A metaphoric exploration of objective constructivism

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Abstract: The dichotomy between constructive and objective accounts of the epistemic process is a persistent problem. This paper describes the theory of reconstruction in quantum physics that may give a metaphor for the possible compatibility of constructivist and realist accounts. The argument works by analog reasoning and suggests that local subjective and global objective descriptions may converge.

Keywords: Constructivism, Realism, QFT, Reconstruction theorems.

I Introduction

There is an ongoing debate between proponents of constructivist and realistic positions in epistemology. A wide spectrum of arguments is presented in the collection by Finkelde and Livingston (2020). In the introductory text, they state that "the concept of objectivity poses both conceptual and methodological problems" (p.1). One of the crucial problems is how to gain insights that are valid beyond the scope of the subjective view and interpretation of a specific individual subject. In more detail, the skeptical questions of constructivists and relativists can be put that way: How can one observer without access to all the things that exist arrive at an objective theory of the world? How can two observers that have different views on the world agree objectively on the world's structure? The question, how the individual, the subjective and the objective interrelate is also a topic discussed by Davidson (2001).

To clarify terminology, in this paper the positions are defined as follows: The constructivist position is characterized by assuming a strictly local nature of the epistemic process: Each single mind (or brain) infers from its restricted local information sources a theory of the world. Due to these restrictions, objectivity cannot be reached, and in radical versions (Glasersfeld 1990) not even the quality of approximation to some objective reality can be assessed. Many other authors – in a sense going back to Kant – can be seen as part of constructivism, although the theoretical underpinning may vary widely. Consider, e.g. (Latour 2005) who reinterprets reality as a construction of a network.

On the other hand, an objectivist position may assume that within the limits of physical laws of relativity and quantum mechanics objective knowledge can be gathered, and different subjects will converge in their approximation of reality (e.g. Weinberg 2001). "Reality" here means an advanced concept of reality, not naïve reality, i.e. one might assume that the quantum state in a Hilbert space is the reality per se. This view is also called "wave function reality" and certainly is not without alternatives, but there are good arguments in favor of it (Nay 2023) and this paper can restrict to it because the argument will be that the constructivist and realistic viewpoints may be compatible after all, and this argument is even better, when applied to a more or less extreme version of realism.

While a lot of philosophers in post-modernism tended to some form of constructivist epistemology, in the last decade, realistic positions have been revitalized (e.g. Ferraris 2015). An interesting approach to mediate between extreme positions was proposed by Giere (2006). In his program of "scientific perspectivism" he suggests that scientific knowledge is always constructed from a perspective, combining elements of realism and constructivism in a pragmatic way. Giere's perspectivism acknowledges that while scientific theories aim to reflect the real world, they do so through the lens of human conceptual frameworks and practices, and hence their local subjective observations. Although this can smooth the hard clash between positions, the problem seems not to be completely resolved. If two perspectives are fully compatible, they may be merged into a single one, and, if they are not, it remains unclear which rules govern the process of selecting an adequate selection.

This paper suggests a different approach: It investigates the idea, that there may be no real clash between constructivist and realistic positions at all, rather it may be just a question of the relative completeness of information.

The method will be an argumentation by analogy and metaphor. It draws on the concept of parallel reasoning as developed by Bartha (2010). In his theory, he supposes that there is a "vertical" relation between two features in the source domain and a "horizontal" relation of one of the features into the target domain. By analogical reasoning, the vertical relation gets transferred to the target domain (Bartha 2010, p. 24). In this paper, the source domain is the algebraic theory of quantum field theory, and the target domain is epistemology.

The paper first describes the reconstruction of the field algebra in the algebraic approach to quantum field theory, and then it uses parallel reasoning to give the main argument.

II Axiomatic algebraic quantum field theory

Physical theories try to give accounts of reality as precise and objective as possible. In traditional quantum physics, the state of a physical system (an atom or the universe) is described by a vector (wave function) in a Hilbert space. This space gives room to all possible states of the system and the vector in it determines the current real state. This vector is the quantum theoretic replacement for reality. This position may be termed as "wave function realism" and it is kind of an extreme position in the set of realistic interpretations of quantum mechanics (see e.g. French & Saatsi 2020). However, these different interpretations of quantum reality seem largely irrelevant to the problem discussed here.

Measurement operations are described by linear operators acting on the Hilbert space. The measurement does not disturb the system only if the current state is an eigenvector of the measurement operator. For quantum field theory, a first axiomatic theory that allowed rigorous proofs was given by Wightman (Streater &Wightman 1964). These axioms started from a Hilbert space and gave axioms for fields, e.g. that there is an action of the Lorentz group of special relativity. This theory allowed to derive some important results rigorously, for example that there is a unique Lorentz invariant state, the vacuum. However, the theory had some limitations and a key for further development was to focus more on the algebras of observables. The laws of physics are – in a way – encoded in the mutual behavior of these operators, e.g. the ladder operators for the harmonic oscillator allow to encode its laws algebraically. The same can be said about the Pauli matrix operators in the case of spin. This motivates a shift of interest from the states (Hilbert space) to the operators (algebras). Details are not important here, as this shall only give some motivation for the Wightman axioms of quantum field theory (QFT) developed further into a set of axioms that form the basis of a rigorous treatment of QFT. These axioms (Haag 1992) center around algebras of observables associated with regions of space-time. In the standard approach to QFT (e.g. Ryder 1985) and also in the older Wightman axioms, operators are functions (or even distributions) over the 4d space of special relativity, e.g. $\widehat{\Psi}(x)$, that fulfill canonical commutation relations $[\widehat{\Psi}(x), \widehat{\Psi}^{\dagger}(x')] = \delta(x - x')$. This has the consequence that these operators are unbounded. However, by smearing them out over some (possible very small) region Ω of space-time one gets operators $\int_{\Omega} \widehat{\Psi}(x)\mathrm{d}x$ that measure local information and that may be bounded so that the operator algebra becomes a C^*-algebra, which has nice mathematical properties. For example, states can be described by vectors in a Hilbert space that the algebra acts on, but equivalently as bounded linear functionals on the algebra. The whole setup is rather technical but well established. The following is a bird's-eye perspective on the theory.

Before going on, it must be acknowledged that in its present state, this axiomatic and algebraic approach to QFT is incapable of fully replacing the standard textbook approach to QFT. Local gauge groups cannot be handled and the need to use bounded operators instead of operator valued distributions makes explicit calculations very difficult. But this is a practical drawback that seems irrelevant to the question posed here.

The total field algebra $\mathcal{F}$ (acting faithfully on a Hilbert space $\mathcal{H}$) is the limit of the fields associated with a region $\mathcal{O}$ of space-time, i.e. $\mathcal{F} = \cup_{\mathcal{O}} \mathcal{F}(\mathcal{O})$. Naturally, $\mathcal{O}_1 \subset \mathcal{O}_2 \Rightarrow \mathcal{F}(\mathcal{O}_1) \subset \mathcal{F}(\mathcal{O}_2)$. The regions considered are usually just double cones, and this suffices. The special theory of relativity then implies that there is an action of the Lorenz-group on the fields, more precisely, if Λ is a Lorenz transformation that maps $\mathcal{O}_1$ to $\mathcal{O}_2$ then there is a transformation that maps $\mathcal{F}(\mathcal{O}_1)$ to $\mathcal{F}(\mathcal{O}_2) = \Lambda\mathcal{F}(\mathcal{O}_1)$.

Furthermore, a global gauge group $\mathcal{G}$ (a Lie group) operates on every $\mathcal{F}(\mathcal{O})$. Those elements $\mathcal{A}(\mathcal{O}) \subset \mathcal{F}(\mathcal{O})$ that are invariant under the action of the gauge group form the subalgebra of observables. These operator algebras are axiomatically assumed to have (among some other more technical points) the following properties:

- There is an action $U(a, \alpha)$ of the Poincare group (a is a translation and α is an element of the Lorentz group of special relativity) such that $U(a, \Lambda)\mathcal{A}(\mathcal{O})U(a, \Lambda)^{-1} = \mathcal{A}(a + \alpha\mathcal{O})$. The generator of time translations is the Hamilton operator, and it is assumed to have a spectrum bounded from below (positivity of energy).
- The algebra of a union of regions is generated from the observables of the regions $\mathcal{A}(\mathcal{O}_1 \cup \mathcal{O}_2) = \mathcal{A}(\mathcal{O}_1) \vee \mathcal{A}(\mathcal{O}_2)$.
- Causality: If $\mathcal{O}_1, \mathcal{O}_2$ are space-like separated then $[\mathcal{A}(\mathcal{O}_1), \mathcal{A}(\mathcal{O}_2)] = 0$.

Note that the causality axiom is not in odds with the Einstein-Podolsky-Rosen states and Bell's inequality, as explained by Haag (1992, p. 107).

The limit of the net of observable algebras is $\mathcal{A} = \cup_{\mathcal{O}} \mathcal{A}(\mathcal{O})$. Under some technical assumptions, it follows that $\mathcal{A}$ and $\mathcal{G}$ have isomorphic representation categories and that the total Hilbert space of $\mathcal{F}$ decomposes into a direct sum of tensor products formed by an irreducible representation of $\mathcal{A}$ and the corresponding irreducible representation of $\mathcal{G}$, i.e. $\mathcal{H} = \bigoplus_{i \in I} V_i \otimes T_i$, where V_i is an irreducible representation of $\mathcal{A}$ and T_i is an irreducible representation of $\mathcal{G}$. These summands that are indexed by I are called super selection sectors. To make this a bit more concrete, consider the example of quantum electro dynamics. In this theory the gauge group is $\mathcal{G} = U_1 = \{e^{i\phi}, \phi \in \mathbb{R}\}$. This commutative group has one-dimensional irreducible representations labeled naturally by the integers $\mathbb{Z}$ and hence the Hilbert space decomposes into direct summands labeled by integers, which, in this case, can be interpreted as the total electric charge of the states in the summand. Observables operate in these summands, hence they never change the total charge – it is a conserved quantity. Summing up, for quantum electrodynamics the theory explains why charge is quantized and why it is conserved.

III Doplicher-Roberts Reconstruction

This analysis of QFT is interesting in itself, but what shall be investigated here is the fact that the whole structure can be reconstructed from $\mathcal{A}(\mathcal{O})$. This is the content of the famous Doplicher-Roberts reconstruction theorem (Doplicher and Roberts 1990). It has been applied to a wide range of theories (Mueger 2000) and generalizations hold under very general assumptions, even if the gauge group is no longer a group at all (Häring-Oldenburg 1996). The chain of ideas leading to this result is approximately the following: By Lorenz invariance $\mathcal{A}(\mathcal{O})$ is essentially independent from $\mathcal{O}$, i.e. knowing the observables for one region, one knows it everywhere and hence one knows $\mathcal{A}$. This algebra defines its category of representations with a certain monoidal structure. Let's label the irreducible objects of this category by $i \in I$. Exploiting the monoidal structure of this category, one can first construct a Hopf algebra with the same representation category and this Hopf algebra is the universal enveloping algebra of a Lie algebra, from which finally, by its exponential map, the gauge group is found. This gauge group $\mathcal{G}$ is determined uniquely up to isomorphism and has the same representation category. Then the total Hilbert space of the theory is the direct sum $\bigoplus_{i \in I} U_i \otimes T_i$ where U_i is an irreducible representation space of $\mathcal{A}$ and T_i is the corresponding representation space of $\mathcal{G}$. Thus, one has the Hilbert space, and the full theory is reconstructed.

As mentioned, it is a drawback of this theory is that it deals only with global gauge invariance, i.e. the additional structure of the observable algebra that accounts for local gauge transformation invariance is not modeled. It is unclear if this limitation will be overcome in the future. However, while this is a problem for the physical applications of the theory, it does not pose any problem for the metaphoric use that will be made in the next section.

IV The Metaphor

Here, I rephrase the above theory from an epistemic perspective. Assume two physicists have access to some far-apart regions of space-time $\mathcal{O}_j, j = 1,2$. By performing various experiments, they can determine the observables $\mathcal{A}(\mathcal{O}_j)$. By applying the Doplicher-Roberts reconstruction, they will arrive at isomorphic Hilbert spaces, isomorphic gauge groups and isomorphic field algebras.

First, concentrate on one of these physicists. Although restricted to a limited set of observables, she can objectively construct unobservable structures like the fields and the gauge group. This is a metaphor for being able to overcome subjective observation. As a second insight, this can form as a model how inter-subjectivity may be realized and how it is related to objectivity.

This example shows that the restriction to local measurements and to a restricted set of observables will not necessarily imply that no objective reality can be (re)constructed. Of course, if the $\mathcal{O}_j$ are separated time-like, one physicist will not know what is actually realized in the region of the other physicist, but they will construct isomorphic theories of the laws of physics. That is, one of them may not know what is actually realized in the other's region, but he can be objectively sure about what is possible in the other's region by the laws of physic and what it not.

Thus, the analogy presented here makes it plausible, that the individual (re)construction of reality from individuals' different experiences does not necessarily make it impossible to discover an objective reality. However, the objective reconstruction (i.e., reconstruction up to isomorphism) described above of the filed algebra is only possible under certain conditions. Besides technical issues that can be left aside, the most important assumption is that both physicists can fully explore their local observable algebra $\mathcal{A}(\mathcal{O}_j)$. Of course, they may fail to do this, e.g. by missing to perform all possible measurements or by neglecting some features due to measurement errors. This is compatible with the fact that many physicists tend to an objective position (despite the existence of entangled states in quantum physics that tweak the traditional concept of reality), while many social scientists adhere to constructivist positions:

Obviously, in physics it is much easier to fully explore the locally available information fully. Equipped with a good measurement of the observables, a good reconstruction of the full theory can be achieved. In social sciences, on the other hand, local observations are much more incomplete and hence leave more freedom for different local observables $\mathcal{A}(\mathcal{O}_j)$, so that an objective global theory is much harder to achieve.

V Conclusion

The argument presented here makes plausible that objective knowledge of global reality may be achieved from local observations if they exhaust the space of possible observations. Incomplete local observations may, however, lead to different global theories. In this sense, the exhaustion quote of local observations maybe seen as a parameter that smoothly connects objectivists and constructivists positions.

VI Literature

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