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Mathematical theory and applications of multiple wave scattering

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Editorial

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One contribution to a special feature "Mathematical theory and applications of multiple wave scattering" organised by guest editors Luke G. Bennetts, Michael H. Meylan, Malte A. Peter, Valerie J. Pinfield and Olga Umnova.

The Isaac Newton Institute for Mathematical Sciences hosted a six-month programme on Mathematical Theory and Applications of Multiple Wave Scattering (MWS) in 2023. The programme was driven by the rapidly growing international research interest in multiple wave scattering, linked to developments in many application areas, from photonics, medical imaging and metamaterials. While it was readily apparent that there are theoretical approaches to multiple wave scattering that are common to various wave phenomena (acoustic, optical, etc.), the research community was widely dispersed in a variety of disciplinary areas, or focused on particular applications, which was holding back the potential for broad scientific advancements. Thus, the MWS programme aimed to bring together these distributed researchers, in order to share their knowledge, bridge the gaps between disciplines and applications and develop a shared language and understanding of the approaches taken by the diverse research communities. In doing so, it was intended to accelerate the progress of mathematical theory for numerous application areas and establish new collaborations between previously disconnected (or merely weakly connected) research communities, to the benefit of enhanced research development. This special feature showcases a cross-section of outcomes of the work initiated or progressed during the MWS programme.

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1. Introduction

This special feature of *Proceedings A* comprises publications authored by researchers spanning a range of career stages and from four continents. The articles cover a range of mathematical approaches, including asymptotic expansions, homogenization, the Wiener–Hopf technique, T-matrix formulations, scattering poles and Green's functions, and a variety of application areas, including sea-ice floes, metamaterials, crystalline structures, ducts, particulate composites and suspensions. As such, it is a faithful representation of the broad range of research conducted at the Isaac Newton Institute programme on Mathematical Theory and Applications of Multiple Wave Scattering (MWS).

A major success of the MWS programme was to foster new collaborations, which is evident in multiple articles in the special feature. One class of collaborations was based on the use of efficient computational techniques for multiple wave scattering to validate and extend theoretical approaches. The field of metamaterials was particularly fruitful, and Hawkins *et al*. [1] summarize four collaborations involving seven programme participants, in which software, based on the T-matrix and self-consistent methods, was applied to four modern metamaterial problems in the context of two-dimensional acoustics. Gower *et al*. [2] also use software based on the T-matrix and self-consistent methods to validate a recently developed theory for effective waves in large clusters of randomly located particles in three dimensions, leveraging the ability to compute large ensembles, such that the necessary statistics could be analysed. Montiel *et al*. [3] use similar computational techniques, but for multiple scattering by floating elastic plates, to create the first model of ocean waves in the marginal ice zone with tens of thousands of ice floes.

The latest developments in state-of-the-art computational techniques for scattering problems were the subject of two further articles. Caetano *et al*. [4] propose a piecewise-constant Galerkin discretization for an integral equation formation of sound-soft wave scattering, including the case of fractal geometries. Meanwhile, Adukova *et al*. [5] develop an efficient condition to determine the stability region for the factorization of 2×2 matrix functions, which is important in vector Wiener–Hopf formulations of scattering problems. Nieves *et al*. [6] and Sharma [7] also focus on the Wiener–Hopf technique, which they use to construct solutions for scattering problems on lattices. Nieves *et al*. [6] consider the dynamic response of a square-cell lattice quadrant having a free lateral boundary that is subjected to a sinusoidal point load, whereas Sharma [7] considers scattering of surface waves on a square-lattice half-plane by inhomogeneities.

Three articles are new collaborations based on novel modal expansions. Chapman & Hawkins [8] are concerned with the scattering of acoustic waves by a compact (small) scatterer in the near field of an acoustic source, based on expanding the wave field in terms of a special representation, which the authors call a 'subsonic wave', defined by the property that in one direction it propagates with subsonic phase speed, while in a perpendicular direction it has exponential amplitude variation. While such an idea has been used successfully in the high-frequency regime, the authors show how it can be applied to low frequencies, i.e. in the Rayleigh scattering regime. The eigenfunction expansion and singularity expansion methods have become popular in recent years, and the new collaboration by Wilks *et al*. [9] solves canonical acoustic multiple wave scattering problems (arrays of cylinders and split-ring resonators) in the time domain using these two frequency-domain-based methods (that is, a discrete version of the generalized eigenfunctions expansion method and a singularity expansion method). Matsushima *et al*. [10] study Rayleigh–Bloch waves using a Green's function that satisfies generalized radiation conditions. The major advantage of this method over a previous approach using a transfer operator is that it allows the point spectrum to be calculated separately from the continuous spectrum of the underlying operator, thus allowing Rayleigh–Bloch waves to be tracked in frequency space after they cut-off or before they cut-on.

Martin [11] provides a collection of fully worked examples for approximations to reflection and transmission coefficients in low-frequency waveguide theory, based on matched asymptotic expansions, in which particular care is taken to ensure that every term at the required order is included. In the most complicated examples, a considerable part of the work involves solving Laplace's equation in situations where the Schwarz–Christoffel method is available, leading to a class of integrals with a family resemblance. Jones *et al*. [12] generalize studies on classical waveguides to elastic chiral waveguides, taking into account the effect of gravity as well as gyroscopic action that couples displacement components. The work obtains results in the two extremes for each of these effects.

Another two contributions focus on inverse wave scattering problems, which were a major theme during the MWS programme. Cakoni *et al*. [13] extend upon previous theoretical ideas about using the duality between scattering poles and transmission eigenvalues to construct an algorithm for calculating the former for an impenetrable obstacle. The approach views scattering poles as a dual to interior eigenvalues by interchanging the roles of incident and scattered fields, and the main idea of the algorithm is then based on computing the interior scattering operator approximately, with details given for two-dimensional problems. As a further point, the theory and the algorithm are extended from Dirichlet to Robin scatterer boundary conditions. Lochner & Peter [14] devise a method for solving a parameter-identification problem in elastodynamics in the long-wavelength regime based on homogenization theory. The method allows them to recover information on subwavelength microstructure features, such as regions of damaged material, from macroscopic boundary measurements, as is of interest in quality inspection of metamaterials.

A further topic of focus in the programme was multiple scattering in random or weakly correlated media. The new collaboration of Pinfield & Valier-Brasier [15,16] explores multiple scattering in elastic random media using the ensemble averaging approach to obtain the effective wavenumber of coherent longitudinal waves, while accounting for longitudinal–shear wave conversions. The first paper [15] establishes the asymptotic behaviour of the analytical model in the long-wavelength limit of the longitudinal waves, while the second paper [16] demonstrates the results of a numerical implementation of the model, validating the asymptotics and demonstrating the impact of weak correlations between scatterers. Similarly, Napal *et al*. [17] use ensemble averaging to obtain the effective T-matrix for a finite two-dimensional circular region randomly populated with scatterers, to represent a physically feasible confined cylinder containing a random medium. Their results are validated with numerical Monte Carlo simulations of multiple wave scattering. Meanwhile, Borcea & Garnier [18] look back on a successful history of collaboration. In their contribution, they analyse the enhancement of wave transmission through random media with mirror symmetry about a reflecting barrier. They explicitly quantify the enhancement in transmission using the asymptotic stochastic theory of wave propagation through random media.

We expect that the outcomes of the work conducted during the MWS programme have helped bring together researchers from the different communities invested in multiple wave scattering. As shown by this collection of 18 articles, it has already led to a number of exciting and beneficial results, which would not otherwise have been obtained by now. We hope the new ideas and collaborations that have arisen from the programme will continue to accelerate advancement of the field.

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