The return on everything and the business cycle in production economies

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The return on everything and the business cycle in production economies[☆]

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ABSTRACT

Motivated by recent empirical evidence on returns on equity, bonds, and housing, we study interactions among an economy's total net worth, consisting of housing and equity, the business cycle, and three specific types of productivity risk: standard, long-run, and disaster. Preferences include habits or follow a generalized recursive form. Procyclical housing adjustments reduce consumption risk as residential investment determines the next-period amount of housing as a fraction of the composite consumption good. The existence of an asset that is safe in real terms and has a positive supply prevents versions with habits or long-run risk from simultaneously replicating risk premia, investment volatility, and housing demand. The disaster risk version replicates these targets. In all versions, a perfectly negative correlation between equity returns and the marginal utility of consumption places the equity Sharpe ratios in the upper bound of any Sharpe ratios (the Hansen–Jagannathan bound). Consequently, replicating Sharpe ratios of housing larger than equity is impossible.

1. Introduction

Consumption capital asset pricing models (CCAPMs) of production economies have made great progress in recent decades in simultaneously explaining asset prices and business cycle statistics. However, most models focus on equity and ignore housing, which accounts for 50% of the net worth in advanced economies. Taking housing into account challenges and strengths the explanatory power of these models. First, empirical evidence indicates a higher Sharpe ratio for housing than for equity (e.g., Jordà et al., 2019a). However, assets with dissimilar Sharpe ratios typically require separate conditions for the Hansen–Jagannathan bound (HJB) and the joint distribution of consumption risk and risk premia. Hitherto unconsidered, these separate conditions challenge existing approaches for reproducing Sharpe ratios. Second, in contrast to equity, housing serves as a consumption good and not as a factor of production. Thus, the return on housing is equivalent to a bond coupon consisting of a certain amount of a specific consumption good—in the case of housing, this is the largest category of the total consumption basket. Yet distinctly unlike bonds, housing net worth is positive, as it is a real asset. Hence, adjustments in the allocation of income between housing and nondurable consumption reduce aggregated consumption risk. Note that the empirical high reward for holding risk (see Mehra and Prescott, 1985) implies a high marginal propensity to reduce consumption risk. This high marginal propensity to reduce consumption risk, on the one hand, may explain the puzzlingly elastic and procyclical demand for housing structures evidenced by the highly volatile, procyclical comovement

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of residential investment and house prices despite smooth rents. On the other hand, the additional opportunity to reduce consumption risk may diminish the ability of existing approaches to generate a sizeable risk reward. This study addresses these two points by considering equity and housing simultaneously within the CCAPM framework in production economies.

Our framework is a standard real business cycle (RBC) model that features housing services as a durable consumption good. Note that housing and equity represent nearly all net-positive investible assets in closed economies and thus represent total domestic net worth. Principal elements for risk premia are the risk process and the model's SDF. Accordingly, we consider versions of both elements that are known to reproduce equity premia. Further, we concentrate on productivity and growth risk as the only sources of uncertainty. In addition to a standard productivity process, we incorporate long-run productivity risk as in Bansal and Yaron (2004) and Croce (2014) and disaster risk as in Rietz (1988), Barro (2006), and Gourio (2012). The SDF follows either from Chen's (2017) version of Campbell and Cochrane's (1999) external habit formation or from a generalized recursive utility introduced by Epstein and Zin (1989) and Weil (1989). Ergo, this study rechecks the compatibility of representative-agent production-based CCAPMs with housing data in addition to the two motives described above.

In response to the latter objects of investigation, i.e., whether these models can replicate sizeable risk premia and the procyclical demand for housing, and to recheck their general compatibility with housing, we challenge the models with stylized facts extracted from data from Jordà et al. (2019a) (JKKST) and OECD.Stats (2019). Business cycle statistics reveal the following well-known characteristics: (i) residential investments are at least moderately more volatile than business investments, (ii) house prices are more volatile than Gross Domestic Product (GDP), rents are less volatile than GDP, and prices fluctuate at least twice as much as rents, and (iii) house prices, business investments, and GDP are positively correlated with residential investments, and house prices and GDP are also positively correlated. Turning to asset prices, the stylized facts are (i) a stable risk-free rate less than 2.25%, (ii) rates of return on equity moderately higher than those of housing, (iii) risk premia all greater than 3%, (iv) rates of return and premia on equity that are at least twice as volatile as rates of return and premia on housing and on total risk, and as a result, (v) a Sharpe ratio for housing significantly larger than for equity.

In response to the first object of investigation, i.e., how a second risky asset with a different Sharpe ratio restricts possible explanations for the observed Sharpe ratio of equity, we decompose the Sharpe ratio into two factors: (1) the SDF's coefficient of variation (the HJB) and (2) the correlation between the asset's risk premia and the SDF. The HJB then defines a common upper bound for the Sharpe ratios of all assets. The second factor is asset-specific and determines the relative size of the Sharpe ratios. Thus, while only the size of the product of the two factors matters for the size of the Sharpe ratio of a single asset, the presence of a second asset with a different Sharpe ratio introduces separate bounds for each factor individually.

We find that a model with standard productivity risk and external habits can replicate housing demand and equity premia but underestimates housing premia and the volatility of business investments. Further, risk premia on both assets are perfectly negatively correlated with the model's SDF. Consequently, the Sharpe ratios of both assets are nearly equal to the HJB. The model's HJB is too small, and the Sharpe ratios are underestimated. Conversely, the equity premium relies on counterfactually volatile rates of return. The same holds for the already too-small housing premia.

A model with long-run productivity risk and Epstein and Zin (1989) preferences replicates equity premia but underestimates housing premia, the volatility of house prices, and the volatility of both investment goods. Yet compared with the previous model, premia are now generated in a different way. The HJB is larger and compensates for less volatile rates of return. Further, the model erases the nearly perfect correlation between rates of return and the SDF. Unfortunately, at odds with the data, housing premia in the model are distinctly less correlated with the SDF than premia on equity. Thus, the Sharpe ratio of housing falls below the Sharpe ratio of equity, and compared with the data, the Sharpe ratio of equity becomes too large while the Sharpe ratio of housing becomes too small.

Finally, the model with disaster risk reproduces the volatility and comovements of GDP, business investments, residential investments, house prices, and housing rents. The model replicates observed housing demand and other business cycle statistics, although the variables are too strongly correlated in part. Disaster risk premia are larger than those of the other risks studied. The model replicates the equity premium observed for the United States. However, housing premia fall short of empirical values by approximately 2 percentage points. Premia in the model rely on an HJB that is similar to the model with long-run risk. The perfect negative correlation between equity premia and the SDF disappears to some degree, yet the correlation remains far too large and rules out additional assets with substantially larger Sharpe ratios. Although the correlation between housing premia and the SDF increases compared with the model with long-run risk, it still falls below the correlation between equity premia and the SDF. As a result, the Sharpe ratio of equity is larger than in the data, whereas the Sharpe ratio of housing is too small. If simulations exclude disasters, decomposition of Sharpe ratios into the HJB and correlations between premia and the SDF do not hold on average since the simulated distribution deviates from the expected distribution. In these simulations, the Sharpe ratio of housing exceeds the Sharpe ratio of equity, although equity premia are more strongly correlated than housing premia with the SDF. Regardless of these correlations, the Sharpe ratio of housing remains too small, and the Sharpe ratio of equity remains too large.

In answering our first question, we identify the following shortcomings of the present CCAPM framework for explaining asset prices. First, equity premia and the SDF are too strongly correlated. Therefore, the Sharpe ratio of equity is close to its upper bound, and any asset with a significantly larger Sharpe ratio is impossible to explain within this setting. Second, the HJB bound is too small to facilitate replicating the Sharpe ratio of housing. Third, having productivity risk as the only source of uncertainty results in stronger correlations between equity premia and the SDF than between housing premia and the SDF. As long as the distribution of the shocks coincides with the agents' expectations, the Sharpe ratio of housing falls below the Sharpe ratio of equity.

Regarding our latter questions, all models retain their ability to generate sizeable equity premia despite introducing housing as a second asset. Additionally, the version with external habits and the version with disaster risk reproduce the empirically observed housing demand solely through productivity uncertainty. Meanwhile, procyclical housing demand provides an additional hedge against consumption risk. All variants underestimate housing premia, and apart from disaster risk, sizeable equity premia can only be explained if investment variability is restricted, resulting in too-smooth business investment activity. Disasters in the model destroy a part of the capital and housing stock, so disaster risk limits the possibilities for reducing risk through housing. Moreover, the time-varying risk of a large depreciation of the housing stock introduces an additional demand effect for housing. An increase in disaster probability increases expectations about future stock depreciation and thus leads households to deinvest. Without changes in productivity, the reduced demand for housing causes house prices to fall. This effect is sizeable and increases the overall volatility and comovement of housing-related prices and quantities.

Extensive literature covers combinations of two of the following three topics: housing, production economies inside a Dynamic Stochastic General Equilibrium (DSGE) framework, and asset prices in CCAPMs. We contribute to the literature by integrating all three topics into one common framework.

First, standard RBC models that feature housing, as in Davis and Heathcote (2005), are doomed to fail in replicating the observed demand for housing structures. This strand of literature adds productivity shocks to the construction sector to account for the high volatility of residential investments. Yet this leads to counterfactual negative comovements of housing-related prices and quantities. Thus, Iacoviello and Neri (2010) conclude that productivity uncertainty alone cannot account for the demand for housing structures and therefore establish housing demand shocks. Yet this implies housing rents that are too volatile.[1] Nguyen (2018) and Fehrle (2019) solve the comovement puzzle by increasing the income effect for housing with adjustment costs for the stock of housing and capital, but neither accounts for high house price volatility. Khan and Rouillard (2018) consider habits in consumption in combination with borrowing constraints and conclude that productivity uncertainty is insufficient to reproduce house price volatility. Dorofeenko et al. (2014) study higher-order productivity uncertainty combined with bankruptcy costs and replicate house price volatility at the expense of underestimated residential and business investment volatility. Chahrour and Gaballo (2020) assume that households use house prices to assess the macroeconomic situation. This assumption creates a channel where rising house prices increase housing demand, which pushes house prices up further. This upward spiral then qualitatively explains the demand effect for housing.[2] Our framework offers simple approaches that can explain the demand for housing solely with productivity or time-varying disaster risk.

Second, several authors have studied CCAPMs with equity and housing in endowment economies. Jordà et al. (2019b) argue that several approaches that successfully reproduce equity premia are less successful once they take housing and total wealth into account. Piazzesi et al. (2007) consider housing an asset that enters the household consumption basket. The authors study the interaction of asset prices and risk in the composition of the consumption basket. Fillat (2009) expands the Piazzesi et al. (2007) framework by generalized recursive utility and long-run risk. However, unlike our framework of a production economy, endowment economies do not attempt to explain the behavior of quantities and exclude any possibility for hedging consumption risk.

Lastly, we find that the progress of CCAPMs in production economies sparked by Jermann (1998) relies on an excessive correlation between equity premia and the SDF. Moreover, procyclical residential investments reduce consumption risk and thereby complicate the explanation of sizeable risk premia. This mechanism is similar to the insurance opportunity against consumption risk provided by endogenous decisions about labor supply in Walrasian labor markets, as previously discussed in the literature. However, hedging consumption risk through the labor market is questionable—households are usually forced into unemployment during downturns and do not voluntarily substitute consumption with leisure to smooth the bundle and reduce consumption risk as in the model. If hedging through the labor market is restricted, labor market statistics and sizeable risk premia can be explained simultaneously (see Boldrin et al., 2001; Uhlig, 2007; Heer and Maußner, 2013). The question of whether housing hedges consumption risk is debatable. On the one hand, Iacoviello (2005), Lustig and van Nieuwerburgh (2005), Mian et al. (2013) and Mian and Sufi (2014) suggest quite the opposite. The authors even motivate consumption risk on imperfect capital markets through declining house prices—declining house prices increase the leverage ratio, and tightened (re)financing options force households on the margin to reduce consumption. On the other hand, the literature provides evidence that households reallocate nondurable consumption and residential investment to keep the composite of nondurable consumption and housing smooth. For example, Piazzesi et al. (2007) and Khorunzhina (2021) argue that households prefer intratemporal substitution between housing and nondurable consumption to intertemporal substitution of the whole composite. In addition, Khorunzhina (2021) outlines that while homeowner expenditures on maintenance, repairs, and improvements of houses are sizeable on average, they are incurred infrequently at the individual household level: average annual expenditures amount to 1.6% of house value, while their within-household coefficient of variation is 108%. The observed elastic and procyclical demand for housing would then be consistent with the assumption that households realize their infrequent but sizeable investments during good times rather than bad. This behavior is like the piece-by-piece construction of houses in developing countries. In those countries, housing serves as a savings stock due to incomplete financial markets (Rosling et al., 2019, Chapter 6). We do not contribute to the debate about which effect prevails. However, the literature on housing has not considered the latter sufficiently even though hedging aggregate consumption risk is a housing-specific characteristic, and leverage effects apply to all collateral—including equity—and depend on nonspecific capital market imperfections.[3]

Jaccard (2011) and Favilukis et al. (2017) already analyze risk premia in production economies with housing. The model of Jaccard (2011) is similar to our model specification with external habits and standard productivity risk. Nevertheless, we revisit the habit formation approach with housing in utility for various reasons. First, the empirical targets of Jaccard (2011) rest on Piazzesi et al. (2007), who assume that the house price index grows with the residential investment price index. However, Davis and Heathcote (2007) and Knoll et al. (2017) show that the main driver for increasing house prices is growth in land prices. JKKST, Campbell et al. (2009), and Demers and Eisfeldt (2021) report similar rates of return on housing and equity but a significantly higher Sharpe ratio on housing than for equity in contrast to Piazzesi et al. (2007), who find that housing returns are markedly smaller than equity returns and that the Sharpe ratios of the two assets are similar. Hence, Jaccard (2011) does not stress our first point, the separate conditions for the HJB and the joint distribution of consumption risk and risk premia due to different Sharpe ratios. Second, Jaccard (2011) does not target the demand effect for housing—it remains undiscovered if a high marginal propensity to reduce consumption risk can explain the puzzling second moments on residential investment, house prices, and rents—the first part of our second point. Third, the model is an extension of the model of Jermann (1998). The literature on habit formation CCAPMs in production economies has improved since—e.g., the model of Chen (2017) solves the risk-free rate volatility puzzle. We want to stress recent improvements in light of the second part of our second point, whether the additional opportunity to reduce consumption risk diminishes the ability to replicate asset prices, and our third point, the general compatibility with housing data. Fourth, we are also interested in total risk premia, which Jaccard (2011) does not account for. Lastly, Jaccard (2011) sets the habit parameter implicitly to one and only calibrates habit persistence. With stationary variables, the steady state surplus of consumption over habit equals one minus the reciprocal of the growth factor ($\approx$0.005), making a general robustness check with standard values from the literature (>0.05) worthwhile.

Favilukis et al. (2017) depart from the representative-agent framework and study a production economy with two sectors and aggregated as well as idiosyncratic income risk. Their model explains the boom-bust cycle in the first decade of this century and matches the empirical

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[1] Various studies extend the Iacoviello and Neri (2010) framework by elaborating on demand shocks and transmission channels. For example, Lambertini et al. (2017) include news shocks, Ge et al. (2020) financial shocks, and Miura (2023) sentiment shocks. However, none of them account for volatility in housing rents.

[2] Chahrour and Gaballo (2020) do not undertake a full quantitative assessment of the model.

[3] In addition to our contribution to hedging consumption risk, Fehrle (2023) gives empirical evidence that housing is superior to equity in hedging against inflation on the business cycle frequency. As we study a real economy, this is outside our scope.

Sharpe ratio of equity, although the mean and standard deviation of the rates of return are moderately too small. Further, the model replicates a sizeable risk premium for housing, yet the authors do not report the Sharpe ratio of the housing index.

From here on, the paper reads as follows. In Section 2, we present the stylized facts on which the remainder of the paper focuses and discuss the suitability of the JKKST data for our purposes. Section 3 presents the basic framework of our RBC model with housing. The following three sections addresses the different specifications of productivity risk and the SDF. Each section addresses one specification and presents the general idea, parameterization, calibration, and results. Section 7 starts with comprehensive model specification comparisons and further discusses the results. The paper concludes with Section 8. The appendix collects additional data work and more detailed derivations.

2. Stylized facts

We start with the presentation of stylized facts that characterize historical data on business cycles, housing, and asset prices and that the literature has identified as main facts that are commonly valid for most countries for extended periods (see JKKST for asset prices and Davis and Nieuwerburgh (2015) for housing and business cycles). In Table 1, we provide a summary of these stylized facts for the United States (1970–2015), the United Kingdom (1969–2015), France (1980–2015), and Japan (1963–2015).[4] Asset price statistics are annual data from the JKKST database. Business cycle statistics are quarterly data from OECD.Stats (2019).

To ensure consistency with our following model economy, we define GDP as the sum of private consumption expenditures (PCE), business investments I and residential investments D.[5] Panel A of Table 1 shows that by this definition, PCE accounts for 67%–78% of GDP, while business and residential investments make up 15%–26% and 6%–8% of GDP, respectively.

Panel B of the table displays the stylized facts from the housing and the business cycle literature. We observe that GDP has a standard deviation of approximately 0.9%–1.2% in the United States, the United Kingdom, and Japan, and 0.6% in France. PCE is less volatile than GDP in all four countries, while business investment is twice as volatile as GDP. Residential investment is even more volatile than business investment. In the United States and Japan, residential investment is twice as volatile as business investment. The difference between the two volatilities is moderately smaller in the United Kingdom and significantly smaller in France.[6] Moreover, house prices P_H are more volatile than GDP, whereas excluding the United Kingdom, rental prices r_H are less volatile than GDP. In all four countries, the standard deviation of house prices is at least twice as large as the standard deviation of rental prices. GDP, house prices, residential and business investments comove. The lowest correlation is observed between business and residential investments. In short, investment quantities and house prices comove procyclically. Usually, the literature also considers lagged cross-correlations with residential investments since residential investments lead the business cycle in the United States. However, Kydland et al. (2016) show that this is unique to the United States and Canada and wherefore we omit lead–lag patterns here.[7]

Table 1
Empirical returns, premiums, and first and second moments.

	USA	UK	France	Japan
Panel A: Expenditure shares (in percent of GDP)				
PCE	77.55	76.92	71.04	67.27
I	16.40	14.68	20.57	25.86
D	6.04	8.40	8.40	6.67
Panel B: Business cycle				
$\sigma(GDP)$	0.87	1.19	0.61	1.25
$\frac{\sigma(PCE)}{\sigma(GDP)}$	0.75	0.90	0.94	0.92
$\frac{\sigma(I)}{\sigma(GDP)}$	2.46	2.87	2.08	1.78
$\frac{\sigma(D)}{\sigma(GDP)}$	5.34	4.35	2.27	3.44
$\frac{\sigma(P_H)}{\sigma(GDP)}$	1.36	2.35	2.42	1.34
$\frac{\sigma(r_H)}{\sigma(GDP)}$	0.72	1.19	0.74	0.60
$\rho(P_H, D)$	0.39	0.26	0.56	0.25
$\rho(I, D)$	0.05	0.05	0.38	0.04
$\rho(GDP, D)$	0.71	0.55	0.52	0.43
$\rho(GDP, P_H)$	0.41	0.55	0.44	0.48
Panel C: Rates of return				
R_g	1.57	1.56	2.24	0.83
R_E^{lev}	7.45	8.00	9.61	5.85
R_H	6.01	7.00	5.78	4.35
R_T^{lev}	6.84	7.47	6.61	5.42
EP	5.88	6.44	7.37	5.02
HP	4.45	5.44	3.54	3.53
TP	5.27	5.91	4.37	4.59
$\sigma(R_g)$	2.31	3.73	2.55	2.58
$\sigma(R_E^{lev})$	16.71	23.41	24.11	21.07
$\sigma(R_H)$	3.78	9.64	5.52	6.00
$\sigma(R_T^{lev})$	6.90	8.44	6.95	8.23
$\sigma(EP)$	16.47	24.27	23.98	20.75
$\sigma(HP)$	4.41	8.88	6.18	6.17
$\sigma(TP)$	7.00	8.62	7.39	8.22
SR_E	0.36	0.27	0.31	0.24
SR_H	1.01	0.61	0.57	0.57
SR_T	0.75	0.69	0.59	0.56

Notes: Periods: USA 1970–2015, United Kingdom 1969–2015, France 1980–2015, and Japan 1963–2015.
Expenditure Shares: Average shares in GDP of private consumption expenditures (PCE), business investments (I), and residential investments (D).
Business cycle moments: Standard deviations $\sigma(.)$ and correlations $\rho(.,.)$ for growth rates of GDP, private consumption expenditures (PCE), business investments (I), residential investments (D), house prices (P_H), and housing rents (r_H). Business cycle statistics are computed from growth rates of quarterly per capita data. Main source: OECD.Stats (2019), own calculations, Appendix A provides more information.
Rates of return: Mean percentage returns on equity (R_E^{lev}), housing (R_H), total risk (R_T^{lev}), and government bonds (R_g), as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP), the corresponding standard deviations $\sigma(.)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). Asset price statistics are computed for annual data. Source: JKKST, own calculations, Appendix A provides more information.

Panel C of Table 1 displays the mean return rates on bills, on the two risky assets (equity and housing), on total risk, and the risk premia. Panel C shows the corresponding standard deviations and the resulting Sharpe ratios of equity, housing, and total risk. We observe a low "risk-free" rate of return on bills between 0.83% and 2.24% with a low standard deviation (2.3–3.7). The returns on equity are between 5.85% and 9.61%, resulting in equity premia between 5.02% and 7.37%. In all countries, the average return on housing is moderately lower than the average return on equity, and housing premia are between 4.35% in Japan and 7.00% in the United Kingdom. The difference between the two risky returns/premia is 1.00, 1.44, 1.50 and 3.83 percentage points in the United Kingdom, the United States, Japan, and France, respectively.[8] Moreover, in the United States and the United Kingdom, the return on total risk is approximately the average of the two risky

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[4] We will only target US data later, but we evidence non-country-specific facts for a broader set of countries here. Next to these four countries, Appendix A shows that we also observe the same stylized facts in most other of the 16 developed countries examined by JKKST. Based on the JKKST database, Rafiq (2022) explores the downturn and the recovery of equity and house prices dependent on different types of recessions.

[5] See also Eq. (11).

[6] For most continental European countries we observe the same relation as in France. However, there is no clear evidence of converging European housing markets in the literature (see Maynou et al., 2021).

[7] Agreeing, Chang (2020) finds related drivers of US and Canadian housing cycles but not those in the United Kingdom.

[8] The difference between the rates of return in France is closer to the value in the other countries in the periods chosen by JKKST (1963–2015 and

rates of return. In France, the return on total risk is close to the smaller return on housing, and in Japan close to the larger return on equity. While returns on equity exceed returns on housing moderately, they are two to four times as volatile: the standard deviation of equity returns lies between 16.7 and 24.11, while the standard deviation of housing returns falls between 3.78 and 9.64. In all countries, the standard deviation of returns on total risk is also significantly lower than the standard deviation of returns on equity. Risk premia are almost identically volatile as rates of return. In all countries, the Sharpe ratio of housing exceeds the Sharpe ratio of equity significantly, and the Sharpe ratio of total risk is close to the Sharpe ratio of housing.

There is some dissent in the literature about housing returns, and some authors have reported lower returns on housing than JKKST. For example, Eichholtz et al. (2021) report a return on housing of 4.0% in Paris during 1809–1943 and 4.8% in Amsterdam during 1900–1979. Chambers et al. (2021) find a return on housing of 2.3% for the residential real estate portfolios of four large Oxbridge colleges during 1901–1983. By contrast, several other studies also support the results of JKKST. Demers and Eisfeldt (2021) find a nominal net return of 8.5% for single-family rentals in the United States during the more recent period 1986–2014; close to the nominal net housing return of 8.86% in the JKKST database during the same period. Further, Demers and Eisfeldt (2021) report a Sharpe ratio of 1.14 for housing and blame the mere focus on rental yields or house price appreciation in previous studies for the lower returns previously reported. Campbell et al. (2009) find housing premia of 3% in the United States during the recent period of 1975–2007, which is somewhat lower than the 4.45% reported in Table 1. Nevertheless, Campbell et al. (2009) report a housing premia standard deviation of 3.13, which results in a similar Sharpe ratio of 0.96.

The JKKST data include national rates of return aggregated from owner-occupied units. This methodology casts doubt on the validity of the reported housing Sharpe ratios for our framework for two reasons. First, it may be questionable whether homeowners are marginal housing investors over the business cycle since they have already invested at an extensive margin. Second, the aggregated data lack information on idiosyncratic risk, although idiosyncratic risk may account for a significant share of the risk of owner-occupied units—particularly compared with easily diversifiable equity. Within the representative agent framework, data on returns of investible housing units to a diversified and deep-pocketed investor may seem more suitable than returns on owner-occupied units.

Concerning the first issue, Khorunzhina (2021) argues that the average cross-sectional and intrahousehold variations in homeowners' residential investments are substantial and so are changes in their housing stock. Hence, homeowners are intensive marginal housing investors.

Regarding the second issue, first, Demers and Eisfeldt (2021) report a Sharpe ratio of 1.14 for single-family rentals. Single-family rentals make up 35% of US rental housing units and thus are investible assets for a diversified and deep-pocketed investor. Hence, their results suggest a similar housing Sharpe ratio that may be consistent with the representative-agent framework. Second, if idiosyncratic risk is the main driver for different Sharpe ratios of housing and equity, there would be a high willingness to pay for hedging and diversifying. Against this backdrop, the reasons for not more predominate housing supply by diversified landlords and for capital markets failing to offer any hedge would be puzzling. For example, a contract of difference or an asset swap could hedge local price risk using a national house price index as underlying, and nationwide acting housing cooperatives could diversify local price risk. Finally, Fig. 1 illustrates the relationship between the excess Sharpe ratio of housing over equity and different indicators for idiosyncratic risk, namely, the homeownership rate, the mortgage debt-to-GDP ratio, and the mortgage repayment-to-GDP ratio. The homeownership ratio serves as an indicator of the portfolio diversification of investors. For example, if all housing units belonged to the same owner (homeownership rate almost zero), this investor would be exposed to no local price risk, and a homeownership rate of one would imply no diversification against local price risk at all. The mortgage debt-to-GDP ratio indicates the average level of leverage. Since a higher leverage ratio amplifies the effects of idiosyncratic risk, we interpret the mortgage debt-to-GDP ratio as an instrument indicating the impact of idiosyncratic risk. Similarly, given that the mortgage repayment-to-GDP ratio indicates debt sustainability and thus debtors' resilience to idiosyncratic shocks, we understand the ratio as an instrument for measuring the consequences of idiosyncratic shocks. If idiosyncratic risk were the only reason for the excess Sharpe ratio of housing, a lower degree of diversification, a larger impact, and more severe consequences of idiosyncratic shocks should be positively correlated with the excess Sharpe. Positive correlations are not evident, however, but instead negative correlations between the excess Sharpe ratio for housing and the idiosyncratic risk indicators.[9]

Summing up, we conclude that factors other than idiosyncratic risk must be substantial for the excessive Sharpe ratio of housing. This conclusion complements (Jordà et al., 2019b). They outline that the sheer size of the Sharpe ratio excess and persistence over different horizons makes it unlikely that idiosyncratic risk is its only driver. One last note—the often-used rates of return from real estate investment trusts are not comparable to the JKKST housing returns. These trusts often invest in commercial real estate and are typically highly leveraged, whereas the JKKST returns apply to unleveraged investments in residential real estate.

3. Basic framework

Our basic framework is a standard RBC model into which we introduce housing services as a durable consumption good. Nondurable consumption and housing have intratemporal nonseparable utility, as evidenced by Khorunzhina (2021). On the supply side, a fixed supply of land depresses the marginal rate of transformation between consumption and new houses, and capital adjustment costs in the manner of Jermann (1998) depress the marginal rate of transformation of consumption and newly installed capital. Housing and capital differ as well in their depreciation rates.

Next, we introduce the parameterized basic framework and describe different levered and unlevered rates of return. After that, we derive conditions for the first and second moments of the SDF, which are fundamental in explaining asset price statistics. The section ends with a calibration exercise for the basic framework.

3.1. The model

We consider an economy that consists of an infinitely lived representative household and a representative firm. Time is discrete and indexed by $t \in \mathbb{N}$.

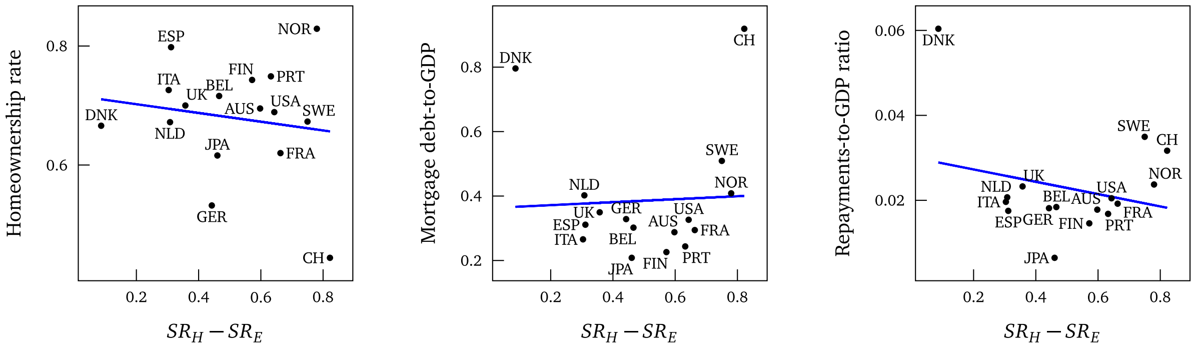
Representative firm. The representative firm produces output Y_t from labor N_t and capital services K_t according to a Cobb–Douglas production function

$$Y_t = Z_t (A_t N_t)^{1-\alpha} K_t^{\alpha}, \quad \alpha \in (0,1). \tag{1}$$

Total factor productivity Z_t and labor-augmenting technical progress A_t may be stochastic; the exact form of the stochastic processes characterizing productivity risk will be pinned down in the particular sections. However, the deterministic component of A_t grows at the rate $a > 0$.

1870–2015). Our French data set begins in 1980 because of missing data for business cycle statistics.

[9] As we do not control for other factors, this does not mean that housing is free of idiosyncratic risk but by definition that the housing Sharpe ratio excess depends on other factors as well and not *only* on idiosyncratic risk.



(a) Homeownership rate (b) Mortgage debt-to-GDP (c) Mortgage repayment-to-GDP

Fig. 1. Idiosyncratic risk and the housing sharpe ratio excess. **Notes:** Scatter plot on the excess Sharpe ratio of housing over equity and various indicators for idiosyncratic risk. The fit line minimizes the squared residuals. Three letters represent countries using ISO 3166-1 Alpha-3 country codes.
Source: JKKST, own calculations, Appendix A provides more information.

The firm's first-order conditions from maximization of profits $Y_t - W_t N_t - r_{K,t} K_t$ under perfect competition and subject to the production function read

$$W_t = (1-\alpha)\frac{Y_t}{N_t}, \tag{2a}$$

$$r_{K,t} = \alpha\frac{Y_t}{K_t}. \tag{2b}$$

Representative household. The representative household derives utility from streams $\{\tilde{C}_t\}_{t\in\mathbb{N}}$ of a composite good

$$U_0 = U(\{\tilde{C}_t\}_{t\in\mathbb{N}}) \tag{3}$$

The composite good consists of consumption C_t, housing H_t, and leisure $1 - N_t$ and will be more concretely specified below.

The household supplies labor services N_t and capital services K_t to the firm and receives wages W_t and capital rents $r_{K,t}$. It buys consumption goods C_t and makes business investments I_t in productive capital and residential investments D_t in new homes. Hence, its budget constraint reads

$$W_t N_t + r_{K,t} K_t = C_t + I_t + D_t. \tag{4}$$

Capital evolves according to

$$K_{t+1} = (1-\delta_K)K_t + \Phi\left(\tfrac{I_t}{K_t}\right)K_t, \tag{5}$$

where $\delta_K \in (0,1)$ is the depreciation rate. The function $\Phi : (0,\infty) \to \mathbb{R}$ describes adjustment costs to the capital stock in the manner of (Jermann, 1998).

$$\Phi(x) = b_1 + \tfrac{b_2}{1-\kappa}x^{1-\kappa}, \quad b_1 \in \mathbb{R}, b_2 \in (0,\infty). \tag{6}$$

On the other hand, residential investments must be combined with a fixed factor L of land (normalized to one) to form new houses, where $\varphi \in (0,1)$ is the share of land

$$H_{\text{new},t} = D_t^{1-\varphi} L^{\varphi}.$$

The stock of houses then evolves according to

$$H_{t+1} = (1-\delta_H)H_t + H_{\text{new},t}, \tag{7}$$

where $\delta_H \in (0,1)$ is the depreciation rate of houses.

Finally, we assume that the consumption bundle $\tilde{C}_t$ is of the Cobb–Douglas form, i.e.,

$$\tilde{C}_t = C_t^{\mu_C}(A_{t-1}^{\varphi}H_t)^{\mu_H}(A_{t-1}(1-N_t))^{\mu_N}, \quad \mu_C + \mu_H + \mu_N = 1. \tag{8}$$

The fact that we multiply housing H_t, which grows at the rate $a^{1-\varphi}$, with A_{t-1}^{φ} and leisure $1 - N_t$ with A_{t-1}, ensures that a balanced growth path exists even if the bundle is a more general CES aggregate. While weighting by the level of productivity is not necessary for the special case of a Cobb–Douglas bundle, it nonetheless helps to increase risk-reward. Hence, we follow Croce (2014) with this assumption and interpret the weighting with adjustments to the standard of living.

The household chooses consumption C_t, its labor supply N_t, business investments I_t, next period's capital stock K_{t+1}, residential investments D_t, and next period's housing stock H_{t+1} to maximize its lifetime utility under the budget constraint (4) and the dynamics (5) and (7). It takes wages W_t and the rental rate $r_{K,t}$ of capital as given. Hence, the first-order conditions of the household can be summarized as

$$W_t = MRS_t^{N,C}, \tag{9a}$$

$$q_t = \mathbb{E}_t\left[M_{t,t+1}\left(r_{K,t+1} + q_{t+1}\left(1 - \delta_K + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right)\right], \tag{9b}$$

$$P_{H,t} = \mathbb{E}_t\left[M_{t,t+1}\left(r_{H,t+1} + P_{H,t+1}(1-\delta_H)\right)\right], \tag{9c}$$

where $MRS_t^{N,C} = \frac{\partial\tilde{C}_t/\partial(1-N_t)}{\partial\tilde{C}_t/\partial C_t} = \frac{\mu_N}{\mu_C}\frac{C_t}{1-N_t}$ is the marginal rate of substitution between leisure and consumption, $r_{H,t} = \frac{\partial\tilde{C}_t/\partial H_t}{\partial\tilde{C}_t/\partial C_t} = \frac{\mu_H}{\mu_C}\frac{C_t}{H_t}$ is the implicit rental rate of housing derived from the marginal rate of substitution between housing and consumption, and $M_{t,t+1}$ is the model's SDF. Moreover, $q_t = \frac{1}{\Phi'(\frac{I_t}{K_t})}$ is Tobin's q and $P_{H,t} = \frac{1}{1-\varphi}D_t^{\varphi}$ is the relative price of new houses.

General equilibrium. In general equilibrium, the first-order conditions (2) and (9) of the firm and the household hold, production is determined by (1), and the stocks of capital and houses evolve according to (5) and (7). Consumption, business investments, and residential investments are homogeneous goods aggregated in output Y_t. Hence, the economy's resource constraint is[10]

$$Y_t = C_t + I_t + D_t. \tag{10}$$

Finally, we follow Davis and Heathcote (2005) and define PCE as consumption plus the implicit rent from housing by

$$PCE_t = C_t + r_{H,t}H_t,$$

so that GDP is

$$GDP_t = Y_t + r_{H,t}H_t = PCE_t + I_t + D_t. \tag{11}$$

Rates of return. The rate of return $R_{E,t+1}$ on equity, the rate of return $R_{H,t+1}$ on housing, and the rate of return $R_{T,t+1}$ on total risk are given

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[10] The economy's resource constraint already implies the budget constraint (4) of the household in equilibrium since the firm makes no profits.

by

$$\begin{aligned} R_{E,t+1} &= \frac{r_{K,t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1-\delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} - 1 \\ &= \frac{r_{K,t+1}K_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}} - 1, \qquad (12a) \\ R_{H,t+1} &= \frac{r_{H,t+1} + P_{H,t+1}(1-\delta_H)}{P_{H,t}} - 1 \\ &= \frac{r_{H,t+1}H_{t+1} - P_{H,t+1}H_{new,t+1} + P_{H,t+1}H_{t+2}}{P_{H,t}H_{t+1}} - 1, \qquad (12b) \\ R_{T,t+1} &= \frac{r_{K,t+1}K_{t+1} - I_{t+1} + q_{t+1}K_{t+2} + r_{H,t+1}H_{t+1} - P_{H,t+1}H_{new,t+1} + P_{H,t+1}H_{t+2}}{q_t K_{t+1} + P_{H,t}H_{t+1}} \\ &\quad - 1 \\ &= \frac{q_t K_{t+1}}{q_t K_{t+1} + P_{H,t}H_{t+1}} R_{E,t+1} + \frac{P_{H,t}H_{t+1}}{q_t K_{t+1} + P_{H,t}H_{t+1}} R_{H,t+1}. \qquad (12c) \end{aligned}$$

Since stock returns provide the basis for the observed return on equity, it includes leverage. This does not hold for housing returns. To be in line with the data, we also consider leveraged rates of return. More precisely, we assume that in each period a constant fraction $m \in [0,1)$ of the firm's capital stock is financed by debt through bonds that all have maturity τ. In addition to these corporate (c) bonds, we consider government (g) bonds.

If there is no risk that bonds may default, the price of such bonds satisfies the recursion

$$Q^{(\tau)}_{j,t} = \mathbb{E}_t[M_{t,t+1} Q^{(\tau-1)}_{j,t+1}], \quad \text{where } Q^{(0)}_{j,t+1} = 1 \text{ and } j \in \{g, c\}. \qquad (12d)$$

Further, the ex post rate of return from holding a bond with maturity τ for one period is defined by

$$R^{(\tau)}_{j,t+1} = \frac{Q^{(\tau-1)}_{j,t+1}}{Q^{(\tau)}_{j,t}} - 1.$$

Since the Modigliani and Miller theorem holds, the leveraged rate of return on equity and total risk are given by

$$R^{lev}_{E,t+1} = \frac{1}{1-m} R_{E,t+1} - \frac{m}{1-m} R^{(\tau)}_{c,t+1} \qquad (12e)$$

$$R^{lev}_{T,t+1} = \frac{q_t K_{t+1}}{q_t K_{t+1} + P_{H,t}H^*_{t+1}} R^{lev}_{E,t+1} + \frac{P_{H,t}H^*_{t+1}}{q_t K_{t+1} + P_{H,t}H_{t+1}} R_{H,t+1}. \qquad (12f)$$

Finally, when talking about the rate of return $R_{g,t+1}$ on a government bond, we mean the return on a bond with a maturity of one period

$$R_{g,t+1} = R^{(1)}_{g,t+1}. \qquad (12g)$$

Fundamental requirements. Using insights from Lucas (1978) and Hansen and Jagannathan (1991), we can derive some fundamental requirements that the model must satisfy to be able to replicate the stylized facts for asset returns summarized in Section 2. First, note that the model's Euler equations (9b) and (9c) together with the pricing formula (12d) for government and corporate bonds implies

$$\mathbb{E}_t\left[M_{t,t+1}(R^{lev}_{E,t+1} - R_{g,t+1})\right] = \mathbb{E}_t\left[M_{t,t+1}(R_{H,t+1} - R_{g,t+1})\right] = 0.$$

Taking unconditional expectations, the equality also holds unconditionally for the model's stationary distribution. Hence,

$$\begin{aligned} \mathbb{E}\left[M_{t,t+1}\right]\mathbb{E}\left[R^{lev}_{E,t+1} - R_{g,t+1}\right] &= -\operatorname{Corr}\left[M_{t,t+1}, R^{lev}_{E,t+1} - R_{g,t+1}\right] \\ &\times \sigma\left[R^{lev}_{E,t+1} - R_{g,t+1}\right]\sigma\left[M_{t,t+1}\right], \end{aligned}$$

and equivalently also for the return on housing $R_{H,t+1}$. The Sharpe ratios, therefore, satisfy

$$\begin{aligned} SR_E &:= \frac{\mathbb{E}\left[R^{lev}_{E,t+1} - R_{g,t+1}\right]}{\sigma\left[R^{lev}_{E,t+1} - R_{g,t+1}\right]} = -\operatorname{CV}[M_{t,t+1}] \\ &\times \operatorname{Corr}\left[M_{t,t+1}, R^{lev}_{E,t+1} - R_{g,t+1}\right] \qquad (13a) \end{aligned}$$

and

$$\begin{aligned} SR_H &:= \frac{\mathbb{E}\left[R_{H,t+1} - R_{g,t+1}\right]}{\sigma\left[R_{H,t+1} - R_{g,t+1}\right]} = -\operatorname{CV}[M_{t,t+1}] \\ &\times \operatorname{Corr}\left[M_{t,t+1}, R_{H,t+1} - R_{g,t+1}\right] \qquad (13b) \end{aligned}$$

where $\operatorname{CV}[M_{t,t+1}]$ is the coefficient of variation of the model's SDF

$$\operatorname{CV}[M_{t,t+1}] =: \frac{\sigma\left[M_{t,t+1}\right]}{\mathbb{E}\left[M_{t,t+1}\right]}, \qquad (13c)$$

commonly known as the HJB. With rational expectations, this bound defines a common upper bound for Sharpe ratios of all assets in the models, while different correlations between the SDF and the asset risk premia are necessary to explain different Sharpe ratios.

More precisely, we can formulate the following quantitative requirements to replicate US data. First, the HJB in the model must be at least as large as the empirical Sharpe ratio of housing (1.01). Second, the correlation between premia on housing and the SDF must be (in absolute value) approximately 3 times as large as the correlation between premia on equity and the SDF to replicate the difference in the size of those Sharpe ratios. It follows that the correlation between premia on equity and the SDF cannot exceed one third.

Finally, note that in the same manner, any risk premium for an uncertain return $\mathbb{E}(R_{t+1})$ equals $-\operatorname{CV}[M_{t,t+1}]\operatorname{Corr}\left[M_{t,t+1}, R_{t+1}\right]\sigma(R_{t+1})$. Thus, the product of the observable $\sigma(R_{t+1})$ with the HJB defines the upper bound for risk premia. Further, given that the model predicts the return volatility (risk premia) correctly, explanations for mispredicted Sharpe ratios also hold for mispredicted risk premia (return volatility).

Stochastic discount factor and productivity risk. Up to this point, we have specified the model's framework apart from the SDF $M_{t,t+1}$ and the processes Z_t and A_t driving productivity risk. However, risk premia in the model depend heavily on these features. We therefore will examine different versions of these elements in the corresponding sections.

3.2. Calibration

We present the numeric calibration of the joint framework. We identify one period in the model with one quarter in the data and summarize the calibration of the joint framework in Table 2. We assume an average quarterly growth rate a of 0.5% as in Jermann (1998) and Gourio (2012), which is also close to the 0.45% rate used by Croce (2014) and Chen (2017). We take the share of capital $\alpha = 0.34$ in the production function from Gourio (2012) and Croce (2014), which is again almost identical to the value of 0.35 in Chen (2017). The depreciation rates of capital $\delta_K = 0.022$ and housing $\delta_H = 0.009$ are taken from Nguyen (2018), who strips down the Davis and Heathcote (2005) model. The share of land in housing matches the upper bound of $\varphi = 0.3$ from Fehrle (2019). We follow Jermann (1998) and set the parameters b_1 and b_2 in the adjustment costs function so that they do not affect the model's balanced growth path. The weight of leisure μ_N in the consumption bundle is determined so that the household works one-third of its time on average, except for the model with external habits, where leisure does not enter the consumption bundle ($\mu_N = 0$). The weight of housing μ_H, in turn, is determined so that on average, 19% of households' total expenditures for consumption and housing are on housing (see Grossmann et al., 2021). Finally, we set the level of leverage for equity to $m = 0.3$ and the maturity of corporate bonds to 10 years.

The original works of Chen (2017), Croce (2014), and Gourio (2012) show that the different forms of productivity risk studied require somewhat different degrees of risk aversion to best explain asset prices: the coefficient of risk aversion varies from 2 in Chen (2017) to 10 used by Croce (2014). Hence, we also allow for different degrees of risk aversion (and EIS) across the following versions of the model. While this limits comparability among model versions, we prefer not to steer the models' performance with regard to risk premia by deciding

Table 2
Calibration.

Parameter	Value or Target	Description
Panel A: Common framework		
a	$\ln(1.005)$	Growth rate
α	0.34	Capital share in production
δ_K	0.022	Depreciation rate of capital
δ_H	0.009	Depreciation rate of housing
φ	0.30	Share of land in housing
b_1	$\Phi(\frac{I}{K}) = e^a - 1 + \delta_K$	Adjustment cost parameter
b_2	$\Phi'(\frac{I}{K}) = 1$	Adjustment cost parameter
μ_C	$1 - \mu_H - \mu_N$	Weight of consumption in bundle
μ_H	$\frac{r_H H}{C + r_H H} = 0.19$	Weight of housing in bundle
μ_N	$N = 0.33$ or $\mu_N = 0$	Weight of leisure in bundle
m	0.30	Leverage level of equity
T	40	Maturity of corporate bonds

Notes: We optimize the elasticity κ of Tobin's q over the range $[0;7]$. The discount factor β is optimized over the range $[\frac{0.99}{a_u}; \frac{0.9999}{a_u}]$, where a_u is the growth factor of utility in the model.

on a common value shared across all versions. Instead, we study the effects of introducing housing when following the original degree of risk aversion (and EIS) from the original studies.

Similarly, somewhat different values for the discount factor β are necessary across the different model versions to match the mean rates of return. While Croce (2014) uses a quarterly value of approximately 0.987, the value is 0.999 in Gourio (2012). Moreover, κ controls capital adjustment costs and therefore the ability to smoothen consumption risk by adjusting investment. Depending on the underlying risk process, different adjustment costs are needed to replicate the empirical volatilities of investment and equity returns as well as the size of the premia. While adjustment costs are small in Croce (2014) with $\kappa = 1/7$, the results of Chen (2017) rely on rather large adjustment costs.[11] To provide the models with the best chance to replicate the data, we do not pin down the values of the discount factor β and the elasticity κ of Tobin's q. Instead, we choose the values for each version separately to minimize the sum of squared deviations between the data and the model-implied values for several targets. The targets are guided by our two main questions, i.e., the reproduction of stylized business cycle statistics together with sizeable risk premia and the explanation of different Sharpe ratios. Hence, we target the business cycle moments from Panel A of Table 1, the rate of return on government bonds, the equity premium, the housing premium, and the Sharpe ratios of the equity and housing premia. We allow values in $[0;7]$ for κ and values in $[\frac{0.99}{a_u}; \frac{0.9999}{a_u}]$ for β, where a_u is the growth factor of utility in the specific model variation. We present the results of model-specific parameter values in the corresponding section.

4. Superficial external habits and standard productivity risk

The first version of the model studies external habits, as in Chen (2017), together with standard productivity risk. Including habit formations in the power utility framework generally increases the slope of the utility function and, therefore, risk aversion. Since the elasticity of intertemporal substitution (EIS) equals the inverse of the relative risk aversion (RRA), the EIS decreases. A higher risk aversion increases the reward for holding risk, and a low EIS increases the preference to smooth the consumption bundle over time. The former increases risk premia, whereas the latter increases the amplitude of procyclical investment demand and thus for housing structures. Further, persistent habits, as in Chen (2017), decrease the consumption surplus over habit formation in downturns, leading to countercyclical risk aversion (procyclical EIS). This nonlinearity in the SDF ends in seemingly excessively volatile asset prices compared with the fluctuation of fundamentals. Thus, persistent habits increase house price volatility for a given rental rate volatility, which becomes visible in ((9c)), where house prices fluctuate due to variations in the present value of present and future rental returns and the SDF.

4.1. The model

Exogenous labor supply. If consumption risk is meant to be amplified by habits, we also shut down the possibility of reducing consumption risk through endogenous adjustments of labor supply. Hence, in this version of the model, we additionally assume that leisure does not enter the consumption bundle, i.e., $\mu_N = 0$ in (8).[12] Consequently, Eqs. (2a) and (9a) from the general framework are replaced by

$$N_t = 1.$$

Stochastic discount factor. The household's preferences are additive time separable, and per-period preferences are described by CRRA utility. The household forms habits $\tilde{C}_{h,t}$ in the consumption bundle. Thus, the utility in (3) becomes

$$U_0 = \mathbb{E}_0 \left[\sum_{t=0}^{\infty} \beta^t \frac{(\tilde{C}_t - \tilde{C}_{h,t})^{1-\gamma} - 1}{1-\gamma} \right].$$

We follow Chen (2017) and assume that the surplus from the consumption bundle over the habit evolves according to

$$\ln\left(\frac{\tilde{C}_{t+1} - \tilde{C}_{h,t+1}}{\tilde{C}_{t+1}}\right) = (1 - \rho_{\tilde{C}}) \ln(\bar{S}_{\tilde{C}}) + \rho_{\tilde{C}} \ln\left(\frac{\tilde{C}_t - \tilde{C}_{h,t}}{\tilde{C}_t}\right) + \left(\frac{1}{\bar{S}_{\tilde{C}}} - 1\right)$$
$$\times \left(\ln\left(\frac{\tilde{C}_{t+1}}{\tilde{C}_t}\right) - a\right),$$

where a is the growth rate of the bundle on the balanced growth path of the model[13] and $\bar{S}_{\tilde{C}}$ is the steady state surplus of the consumption bundle. The corresponding SDF is then pinned down to

$$M_{t,t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{\mu_C - 1} \left(\frac{H_{t+1}}{H_t}\right)^{\mu_H} \left(\frac{\tilde{C}_{t+1} - \tilde{C}_{h,t+1}}{\tilde{C}_t - \tilde{C}_{h,t}}\right)^{-\gamma}.$$

Productivity risk. We assume standard productivity risk, i.e., the log of total factor productivity follows a standard AR(1) process

$$\ln Z_{t+1} = \rho_z \ln Z_t + \epsilon_{z,t+1}, \quad \epsilon_{z,t+1} \sim iid\mathcal{N}(0, \sigma_z^2).$$

Further, labor-augmenting technical progress grows deterministically at the rate $a > 0$, i.e.,

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a.$$

Lastly, there is no disaster risk, i.e., $b_t \equiv 0$ for all t so that ω_t and $\Gamma_{j,t}$ are dropped from the model.

4.2. Calibration

Here, we pin down the additional parameters of the present model version, i.e., the parameters of the SDF and the process driving pro-

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[11] Chen (2017) uses quadratic adjustment costs of the form $\frac{\phi}{2}(\frac{I}{K} - (e^a - 1 + \delta_K))^2$ and sets $\phi = 100$.

[12] On the one hand, different labor market assumptions are problematic for comparison across models. On the other hand, we do not want to investigate versions that already fail to replicate the equity premium in the absence of housing. While the literature also offers other solutions, e.g., additional habits in leisure (see Uhlig (2007)) or predetermined working hours (see Boldrin et al. (2001)), we follow Chen (2017) from whom we borrow our habit specification and who assumes exogenous labor decisions. The habit specification of Chen (2017) has the advantage that it does not lead to an excessively volatile risk-free rate. In an earlier version of the paper (Fehrle and Heiberger, 2020), we included endogenous labor supply as in Boldrin et al. (2001). The contribution was minor, which is why we do not consider it here.

[13] This is ensured by the weighting of housing with A_{t-1}^{φ} in the bundle (8).

Table 3
Calibration: External habits and standard productivity risk.

Panel B: External habits and standard productivity risk		
γ	2	Coefficient of relative risk aversion
$\bar{S}_{\tilde{C}}$	0.07	Steady state surplus over habit
$\rho_{\tilde{C}}$	0.98	Habit persistence
ρ_z	0.98	TFP persistence
σ_z	0.012	Standard deviation of TFP innovations
Optimized values		
β	1.0033	Discount factor
κ	4.4964	Elasticity of Tobin's q

Notes: We optimize the elasticity κ of Tobin's q over the range $[0;7]$. The discount factor β is optimized over the range $[\frac{0.99}{a_u};\frac{0.9999}{a_u}]$, where a_u is the growth factor of utility in the model.

ductivity risk. Risk aversion, habit formation, and the process driving total factor productivity are chosen identically as Chen (2017) and collectively summarized in Table 3. The optimized values of the two free parameters are $\beta = 1.0033$ and $\kappa = 4.4964$.

4.3. Results

The second column of Table 6 presents the results for the model with external habits. Panel A of the table reports average expenditures as percentage shares of GDP, Panel B shows business cycle statistics, and Panel C displays the rates of return generated by the model. All moments reported are the mean outcomes from 100 model simulations, where each simulation includes 180 periods after 1000 burn-in periods. Business cycle moments are computed from the growth rates of simulated time series. The model solutions we use for simulations are from projection methods (see the Appendix for details).

The model replicates the average share of residential investments in GDP fairly well yet overpredicts the share of business investments and underpredicts the share of PCE. Note, however, that the latter two are within the range of values for the countries in Table 1.

The model overpredicts the volatility of GDP (1.10 in the model vs. 0.87 in the data) and substantially underpredicts the relative volatilities of PCE and business investment (0.29 and 0.93 in the model vs. 0.75 and 2.46 in the data). Yet the model accounts for housing-related characteristics well. Residential investment and house prices are procyclical and volatile, and the rental rate of housing is less volatile than GDP.

Turning to the rates of return, the model replicates the return on government bonds fairly well (1.70% in the model vs. 1.57% in the data). However, the return on equity in the model is too low compared with the data (5.58% vs. 7.45%). Consequently, while the model can explain a sizeable equity premium of 3.82%, it remains approximately 2 percentage points below the value given by the data. The return on housing in the model (2.66%) remains even further below its empirical counterpart (6.01%). Combined with the slightly too large risk-free rate in the model, the model can only generate a housing premium of 0.95%, which is significantly below the 4.45% found in US data. Accordingly, the total risk premium in the model is also too low compared with the data (2.27% vs. 5.27%).

The external habit of Chen (2017) allows simultaneous explanation of the high volatility of equity returns and the low volatility of the risk-free rate. In the present model, while the risk-free rate is less volatile in the model (standard deviation of 1.11) than in the data (standard deviation of 2.31), the return on equity in the model is almost twice as volatile as in the data (standard deviation of 24.26 in the model vs. 16.71 in the data), and the return on housing is more than twice as volatile as in the data (standard deviation of 9.34 in the model vs. 3.78 in the data). The standard deviations of the premia are similar to the standard deviations of the rates of return, and consequently, the model fails to explain the observed Sharpe ratios. The Sharpe ratio of equity in the model (0.15) becomes too low compared with the data (0.36), and the Sharpe ratio of housing in the model (0.11) remains below the empirical value (1.01) by a factor of even more than 9.

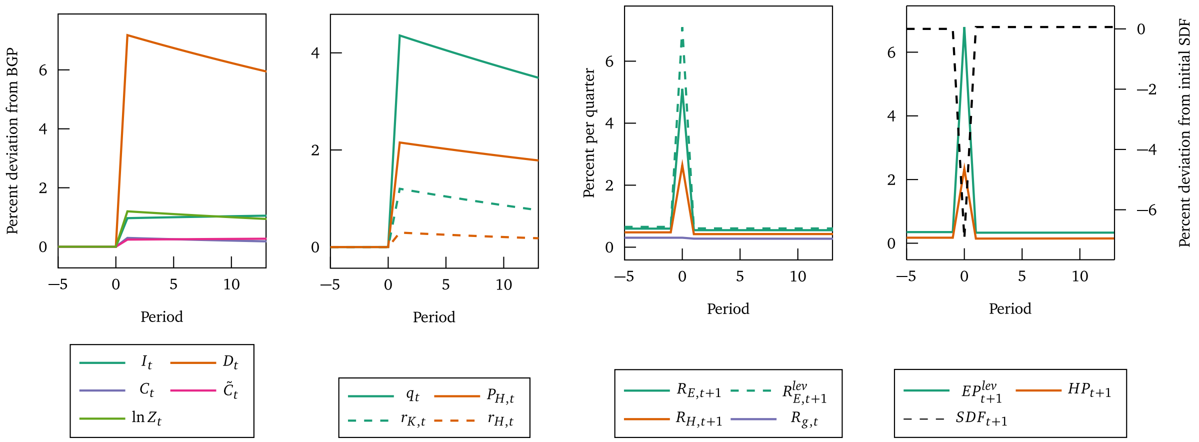
To provide additional reasoning for the model's failures with regard to asset price statistics, we summarize the (annualized) decomposition of the Sharpe ratios provided by Eq. (13) in the second row of Table 7. Concerning our fundamental requirements discussed in Section 3, the decomposition first reveals that the HJB is far too small at 0.14. Although the risk premia are perfectly negatively correlated with the SDF, the Sharpe ratios in the model remain far too small. Second, the model does not generate different correlations between the SDF and premia on equity or housing. In contrast, the premia on both assets are perfectly correlated so that their Sharpe ratios—at odds with the data—are identical. Since the Sharpe ratios are far too small, the model must rely on the too-large volatilities of risky assets to replicate sizeable premia. The counterfactually high volatility of risky returns compensates for the lack of volatility in the SDF.

Finally, we show the impulse response functions to a one-time shock $\epsilon_{z,t+1}$ to total factor productivity Z_{t+1} in Fig. 2. The variables' response to the "classic" technology shock is standard, and business investments, residential investments, and consumption increase in the period during which the shock hits the economy. Increasing business investments imply an increasing Tobin's q, $q_t = (1/b_2)(I_t/K_t)^{\kappa}$, and increasing residential investments imply increasing house prices, $P_{H,t} = (1/(1-\varphi))D_t^{\varphi}$. Although D_t increases more than I_t, the elasticity κ of Tobin's q exceeds the elasticity φ of house prices and Tobin's q expands significantly more than house prices. Moreover, increasing productivity yields an increasing marginal product of capital, and increasing consumption implies an increasing marginal rate of substitution between housing and consumption. Consequently, the returns on unlevered equity and housing increase but mainly due to the higher elasticity of Tobin's q—the return on unlevered equity dominates, and leverage further multiplies the effect. By the same token, the (ex post realized) SDF drops in response to the shock because consumption increases while housing is predetermined.

The initial response of the SDF to the unanticipated shock deviates from expectations and is therefore not reflected in the return on government bonds. This changes in subsequent periods where expectations meet. Households expect consumption to decline in the periods following its peak but only very slowly. Therefore, the SDF moves only slightly above its initial value, the price of bonds increases very moderately, and the return on government bonds declines only marginally. In the following periods, Tobin's q and house prices decrease from their peak, but the rental rates of capital and housing remain above their initial values. Those effects cancel out, the rates of return of risky assets drop to their initial values, and the same holds for premia.

Premia on equity and housing react similarly. Both are perfectly negatively correlated with the SDF, and the model cannot explain different Sharpe ratios. The equity premium in the model is higher than the housing premium due to the higher volatility of returns on equity compared with housing. Yet rates of return are already too volatile.

To summarize the results in light of our research questions: the habit formation specification accounts well for the housing-related business cycle patterns, i.e., the model matches the volatility of residential investment and house prices and rents via the increased preference for smooth consumption and the nonlinearity of the SDF and thus also matches housing-related comovements. Similarly, the model accounts for the equity premia. However, housing premia are too low. Further, the model's success in accounting for equity premia and housing demand relies on a too-restricted flexibility of business investment and a too-high preference for smooth consumption, becoming apparent in too-low standard deviations of business investment and consumption. Lastly, the HJB is too low by a factor of 7—the model specification cannot account for Sharpe ratios in the region of the observed Sharpe ratio of housing. Additionally, the relation between the SDF and the return on equity is too linear.



(a) TFP shock $\epsilon_{z,t+1}$ (I) (b) TFP shock $\epsilon_{z,t+1}$ (II) (c) TFP shock $\epsilon_{z,t+1}$ (III) (d) TFP shock $\epsilon_{z,t+1}$ (IV)

Fig. 2. Impulse response functions for habit formation. **Notes:** Impulse response functions to a one-standard-deviation innovation of total factor productivity as a percentage deviation from the stochastic balanced growth path (BGP) of business investments (I), residential investments (D), consumption (C), the composite ($\tilde C$), total factor productivity (Z), equity and house prices (q_t, P_H), capital and housing rents (r_K, r_H), unlevered and levered percentage returns on equity (R_E, R_E^{lev}), on housing (R_H), on government bonds (R_g), levered equity premium (EP^{lev}), housing premium (HP), and the SDF (SDF).

5. Epstein-Zin preferences and long-run risk

The preference structure from the previous model linked the household's attitude toward uneven consumption paths over time, measured by the EIS, and its attitude toward risk, measured by the RRA, in an inverse way. Even though there is no theoretical rationale for this linkage and empirical estimates cannot confirm the reciprocity of the two measures,[14] the inherent restriction may limit the model's performance. The generalized recursive preferences introduced by Epstein and Zin (1989) (EZ) and Weil (1989) allow us to dissolve this linkage. Moreover, with EZ preferences, next period's marginal utility depends not only on the lottery over the next period's consumption—as is the case with standard preferences—but also on the lottery over the next period's lifetime utility. Hence, EZ preferences are particularly promising when combined with risk progressing over longer periods. Here we consider a long-run risk component of productivity that shifts the mean of future growth rates similar to Bansal and Yaron (2004). Changing prospects about future growth substantially changes the household's marginal rate of substitution between current and future consumption and implies a volatile SDF. This demands higher risk premia even when rates of return are less volatile and less negatively correlated with future consumption. Changing expectations about future productivity growth directly affect expected returns on equity, whereas effects on housing returns appear only indirectly through the impact of investment adjustments to the marginal rate of substitution between housing and consumption and prices.

5.1. The model

Stochastic discount factor. Here, we assume that the household's preferences over streams of the composite good are described by a recursive utility function, as introduced by Epstein and Zin (1989) and Weil (1989), of the form

$$\tilde V_t = \left[(1-\beta)\tilde C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t \tilde V_{t+1}^{1-\gamma}\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right]^{\frac{1}{1-\frac{1}{\psi}}},$$

where ψ is the household's EIS and γ the coefficient of RRA. Note, however, that γ and ψ describe the household's RRA and EIS with respect to the composite good $\tilde C$. Since the composite good is of the Cobb–Douglas type, the consumption-based RRA is given by $\mu_c\gamma$, and the consumption-based EIS reads $\frac{1}{1-\mu_c(1-1/\psi)}$.[15] For easier notation we define $V_t := \tilde V_t^{1-1/\psi}$ which satisfies the recursion

$$V_t = (1-\beta)\tilde C_t^{1-\frac{1}{\psi}} + \beta\left(\mathbb{E}_t V_{t+1}^{1-\theta}\right)^{\frac{1}{1-\theta}}, \tag{14}$$

where we use, similar to Caldara et al. (2012), the notation

$$\theta := 1 - \frac{1-\gamma}{1-\frac{1}{\psi}}.$$

In the case where $\theta = 0$, the RRA equals the reciprocal of the EIS, and the household's utility reduces to the "classical" expected discounted sum of within-period CRRA utilities. Hence, θ can also be interpreted as a deviation from this "classic" case.

With these assumptions, the model's SDF is

$$M_{t,t+1} = \beta\left(\frac{\tilde C_{t+1}}{\tilde C_t}\right)^{1-\frac{1}{\psi}} \frac{C_t}{C_{t+1}} \left(\frac{V_{t+1}}{(\mathbb{E}_t V_{t+1}^{1-\theta})^{1/(1-\theta)}}\right)^{-\theta}. \tag{15}$$

Productivity risk. In this version, total factor productivity is nonstochastic and normalized to unity

$$Z_t = 1,$$

Labor-augmenting technical progress A_t grows stochastically as in Croce (2014) according to the process

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_t + \epsilon_{a,t+1}, \tag{16a}$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \tag{16b}$$

where

$$\begin{pmatrix}\epsilon_{a,t+1}\\ \epsilon_{x,t+1}\end{pmatrix} \sim iid\,\mathcal{N}(0,\Sigma), \quad \text{and } \Sigma = \begin{pmatrix}\sigma_a^2 & \rho_{a,x}\sigma_a\sigma_x\\ \rho_{a,x}\sigma_a\sigma_x & \sigma_x^2\end{pmatrix}.$$

Shocks $\epsilon_{a,t+1}$ affect the growth rate only once in the period of occurrence and describe short-run growth risk. On the other hand, x_t describes persistent changes in the growth rate and is therefore interpreted as a long-run risk component of productivity.

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[14] For example, Hall (1988) estimates the EIS to be close to zero, whereas Hansen and Singleton (1982) estimate an EIS above 1. On the other hand, Mehra and Prescott (1985) argue for an RRA not larger than 10, and the most commonly employed values lie between 1 and 3.

[15] See Swanson (2012) and Heiberger and Ruf (2019).

Table 4
Calibration.

Parameter	Value or Target	Description
Panel C: EZ utility and long-run risk		
γ	10	Coefficient of relative risk aversion
ψ	2	Elasticity of intertemporal substitution
ρ_x	$0.8^{\frac{1}{4}}$	Persistence of long-run component of productivity
$\rho_{a,x}$	0	Correlation of shocks to short- and long-run component of productivity
σ_a	$\frac{0.0335}{2}$	Conditional standard deviation of long-run component of productivity
σ_x	$0.1\sigma_a$	Conditional standard deviation of short-run component of productivity
Optimized values		
β	0.9942	Discount factor
κ	3.3258	Elasticity of Tobin's q

Notes: We optimize the elasticity κ of Tobin's q over the range $[0; 7]$. The discount factor β is optimized over the range $[\frac{0.99}{a_u}; \frac{0.9999}{a_u}]$, where a_u is the growth factor of utility in the model.

5.2. Calibration

The calibration of the additional parameters for Epstein and Zin (1989) preferences and long-run productivity risk is summarized in Panel C of Table 4 and closely follows Croce (2014).

More concretely, the coefficient of relative risk aversion and the elasticity of intertemporal substitution are chosen as Croce (2014), i.e., we set $\gamma = 10$ and $\psi = 2$. Further, we adjust the calibration in Croce (2014) for the stochastic process governing productivity to quarterly values. Finally, the values for β and κ, which provide the best fit of the model to data, are $\beta = 0.9942$ and $\kappa = 3.3258$.

5.3. Results

We summarize the results for the model with long-run risk in the third column of Table 6. The volatility of GDP remains somewhat too large (1.20 in the model and 0.87 in the data). The model substantially underpredicts the relative volatility of business investment (0.80 vs. 2.46). The volatilities of residential investment (3.35 vs. 5.34) and house prices (1.00 vs. 1.36) are matched better yet also underpredicted by the model. On the other hand, PCE (0.91 vs. 0.75) and rents (0.90 vs. 0.72) are moderately too volatile relative to GDP in the model.[16] As in the data, rents fluctuate less than house prices. Residential investment and house prices move procyclically, but the correlations are too large.

The risk-free rate, 1.94% in the model, and the return on equity, 7.78% in the model, are both moderately larger than in the data (1.57% and 7.45%, respectively). Hence, the model can closely match the empirically observed equity premium (5.75% in the model and 5.88% in the data). However, the return on housing in the model (2.87%) again is too low compared with the value from the data (6.01%). Therefore, the model also fails to explain the housing premium of 4.45% found in the data, as the premium in the model is only 0.91%. Consequently, the total risk premium in the model also remains too low compared with the data (3.19% vs. 5.27%).

While the rates of return are similar to the previous model with external habits, the volatilities of the rates of return are significantly different. Whereas all rates of return were too volatile before, volatilities are now too small throughout: the standard deviations of the risk-free rate, the return on equity, and the return on housing are 0.98, 9.52, and 2.39, respectively, and therefore all remain below their empirical values of 2.31, 16.71, and 3.78. The volatilities of premia are again close to the volatilities of the rates of return. Since the model can replicate the empirical equity premium but generates too involatile rates of return, the Sharpe ratio of equity in the model (0.61) exceeds the Sharpe ratio in the data (0.36). The housing premium in the model is by a factor of almost five too low compared with the data, while its standard deviation in the model is only too low by a factor of two. Hence, the Sharpe ratio of housing in the model (0.42) remains too low by a factor of more than 2 compared with the data (1.01). Contrary to the data, the model generates a higher Sharpe ratio for equity than for housing.

The third row of Table 7 summarizes the (annualized) decomposition (13) of the Sharpe ratios for the present variant. Compared with the model with external habits, this model generates risk premia differently. More specifically, the HJB increases by nearly a factor of 5, from 0.14 to 0.62. Premia on equity are still almost perfectly negatively correlated with the SDF so that the Sharpe ratio of equity equals the HJB. However, premia on housing and the SDF are less linearly related. The Sharpe ratio of housing amounts to only approximately 2/3 of the Sharpe ratio of equity, with a correlation of −0.63. Unlike the model with external habits, the present model relies on a more volatile SDF that implies larger Sharpe ratios. Premia therefore rest on less volatile rates of return.

However, the decomposition again reveals two principal failures of the model. First, although the HJB increases substantially compared with the model with external habits, it remains below the value of 1—the minimum value necessary to allow for a Sharpe ratio as observed for housing in the US data. Second, the correlation between the SDF and premia on housing amounts only to $2/3$ of the correlation between the SDF and premia on equity, while the data demand that the former must exceed the latter by a factor of 3. The model's structure separates the correlations between rates of return in the wrong direction. The premium on equity rests on a too-strong correlation between rates of return and the SDF instead of more volatile rates of return. On the other hand, the correlation between rates of return on housing and the SDF is too small. The resulting Sharpe ratio remains too small, and the rates of return do not fluctuate enough, so the premium on housing is considerably smaller than in the data.

We show the effects in response to a shock $\epsilon_{a,t+1}$ to the short-run component of the growth rate in panels (a)–(d) of Fig. 3, and the effects of a shock $\epsilon_{x,t+1}$ to the long-run component in panels (e)–(h). The variables' responses to the short-run shock are similar to the effects of a TFP shock in Fig. 2 for the model with external habits: business investments, residential investments, and consumption increase during the period when the shock hits the economy. However, in the present model, consumption increases more, and business and residential investments increase less, than in the habit model. As before, residential investments increase significantly more than business investments, but the effect turns for Tobin's q and house prices due to the larger elasticity of Tobin's q. Consequently, returns on equity and housing both increase, but the increase is stronger for equity. The (ex post realized) SDF decreases in the period the shock hits the economy: consumption increases even more than the bundle since housing is fixed, and additionally, lifetime utility increases.

In the next period, business and residential investments drop from their initial peak, which results in a declining Tobin's q and falling

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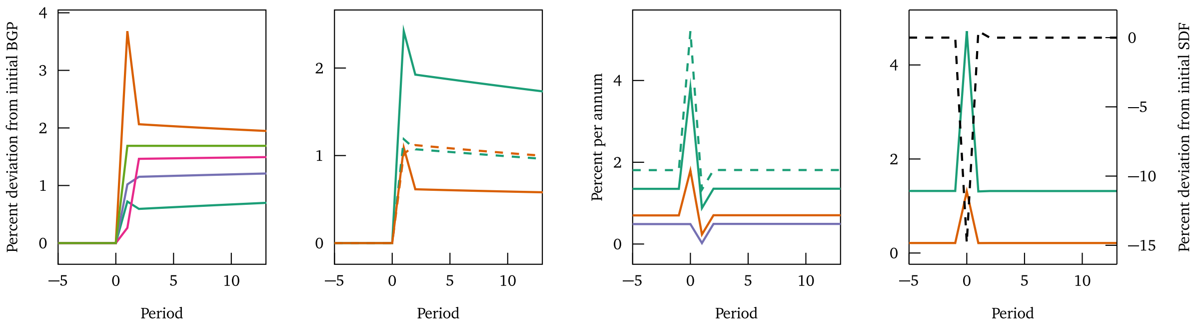
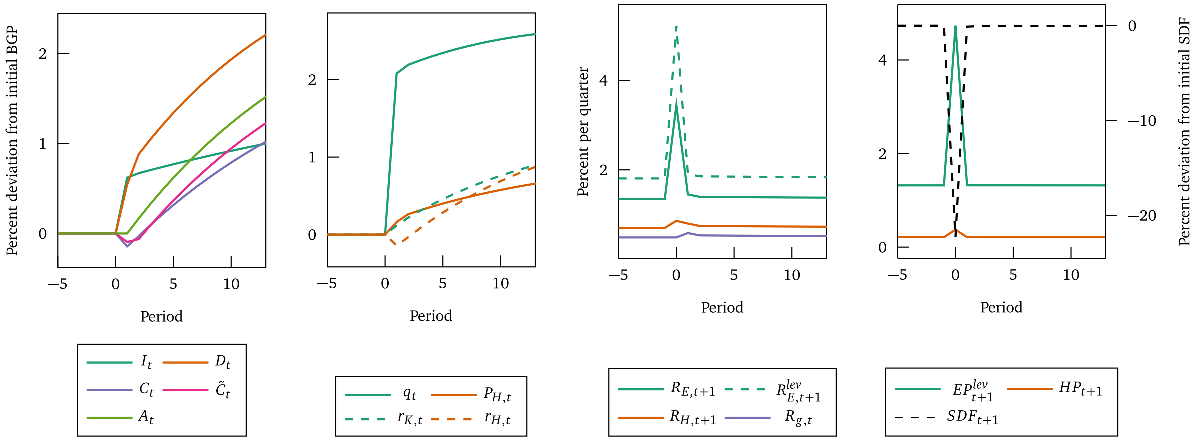
[16] Note that the predicted standard deviations of PCE and rents are not outlandish, as they are within the standard deviation range of Table 1.



(a) Short-run shock ϵ_a I (b) Short-run shock ϵ_a II (c) Short-run shock ϵ_a III (d) Short-run shock ϵ_a IV



(e) Long-run shock ϵ_x I (f) Long-run shock ϵ_x II (g) Long-run shock ϵ_x III (h) Long-run shock ϵ_x IV

Fig. 3. Impulse response functions for long-run risk. **Notes:** Impulse response functions to a one-standard-deviation short- and long-run (ϵ_a, ϵ_x) innovation of labor-augmenting technical progress as the percentage deviation from initial stochastic balanced growth path (BGP) of business investments (I), residential investments (D), consumption (C), the composite ($\tilde{C}$), labor-augmenting technical progress (A), equity and house prices (q_t, P_H), capital and housing rents (r_K, r_H), unlevered and levered percentage returns on equity (R_E, R_E^{lev}), on housing (R_H), on government bonds (R_g), levered equity premium (EP^{lev}), housing premium (HP), and the SDF (SDF).

house prices. Although the rental rates of capital and housing remain higher than before the shock, decreasing prices imply that rates of return on equity and housing drop below their initial values. Further, consumption now increases less than the composite since residential investments from the previous period result in more houses. Without unexpected changes in lifetime utility, this implies that the SDF now increases to slightly above its initial value. The price of government bonds increases so that the return on government bonds slightly decreases. Decreasing returns on risky assets and bonds cancel out, and premia return to their initial values.

We now turn to the effects of a shock to long-run components of the growth rate, which are pictured in panels (e)–(h). This shock has no immediate effect on productivity, but expectations of future productivity increase. Hence, there is an incentive to reallocate output from consumption to investments. While business and residential investments increase, consumption now decreases. Increasing business investments yield an increasing Tobin's q, and the return on equity also rises. However, decreasing consumption and fixed housing imply that housing rentals also decrease. While house prices increase due to more residential investments and higher expected rents, the current drop in rents almost cancels out the former effect. Thus, there is only a tiny increase in the return on housing. Despite consumption declining, the (ex post realized) SDF drops in response to the shock—lifetime utility increases and is part of the SDF under Epstein and Zin (1989) preferences. The effect on the SDF becomes even stronger in the case of a long-run shock than for a short-run shock since lifetime utility reacts more strongly.

In subsequent periods, Tobin's q increases only moderately, and the return on equity falls to near its initial value. The same holds for house prices and returns on housing. Moreover, consumption increases, and with no unexpected change in lifetime utility, the SDF remains slightly below its initial value so that the return on government bonds rises marginally above the initial value. Premia essentially return to the values realized before the shock.

In terms of our research questions, the long-run risk predictions for housing-related business cycle patterns—i.e., the volatility of residential investment and house prices and rents and housing-related comovements—are partially improved compared with the literature. Further, long-run risk shocks disentangle house prices and rents due to different discounting. Yet the volatility of residential investment and house prices is still too low and too high for rents. The model predicts equity premia well but not housing premia. Housing provides a better hedge for consumption risk—i.e., returns are less (negatively) correlated with the SDF—and contrary to the empirical findings, the Sharpe ratio of housing is smaller than for equity. Interrelatedly, the HJB is too low by a factor of 2. Lastly, the model's success in accounting for equity premia relies on the too-restricted flexibility of business investment apparent in the too-low standard deviations of business investment.

6. Epstein–Zin preferences and disaster risk

The previous model assumed shocks to the mean of future growth that were symmetric. Economic disasters imply large, transitory economic downturns. Since consumption risk increases disproportionately, disaster risk helps explain large equity premia.[17] We consider a model with time-varying disaster risk. When the risk of entering an economic downturn increases, investments drop. Productivity risk directly affects equity returns, while the effects on housing develop only indirectly through the impact of adjustments to the marginal rate of substitution between housing and consumption. We additionally introduce a direct effect of disaster risk on housing returns, since disasters lead to the destruction of housing and productive capital.

6.1. The model

Stochastic discount factor. The household's preferences are described again by a recursive utility function as in Eq. (14), so the model's SDF remains as in (15).

Productivity risk. The last version of productivity risk in our analysis introduces disaster risk. Disasters are introduced through an exogenous shock in the form of a binary variable b_t that indicates disasters in the case of $b_t = 1$, while $b_t = 0$ in normal times.

Disasters result in a decline of productivity by the factor $1 - e^{\omega_{t+1}}$ so that technology grows stochastically according to

$$\ln\left(\frac{A_{t+1}}{A_t}\right) = a + x_{t+1} + \omega_{t+1} b_{t+1}, \tag{17a}$$

$$x_{t+1} = \rho_x x_t + \epsilon_{x,t+1}, \quad \epsilon_{x,t} \sim iid\mathcal{N}(0, \sigma_x^2). \tag{17b}$$

There are two differences compared with the previously introduced case of long-run risk in (16). First, the effect of the "long-run" component x on the growth rate is no longer lagged by one period but appears immediately. For our calibration with $\rho_x = 0$ used below for disaster risk, $\epsilon_{x,t+1}$, therefore, acts just as the short-run risk $\epsilon_{a,t+1}$ in (16). Second, instead of the normally distributed short-run risk $\epsilon_{a,t+1}$, the process now includes the possible effect ω_{t+1} on the growth rate in case a disaster $b_{t+1} = 1$ occurs.

In addition to the effect on productivity, disasters destroy a fraction $1 - e^{\omega_{t+1}}$ of the stocks of capital and residential structures, i.e., the dynamics (5) and (7) from the general framework are replaced by

$$K_{t+1} = e^{\omega_{t+1} b_{t+1}} \underbrace{\left((1-\delta_K)K_t + \Phi\left(\tfrac{I_t}{K_t}\right)K_t\right)}_{=:K^\star_{t+1}},$$

$$H_{t+1} = e^{\omega_{t+1} b_{t+1}(1-\phi)} \underbrace{\left((1-\delta_H)H_t + H_{new,t}\right)}_{=:H^\star_{t+1}}.$$

In consequence, the first-order conditions (9b) and (9c) also must be adjusted accordingly to

$$q_t = \mathbb{E}_t\left[e^{\omega_{t+1} b_{t+1}} M_{t,t+1}\left(r_{K,t+1} + q_{t+1}\left(1 - \delta_K + \Phi\left(\frac{I_{t+1}}{K_{t+1}}\right) \right.\right.\right.$$
$$\left.\left.\left. - \Phi'\left(\frac{I_{t+1}}{K_{t+1}}\right)\frac{I_{t+1}}{K_{t+1}}\right)\right)\right],$$
$$P_{H,t} = \mathbb{E}_t\left[e^{(1-\varphi)\omega_{t+1} b_{t+1}} M_{t,t+1}\left(r_{H,t+1} + P_{H,t+1}(1-\delta_H)\right)\right],$$

The destruction of the stocks of capital and residential structures during disasters hence acts as an additional shock on the model's SDF.

Following Gourio (2012), disasters appear with time-varying probability and size. More specifically, we assume that

$$P(b_{t+1} = 1 | b_t = 0) = \min\{p_t, 1\} \quad \text{and}$$
$$P(b_{t+1} = 0 | b_t = 0) = 1 - \min\{p_t, 1\}$$

where the log of p_t follows an AR(1)-process

$$\ln p_{t+1} = (1-\rho_p)\ln \bar{p} + \rho_p \ln p_t + \epsilon_{p,t+1}, \ \epsilon_{p,t} \sim iid\mathcal{N}(0, \sigma_p^2).$$

Moreover, disasters remain persistent, with a probability no less than $q \in (0, 1)$, so that

$$P(b_{t+1} = 1 | b_t = 1) = \max\{q, \min\{p_t, 1\}\},$$
$$P(b_{t+1} = 0 | b_t = 1) = 1 - \max\{q, \min\{p_t, 1\}\}.$$

Finally, disaster size $1 - e^{\omega_{t+1}}$ also evolves stochastically according to

$$\omega_t := \bar{\omega} e^{\hat{\omega}_t},$$
$$\hat{\omega}_{t+1} = \rho_\omega \hat{\omega}_t + \epsilon_{\omega,t+1}, \quad \epsilon_{\omega,t} \sim iid\mathcal{N}\left(0, \sigma_\omega^2\right),$$

where $\bar{\omega} < 0$. We slightly deviate from the treatment in Gourio (2012) in specifying the process governing disaster size and allow autocorrelation, but we restrict outcomes to $\omega_t < 0$ so that disasters always have negative effects. This specification is similar to that of Fernández-Villaverde and Levintal (2018).[18] It is assumed that the shocks ϵ_x, ϵ_p, and ϵ_ω are stochastically independent.

Rates of return. With disasters, the rates of return on equity, housing, and total risk in (12) now become

$$R_{E,t+1} = e^{\omega_{t+1} b_{t+1}} \frac{r_{K,t+1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}(1 - \delta_k + \Phi(\frac{I_{t+1}}{K_{t+1}}))}{q_t} - 1$$
$$= \frac{r_{K,t+1}K_{t+1} - I_{t+1} + q_{t+1}K^\star_{t+2}}{q_t K^\star_{t+1}} - 1,$$
$$R_{H,t+1} = e^{(1-\varphi)\omega_{t+1} b_{t+1}} \frac{r_{H,t+1} + P_{H,t+1}(1-\delta_H)}{P_{H,t}} - 1$$
$$= \frac{r_{H,t+1}H_{t+1} - P_{H,t+1}H_{new,t+1} + P_{H,t+1}H^\star_{t+2}}{P_{H,t}H^\star_{t+1}} - 1,$$
$$R_{T,t+1} = \frac{r_{K,t+1}K_{t+1} - I_{t+1} + q_{t+1}K^\star_{t+2} + r_{H,t+1}H_{t+1} - P_{H,t+1}H_{new,t+1} + P_{H,t+1}H^\star_{t+2}}{q_t K^\star_{t+1} + P_{H,t}H^\star_{t+1}}$$
$$- 1$$
$$= \frac{q_t K^\star_{t+1}}{q_t K^\star_{t+1} + P_{H,t}H^\star_{t+1}} R_{E,t+1} + \frac{P_{H,t}H^\star_{t+1}}{q_t K^\star_{t+1} + P_{H,t}H^\star_{t+1}} R_{H,t+1}$$

We follow Barro (2006) and Gourio (2012) and allow that bonds in the model may default during disasters. More concretely, government (g) and corporate (c) bonds differ by their recovery rates $\Gamma_{g,t}$ and $\Gamma_{c,t}$ during disasters. Instead of (12d), the price of a bond with maturity τ and recovery rate Γ_{t+1} then follows the recursion

$$Q^{(\tau)}_{j,t} = \mathbb{E}_t[M_{t,t+1}(1 - b_{t+1} + b_{t+1}\Gamma_{j,t+1})Q^{(\tau-1)}_{j,t+1}], \quad \text{where } Q^{(\tau)}_{j,t+1} = 1 \text{ and}$$
$$j \in \{g, c\}.$$

Further, the ex post rate of return from holding a bond with maturity τ for one period is defined by

$$R^{(\tau)}_{j,t+1} = \frac{(1 - b_{t+1} + b_{t+1}\Gamma_{\tau-1,t+1})Q^{(j)}_{j,t+1}}{Q^{(\tau)}_{j,t}} - 1.$$

Finally, we assume that the loss given default during disasters is coupled to the disaster size $1 - e^{\omega_{t+1}}$ via constant fractions $\chi_g, \chi_c \in [0, 1]$ so that

$$1 - \Gamma_{g,t+1} = \chi_g(1 - e^{\omega_{t+1}}) \quad \text{and} \quad 1 - \Gamma_{c,t+1} = \chi_c(1 - e^{\omega_{t+1}}).$$

6.2. Calibration

Panel D of Table 5 shows the calibration of the parameters that are specific to the model version with disaster risk, which now closely follows Gourio (2012).

The elasticity of intertemporal substitution $\psi = 2$ remains at the same value as before as it already matches the value used in Gourio

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[17] See, for example, Rietz (1988), Barro and Ursúa (2008), and Gourio (2012).

[18] Gourio (2012) additionally considers a transitory component of disasters. We checked the effects of a transitory shock component as well. Since we find that the effects for our targets are marginal, we omit the transitory component for the sake of simplicity.

Table 5
Calibration.

Panel D: EZ utility and disaster risk		
γ	3.8	Coefficient of relative risk aversion
ψ	2	Elasticity of intertemporal substitution
ρ_x	0.00	Autocorrelation of log technology shock
ρ_ω	0.00	Autocorrelation of log disaster size
ρ_p	0.90	Autocorrelation of log disaster probability
σ_x	0.01	Conditional standard deviation of log technology shock
$\frac{\sigma_p}{\sqrt{1-\rho_p^2}}$	2.80	Unconditional standard deviation of log disaster probability
$\bar{p}\exp(\frac{\sigma_p^2}{2(1-\rho_p^2)})$	0.0072	Mean disaster probability
q	0.91	Probability for disaster persistence
χ_g	0.20	Default loss of government bonds as fraction of disaster size
χ_c	0.38	Default loss of corporate bonds as fraction of disaster size
Optimized values		
β	0.9939	Discount factor
κ	1.6318	Elasticity of Tobin's q
$\bar{\omega}$	−0.0404	Disaster size
σ_ω	0.2541	Conditional standard deviation of log disaster size

Notes: We optimize the elasticity κ of Tobin's q over the range $[0;7]$. The discount factor β is optimized over the range $[\frac{0.99}{a_u};\frac{0.9999}{a_u}]$, where a_u is the growth factor of utility in the model.
With disaster risk, we additionally optimize $\bar{\omega}$ over $[-0.055;-0.025]$ and σ_ω over $[0.05;0.55]$.

(2012). We adjust the coefficient of relative risk aversion to the value of $\gamma = 3.8$ from Gourio (2012) for the model with disaster risk.

We set $\rho_x = 0$ and $\sigma_x = 0.01$ so that during normal times the stochastic process governing technological progress is identical to the process for the permanent component of productivity in Gourio (2012). We choose the same autocorrelation ($\rho_p = 0.90$) and standard deviation ($\sigma_p = 2.8\sqrt{1-\rho_p^2}$) as Gourio (2012) for the disaster probabilities. Further, we set $\bar{p}$ such that the average probability of entering a disaster is 0.72 percent, and pin down the persistence of disasters to $q = 0.91$—the same values used by Gourio (2012). Following Gourio (2012), we assume an iid process for disaster size, i.e., $\rho_\omega = 0$. Yet while Gourio (2012) considers a permanent and a transitory effect of disasters on productivity, our model specification only includes a permanent effect. We, therefore, deviate from Gourio (2012) and add the mean disaster size $\bar{\omega}$ and the standard deviation σ_ω to the list of parameters over which we optimize the model's fit to the data. The resulting values are $\beta = 0.9939$, $\kappa = 1.6318$, $\bar{\omega} = -0.0404$ and $\sigma_\omega = 0.2541$. For comparison, Gourio (2012) assumes a mean of −0.007 and −0.055 for the effects of disasters on the permanent and transitory components of productivity, respectively.

6.3. Results

Finally, the results for the model with disaster risk are shown in the fourth column of Table 6 for samples that do not include disasters and in the fifth column of Table 6 for samples where we allow disasters in the sample.

Within samples that do not include disasters, the volatility of GDP in the model (0.76) is close to the value from the data (0.87). In the data, business investment (2.23 vs. 2.46), residential investment (6.52 vs. 5.34), and house prices (1.91 vs. 1.36) are more volatile than GDP, and all relative volatilities are matched fairly well. PCE (0.97 vs. 0.75) and housing rents (0.96 vs. 0.72) are less volatile than GDP, but the model slightly overpredicts the relative volatilities.[19] The model correctly replicates the fact that rents fluctuate only approximately half as much as house prices. Considering samples that include disasters, the volatility of GDP in the model (1.05) now slightly exceeds the volatility in the data (0.87), and all relative volatilities decrease moderately. Business investments become somewhat too involatile (1.81 vs. 2.46), but the model matches the relative volatility of rents closer (0.79 vs. 0.72). House prices are again approximately twice as volatile as rents.

Residential investments are nearly perfectly correlated with both house prices and business investments in both cases. Further, the correlations of residential investments and GDP (0.76 without disasters and 0.67 with disasters) are close to the empirical value (0.71). The same holds for the correlations of house prices and GDP: 0.76 without disasters, 0.69 with disasters, and 0.41 in the data.

All rates of return and premia are higher in samples without disasters than in samples that include disasters. In both cases, the model can replicate the return on government bonds fairly well (1.89% without disasters, 1.65% with disasters, and 1.57% in the data). The model also closely matches the return on equity in samples without disasters (7.90% vs. 7.45%) and in samples that include disasters (7.06% vs. 7.45%). Consequently, the model generates a premium on equity (5.94% without disasters and 5.35% with disasters) that is similar to the empirical equity premium (5.88%). On the other hand, the return on housing in the model (3.94% without disasters and 3.42% with disasters) remains below the empirical value (6.01%). Hence, the housing premium in the model (2.03% without disasters and 1.75% with disasters) falls approximately 2.5 percentage points below the value from the data (4.45%). Compared with the model with long-run risk, premia on equity remain similar while premia on housing increase by more than 1 percentage point.

Rates of return are moderately more volatile in samples with disasters than without disasters. In both cases, the model can replicate the low volatility of government bonds (1.39 without disasters and 1.54 with disasters) close to the data (2.31). Moreover, rates of return on equity are somewhat less volatile in the model than observed in the data (11.41 without disasters and 11.26 with disasters compared with 16.71 in the data). The same holds for the volatility of equity premia (11.85 without disasters, 11.53 with disasters, and 16.47 in the data). Since premia on equity are less volatile, the Sharpe ratio of equity (0.50 without disasters and 0.46 with disasters) exceeds the value (0.36) from the data.

Further, the model can closely match the volatility of returns on housing (2.94 without disasters, 3.83 with disasters compared with 3.78 in the data) as well as the volatility of housing premia (3.47 without disasters, 4.02 with disasters compared with 4.41 in the data). However, since the housing premium in the model remains too low, the obtained Sharpe ratio of housing is also too small (0.59 without disasters, 0.46 with disasters, and 1.01 in the data). Compared with the model with long-run risk, the Sharpe ratio of equity decreases while the Sharpe ratio of housing increases.

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[19] Again, the predicted standard deviations of PCE and rents are not outlandish, as they are within the standard deviation range of Table 1.

Finally, the fourth and fifth rows of Table 7 show again the (annualized) decomposition (13) of the Sharpe ratios. Note, however, that the decomposition only holds for simulations that include disasters. In simulations without disasters, the Lucas (1978) asset pricing equations do not hold on average since the simulated distribution deviates from assumed expectations in the model solution. Compared with the previous model, the coefficient of variation decreases and falls substantially below the minimum value of approximately 1 that would be necessary to replicate the empirical Sharpe ratio of housing. While the perfect correlation between the SDF and the equity premium is further dissolved, it remains significantly higher than the upper bound of 1/3 that could potentially allow for a Sharpe ratio of housing that exceeds the Sharpe ratio of equity by a factor of 3. Moreover, although the correlation between the SDF and the housing premium is again raised compared with the previous model, the present model still disentangles the correlations in the wrong way. The correlation between the SDF and premia on housing is smaller than the correlation between the SDF and premia on equity—while the former must exceed the latter by a factor of 3. Consequently, housing's Sharpe ratio is found to be slightly smaller than equity's in samples with disasters where expectations meet. The Sharpe ratio of housing can exceed the Sharpe ratio of equity in samples without disasters, which rests on the deviation of the simulated distribution from expectations.

The effects in response to a growth-rate shock $\epsilon_{x,t+1}$, in response to a disaster risk shock $\epsilon_{p,t+1}$, and in response to a disaster shock b_{t+1} are pictured in panels (a)–(d), (e)–(h) and (i)–(l) of Fig. 4.[20] First, note that our calibration without autocorrelation of the growth-rate process in normal times, $\rho_x = 0$, implies that the growth-rate shock $\epsilon_{x,t+1}$ acts the same way as the short-run shock in the model with long-run risk. In consequence, the implications of a standard growth-rate shock in the present model are nearly identical to the effects of a short-run productivity shock from the previous model. The effects in the present model become somewhat more moderate since the size of the shock in the present model ($\sigma_x = 0.01$) is smaller than in the model with long-run risk ($\sigma_a = \frac{0.0355}{2}$).

An increase in the probability of the economy entering a disaster has the following effects (panels e–h). Positive autocorrelation ($\rho_p > 0$) implies an increased risk for a drop in productivity and destruction of capital and housing in the next period. In consequence, output is shifted from investments in productive capital and from investments in residential structures to consumption.[21] Decreasing investments entail drops in Tobin's q and house prices. The demand effect on house prices and residential investments triggered by the disaster-probability shock is almost identical in size to the effects in response to productivity shocks. Time-varying disaster risk therefore significantly contributes to explain the volatility of housing-related prices and quantities that business cycle models typically fail to replicate. Similar to the previous models, investments in residential structures react more strongly than investments in productive capital. Yet the different elasticities again imply that the effect on Tobin's q dominates the effect on house prices. Moreover, a reduction in working hours implies a decreasing marginal product of capital $r_{K,t}$, whereas increasing consumption implies that the rent $r_{H,t}$ of houses increases. The more pronounced drop in Tobin's q compared with the drop in house prices combined with an increasing rent $r_{H,t}$ of houses yields a larger contraction of the return on unlevered equity than of the return on housing and the effect is further amplified by leverage. Finally, increased disaster risk lowers the household's expected future utility and therefore its lifetime utility. Despite the present increase in the consumption bundle, the realized SDF rises substantially under Epstein and Zin (1989) preferences.

In the subsequent periods, the risk for a disaster declines toward its initial value, and without any actual disaster unfolding, business and residential investments begin to increase again from the initial bottom. In consequence, Tobin's q and house prices also increase in the following periods, and rates of return on risky assets raise slightly above the rates before the shock. Further, the still increased risk of disasters causes bond prices to increase so that the return on the government bond declines. Premia raise above their initial values. On the other hand, the fact that no disaster occurs despite the increased risk improves the household's lifetime utility relative to expectations and lowers the ex post realized SDF below the initial value.

Lastly, an occurrence of a disaster (panels ((i)–(l)) implies that technology A_t drops by the factor $e^{\bar{\omega}}$ as long as the disaster continues. In the period the disaster starts, a second effect appears. The probability that the disaster remains persistent raises to $q = 0.91$, whereas the probability to enter a disaster was initially only $p \approx 0.0072$. The massive increase in probability for the continued destruction of technology, capital, and residential structures in the subsequent period has the previously described effects-amplified by a multitude. The two effects combined—a drop in productivity and increased risk for the disaster to persist—cause extraordinary drops in business and residential investment in the initial period of the disaster. In the following disaster periods, expectations do not change anymore until the disaster ends so investments are only affected by decreasing technology, capital, and residential structures. The initial drop in business investments exceeds the destruction of productive capital so Tobin's q also collapses. In the following periods, the effect turns and business investments decline by less than the rate at which capital is destructed so that Tobin's q begins to slowly recover. On the other hand, since land is not destructed, house prices continue to decline as long as D_t declines. Finally, once the disaster ends, the probability for the economy to be hit by a disaster again jumps back to $p \approx 0.0072$. The massive change in expectations leads to a boom immediately after the disaster. Both investments increase and so do Tobin's q and house prices. The enormous drops in Tobin's q and house prices at the start of the disaster yield massive drops in rates of return, while the boom after the disaster ends implies enormous yields for both risky assets. The realized SDF reacts the opposite way.

With a focus on the research questions, the rare-disaster risk specification accounts well for housing-related and general business cycle patterns. Disaster-risk shocks disentangle house prices and rents due to different discounting and increase the volatility of housing-related prices and quantities, which business cycle models typically fail to replicate. The model accounts for the different volatilities of the asset rates of return and predicts sizable risk premia. However, the housing premia is still too small by a factor of two, resulting in a too-small Sharpe ratio by the same factor. In line with this, the HJB is also too small by a factor of two, and therefore, the SDF and the return on equity must be strongly correlated to match the Sharpe ratio of equity.

7. Model comparison and discussion

Finally, we comprehensively compare the model specification and briefly discuss our conclusion as for why the present framework with productivity risk only fails to replicate the observed Sharpe ratios in their relative size. While we argue that different specifications of the composite good or increased technological frictions do not improve the model's fit, the section concludes with further thoughts on mechanisms that may help.

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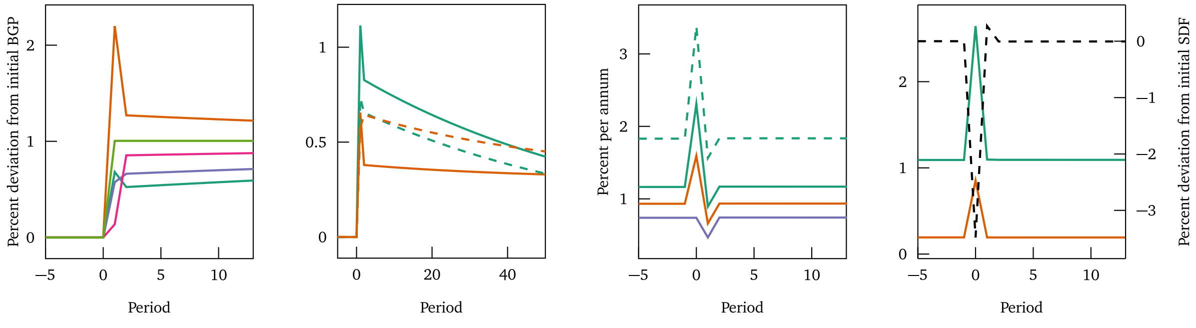
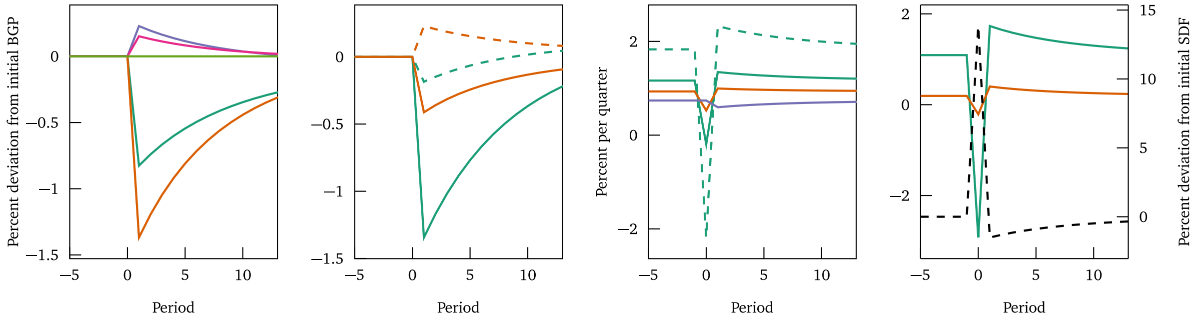
[20] A shock to the disaster size does not trigger any effects on the economy in normal times. If there is currently no disaster, the size of the disaster is irrelevant in the present period and—without autocorrelation, $\rho_{\omega} = 0$—the present disaster size neither affects expectations about potential future disaster sizes. Therefore, we do not picture the impulse response functions as a disaster-size shock.

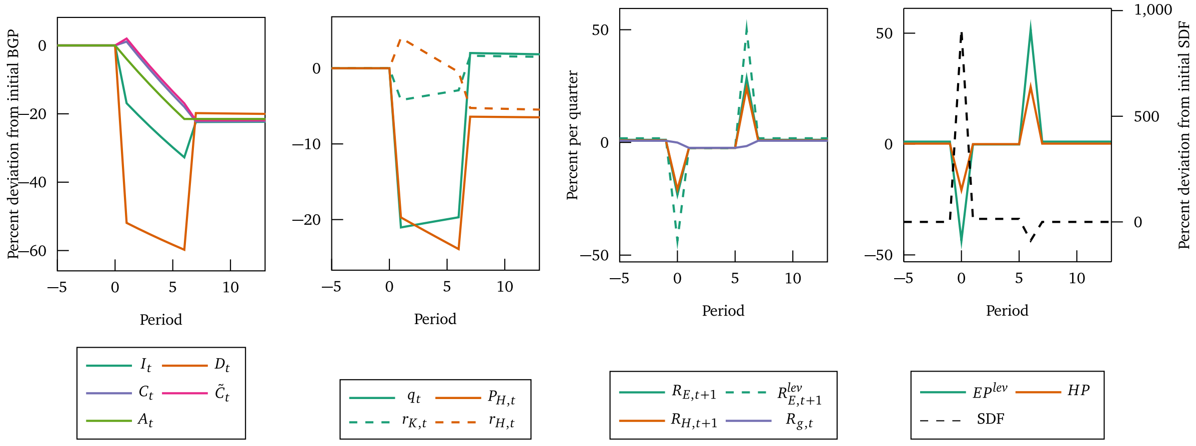
[21] The imperfect comovement of consumption is discussed by Gourio (2012). Gourio (2012) solves this problem with complementarity between consumption and hours worked and countercyclical wage markups. As this study focuses on investment and asset price behavior, we abstain from this extension.



(a) Technology shock ϵ_x I (b) Technology shock ϵ_x II (c) Technology shock ϵ_x III (d) Technology shock ϵ_x IV



(e) Disaster risk shock ϵ_p I (f) Disaster risk shock ϵ_p II (g) Disaster risk shock ϵ_p III (h) Disaster risk shock ϵ_p IV



(i) Disaster $b_t = 1$ I (j) Disaster $b_t = 1$ II (k) Disaster $b_t = 1$ III (l) Disaster $b_t = 1$ IV

Fig. 4. Impulse response functions for disaster risk. **Notes:** Impulse response functions to a one-standard-deviation innovation of labor-augmenting technical progress (ϵ_x), of disaster risk (ϵ_p), and to a disaster ($b_t = 1$) as the percent deviation from initial stochastic balanced growth path (BGP) of business investments (I), residential investments (D), consumption (C), the composite ($\tilde{C}$), labor-augmenting technical progress (A), equity and house prices (q_t, P_H), capital and housing rents (r_K, r_H), unlevered and levered percentage returns on equity (R_E, R_E^{lev}), on housing (R_H), on government bonds (R_g), levered equity premium (EP^{lev}), housing premium (HP), and the SDF (SDF).

7.1. Model comparison

To compare the versions of the model, Table 8 lists the models' score statistics (the sum of squared differences between various moments of the variables in the model and their empirical counterparts). First, we list the score statistics concerning the housing-related targets from our minimization exercise (all housing-related business cycle statistics plus housing premia and the Sharpe ratio of housing), the business cycle-related targets (Panel B from Table 6), asset pricing-related targets (risk-free rate, equity and housing premia, and the concerning Sharpe ratios), and all targets (sum of business cycle and asset pricing-related targets).

Given the listed scores, the disaster version has the most explanatory power of the three model versions. The disaster model total score is 9.35 in the no-disaster sample and 10.23 in the disaster sample, the score of the long-run risk specification equals 21.39, and the habit formation score is 24.37. Likewise, the disaster specifications also match the particular housing, business cycle, and asset price targets considerably better, the habit formation version has a better score compared with the long-run risk version concerning the housing and

Table 6
Simulated returns, premiums and second moments.

	USA	External habits	Long-run risk	Disaster risk	
				No disaster sample	Disaster sample
Panel A: Expenditure shares (in percent of GDP)					
PCE	77.55	63.20	69.77	69.73	69.87
I	16.40	28.24	22.26	23.47	23.38
D	6.04	8.56	7.96	6.80	6.76
Panel B: Business cycle moments					
$\sigma(GDP)$	0.87	1.10	1.20	0.76	1.05
$\frac{\sigma(PCE)}{\sigma(GDP)}$	0.75	0.29	0.91	0.97	0.95
$\frac{\sigma(I)}{\sigma(GDP)}$	2.46	0.93	0.80	2.23	1.81
$\frac{\sigma(D)}{\sigma(GDP)}$	5.34	6.95	3.35	6.52	6.05
$\frac{\sigma(P_H)}{\sigma(GDP)}$	1.36	1.62	1.00	1.91	1.72
$\frac{\sigma(r_H)}{\sigma(GDP)}$	0.72	0.39	0.90	0.96	0.79
$\rho(P_H, D)$	0.39	0.99	1.00	1.00	0.99
$\rho(I, D)$	0.05	0.97	0.82	0.96	0.91
$\rho(GDP, D)$	0.71	1.00	0.93	0.76	0.67
$\rho(GDP, P_H)$	0.41	0.99	0.93	0.76	0.69
Panel C: Rates of return					
R_g	1.57	1.70	1.94	1.89	1.65
R_E^{lev}	7.45	5.58	7.78	7.90	7.06
R_H	6.01	2.66	2.87	3.94	3.42
R_T^{lev}	6.84	4.00	5.19	5.95	5.31
EP	5.88	3.82	5.75	5.94	5.35
HP	4.45	0.95	0.91	2.03	1.75
TP	5.27	2.27	3.19	4.01	3.62
$\sigma(R_g)$	2.31	1.11	0.98	1.39	1.54
$\sigma(R_E^{lev})$	16.71	24.26	9.52	11.41	11.26
$\sigma(R_H)$	3.78	9.34	2.39	2.94	3.83
$\sigma(R_T^{lev})$	6.90	16.95	5.57	7.04	7.49
$\sigma(EP)$	16.47	23.71	9.48	11.85	11.53
$\sigma(HP)$	4.41	16.52	2.18	3.47	4.02
$\sigma(TP)$	7.00	9.10	5.48	7.50	7.75
SR_E	0.36	0.15	0.61	0.50	0.46
SR_H	1.01	0.11	0.42	0.59	0.44
SR_T	0.75	0.14	0.58	0.54	0.47

Notes: Expenditure Shares: Average shares in GDP of private consumption expenditures (PCE), business investments (I), and residential investments (D).
Business cycle moments: Standard deviations $\sigma(.)$ and correlations $\rho(.,.)$ for growth rates of GDP, private consumption expenditures (PCE), business investments (I), residential investments (D), house prices (P_H), and housing rents (r_H). Business cycle statistics are reported for growth rates of the time series.
Rates of return: Mean percentage returns on equity (R_E^{lev}), housing (R_H), total risk (R_T^{lev}), and government bonds (R_g), as well as the equity premium (EP), the housing premium (HP), and the total risk premium (TP), the corresponding standard deviations $\sigma(.)$ as well as the Sharpe ratios of equity (SR_E), of housing (SR_H) and of total risk (SR_T). All rates of return are annualized.
We report the mean outcome from 100 simulations, each for 180 periods after 1000 burn-in periods. For the version with disaster risk, we report the moments from samples without disasters and samples with disasters.

Table 7
Risk premia: Components.

	SR_E	SR_H	CV$(M)/HJB$	$\rho(M, EP)$	$\rho(M, HP)$
USA	0.36	1.01	>1.01	$\in (-0.36;\ 0)$	$2.81 \cdot \rho(M, EP)$
External habits	0.15	0.11	0.14	−0.99	−0.99
Long-run risk	0.61	0.42	0.62	−0.95	−0.63
Disaster risk					
-no disaster samples	0.50	0.59	0.48	−0.91	−0.74
-disaster samples	0.46	0.44	0.51	−0.87	−0.75

Notes: Sharpe ratios of equity (SR_E) and of housing (SR_H), $CV(M)$ Coefficient of variation of the SDF M (Hansen–Jagannathan bound (HJB)), and $\rho(.,.)$ the correlations. We report the mean outcome from 100 simulations, each for 180 periods after 1000 burn-in periods. For the version with disaster risk, we report the moments from samples without disasters and samples with disasters.

Table 8
Scores statistics.

	Housing	Business cycle	Asset pricing	Total
External habits	17.45	7.00	17.36	24.36
Long-run risk	18.28	8.30	13.1	21.39
Disaster risk				
-no disaster samples	9.11	3.19	6.16	9.35
-disaster samples	9.43	2.31	7.91	10.22

Notes: Score equals the sum of various squared differences between empirical and model predicted moments. Housing score: $\frac{\sigma(D)}{\sigma(GDP)}$, $\frac{\sigma(P_H)}{\sigma(GDP)}$, $\frac{\sigma(r_H)}{\sigma(GDP)}$, $\rho(P_H, D)$, $\rho(I, D)$, $\rho(GDP, D)$, and $\rho(GDP, P_H)$. Business cycle score: $\sigma(GDP)$, $\frac{\sigma(PCE)}{\sigma(GDP)}$, $\frac{\sigma(I)}{\sigma(GDP)}$, $\frac{\sigma(D)}{\sigma(GDP)}$, $\frac{\sigma(P_H)}{\sigma(GDP)}$, $\frac{\sigma(r_H)}{\sigma(GDP)}$, $\rho(P_H, D)$, $\rho(I, D)$, $\rho(GDP, D)$, and $\rho(GDP, P_H)$. Asset pricing score: R_g, EP, HP, SR_E, and SR_H. Total score: Business cycle + Asset prices, targets of minimization.

business cycle targets. However, the long-run risk version has a better score concerning asset pricing. The difference between the asset pricing scores is larger than between the business cycle scores ending in the better total score of the long-risk version.

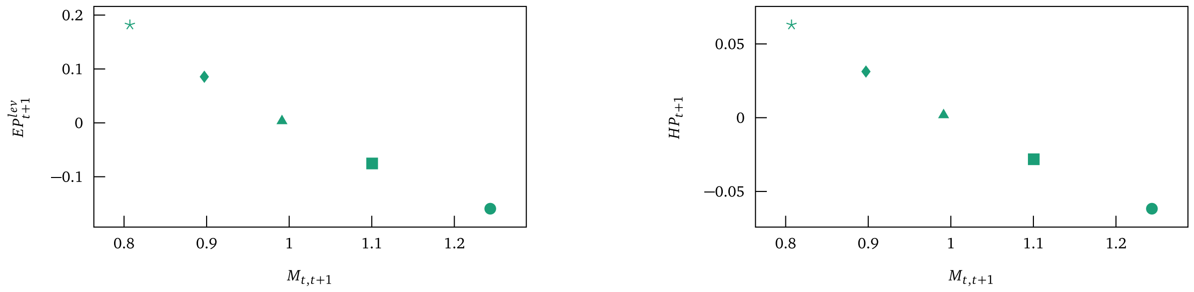
Beyond these scores, our study shows that both utility paramtrizations—habit formations and generalized recursive utility—allow us to account for the housing demand pattern, given sufficient consumption risk. However, a safe net-positive asset like housing challenges the CCAPM-in-production-economies framework to contain sufficient consumption risk. This problem becomes apparent in the high optimal elasticities of Tobin's q to account for our targets, resulting in too-smooth business investment behavior in the habit formation and long-run risk versions. As the disaster-risk version includes housing depreciation risk, housing becomes a risky asset in real terms. Consequently, adjustments to business investments must be less restricted to generate sufficient consumption risk (lower elasticity of Tobin's q), which results in predicted business investment volatility as observed. Additionally, housing deprecation risk generates a higher reward for holding housing risk than the two other versions. Finally, all specifications suffer from a too-low HJB (with the habit formation framework the lowest) and a too-linear relationship between equity returns and the SDF. The low HJB combined with the linear relation between equity returns and the SDF makes it impossible for all versions to account for the difference between the Sharpe ratios or the Sharpe ratio of housing at all.

Due to the rich uncertainty structure of the long-run and disaster-risk version and recursive utility, house prices can rise even when rents fall as the SDF rises. The lean habit formation version does not allow for such a decoupling of present rents and the net present value of future rents. However, the explanatory power of the cyclical housing demand is high, especially given the lean structure. In different frameworks, the habit formation explanatory power could be a sufficient utility parametrization to account for housing demand and risk premia, e.g., in cases where agents only invest in housing, which holds for a majority of homeowners even in countries with highly developed capital markets.

7.2. Discussion

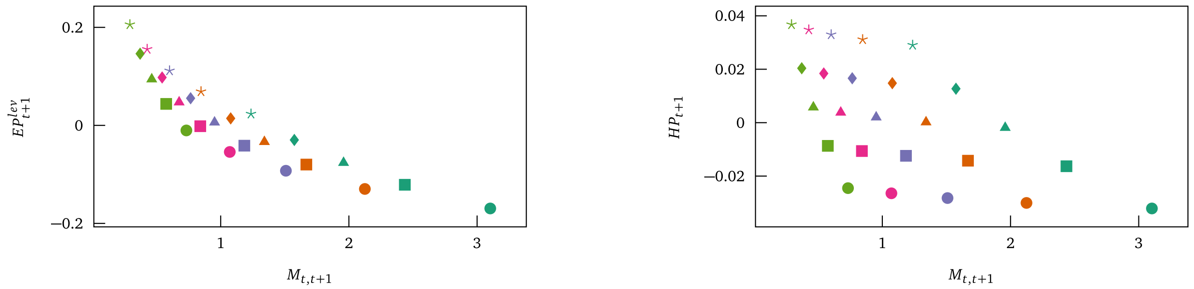
Insufficient explanatory power of technology shocks:. The results from the preceding sections revealed two failures concerning asset prices that are common to our framework: (i) the relation between returns on stocks and the SDF is too linear, and (ii) the relation is, contrary to what is required, less linear between returns on housing and the SDF. This result is further illustrated in Fig. 5, which shows scatter plots of these variables. More specifically, we numerically compute expectations by Gauss–Hermite quadrature with five nodes for each shock. Conditional on the state in period t being on the deterministic growth path, the figure shows the corresponding realizations of the tuples $(M_{t,t+1}, EP_{t+1}^{lev})$ and $(M_{t,t+1}, HP_{t+1})$ at these nodes.

The first row of the figure pictures the scatter plots for the model with external habits. The marks distinguish different realizations of the total factor productivity. As can be seen, equity and housing premia are strongly negatively and linearly related to the SDF. Hence, their



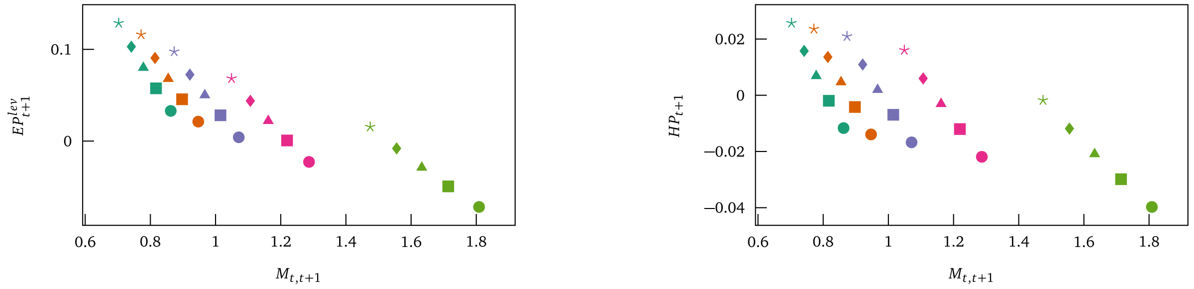
(a) External Habits: Equity

(b) External Habits: Housing



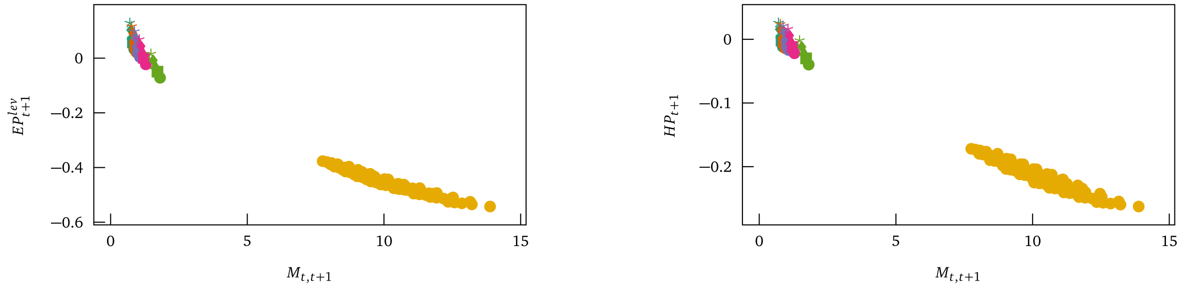
(c) Long-Run Risk: Equity

(d) Long-Run Risk: Housing



(e) Disaster Risk: Equity (no disaster)

(f) Disaster Risk: Housing (no disaster)



(g) Disaster Risk: Equity (disaster)

(h) Disaster Risk: Housing (disaster)

Fig. 5. Correlation: Returns and SDF. **Notes:** Numerically computed expectations by Gauss–Hermite quadrature with five nodes for each shock. Conditional on the state in period t being on the deterministic growth path, the figure shows the corresponding realizations of the tuples $(M_{t,t+1}, EP_{t+1}^{lev})$ and $(M_{t,t+1}, HP_{t+1})$ at these nodes. External Habits: The marks distinguish different realizations of the total factor productivity. Long-Run Risk: Different marks distinguish different realizations of the short-run risk component, different colors identify the long-run risk component. Disaster Risk: First row, no disaster occurs, different marks distinguish particular realizations of the technology shock, different colors identify the disaster risk. Second row, no different colors for disaster risk shocks in disasters.

correlations with the SDF are close to −1, and consequently, the model predicts similar Sharpe ratios of both assets.

The second row of the figure pictures the scatter plots for the model with long-run risk. Different marks distinguish different realizations of the short-run risk component, and different colors identify the long-run risk component. The plot on the left-hand side shows that the effect on the return on equity relative to the effect on the SDF is similar between the two shocks (see also Fig. 3 for the details). Consequently, there is a strong linear relationship between the two variables, and the correlation is close to -1.[22] The plot on the right-hand side reveals that a long-run productivity shock has significantly less effects on returns on housing than a short-run productivity shock, while the opposite is true for the effects on the SDF (see again Fig. 3 for the details). Hence, the effect on the return on housing relative to the effect on the SDF differs significantly between the two shocks. Although there is a strong linear relation between $M_{t,t+1}$ and HP_{t+1} for each shock individually when keeping the other shock constant, the slopes are substantially different. The relation becomes more scattered with a less perfect negative correlation.

Finally, the scatter plots for the model with disaster risk are shown in the third and fourth rows of the figure. If no disaster occurs, the disaster size shock is irrelevant and therefore not pictured. In this case, different marks distinguish particular realizations of the technology shock, while different colors identify the disaster risk. In case a disaster occurs, the disaster size shock becomes active, and we abstain from distinguishing the then 125 shock realizations by colors or marks. The picture on the left-hand side now shows that the technology shock has larger effects on the return on equity compared with the disaster risk shock, while the disaster risk shock has larger effects on the SDF than the technology shock (see also Fig. 4). While the relation between returns on stocks and the SDF is again highly linear in each shock individually, the slopes become somewhat different between the two shocks, and the high (negative) correlation can be moderately reduced. However, the relation remains far too linear. Further, the plot on the right-hand side shows that the effect of a disaster risk shock on the return on housing is even smaller (see again Fig. 4). As in the model with long-run risk, the slopes in the relation between returns on housing and the SDF develop differently. Contrary to what is required, returns on housing are less correlated with the SDF.

In summary, a possible first step to simultaneously explain premia on equity and housing could be to invert the effects seen for the model with long-run productivity risk or disaster risk. This would require two shocks: the first would have larger effects on the SDF but less effects on the return on equity, while the second shock would affect the SDF less but have larger effects on stock returns. Additionally, the relative effects for returns on housing and the SDF must be similar between the two shocks. We could not meet these requirements with productivity risk.

Specification of the composite good:. While the literature generally agrees about nonseparable preferences over housing and nondurable consumption, there is less consensus about the intratemporal elasticity of substitution between the two goods. We choose a constant intratemporal elasticity of substitution of one. We were not successful in improving the model's fit by varying the intratemporal elasticity of substitution between housing and nondurable consumption. Increasing the substitutability would reduce the already too-low housing premia. Reducing the intratemporal substitutability would increase risk premia but at the cost of the demand effect for housing. Since the demand effect may track the business cycle statistics, an increase in housing premia comes with insufficient volatility in house prices and residential investments.

In our specification of habits, habits are superficial, i.e., habits are formed jointly over the composite good $\tilde{C}_t$. A different and common specification is a deep habit formation, where habits are formed separately for the individual goods C_t and H_t. In the present context, deep habits would compromise the household's possibility to substitute housing for consumption, which could potentially improve asset price statistics. However, in our checks improvements to asset price statistics were only marginal, but the rental rate of housing becomes excessively volatile with deep habits, implying also excessively volatile PCE and GDP.

Lastly, changes to the intratemporal substitutability or the habit formation did not allow the replication of a housing Sharpe ratio surplus.

Technological restrictions:. In our framework, the marginal rate of transformation between residential investment and consumption is one. The literature on housing and the business cycle restricts this possibility often. Unfortunately, this cannot improve the model's fit. However, a concave productivity possibility frontier limits the household to smooth its composite good by substituting housing for consumption and thus would increase housing premia. In addition, the limited substitutability would reduce the volatility of residential investment and thereby limit the demand effect for housing, which runs contrary to the data. Concerning the too-low business investment volatility, the productivity possibility frontier is too strict. Loosening this restriction would not improve the business cycle statistics as with the lower reward for holding risk, procyclical demand for housing vanishes.

Again, neither changes in the productivity possibility frontier of residential nor business investment allow replicating a housing Sharpe ratio surplus.

Further thoughts:. The main deficit of the model is obvious. The rates of return and the premia of equity are almost perfectly correlated with the SDF. Any mechanism that increases the volatility of the return on equity and decreases, in absolute terms, the correlation of the return on equity with the SDF would improve the model's fit.

Assuming that corporate bonds could additionally default in normal times meets these requirements.[23] The additional source of uncertainty increases the volatility of the return on equity while the assumption of independence decreases in absolute terms the correlation between the SDF and the return on equity.

Other mechanisms concerning housing-specific characteristics could improve the model's fit in general. For example, due to the poor divisibility of housing, there may be credit-constrained households that can only invest in equity. For them, it would be impossible to smooth the consumption bundle by adjusting consumption and residential investment and subsequently, the equity risk would increase. Housing investment participation, however, is distributed far more broadly and is less concentrated toward the top quantiles than participation in equity stock markets, as Kuhn et al. (2020) show. This participation distribution indicates that the effect is minimal at best.

Among others, Mian et al. (2013) find a strong effect of housing wealth on consumption. Modeling such a channel would increase in absolute terms the correlation between house prices and the SDF and thus between the return on housing and the SDF, which would separate the Sharpe ratios. Theoretical foundations for strong causal effects are given, e.g., by Berger et al. (2017) and Guerrieri and Lorenzoni (2017). Gertler and Gilchrist (2018) summarize this channel in a review as follows: Mortgages are the household's most common structure of debt. Hence, declining house prices increase the household's leverage ratio, and the resulting tightened (re)financing options force the household to reduce its consumption spending. However, Khan and Rouillard (2018) show that household borrowing constraints alone cannot account for house prices' volatility.

Finally, the risk for housing wealth is potentially more idiosyncratic, which increases the volatility of the return on housing on an individual level and thus helps to explain differences in the Sharpe ratios at the aggregated level. However, we argued that other determinants must

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[22] Note that while the return on equity and the SDF may be nonlinear in shocks, they display similar nonlinearities, and the relation between the two variables is linear again.

[23] Gourio (2012) argues the Great Recession was not a great disaster and US treasury bonds and bills did not default. Nevertheless, many corporate bonds defaulted.

also be at work and that the main shortcoming of the model is the almost perfect correlation of the SDF with equity premia. For this reason, we agree with Jordà et al. (2019b) and conclude that a putative solution via idiosyncratic housing falls short.

8. Conclusion

We confront existing approaches to solve the equity premium puzzle with the presence of a second asset that is safe in real terms and has a positive supply, namely housing, and the data from Jordà et al. (2019a) on returns on equity, housing, and total wealth. Our framework is a standard RBC model with housing as a durable consumption good. The model features different types of productivity risk: standard productivity risk, long-run productivity risk as in Croce (2014), or disaster risk similar to Gourio (2012). The SDF follows either from preferences with external habits as in Chen (2017) or from generalized recursive Epstein and Zin (1989) preferences.

The main results of our study concerning asset pricing statistics are as follows. First, the models retain their previously documented ability to generate sizeable equity premia with the introduction of housing as a second asset. Second, as in the data, housing premia in all models are smaller than premia on equity. Yet while only a small difference is observed between the premia of the two assets in the data, housing premia in the model are too small. Third, in all models considered, the mechanism to generate sizeable premia on equity relies on a far too high correlation between returns on equity and the marginal utility of consumption. The high correlation already excludes a second asset with a significantly larger Sharpe ratio, as empirically observed for housing. Fourth, the mechanism relies insufficiently on a sizeable HJB. This quantity remains too small to explain the size of the Sharpe ratio of housing. Fifth, returns on housing are less correlated than equity with the marginal utility of consumption. Contrary to the data, the Sharpe ratio of housing in the model falls below the Sharpe ratio of equity.

Additionally, we examine the model's ability to reproduce business cycle statistics. The model with disaster risk can replicate the volatilities of GDP, business investments, residential investments, and house prices and rents. The model with habit formation accounts well for the procyclical demand for housing, yet the volatility of business investment and consumption is far too low. In the model with long-run productivity risk, the volatility of business investments, residential investments, and house prices remains too low and in the model with habits and standard productivity risk the volatility of business investments and PCE. The model with disaster risk can further explain the empirically observed correlations between GDP and residential investments and between GDP and house prices. Otherwise, the correlations are only matched in sign but are too large. The increased volatility in the SDF disentangles the excessively proportional relationship between house prices and rents of previous models.

The high correlation between equity returns and the SDF is the weak point in the explanatory power of all variants in accounting for different Sharpe ratios and risk premia. Disentangling this relationship points the way for future research.

Declaration of competing interest

The author declares that he has no relevant or material financial interests that relate to the research described in this paper.

Data availability

The code is uploaded on mendeley data: http://dx.doi.org/10.17632/mw9p4wnzpc.1

The return on everything and the business cycle in production economies (Original data) (Mendeley Data).

Appendix A. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.econmod.2024.106742.

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