' = a(x)))$, and hence: $\exists x \exists x'(H(x) \wedge I(x') \wedge x' = a(x))$; hence by C1, D6: $\exists x \exists x'(M(x) \wedge \neg M(x') \wedge x' = a(x))$; hence by T1, D3: $\exists x \exists x'(M(x) \wedge \neg M(x') \wedge x' = a(x') \wedge x' = a(x))$, and hence: $\exists x \exists x'(a(x) = a(x') \wedge \neg x = x')$.

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35. $B_{con}[\varphi[\upsilon] = \varphi[\upsilon']]$ is a SL of T* (with the free OVs υ and υ') which either is identical to the PSL $\varphi[\upsilon] = \varphi[\upsilon']$ or includes it as a conjunct. Note that the most important specialization of $A[\upsilon, \upsilon']$ is $\upsilon = \upsilon'$.

The actuating form of the soul of a human being, which is an immaterial object, is the soul itself, which, in turn, is the actuating form of the human being. Thus, the actuating form of the soul and the actuating form of the human being are identical. But the soul of the human being is not identical to the human being since the former is an immaterial object, the latter a material one. Thomas himself says:

47. Plato posuit quod homo non sit aliquid compositum ex anima et corpore: sed quod ipsa anima utens corpore sit homo; sicut Petrus non est aliquid compositum ex homine et indumento, sed homo utens indumento. Hoc autem esse impossibile ostenditur (*Summa contra gentiles*, 2, 57).

It is in keeping with the purely auxiliary character of empty aspects to adopt $\forall x \forall x'(x = x \wedge x' = x' \supset \mathrm{c}(x) = \mathrm{c}(x'))$ – or rather, for the sake of brevity, its logical equivalent $\forall x \forall x'(\mathrm{c}(x) = \mathrm{c}(x'))$ – as a Thomasic individuation axiom:

I1 $\forall x \forall x'(\mathrm{c}(x) = \mathrm{c}(x'))$
(The empty aspects of all objects are identical)

I1 has to be adopted without exegetical justification since there is no evidence for it in the writings of Thomas Aquinas. Aquinas had no idea of empty aspects; their sole Thomasic justification – an indirect one – is their great usefulness for the systematic formulation of Thomas's ontological doctrines. Note that I1 can be taken to justify the introduction of an aspect-constant: 'c*' – '*the* empty aspect'. But if such a constant is introduced, then it has to be kept in mind that it would not be another OD of T* (or rather of T**): 'f(c*)', 's(c*)', etc. are as syntactically nonsensical as are 'f(c(*x*))', 's(c(*x*'))', etc. Given I1 we can easily prove:

T51 $\exists x \exists x'(\mathrm{m}(x) = \mathrm{m}(x') \wedge \neg x = x')$
(There are objects that are different although their matters are identical)

I present the proof in an informal way: Both **d** (God) and a human soul x' (it is provable in TO** that there are such objects) are immaterial objects (as is also provable); hence by the definition of immateriality: $\mathrm{m}(\mathbf{d}) = \mathrm{c}(\mathbf{d})$ and $\mathrm{m}(x') = \mathrm{c}(x')$, and therefore by I1: $\mathrm{m}(\mathbf{d}) = \mathrm{m}(x')$ –the "matters" of **d** and x' are identical. But **d** and x' are nevertheless different, since x' is a created object, but **d** is not. Obviously, T51 shows, like T50, that $\forall \upsilon \forall \upsilon'(\varphi[\upsilon] = \varphi[\upsilon'] \supset \upsilon = \upsilon')$ cannot be adopted as an *axiom schema* (so that all of its instances would be Thomasic individuation axioms).

Aquinas *does not* assert $\forall x \forall x'(\mathrm{m}(x) = \mathrm{m}(x') \supset x = x')$. When he says

48. individuationis principium est materia (*De ente et essentia*, 2, 7),

he is not asserting an individuation principle for all objects (as is clear from the context), but an individuation principle only for all material objects:

I2 $\forall x \forall x'(\mathrm{M}(x) \wedge \mathrm{M}(x') \wedge \mathrm{m}(x) = \mathrm{m}(x') \supset x = x')$
(Material objects that have the same matter are identical)

However, if 'object' ('substance or quasi-substance') is taken in a broad sense, then there are obvious counterexamples even to I2 – for example, the statue x and the lump of bronze x' that x is made of. Thus, if I2 is to be plausible, and not just Thomasically valid because Thomas asserted it (more or less) *verbatim*, the word "object" must be understood in an appropriately narrow sense. But it is difficult to characterize that sense in a principled way, and not just in an *ad hoc* way. Why does a statue count as an object, but not the lump of bronze that the statue is made of? Why does a human being (and the human being's soul) count as an object, but not the human being's body ("body" taken in the sense of *corpus potentia vitam habens*, not in the sense of *corpus vivens*; regarding this important distinction, see section XIII)? Note that both the lump of bronze and the human body (that is, the *corpus potentia vitam habens*) can exist on their own (like the human soul), without being integrated into a statue, respectively a human being; for the lump of bronze, this is even its natural way of existence. But, nevertheless, neither the human body (in abstraction form the human being) nor the lump of bronze are counted as objects by Aquinas (if they were, he could not – in reason – assert I2). A rationale that suggests itself is this: Among entities *with the same matter*, only the one with the relatively highest degree of inner organization and completeness is (or is counted as) an object.

Just as $\forall x \forall x'(\mathrm{m}(x) = \mathrm{m}(x') \supset x = x')$ is not Thomasically valid but $\forall x \forall x'(\mathrm{M}(x) \wedge \mathrm{M}(x') \wedge \mathrm{m}(x) = \mathrm{m}(x') \supset x = x')$ *is*, so $\forall x \forall x'(\mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$ is not Thomasically valid but the following *is*:

T52 $\forall x \forall x'(\neg \mathrm{M}(x) \wedge \neg \mathrm{M}(x') \wedge \mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$
(Immaterial objects that have the same actuating form are identical)
Proof: Assume $\neg \mathrm{M}(x) \wedge \neg \mathrm{M}(x') \wedge \mathrm{a}(x) = \mathrm{a}(x')$; hence by D3, T1: $x = \mathrm{a}(x) \wedge x' = \mathrm{a}(x') \wedge \mathrm{a}(x) = \mathrm{a}(x')$, and hence $x = x'$.

Moreover, Aquinas says:

49. animae humanae multiplicantur secundum multiplicationem corporum, ut supra ostensum est (*Summa contra gentiles*, 2, 80).

And we can add $\forall x \forall x'(\mathrm{H}(x) \land \mathrm{H}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$ – *human beings whose souls are identical are themselves identical* – to the Thomasic individuation axioms. That is, we can do so if we take the word "corpus" to mean *corpus vivens* in the above quotation; if we take it to mean *corpus potentia vitam habens*, we rather ought to add $\forall x \forall x'(\mathrm{H}(x) \land \mathrm{H}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset \mathrm{m}(x) = \mathrm{m}(x'))$ to those axioms. However, which of the two sentences we choose for axiomatization does not matter after all since, they are provably equivalent on the basis of C1 and I2. And so are $\forall x \forall x'(\mathrm{M}(x) \land \mathrm{M}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$ and $\forall x \forall x'(\mathrm{M}(x) \land \mathrm{M}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset \mathrm{m}(x) = \mathrm{m}(x'))$.

As a matter of fact, I posit as a Thomasic individuation axiom

I3 $\forall x \forall x'(\mathrm{M}(x) \land \mathrm{M}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$
(Material objects that have the same actuating form are identical).

I3 is more general than $\forall x \forall x'(\mathrm{H}(x) \land \mathrm{H}(x') \land \mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$ (in view of C1), and indeed Thomas says:

50. Impossibile est enim plurium numero diversorum esse unam formam, sicut impossibile est quod eorum sit unum esse (*Summa theologiae*, 1, 76, 2).

This cannot be represented by $\forall x \forall x'(\mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$, which is contradicted by T50; and it cannot be represented by $\forall x \forall x'(\mathrm{f}(x) = \mathrm{f}(x') \supset x = x')$, since Aquinas clearly takes 'forma' to mean *actuating form* in the context from which quotation 50 is taken:

51. Respondeo dicendum quod intellectus esse unum omnium hominum, omnino est impossibile [...] Similiter etiam patet hoc esse impossibile, si, secundum sententiam Aristotelis, intellectus ponatur pars, seu potentia, animae quae est hominis forma. *Impossibile est enim plurium numero diversorum esse unam formam, sicut impossibile est quod eorum sit unum esse* [quotation 50; my italics]: nam forma est essendi principium (*Summa theologiae*, 1, 76, 2).

The argument in quotation 51, of which quotation 50 is a part, is used by Aquinas to decide the question 'Utrum intellectivum principium multiplicetur secundum multiplicationem corporum' in the positive. For deciding this question affirmatively, it is sufficient to suppose that different *material objects* have different actuating forms, that is: it is sufficient to accept I3, which, of course, is logically equivalent to its contrapositive: $\forall x \forall x'(\mathrm{M}(x) \wedge \mathrm{M}(x') \wedge \neg x = x' \supset \neg \mathrm{a}(x) = \mathrm{a}(x'))$. This is what Aquinas intends by 'impossibile est enim plurium numero diversorum esse unam formam'. In consequence, different human beings have different souls [or in other words: $\forall x \forall x'(\mathrm{H}(x) \wedge \mathrm{H}(x') \wedge \neg x = x' \supset \neg \mathrm{a}(x) = \mathrm{a}(x'))$], and therefore also the intellects of different human beings are different. (One may doubt this last inference, but Aquinas certainly did not.)

Quotation 51 (and quotation 50) contains more propositional information than is pertaining to I3 alone, namely, also propositional information that is represented by the principle $\forall x \forall x'(\mathrm{M}(x) \wedge \mathrm{M}(x') \wedge \mathrm{s}(x) = \mathrm{s}(x') \supset x = x')$. If we say that part of what is asserted in quotation 51 is represented by I3, and *not* – in spite of the literal wording of that quotation – by $\forall x \forall x'(\mathrm{a}(x) = \mathrm{a}(x') \supset x = x')$, it is only appropriate to say that another part of what is asserted in quotation 51 (i.e., what follows after "sicut") is represented by $\forall x \forall x'(\mathrm{M}(x) \wedge \mathrm{M}(x') \wedge \mathrm{s}(x) = \mathrm{s}(x') \supset x = x')$, and *not* – again in spite of the literal wording – by $\forall x \forall x'(\mathrm{s}(x) = \mathrm{s}(x') \supset x = x')$. However, there is independent support in the works of Thomas Aquinas for this latter, logically stronger principle:

52. esse diversum est in diversis (*De ente et essentia*, 5, 30).

Thus, I accept as a fourth Thomasic individuation axiom

I4 $\forall x \forall x'(\mathrm{s}(x) = \mathrm{s}(x') \supset x = x')$.

XVIII. Pure form and universal form

I4 shows that, while not every sentence of T* having the form $\forall \upsilon \forall \upsilon'(\varphi[\upsilon] = \varphi[\upsilon'] \supset \upsilon = \upsilon')$ is a Thomasic individuation axiom, at least one sentence of T* with that form is indeed a Thomasic individuation axiom. Is $\forall x \forall x'(\mathrm{f}(x) = \mathrm{f}(x') \supset x = x')$ another one? It seems the question has to be answered in the negative. There are in fact not just one but many human beings, and they all have the same pure (substantial) form: humanity (*whereas* each of the many human beings has a different actuating (substantial) form, i.e., a different soul); hence there are different objects that have the same pure form.

This argument presupposes that the pure form of a human being x is *humanity.* But alternatively we could say that the pure form of a human being x is not humanity but rather *the humanity of* x, and that if x and x' are different human beings, then *the humanity of* x – the pure form of x – *is different from the humanity of* x' – the pure form of x'; and that consequently there is no counterexample to $\forall x \forall x'(\mathrm{f}(x) = \mathrm{f}(x') \supset x = x')$ in the realm of human beings. Now, which of these two arguments is in the spirit of Thomas Aquinas?

In Thomasic ontological doctrine there are several distinctions concerning *forms* (the word "form" is still to be taken in the sense of 'substantial form'). The distinction between subsistent forms and non-subsistent forms has already been considered in this treatise, and so has been the distinction between pure forms and actuating forms (a distinction indispensable for Thomasic ontological doctrine but unfortunately not recognized by Aquinas himself). Now, concerning forms, yet another Thomasic distinction becomes crucial. It is the distinction between *universal* and *individual forms*. In the following passages Thomas is speaking about universal forms, and *implicitly* also about individual forms:

53. formae quae sunt receptibiles in materia, individuantur per materiam, quae non potest esse in alio, cum sit primum subiectum substans: forma vero, quantum est de se, nisi aliquid aliud impediat, recipi potest a pluribus (*Summa theologiae*, 1, 3, 2).

54. Forma vero finitur per materiam, inquantum forma, in se considerata, communis est ad multa: sed per hoc quod recipitur in materia, fit forma determinate huius rei (*Summa theologiae*, 1, 7, 1).

Universal forms that are receivable in matter 'can be received by many' and are individuated by matter. But individuated universal forms are, of course, no longer universal forms, they are individual forms. For example, as a consequence of being individuated by the matter of the human being x, the universal form *humanity* becomes an individual form: *the humanity of* x.

According to Thomasic ontological doctrine, every object has exactly one individual (substantial) form, and, *normally* (for more information see section XXIV), exactly one universal (substantial) form. The universal form of an object is simply its (natural) kind (*species*), which is particularized in the object to constitute the individual form of the object. Thus, for every human being x, *the universal (substantial) form of* x is *humanity*, and *the individual (substantial) form of* x is *the [particular] humanity of* x. We already know that the actuating form of a human being x – $\mathrm{a}(x)$ – is *the soul of* x. But now, what is the pure form of x: $\mathrm{f}(x)$?

I proceed on the assumption that the functions assigning the universal form and the individual form to objects are each identical (on the basis of Thomasic doctrine) with one of the six fundamental aspect-functions treated in this treatise. From these six, we can rule out *the matter of*, *the empty aspect of*, and *the being of*; obviously, neither *the universal form of* nor *the individual form of* can be identified with one of *those* three. Then we are left with the following possibilities of identification:

	f	a	w
(i)	u	i	
(ii)	i	u	
(iii)	u		i
(iv)	i		u
(v)		u	i
(vi)		i	u

Possibilities (ii) and (v) can be ruled out because *the universal form of* is not identical with *the actuating form of*. These aspect functions differ because *the universal form of* a human being x – humanity – is identical with the universal form of a human being x' – again humanity – even if x and x' are different; whereas *the actuating form of* x – the soul of x – differs from the actuating form of x' – the soul of x' – if x and x' are different, as Thomas explicitly asserts (see quotation 49).

And possibilities (iv) and (vi) can be ruled out because *the universal form of* is not identical with *the essence of*. Thomas Aquinas says:

55. Dato enim quod esset aliquod corpus infinitum secundum magnitudinem, utpote ignis vel aer, non tamen esset infinitum secundum essentiam: quia essentia sua esset terminata ad aliquam speciem per formam, et ad aliquod individuum per materiam (*Summa theologiae*, 1, 7, 3).

I take this quotation in its second part (after the colon) to make a statement not only about material objects that have infinite magnitude, but about all material objects. Consequently, the essence of any human being x is (uniquely) determined (*per materiam*) to some 'individuum', that is: to some (individual) object: x itself; the universal form of x, however, is not thus determined, since it is common to several human beings. Thus, if x and x' are different human beings (a fulfilled condition!), then the essence of x is different from the essence of x', but the universal form of x is nevertheless identical with the universal form of x'. And therefore *the essence of* and *the universal form of* are not one and the same function. (From quotation 55, we may also conclude that $\forall x \forall x'(M(x) \wedge M(x') \wedge w(x) = w(x') \supset x = x')$ – *Material objects are identical if their essences are identical*

– ought to be provable in TO**, either trivially, as an axiom, or more or less nontrivially, as a theorem. Concerning this matter, see I5 in section XX.)

We are now left with the possibilities (i) and (iii). Whichever of the two we choose, *the universal form of* turns out to be identical with *the pure form of*. Therefore, if *the universal form of* and *the individual form of* are each assumed to be identical with one of the fundamental aspect-functions treated in this treatise, then for every object *x*, the pure form of *x* turns out to be identical with the universal form of *x*.

However, the basic supposition that led us to this result, the supposition that *the universal form of* and *the individual form of* are each identical with one of the fundamental aspect-functions, may seem to be insufficiently supported by Thomasic evidence. Consider, therefore, also a different line of argument for the conclusion that the function *the pure form of* and the function *the universal form of* are the *same* function. The word "forma" in its occurrence in quotation 55 means *pure form*, since it is used to speak about a constituent of *essence*. Thus, quotation 55 says, in its second part, that the essence of a material object is determined by the object's pure form to a certain species: the universal form of the object; but that it is not determined by the object's pure form (alone) to a certain 'individuum': the object itself; rather, this latter determination is effected by (with the essential help of) the matter of the object. Therefore, the pure form cannot be the individual form of the object, since the individual form of the object (if it were the pure form and, as such, a constituent of essence) would already *by itself* determine the essence of the object to a certain 'individuum', because the individual form cannot be common to several 'individua' (i. e., several objects); hence we can very plausibly assert that, according to Aquinas, the pure form of a material object (i.e., the aspect that determines its species) is its universal form (i.e., the aspect that *is* its species).

And this assertion about material objects can be generalized: *the pure form of any object is its universal form*. This generalization is not supported by any direct textual evidence I am aware of (but there is also no textual evidence against it I am aware of) but strongly recommends itself by its positive effect on the systematization of other Thomasic doctrines, as we shall see.

Given as determined Thomasic doctrine that the pure form of any object is its universal form, it is, after all, clear that $\forall x \forall x'(\mathrm{f}(x) = \mathrm{f}(x') \supset x = x')$ cannot be regarded as a Thomasic individuation axiom or principle, since, obviously, there are human beings who are different although their universal forms (species) – coinciding with their pure forms – are the same form: humanity.

XIX. Individual form and essence

The following considerations show (on the basis of Thomasic doctrine) that *the actuating form of* and *the individual form of* are not identical aspect-functions. There are different objects that have the same actuating form (see T50); but there are no different objects that have the same individual form: different objects have different individual forms. Therefore, the only possibility left – under the assumption that allowed us to draw up the list (i) – (vi) in the previous section, and under the already established exclusion of possibilities (ii), (iv), (v), and (vi), and under the just presented exclusion of possibility (i) – is item (iii): *the individual form of* is identical with *the essence of (*together with the already established finding that *the universal form of* is identical with *the pure form of).* There is independent evidence for this:

(a) On the one hand, according to quotations 53 and 54, the individual form of a material object is determined by *the universal form and the matter of the object*. On the other hand, the essence of a material object is determined (more precisely speaking, is composed) by the pure form and the matter of the object (see quotation 1), that is, by *the universal form and the matter of the object*; hence it is plausible (although not inevitable) to conclude that the individual form of a material object is its essence.

(b) According to quotation 55, the essence of a material object is determined *to* a certain species ('*ad* aliquam speciem') and *to* a certain (individual) object ('*ad* aliquod individuum'); but this is also true of the material object's individual form: it is determined to a certain object, and hence also to a certain species (namely, the species of the object). It is again plausible to conclude that the individual form of a material object is its essence.

As in the case of the identity of pure form and universal form, so also in the case of the identity of individual form and essence: the generalization from material objects to *all* objects is not supported by any direct textual evidence – unless we count *Summa theologiae,* 1, 3, 3, where Aquinas argues for the identity of God and God's essence and finally concludes in the *responsio*:

56. Et sic, cum Deus non sit compositus ex materia et forma, ut ostensum est, oportet quod Deus sit sua deitas.

The reason given for the identity of God with God's essence (i.e., for the identity of God with God's divinity) is insufficient (since there are objects which are not composed of matter and actuating form but which are, nevertheless, not their essences). But this is not what is important here. What is important here is that the identity of God with his divinity – the identity of God with his individual form – is obviously meant to be the identity of God with his essence. For Thomas, the individual form of God is the essence of God.

In the *sed contra* of the same article (*Summa Theologiae*, 1, 3, 3) it is concluded, *not* that God is *his* divinity, but rather that God is divinity itself:

57. Deus est ipsa deitas.

From this we can infer that the identity of God and divinity itself – the identity of God and his universal form – is *also* meant to be the identity of God and his essence (since the article in question is about the identity of God and his essence); hence not only the individual but also the universal form of God is the essence of God. This is not surprising: in an immaterial object, essence – i.e., individual form – and pure form – i.e., universal form – are identical (see T3). The individual form of God *is* the universal form of God, and consequently some individual form is a universal form, which means that *universal* must not be equated with *non-individual.* (Not every universal form is a non-individual form; nor is every non-individual form a universal form, as will be seen in the last section of this treatise.)

I append one more reason for the identity of essence and individual form (according to Thomasic doctrine): The second argument in *Summa theologiae*, 1, 3, 3, for the opposite of what Thomas seeks to establish is as follows:

58. Praeterea, effectus assimilatur suae causae: quia omne agens agit sibi simile. Sed in rebus creatis non est idem suppositum quod sua natura: non enim idem est homo quod sua humanitas. Ergo nec Deus est idem quod sua deitas.

In his refutation of this *argument by analogy*, Thomas denies that the similarity it appeals to is sufficient for its conclusion. What he does not deny is that a human being *is not* its (i.e., his or her) humanity, that is, its individual form, and the non-identity of a human being and its individual form is clearly meant to be the non-identity of the human being and its essence (given the context of *Summa theologiae*, 1, 3, 3). Thus, the essence of a human being appears to be for Aquinas its individual form (*sua humanitas*). In the same article, however, Aquinas also quite generally identifies essence and pure form (= universal form), as was mentioned in section

VIII, footnote 9. If we followed *that* identification, then the essence (*natura*) of a human being (*quo homo est homo*) would be *humanitas* tout court, as Thomas explicitly says in the article. (See also *Summa contra gentiles*, 1, 21.) Now, the essence of a human being cannot be both the human being's individual form and the human being's universal form, because a human being is a material object: consider quotations 53 and 54, which clearly imply that individual form and universal form are non-identical in a material object. In order to avoid the inconsistency, I ignore Thomas's assertion to the effect of a general identification of pure form and essence, which assertion also contradicts what he says elsewhere (see quotation 5). (Perhaps there is no real inconsistency: Thomas may simply be using one word – "essentia", or "natura" – in two different senses: *individual essence* and *universal essence*.)

XX. A further individuation axiom

The upshot of sections XVIII and XIX is that we can read f(t) both as 'the pure form of t' and as 'the universal form of t', and that we can read w(t) both as 'the essence of t' and as 'the individual form of t'. We know, then, quite well what the pure form and the essence of an object is, since the universal (substantial) form of an object is its *kind or species,* and the individual (substantial) form *the species of the object in the object* (i.e., as particularized in the object).

The identity of essence and individual form leads to the acceptance of yet another Thomasic individuation axiom:

I5 $\forall x \forall x'(\mathrm{w}(x) = \mathrm{w}(x') \supset x = x')$
(Objects that have the same essence – the same individual form – are identical)

Using I5 and T3, we obtain

T53 $\forall x \forall x'(\neg \mathrm{M}(x) \wedge \neg \mathrm{M}(x') \wedge \mathrm{f}(x) = \mathrm{f}(x') \supset x = x')$
(Immaterial objects that have the same pure form are identical).
Proof: Assume $\neg \mathrm{M}(x) \wedge \neg \mathrm{M}(x') \wedge \mathrm{f}(x) = \mathrm{f}(x')$; hence by D3 and T3: $\mathrm{w}(x) = \mathrm{f}(x) \wedge \mathrm{w}(x') = \mathrm{f}(x')$, and therefore $\mathrm{w}(x) = \mathrm{w}(x')$ [because of $\mathrm{f}(x) = \mathrm{f}(x')$]; hence by I5: $x = x'$.

According to T53, there are no two immaterial objects of the same species; there are as many species of immaterial objects as there are immaterial objects. This agrees with Thomasic doctrine:

59. *Secunda* differentia [inter essentiam substantiae compositae et essentiam substantiae simplicis] est quia essentiae rerum compositarum ex eo quod recipi-

untur in materia designata multiplicantur secundum divisionem eius, unde contingit quod aliqua sint idem specie et diversa numero. Sed cum essentia simplicis non sit recepta in materia, non potest ibi esse talis multiplicatio; et ideo oportet ut non inveniantur in illis substantiis plura individua eiusdem speciei, sed quotquot sunt ibi individua, tot sunt species, ut Avicenna expresse dicit (*De ente et essentia*, 4, 25).

In the *Summa theologiae,* however, Aquinas excludes human souls from the general principle expressed by T53. According to him, there are many *human souls* of the same species, albeit there are not many (in fact, not two) *angels* of the same species:

60. licet anima intellectiva non habeat materiam ex qua sit, sicut nec angelus, tamen est forma materiae alicuius; quod angelo non convenit. Et ideo secundum divisionem materiae sunt multae animae unius speciei: multi autem angeli unius speciei omnino esse non possunt (*Summa theologiae*, 1, 76, 2).

What forces Aquinas to exclude human souls from the general principle that, in immaterial objects, there are as many species as there are objects ('individua') is the following opposing (Averroistic) argument:

61. [1] Nulla enim substantia immaterialis multiplicatur secundum numerum in una specie. [2] Anima autem humana est substantia immaterialis [...] [3] Non ergo sunt multae in una specie. [4] Sed omnes homines sunt unius speciei. [5] Est ergo unus intellectus omnium hominum (*Summa theologiae*, 1, 76, 2).

Aquinas cannot accept [5]; his way out is to deny [3] (see quotation 60), and therefore [1] (given [2]), thus contradicting T53 *and* what he says in one place of *De ente et essentia*. In another place of *De ente et essentia*, however, he states the same as in the *Summa theologiae*:

62. Et ideo in talibus substantiis [substantiis creatis intellectualibus] non invenitur multitudo individuorum in una specie, ut dictum est, nisi in anima humana propter corpus cui unitur (*De ente et essentia*, 5, 31).

But to deny [3] in quotation 61 is certainly not the most reasonable way out for Thomas Aquinas (who, of course, does not wish to accept Averroism: see quotation 51). For how does [5] 'There is one intellect of all human beings' follow from the conjunction of [4] 'All human beings are of the same species' and [3] 'Each human soul is the only one in its species'? Only by tacitly supposing that 'All human beings are of the same species' *validly* *entails* 'All human souls are of the same species'[36] – and the validity of this entailment is surely not beyond reasonable doubt. In fact, we are well-advised *not to* accept its validity if we, unlike Thomas in some places of his works, *accept* T53 just as it stands. For, if we accepted the validity of the entailment in question, we could deduce, with the help of T53, the absurd conclusion that there is *at most one* human being:

Suppose (for *reductio*) that x and x' are *two* human beings; though they are two, they are of the same species. Consider (making use of C2) the souls x'' and x''' of these human beings: $x'' = a(x)$ and $x''' = a(x')$. Suppose (as is tacitly – and problematically – supposed in the Averroistic argument of quotation 61) that if x and x' are of the same species, x'' and x''' must be of the same species, too. Then, according to T53, it follows that x'' and x''' are identical (since both x'' and x''' are immaterial objects and $f(x'') = f(x''')$, x'' and x''' being of the same species). Therefore: $a(x) = a(x')$, and therefore because of I3 (since x and x' are material objects): $x = x'$ – contradicting the initial assumption.

XXI. The interpretation of the being-aspect (*esse*) of objects

Essence, pure form, and actuating form of an object are *formal* aspects of it. Of the essence and pure form of an object, we now have a fairly precise understanding; our understanding of its actuating form depends on the understanding of its being. What is the being (or *esse*) of an object?

It is another formal aspect of it. According to Aristotelian-Thomasic doctrine every universal form F (substantial or not) that applies to an object x is individualized in x: the F of x. Now, *existence* is a universal form that applies to every object (recall that "object" was stipulated to mean as much as '*existing* object' in section III, footnote 1); hence existence is individualized in every object x: *the existence of* x. Like the whiteness of x is that by which x, if it is white, is white, so the existence of x is that by which x exists; and, for Thomas, *the being (esse) of* x is that by which x *is*:

63. Unumquodque est per suum esse (*Summa contra gentiles*, 1, 22).

36. From 'All human souls are of the same species' and 'Each human soul is the only one in its species', it follows that there is at most one human soul, and hence that there is *precisely one* human soul (since there is *at least one* human soul), and therefore, finally, that there is precisely one human intellect (since there are as many human intellects as there are human souls).

Therefore, accepting 'x is' as a synonym of 'x exists', we may safely conclude: the being of x is, for Thomas, its existence.

Like the whiteness of x is different from the whiteness of x' if x and x' are different white objects, so the existence of x is different from the existence of x' if x and x' are different objects; hence it is clear *why* I4 is a Thomasic individuation axiom (the *esse* of any object being its existence, as has just been established for the interpretation of Thomasic doctrine).

I4 and I5, in their intended interpretation, can be regarded as consequences of a more general principle, which I state informally as follows:

IP If F is a universal form (substantial or not) that applies to object x and F' a universal form that applies to object x' and *the* F *of* x [or: F *in* x] is *the* F' *of* x' [or: F' *in* x'), then x is identical to x'.

Given IP, I4 can be deduced as follows (making use of the identification of the being of an object with the object's existence): Assume x is an object and x' is an object; hence existence is a universal form that applies to both; assume, moreover, the being of x [$s(x)$] is the being of x' [$s(x')$]; hence the existence of x is the existence of x'. Consequently by IP: x is identical to x'.

And I5 can be deduced as follows (making use of the identification of the essence of an object with the object's individual [substantial] form): Assume x is an object and x' is an object; assume, moreover, the essence of x is the essence of x'; hence the individual (substantial) form of x is the individual (substantial) form of x'. The species of x, F_1, is a universal form that applies to x, and the species of x', F_2, is a universal form that applies to x'; moreover, the individual form of x is F_1 – the species of x – *in* x; the individual form of x' is F_2 – the species of x' – *in* x'; hence F_1 *in* x is F_2 *in* x'. Consequently by IP: x is identical to x'.

XXII. The interpretation of the actuating form

The actuating form of an object is determined by the pure form *and* the being of the object, that is, by its species *and* its existence. In which manner? For answering this question, it must first be noted that the sentence $\exists x \exists x'(a(x) = a(x') \wedge \neg f(x) = f(x') \wedge \neg s(x) = s(x'))$ becomes provable in TO** if we add as an axiom the following sentence:

I6 $\forall x \forall x'(M(x) \wedge \neg M(x') \supset \neg f(x) = f(x'))$
(Material and immaterial objects are not of the same species)

Axiom I6 can easily be brought into the form of an individuation principle,[37] and Aquinas would certainly have agreed to it. Before turning to the theorem mentioned just before the introduction of I6, let it be noted that I6 allows us to deduce 'Immaterial objects are singular in their species' (in other words, 'There is no immaterial object that is of the species of an immaterial object x in addition to x [on top of: There is no *material* object that is of the species of x]') to be deduced from 'Immaterial objects that are of the same species are identical' (i.e., T53):

T54 $\forall x(\neg M(x) \supset \forall x'(f(x') = f(x) \supset x' = x))$
Proof: Assume $\neg M(x)$ and $f(x') = f(x)$; hence by I6: $\neg M(x')$; hence by T53: $x' = x$.

And now the already announced theorem:

T55 $\exists x\exists x'(a(x) = a(x') \wedge \neg f(x) = f(x') \wedge \neg s(x) = s(x'))$
Proof: By C3: $\exists xH(x)$; hence by C2: $\exists x(H(x) \wedge \exists x'(I(x') \wedge x' = a(x)))$, hence by C1 and D6: $\exists x\exists x'(M(x) \wedge \neg M(x') \wedge x' = a(x))$; hence by D3 and T1: $\exists x\exists x'(M(x) \wedge \neg M(x') \wedge x' = a(x') \wedge x' = a(x))$; hence by I6 and I4: $\exists x\exists x'(a(x) = a(x') \wedge \neg f(x) = f(x') \wedge \neg s(x) = s(x'))$.

T55 shows that the actuating form of an object is determined by the object's species [pure form] and existence [being] in a manner that is different from the manner in which the individual form [essence] of it is determined by its species [pure form] and its matter. In contrast to T55, we have because of I5:

T56 $\forall x\forall x'(w(x) = w(x') \supset f(x) = f(x') \wedge m(x) = m(x'))$

However, the principle for the actuating form of an object that would be precisely analogous to T56 (but is falsified by T55) is preserved to a certain extent, since its restrictions

T57 $\forall x\forall x'(M(x) \wedge M(x') \wedge a(x) = a(x') \supset f(x) = f(x') \wedge s(x) = s(x'))$
T58 $\forall x\forall x'(\neg M(x) \wedge \neg M(x') \wedge a(x) = a(x') \supset f(x) = f(x') \wedge s(x) = s(x'))$

are indeed theorems of TO** because of I3 and T52.

We do know a considerable amount about the behaviour of the actuating form of an object in relation to other object-aspects, but it is doubtful whether an object's actuating form can *in itself* be satisfactorily described in ontological terminology that is familiar and (relatively) clear

37. The relevant form is $\forall \upsilon\forall \upsilon'(B_{con}[\varphi[\upsilon] = \varphi[\upsilon']] \supset A[\upsilon, \upsilon'])$; see the beginning of section XVII.

to us. We have seen that, according to Thomasic doctrine, the actuating form of a human being is its (his or her) soul, and that the immaterial objects are the subsistent actuating forms. But what has become clearer by this? What is a human soul? What is an immaterial object? To the latter question Aquinas would answer: God, or an angel, or a human soul. It seems we must rest content with this. But of course we can add (with Aquinas): the non-subsistent actuating forms – the actuating forms that are not objects – are the souls of animals and plants, and the actuating forms of inanimate material objects.

Vaguely, the actuating form of an object is that aspect of it that makes it exist (that is, *subsist*) as an object of a certain species (regarding merely the s(x) in a(x) – that is, in f(x) + s(x) – and its role for the existence of the object, see quotation 63). For example, the actuating form of this particular horse is what makes it exist as a horse. And *vivere est esse viventis* (cf. quotation 39); hence the actuating form of this horse is what makes it *live* as a horse. But what makes the horse live as a horse is also the horse's soul; for the soul is the principle of life:

64. anima dicitur esse primum principium vitae in his quae apud nos vivunt; *animata* enim viventia dicimus, res vero *inanimatas* vita carentes (*Summa theologiae*, 1, 75, 1).

Thus, the actuating form of the horse is its soul. And thus, the line of Thomasic thinking that leads to the general identification of soul and actuating form in living material objects is apparent.

XXIII. The interpretation of the matter-aspect of objects

The matter of an object is (leaving aside its empty aspect) the one non-formal aspect of the object. There are two difficulties concerning that aspect:

The matter of an object is what it materially consists of. *When?* Living material objects do not materially consist of the same at each instant of their existence. Let x be a human being. Which instant of the existence of x shall we select so that what x materially consists of at that instant is *the* matter of x? This is the first difficulty.

The second difficulty is this: *What* does a material object (at a given time) materially consist of? There are many levels of decomposition relative to which an answer can be given to this question. We may say that human being x materially consists of this entire *corpus potentia vitam habens*; or of this head and trunk, these arms and legs; or of this flesh and bones; or of these cells; or of these protein-molecules; and so on. Which level of decomposition shall we select so that what *x* materially consists of at that level is *what x materially consists of* (at the given time)?

The second difficulty can be resolved as follows: The matter of *x* (at a given time), that what *x* materially consists of, is not the collection of the material parts of *x* at a *certain* level of decomposition (in other words, is not what *x* materially consists of *at that level of decomposition*). The matter of *x* cannot be reached at any level of decomposition: there is no level of decomposition such that the matter of *x* is the collection of the material parts of *x* at that level of decomposition. Rather, for every level of decomposition, the matter of *x* is also the (collective) matter of the collection of the material parts of *x* at that level of decomposition. Thus, the matter of *x* is an intangible entity, no less intangible, in a different way, than the species of *x*. However, in an analogical sense, the *corpus potentia vitam habens* of *x* can be called "the matter of *x*" since this *corpus* is certainly *the first (tangible) representative* of the matter of *x*. *Nota bene*: Not the *corpus potentia vitam habens* of *x*, and not the collections of material parts of *x* at any *other* (i.e., higher than *zero*) level of decomposition for *x*, are counted as material objects. Otherwise we would have as many counter-instances to I2 as there are levels of decomposition for *x*.

Concerning the first of the above-described difficulties, relevant passages can be found in the *Summa contra gentiles*, 4, 81. Thomas is confronted with the problem of determining with which matter the soul is reunited at the resurrection to make up the resurrected human being. This problem arises because at different times of life different matter was in the human being. He rejects the idea that the soul is reunited with the totality of matter that was in the human being while alive; rather, the soul is reunited with a sufficient part of this totality. *Which* part? Thomas suggests: that part which was existing in a 'more perfect' manner ('perfectius') under the form (species) of humanity:

65. non requiritur ad hoc quod resurgat homo numero idem, quod quicquid fuit materialiter in eo secundum totum tempus vitae suae resumatur: sed tantum ex eo quantum sufficit ad complementum debitae quantitatis; et praecipue illud resumendum videtur quod perfectius fuit sub forma et specie humanitatis consistens (*Summa contra gentiles*, 4, 81).

Following Aquinas's suggestion, I select an instant of time in the prime of life of human being *x* and determine that *the* matter of *x* is what *x* materially consists of at *that* instant.

XXIV. Synopsis of the Thomasic theory of forms

A synopsis of Thomas Aquinas's theory of forms concludes this paper. The *universal forms* are the entities designated by abstract nominalizations ('beautiful' – '*beauty*', 'human' – '*humanity*', 'just' – '*justice*', 'woman' – '*femininity*', 'exist' – '*existence*'). Some universal forms are *substantial forms,* but most are not; *substantial universal forms* are, for example, (exemplified) species or kinds (for example, *humanity, divinity, caninity*). A substantial universal form is a substantial universal form *of some object*, and a universal form F is a substantial universal form of object *x iff* F is substantial for *x* and *x* has F.

[I] Every object (i.e., substance or quasi-substance: subsistent entity) has a unique universal form which is *representatively* substantial for it; that form is *its* [representative] *universal* [substantial] *form*, or in other words: *its species*, or: *its pure* [substantial] *form*. (The square brackets indicate that what they enclose is usually left implicit in this treatise.)

A special universal form is *existence.* Though it is not a species, *existence*, too, is in a sense a substantial form: it is a substantial universal form *of some object* (certainly not *of every object*, though *existence* is, indeed, a form had by every object). This will be shown later in this section.

Every universal form is individuated in the object which has it:

[II] There is at most one individuation of a universal form F in an object *x* (whatever the universal form F and whatever the object *x*).

[III] For all universal forms F and objects *x*: If *x* has F, then there is an individuation of F in *x*: *the* individuation of F in *x*,[38] or in other words: F in *x*, the F of *x*.

And vice versa:

[IV] For all universal forms F and objects *x*: If there is an individuation of F in *x*, then *x* has F.

Furthermore:

[Va] For all objects *x* and all *f*: *f* is an individual form of *x iff* there is a universal form F such that *f* is an individuation of F in *x*.

[Vb] For all objects *x* and all *f*: *f* is a *substantial individual form* of *x iff* there is a universal form F such that F *is substantial for x* and *f* is an individuation of F in *x*.

38. The transition from 'an' to 'the' can be made in consideration of [II].

[VI] Every object has *exactly one* substantial individual form: *its individual* [substantial] *form*, or: *its essence*.

Normally, there is just one universal form that is substantial for an object and that the object has: its species (so that, normally, the *representative* substantial universal form of the object – see [I] – is at the same time the *only* substantial universal form of it). It may, however, happen that more than one universal form F is substantial for object x and such that f is an individuation of F in x; *still*, [VI] requires that all substantial individual forms f of x that result in this case – in accordance with [Vb] – are identical to each other: they coalesce in *the* substantial individual form of x: in the essence of x. We shall soon have occasion to come back to this unusual, non-normal ontological situation.

Quite generally, however, the essence of x – the substantial individual form of x – is the individuation of the species of x in x. For the species of x is a universal form which is substantial for x and which x has, that is (in accordance with [III]): there is an individuation of the species of x in x, and hence (in accordance with [II]) there is *exactly one* individuation of the species of x in x. It follows, according to [Vb], that *the* individuation of the species of x in x is a substantial individual form of x, and hence, according to [VI], that the individuation of the species of x in x is *the* substantial individual form of x: the essence of x.

In material objects, the individuation of the species of x in x [w(x)] is different from the species of x [f(x)]. In immaterial objects, however, the individuation of the species of x in x is the species of x itself. Therefore, if there is an immaterial object (which Thomas does not doubt), then some universal (and substantial) form, namely its species, is an individual (and substantial) form.

Concerning existence there are the following two *theorems*:

[VII] For every object x: the existence of x [s(x), the being of x] is an individual form.
Proof: Assume x is an object; *existence* is a universal form had by x (since, in this treatise, objects are understood to be *existing* objects); hence by [III]: there is an individuation of *existence* in x; hence by [II]: *the* individuation of *existence* in x is an individuation of *existence* in x; hence by [Va]: the individuation of *existence* in x – in other words, the existence of x – is an individual form.

[VIII] The existence of God is a *substantial* individual form.
Proof: By T44: the existence of God is the essence of God [s(**d**) = w(**d**)]; the essence of God – like the essence of every other object – is a substantial indi-

> vidual form (in accordance with [VI]); hence the existence of God is not only an individual form, but also a *substantial* individual form.

The existence of God is a substantial individual form *of* God. This follows by [Vb] because the universal form *existence* is substantial for God *and* because there is precisely one individuation of *existence* in God, according to [III] and [II], since God has *existence*; the existence of God is nothing else than *that* individuation and it is as much a substantial individual form of God as *that* individuation is (according to [Vb]). Thus, not only the species of God – *divinity* – but also *existence* is substantial for God, and both *divinity* and *existence* are substantial universal forms of God. It follows (by [Vb] and [VI]) that the corresponding substantial individual forms of God are identical: the existence of God is the divinity of God [s(**d**) = w(**d**) = f(**d**)].

This is the unusual, non-normal ontological situation described above in a general manner. There is no, indeed, object other than God whose existence is not only an individual form but also a *substantial* individual form (namely, of God).

It is entirely in the spirit of Aquinas to postulate principle IP here, which, however, is already to be found in section XXI; so I do not repeat it here.

A *substantial actuating form* is the composite of the species of an object – that is, of the representative substantial *universal form* of the object (see [I]) – *and* the existence of the object: an *individual form*. Every object has exactly one substantial actuating form: *its actuating* [substantial] *form*, since there is exactly one substantial actuating form which is the composite of the object's species and the object's existence. The immaterial objects are the *subsisting* substantial actuating forms. But no individual or universal form *in itself* (and not *in composition*, like a universal form and an individual form in a substantial actuating form) *subsists* – with one exception: *divinity* (= the species of God = the divinity of God = the individuation of the species of God in God [f(**d**) = w(**d**)] = the existence of God [w(**d**) = s(**d**)] = God [s(**d**) = **d**]), which is both a subsisting universal form and a subsisting individual form. Since no individual or universal form, except *divinity*, subsists, every subsisting substantial actuating form that is not *divinity* (= the substantial actuating form of God, since f(**d**) = a(**d**)) is neither a universal nor an individual form. (A subsisting substantial actuating form – different from *divinity* – is not an individual form in the sense that fits [Va], although, of course, it is *an individual – object – which* *is also a form*.) Thus, there are non-individual forms that are not universal forms, in other words: not every non-individual form is a universal form.

Divinity, and no other entity, is at once an object – something subsisting – and a form that is universal, individual, and actuating; it is, we may say, a form in the original Platonic sense. As far as God and divinity is concerned, Thomas Aquinas adheres to Platonism. (It is a distortion to see him as a pure Aristotelian.) As the Beautiful in itself, subsistent Beauty, is the object of intense emotion for Plato (as is apparent in the climax of Socrates's rendering of Diotima's speech in the *Symposion*), so subsistent Divinity – God – is for Thomas the object of intense emotion: *Adoro te devote, latens Deitas* ...[39]

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39. The evidence this hymn provides for the Platonism in Thomas's Christian devotion is slightly less perfect than the quoted words suggest. For, *originally* (as formulated by Thomas himself), the first line of the hymn read "Te devote laudo, latens veritas". See Robert Wielockx, "*Adoro te devote*. Zur Lösung einer alten Crux", *Annales Theologici* 21 (2007), 101-138, and see *ibid.* especially 136-138. Of course, *subsistent Truth* had been officially identified with God ever since the time of St. Augustine. Thus, *subsistent Truth* was to Thomas's mind nothing else than *subsistent Divinity*. But note the difference: *subsistent Divinity* is identical to *divinity*, but *subsistent Truth* is not identical to *truth* (just as *subsistent Esse* – God again – is not identical to *esse*).