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von

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Abstract

A simple linear model with product differentiation is used to examine the effects of deterministic and stochastic changes of the exchange rate on an international duopoly or oligopoly under both quantity and price competition. Results concerning exchange rate pass-through, the effects of exchange rate uncertainty, and the incentives for non-strategic real or financial hedging are found to be independent of the kind of oligopolistic interaction. If hedging is used as a strategic device either by the exporting firm or by its government, the type of oligopolistic interaction matters, creating an incentive for over-hedging under quantity competition and an incentive for under-hedging under price competition. In the appendix it is shown how the results can be extended to a more general framework.

Key words: oligopoly, exchange rate, uncertainty, pass-through

JEL classification: F12, F31, L13

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1 Introduction

The last decade has seen the emergence of a considerable literature on the effects of flexible exchange rates on international oligopolies. This research interest was primarily stimulated by the large exchange rate fluctuations in the world economy since the 1980s. Market structure was recognized to be important for an explanation of the reaction of prices of imported goods to a change in the exchange rate. In particular, oligopoly models provide one way to explain why, for example, the massive appreciation of the U.S. dollar during the first half of the 1980s did not or did only weakly reduce the dollar prices of many goods imported to the U.S. (cf. *Dornbusch, 1987a*). This is the well-known question of exchange rate pass-through in the presence of oligopoly, i.e., to what extent do fluctuations in the exchange rate end up changing prices paid by the buyers of an oligopolistic good?

There are, however, other aspects of international oligopolies and exchange rates which so far have been less closely examined. This paper will address one issue: the effect of exchange rate uncertainty in a static international oligopoly. Notice the emphasis on uncertainty. Whereas *Dornbusch (1987a)* and the typical pass-through papers analyze a temporary or permanent shift of the exchange rate from one level to another, the role of the variance of the exchange rate is to be considered here. In a one-period framework the impact of risk aversion on conduct and performance and the incentives for hedging against adverse changes in the exchange rate when firms earn all or part of their revenues in a foreign currency will be examined. Hedging has been analyzed in numerous papers by *Broll* and co-authors (e.g. *Broll, 1992*, *Broll and Eckwert, 1996*, *Broll and Wahl, 1992a, 1992b*), but these analyses are confined to price-taking firms and have yet to be extended to the case of oligopoly. In addition, a government's incentives for providing a hedge against exchange rate changes faced by an exporting oligopolist will be analyzed.

The plan of the paper is as follows: In section 2 the basic effects of the exchange rate on a one-period duopoly are considered. This is done under the assumption of exchange rate certainty, i.e., non-stochastic shifts of the exchange rate are used to work out the basics. Section 3 then introduces exchange rate uncertainty and examines the role of risk aversion and the incentives for hedging against unfavorable shifts in the exchange rate. Section 4 sums up. Most of the analysis will be performed in a duopoly framework. Extensions to more than two firms are possible, albeit cumbersome for the case of price-competition.

2 Exchange rate and international oligopoly

Consider an international oligopoly with n_1 identical firms in country 1 (called home) and n_2 identical firms in country 2 (called foreign) competing in the home market. Outputs, of which each firm produces just one, are identical within countries but may differ among countries. Demand originates from a quadratic utility function

$$u(x_i, x_j) = a(x_i + x_j) - (x_i^2 + 2kx_i x_j + x_j^2)/2 + x_0 \tag{1}$$

expressing the „love of variety"-approach, where the numéraire x_0 denotes the composite of all other goods (see e.g. *Singh and Vives, 1984*). Inverse demand functions are thus given by

$$p_i = a - x_i - kx_j \quad i = 1,2, \quad j \neq i, \tag{2}$$

with a being an indicator of market size, k a parameter capturing the substitutability of the two products, and x_i the aggregate output $n_i y_i$ of all country i firms. If $k = 1$, all $n_1 + n_2$ firms produce a homogeneous product, if $k = 0$, the domestic and the foreign goods are not substitutable implying the existence of two separate markets with n_1 and n_2 producers. Inputs are supplied nationally and firms are assumed to have constant marginal costs c_i which are identical for all firms in country i.[1]

For further reference we examine first the case of no exchange rate uncertainty. Denote by e the price of one unit of domestic currency in units of foreign currency. If, for example, we take the U.S. as the domestic country, e is the value in DM earned by a German firm exporting the equivalent of one dollar to the U.S.

If firms compete in quantities, their profit functions are given by

$$\pi_1 = (a - x_1 - kx_2 - c_1)y_1 = (a - (n_1 - 1)y_1 - y_1 - kn_2 y_2 - c_1)y_1, \tag{3}$$

$$\pi_2 = [e(a - x_2 - kx_1) - c_2]y_2 = [e(a - (n_2 - 1)y_2 - y_2 - kn_1 y_1) - c_2]y_2, \tag{4}$$

where we assume $a > c_1$ and $ea > c_2$. Note that $(n_i - 1)y_i$ is the aggregate output of all other country i firms, i.e., the quantity y_i involved in this expression is not a decision variable of the producer under consideration. For a given exchange rate e individual first-order conditions for profit maximization of firms in country 1 and 2 are

$$a - (n_1 + 1)y_1 - kn_2 y_2 - c_1 = 0, \tag{5}$$

$$e(a - (n_2 + 1)y_2 - kn_1 y_1) - c_2 = 0. \tag{6}$$

1 Under the assumption of constant marginal costs, a firm's decision concerning the home market can be analyzed in isolation, no matter where else the firm is active.

Using symmetry and adding up the conditions for all domestic and all foreign producers, respectively, yields two expressions in aggregate outputs

$$n_1 a - (n_1 + 1)x_1 - n_1 k x_2 - n_1 c_1 = 0, \tag{7}$$

$$n_2 a - (n_2 + 1)x_2 - n_2 k x_1 - n_2 c_2 / e = 0. \tag{8}$$

which in turn lead to the equilibrium values of x_1 and x_2

$$x_1 = n_1\left[e\left((1-k)n_2 + 1\right)a - e(n_2 + 1)c_1 + n_2 k c_2\right] / (eA), \tag{9}$$

$$x_2 = n_2\left[e\left((1-k)n_1 + 1\right)a - e(n_1 + 1)c_2 + e n_1 k c_1\right] / (eA), \tag{10}$$

where $A = \left(n_1 + n_2 + 1 + (1-k^2)n_1 n_2\right) > 0$. Individual outputs are then given by $y_i = x_i / n_i$. Straightforward calculations show that $dx_i / dn_i > 0$ and $dx_i / dn_j < 0$ as long as outputs are strictly positive. I.e., an increase in concentration in country *i* raises the market share of its firms.

Equilibrium quantities and prices are affected by the exchange rate as follows:

$$\frac{dx_1}{de} = \frac{-n_1 n_2 k c_2}{e^2 A} \le 0, \qquad \frac{dp_1}{de} = \frac{-n_2 k c_2}{e^2 A} \le 0, \tag{11}$$

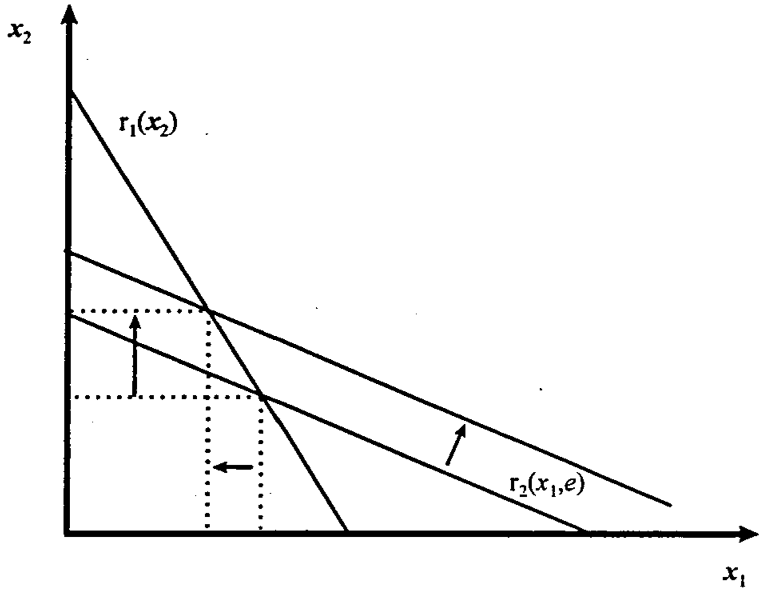
$$\frac{dx_2}{de} = \frac{(n_1 + 1)n_2 c_2}{e^2 A} > 0, \qquad \frac{dp_2}{de} = \frac{-n_2\left(1 + (1-k^2)n_1\right)c_2}{e^2 A} < 0. \tag{12}$$

For values of $k \in]0,1]$, an appreciation of the domestic currency, i.e. an increase of e, leads to a reduction in aggregate domestic output x_1 and to an increase in total imports x_2 from the foreign firms which find exporting now more attractive. Both output levels are influenced by e despite the fact that the exchange rate does not show up in the first-order conditions of country 1 firms. As long as there is some substitutability between the home and the foreign product, there is a strategic effect of e on the oligopoly. Only if $k = 0$ we observe no change in domestic output, but there is still the import increase. Note the decrease in p_1 even if the quantity x_1 is reduced as a consequence of the exchange rate appreciation. The expansion of foreign output x_2 dominates the reduction of domestic output x_1. Furthermore, a comparison of the effect on the two prices shows that $dp_2 / de - dp_1 / de < 0$ for $k < 1$. I.e., foreign goods become relatively less (more) expensive as the domestic currency appreciates (depreciates).

Consider Figure 1 for the intuition on the level of aggregate home and domestic outputs. An increase of e leads to an outward shift of the aggregate reaction curve r_2 of foreign firms, affecting only the intercept but not the slope. Notice that domestic and foreign outputs are strategic substitutes, i.e., the reaction functions are downward sloping in (x_1, x_2)-space.[2]

Figure 1

Appreciation of the importing country's currency under quantity competition



In order to address the question of exchange rate pass-through we calculate the elasticity of the domestic price p_2 of foreign output with respect to a change in e as

$$\varepsilon_{p_2,e} = -\frac{n_2\left(1+(1-k^2)n_1\right)c_2}{e\left((1-k)n_1+1\right)a + n_2\left(1+(1-k^2)n_1\right)c_2} < 0. \tag{13}$$

The absolute value of this expression is clearly below 1, i.e., an appreciation of the domestic currency is only partially passed through to domestic consumers. Using the equilibrium price p_2, we can alternatively write for the price elasticity

$$\varepsilon_{p_2,e} = -\frac{n_2+(1-k^2)n_1n_2}{n_1+n_2+1+(1-k^2)n_1n_2}\frac{c_2/e}{p_2} < 0. \tag{14}$$

The first term is an indicator of the relative number of foreign firms active in the domestic market. It is increasing in n_2, decreasing in n_1 and decreasing in k which means that stronger substitutability between domestic and foreign output reduces the relative importance of foreign firms. The second term is the ratio of marginal cost to price of

2 For the slopes we get $-(n_1+1)/(n_1k) \in]-\infty,-1[$ and $-(n_2k)/(n_2+1) \in]-1, 0]$, respectively, where orthogonal reaction curves result from $k=0$ and parallel reaction curves result from $k=1$ and $n_1, n_2 \to \infty$.

foreign producers expressed in domestic currency. For $k = 1$ the elasticity formula simplifies to the one given in *Dornbusch (1987a, p. 97)*. As for effects of market structure parameters of the model on pass-through note the following:

$$\frac{d\varepsilon_{p_2,e}}{dk} \gtrless 0, \quad \frac{d\varepsilon_{p_2,e}}{dn_1} \gtrless 0, \quad \frac{d\varepsilon_{p_2,e}}{dn_2} < 0. \tag{15}$$

The ambiguity of the reaction of $\varepsilon_{p_2,e}$ to an increase in k results from the indeterminate sign of dp_2/dk. If the latter is non-negative, we have a positive reaction of the elasticity, implying that pass-through gets lower as substitutability increases. An increase of the number n_1 of domestic firm also has an ambiguous effect on pass-through. Note, however, that for $k = 0$ or a, i.e., market size, sufficiently high, the change in the elasticity is negative. In this case an increase of n_1 increases pass-through. For the number of foreign firms we get the unambiguous result that a higher n_2 increases pass-through. Higher concentration in the foreign export industry therefore reduces pass-through (for a similar result from a different model see *Kirman and Phlips, 1996, p. 142*).[3]

Since we know already that an increase of e may also lower the price of domestic output, we calculate a second elasticity as

$$\varepsilon_{p_1,e} = -\frac{n_2 k c_2}{e\big((1-k)n_2+1\big)a + n_1 e\big(1+(1-k^2)n_2\big)c_1 + n_2 k c_2} \le 0, \tag{16}$$

which for $k > 0$ has an absolute value below 1. Market structure has unambiguous effects on $\varepsilon_{p_1,e}$

$$\frac{d\varepsilon_{p_1,e}}{dk} < 0, \quad \frac{d\varepsilon_{p_1,e}}{dn_1} \ge 0, \quad \frac{d\varepsilon_{p_1,e}}{dn_2} \le 0, \tag{17}$$

with strict inequality if $k > 0$. Higher substitutability therefore leads to stronger effects of the exchange rate on the price of the domestic good. If there is some substitutability, an increase in the relative number of domestic (foreign) firms, reduces (increases) this indirect form of pass-through.

The previous analysis covered the range from no substitutability to perfect substitutability between the domestic and the foreign good. The alternative case of complementarity is easily analyzed by using $k < 0$ when interpreting the expressions derived above. Whenever the sign of k determines the sign of an expression, we get a different

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3 If changes in the price reaction instead of changes in the reaction of the elasticity are considered, we unambiguously get $d^2 p_2/dedn_1 \ge 0$, $d^2 p_2/dedn_2 < 0$ and $d^2 p_2/dedk \ge 0$ with strict inequality if $d > 0$.

sign for the case of complementary goods. In particular, reaction curves are upward sloping in (x_1, x_2)-space, an appreciation increases both quantities, increases the price of the domestic variety and reduces the price of the foreign variety. The sign of $\varepsilon_{p_2,e}$ remains negative, whereas $\varepsilon_{p_1,e}$ now has a positive sign.

So far we assumed that oligopolists compete in quantities. For an analysis of price competition we would have to write down a full system of $n_1 + n_2$ inverse demand functions of type (2) and solve it for quantities in order to formulate individual profit functions depending on individual prices. Solving this system, however, turns out to be impossible under the maintained assumption of perfect substitutability of outputs from one country. To still be able to get an idea about pass-through under price competition, we restrict ourselves to the case of $n_1 = n_2 = 1$, where we can write demand as[4]

$$x_i = a - p_i + kp_j, \quad i = 1,2, \quad i \neq j. \tag{18}$$

Profit functions are then given by

$$\pi_1 = (p_1 - c_1)(a - p_1 + kp_2), \tag{19}$$

$$\pi_2 = (ep_2 - c_2)(a - p_2 + kp_1). \tag{20}$$

Maximization with respect to p_1 and p_2, respectively, leads to first-order conditions

$$(a + c_1 - 2p_1 + kp_2) = 0, \tag{21}$$

$$(ea + c_2 - 2ep_2 + ekp_1) = 0, \tag{22}$$

and to *Nash* equilibrium prices

$$p_1 = \frac{e(2+k)a + 2ec_1 + kc_2}{e(4-k^2)}, \tag{23}$$

$$p_2 = \frac{e(2+k)a + ekc_1 + 2c_2}{e(4-k^2)}. \tag{24}$$

The influence of the exchange rate on quantities and prices is given by

$$\frac{dx_1}{de} = \frac{-kc_2}{e^2(4-k^2)} \leq 0, \quad \frac{dp_1}{de} = \frac{-kc_2}{e^2(4-k^2)} \leq 0, \tag{25}$$

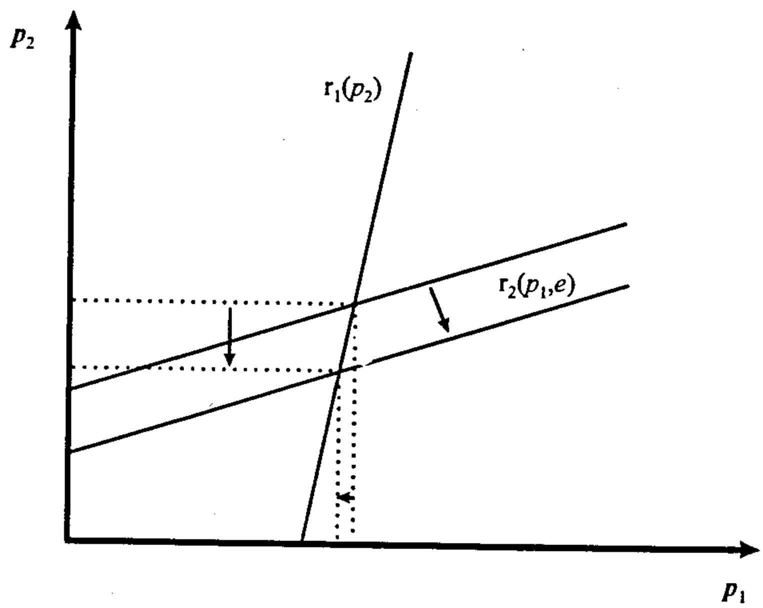
4 This results from solving the system of indirect demands for output quantities, normalizing quantity units by a factor $(1-k^2)$, and writing a for $(1-k)a$. Doing this, we have to assume $k < 1$. If the two products were perfect substitutes, we would have a *Bertrand* oligopoly with a homogeneous good and $p_1 = p_2 = \min(c_1, c_2/e)$.

$$\frac{dx_2}{de} = \frac{(2-k^2)c_2}{e^2(4-k^2)} > 0, \quad \frac{dp_2}{de} = \frac{-2c_2}{e^2(4-k^2)} < 0. \qquad (26)$$

For $k \in]0,1[$ an appreciation of the domestic currency reduces domestic output x_1, increases foreign output x_2, and leads to a lower price for both goods. The directions of responses of equilibrium prices and quantities to changes in the exchange rate are therefore the same as under quantity competition.

Figure 2

Appreciation of the importing country's currency under price competition



Again there is a strategic effect of e as long as there is some substitutability. If $k = 0$, only foreign variables change in the way indicated above. Note the drop in x_1 even if p_1 falls as a consequence of the exchange rate appreciation. The reduction of the foreign price p_2 is strong enough to dominate the reduction of p_1 and lead to a lower domestic output. Figure 2 depicts an increase in e. It leads to a downward parallel shift of the foreign reaction curve r_2. Domestic and foreign prices are strategic complements, i.e., the reaction functions are upward-sloping in (p_1, p_2)-space.[5]

To evaluate exchange rate pass-through we again calculate the elasticity

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5 An increase in k does not affect the p_1-intercept of r_1 and lowers the slope from infinity (at $k = 0$) towards 2 (as $k \to 1$). Similarly, an increase in k has no effect on the p_2-intercept of r_2 and increases the slope from zero (at $k = 0$) towards 1/2 (as $k \to 1$).

$$\varepsilon_{p_2,e} = -\frac{2c_2}{e(2+k)a + ekc_1 + 2c_2} < 0 \tag{27}$$

which has an absolute value below 1, indicating less than perfect pass-through. For the impact of substitutability on pass-through we now get the unambiguous result

$$\frac{d\varepsilon_{p_2,e}}{dk} > 0, \tag{28}$$

i.e., if the domestic and foreign products become closer substitutes, pass-through decreases. This is the result of two opposing forces: if substitutability is higher, there is a stronger effect of e on p_2; this, however, is more than compensated by the fact that an increase in substitutability leads to a higher price p_2 under price competition. Both forces can be visualized in Figure 2, recalling that an increase in k reduces the slope of r_1 and increases the slope of r_2.

For the indirect pass-through, i.e., the effect of the exchange rate on the price of the domestic product, we get

$$\varepsilon_{p_1,e} = -\frac{kc_2}{e(2+k)a + 2ec_1 + kc_2} \leq 0, \tag{29}$$

the absolute value of which is also below 1. Note finally that

$$\frac{d\varepsilon_{p_1,e}}{dk} < 0, \tag{30}$$

i.e., an increase in substitutability increases indirect pass-through.

Conclusions from this simple oligopoly model with product differentiation between domestic and foreign outputs are the following: An appreciation of the importing country's currency leads to increased sales of the foreign good and - given some substitutability - reduced sales of the domestic good. The price of the imported good falls as does - given some substitutability - due to a strategic effect the price of the domestic good, i.e., the change of the import price is transmitted to the price of the domestic variety. The percentage decrease in the price of the imported good, however, is smaller than the percentage appreciation of the importing country's currency. This is the phenomenon of less than perfect pass-through. Note that the effects outlined here are independent of the nature - quantity-setting or price-setting - of oligopolistic interaction.

The model used above dealt with the direction and magnitude of price changes in reaction to a change in the exchange rate. In the literature there are both empirical papers such as, for example, *Mann (1986)*, *Hooper and Mann (1989)*, *Knetter (1989)* and the comprehensive survey in *Menon (1995, pp. 208-226)*, and theoretical papers such as e.g.

Dornbusch (1987a, 1987b), Caselli (1991), Baniak and Phlips (1995), Phlips and Kirman (1996) on this issue. Other questions which require at least a two-period framework and were not discussed here concern the consequences of temporary as opposed to permanent changes in the exchange rate (see e.g. *Froot and Klemperer, 1989, Tivig 1996*) or the duration of price changes due to temporary changes in the exchange rate when firms have to decide on costly entry or exit (see e.g. *Baldwin, 1988, Baldwin and Krugman, 1989; Dixit, 1989*).

3 Exchange rate uncertainty

So far we examined the effect of a deterministic and permanent shift of the level of the exchange rate on prices and quantities in an international oligopoly with differentiated goods. We now turn our attention to the consequences of exchange rate uncertainty for the equilibrium. Producers decide on outputs (or prices) before the realization of e which is now a random variable. To keep the analysis simple, we focus on the case of only one domestic and one foreign firm, i.e., $n_1 = n_2 = 1$, which implies $x_i = y_i$. For an agent i who faces uncertainty constant (absolute) risk aversion ρ_i is assumed.[6] To analyze rational b·havior in this framework, we use the (μ,σ)-rule, i.e., agent i maximizes $u(\pi_i) = \mathrm{E}(\pi_i) - (\rho/2)\mathrm{Var}(\pi_i)$.[7] Considering first a quantity-setting duopoly, objective functions of the two firms can be written as

$$\pi_1 = (a - x_1 - kx_2 - c_1)x_1, \tag{31}$$

$$u(\pi_2) = \left[\mu_e(a - x_2 - kx_1) - c_2\right]x_2 - \frac{\rho_2}{2}(a - x_2 - kx_1)^2 x_2^2\sigma_e^2, \tag{32}$$

where μ_e and σ_e^2 denote the expected value and the variance of the exchange rate e. Optimality requires

$$a - 2x_1 - kx_2 - c_1 = 0, \tag{33}$$

$$\mu_e(a - 2x_2 - kx_1) - c_2 + \rho_2(a - x_2 - kx_1)x_2^2\sigma_e^2 - \rho_2(a - x_2 - kx_1)^2 x_2\sigma_e^2 = 0. \tag{34}$$

The first-order condition (34) of the foreign firm is now non-linear in outputs. While the explicit solution for x_2 turns out not to be very illuminating, a number of conclusions on the reaction function can be drawn from inspecting (34). Notice first that for $x_2 = 0$ we have $x_1 = (a - c_2/\mu_e)/k$ which is independent of ρ_2 and σ_e^2, i.e., the attitude to-

6 I.e., the *von Neumann-Morgenstern* utility functions are from the family $v(\pi_i) = 1 - e^{-\rho_i \pi_i}$.

7 A sufficient condition for compatibility of this decision rule with expected utility maximization under constant risk aversion is that π_i be normally distributed.

wards risk and the variance of the exchange rate do not affect the x_1-intercept of firm 2's reaction curve. Second, for $\sigma_e^2 = 0$ we are back to the case of a deterministic exchange rate examined earlier. Third, if duopolist 2 is risk-neutral, i.e., $\rho_2 = 0$, (34) simplifies to $\mu_e(a - 2x_2 - kx_1) - c_2 = 0$ which is linear in outputs and differs from the first-order condition under certainty only by including μ_e instead of e. In fact, we have a certainty equivalence result here: The reaction function of firm 2 and the duopoly equilibrium are as if the exchange rate were certain and identical to μ_e. On average, therefore, the same equilibrium as under certainty is reached. An increase or a decrease of σ_e^2 does not affect individual decisions as long as it is mean-preserving. In particular, there is no negative effect of exchange rate uncertainty on the volume of trade, if duopolist 2 is risk-neutral. Changes in the expected level μ_e of the exchange rate can be analyzed analogously to the previous section with Figure 1 providing the appropriate diagrammatic exposition.[8]

For further insights we rewrite duopolist 2's first-order condition (34) as

$$\mu_e MR_2 - c_2 - \rho_2 TR_2 MR_2 \sigma_e^2 = 0, \tag{35}$$

where TR_2 is total revenue and MR_2 $(= a - kx_1 - 2x_2)$ is (perceived) marginal revenue of firm 2. Let ρ_2, TR_2 and σ_e^2 be strictly positive. If also $MR_2 > 0$, $MR_2 > c_2/\mu_e$ must hold. Since $\partial MR_2/\partial x_2 < 0$ and under certainty we have $MR_2 = c_2/\mu_e$, x_2 is smaller in the uncertainty compared to the certainty case. This conclusion hinges on the assumption of a strictly positive MR_2. Note, however, that (35) could also hold for $MR_2 < 0$. If we consider the introduction of (marginal) uncertainty σ_e^2 starting from $\sigma_e^2 = 0$, we can by implicitly differentiating the system of first-order conditions rule out the possibility of a negative marginal revenue.[9] At $\sigma_e^2 = 0$ MR_2 is increasing in σ_e^2 which implies that under uncertainty it is higher than the value of c_2/μ_e reached under certainty. This result is quite intuitive: When there is exchange rate uncertainty, a risk-averse duopolist 2 demands a higher marginal revenue to compensate for the risk.

Figure 3 summarizes the effects of exchange rate uncertainty on the equilibrium. We have (starting from $\sigma_e^2 = 0$)

$$\frac{dx_1}{d\sigma_e^2} = \frac{k\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} \geq 0, \tag{36}$$

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8 If we were to draw iso-profit contours, they would have to be interpreted in terms of expected profit.

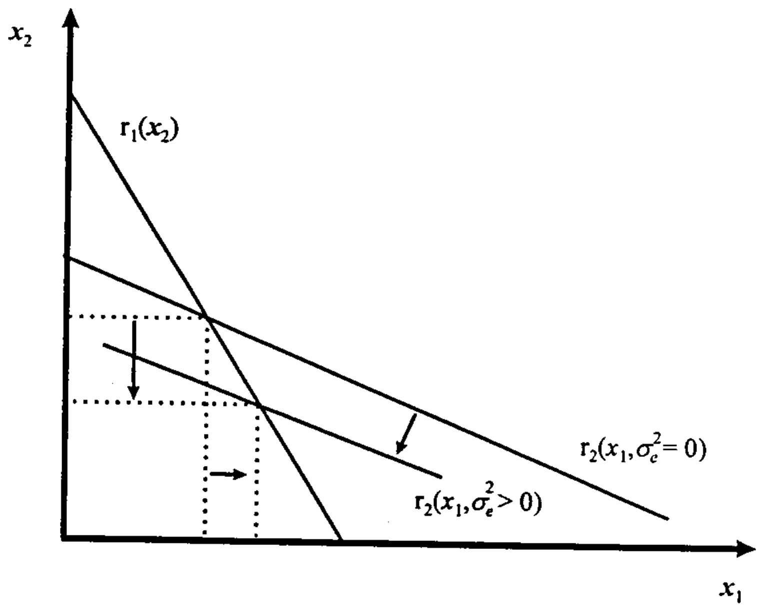
9 Introducing positive risk aversion starting from $\rho_2 = 0$ yields analogous results.

$$\frac{dx_2}{d\sigma_e^2} = \frac{-2\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} < 0. \tag{37}$$

An increase in exchange rate uncertainty leads to a flatter reaction curve r_2. It reduces the output of the foreign exporter, and increases the output of the domestic import-competing firm, if there is some substitutability between the domestic and the foreign product.

Figure 3

Increase in exchange rate uncertainty under quantity competition



Price effects are given by

$$\frac{dp_1}{d\sigma_e^2} = \frac{k\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} \geq 0, \tag{38}$$

$$\frac{dp_2}{d\sigma_e^2} = \frac{(2-k^2)\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} > 0, \tag{39}$$

i.e., exchange rate uncertainty worsens performance of this duopoly market.

If the two firms compete in prices, objective functions are given by

$$\pi_1 = (p_1 - c_1)(a - p_1 + kp_2) = TR_1 - c_1 x_1, \tag{40}$$

$$\begin{aligned} u(\pi_2) &= (\mu_e p_2 - c_2)(a - p_2 + kp_1) - \frac{\rho_2}{2}(a - p_2 + kp_1)^2 p_2^2 \sigma_e^2 \\ &= \mu_e TR_2 - c_2 x_2 - \frac{\rho_2}{2} TR_2^2 \sigma_e^2. \end{aligned} \tag{41}$$

As before, first-order conditions can be written as

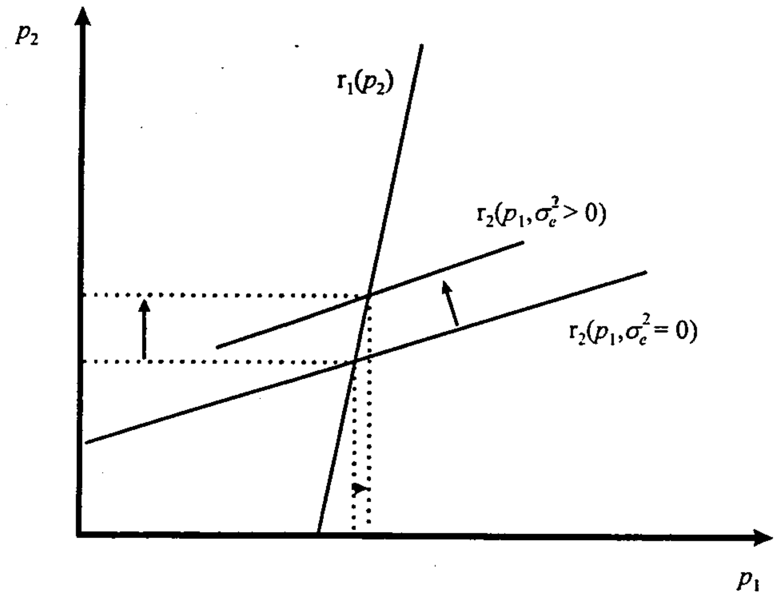
$$MR_1 + c_1 = 0\,, \tag{42}$$

$$\mu_e MR_2 + c_2 - \rho_2 TR_2 MR_2 \sigma_e^2 = 0\,, \tag{43}$$

with $MR_i = a - 2p_i + kp_j$.

Figure 4

Increase in exchange rate uncertainty under price competition



Our reasoning about firm 2's reaction function and the comparative-static properties of the equilibrium exactly parallels the one for the case of quantity-setting: The p_1-intercept of r_2 is independent of ρ_2 and σ_e^2; there is certainty equivalence for $\rho_2 = 0$; MR_2 which is again decisive for the sign is negative and decreasing in σ_e^2, if we introduce (marginal) uncertainty, which also leads to a steeper reaction curve r_2. Figure 4 depicts the effects on prices which are given by

$$\frac{dp_1}{d\sigma_e^2} = \frac{-k\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} \geq 0\,, \tag{44}$$

$$\frac{dp_2}{d\sigma_e^2} = \frac{-2\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} > 0\,, \tag{45}$$

whereas quantities change according to

$$\frac{dx_1}{d\sigma_e^2} = \frac{-k\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} \geq 0\,, \tag{46}$$

$$\frac{dx_2}{d\sigma_e^2} = \frac{(2-k^2)\rho_2 TR_2 MR_2}{\mu_e(4-k^2)} < 0\,. \tag{47}$$

The foreign exporter will sell less and charge a higher price in domestic currency. If there is some substitutability, the domestic firm will sell more and also receive a higher price.

Note that from the perspective of market performance exchange rate uncertainty has the same unfavorable effects on the duopoly equilibrium irrespective of the nature of the duopolistic interaction. Both under quantity and under price competition prices will rise and the volume of trade will drop if exchange rate uncertainty increases.

Our analysis was based on the assumption of constant (absolute) risk aversion. An approach for the analysis of risk-averse duopolists from a recent working paper by *Asplund (1995)* can be used to extend our results to the more general case of non-constant risk aversion (for details see the appendix). Note also that the results derived above are similar in spirit to the fairly general insights of *Leland (1972)* for a firm facing uncertain demand.

When we examined the effects of exchange rate uncertainty on an international duopoly in this section, we implicitly ruled out the possibility of hedging. In reality the foreign firm will quite often be able to hedge against adverse movements of the exchange rate. The simplest way to achieve this is through buying some or all of firm 2's inputs in country 1, thereby reducing the exporter's exposure to the exchange rate. A more sophisticated hedging strategy would be to sell some or all of the revenue earned in country 1's currency in a forward market and receive a certain amount of country 2's currency.

Consider first the hedging strategy of buying inputs in the country where outputs are sold, and assume that firm 2 buys a share h of its inputs in country 1. For the case of quantity-setting firm 2's profit function and objective function can then be written as

$$\pi_2 = eTR_2 - ehc_2x_2 - (1-h)c_2x_2, \tag{48}$$

$$u(\pi_2) = TR_2 - c_2x_2 - \frac{\rho_2}{2}(TR_2 - hc_2x_2)^2\sigma_e^2. \tag{49}$$

To simplify the analysis we set $\mu_e = 1$ and assumed factor price equalization „on average" between the two countries.[10] Maximization with respect to h yields

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10 Suppose, firm 2 buys only one input which costs w_2 and w_1 in country 1 and 2, respectively. The assumption on factor price equalization is $w_2 = \mu_e w_1$. With $\mu_e = 1$ and an input coefficient a_2 firm 2' marginal cost is $a_2 w_1$, if it buys in country 2, and $ea_2 w_1$, if it buys in country 1. This

$$h = \frac{TR_2}{c_2 x_2}, \tag{50}$$

which equals 1, if the duopolist just covers its costs, and exceeds 1, if it makes a profit. If we assume for a moment that the producer is not confined to $h = 1$, but can „over-hedge" by optimally choosing $h > 1$, substituting (50) into (48) yields

$$\pi_2 = eTR_2 - eTR_2 - c_2 x_2 + TR_2 = TR_2 - c_2 x_2, \tag{51}$$

i.e., the firm buys inputs for the same value TR_2 in domestic currency which it will receive in revenue, and it holds a „resale" of the superfluous inputs in its own home country. This totally eliminates exchange rate risk and turns firm 2 into a maximizer of the difference between revenue exchanged at a certain exchange rate $\mu_e = 1$ and cost denoted in its home currency. Maximization with respect to output x_2 then leads to the reaction curve and duopoly equilibrium as if there were risk neutrality or certainty with an exchange rate fixed at a value of $\mu_e = 1$.

For a number of reasons this kind of „over-hedging" can hardly be expected to exist in the real world. In particular, firms may not be able to organize the reselling of inputs involved here, and various transaction costs will diminish the incentive for doing so. If we therefore consider a reduction of h and examine dx_2/dh for a given x_1, it turns out that a lower share of inputs bought abroad leads to less output of duopolist 2. The reaction curve shifts downward towards the one analyzed for the case of full uncertainty. If we confine the firm to $h = 1$, we find that x_2 has to be higher compared to the case of no hedging through buying inputs abroad. This amounts to an upward shift of the reaction curve in the direction of the curve which is relevant under risk-neutrality with $\mu_e = 1$ or certainty with $e = 1$. Analogous results can easily be derived for a price-setting duopoly.

We now proceed to the more sophisticated way of hedging through the use of capital markets. This has been examined for the perfectly competitive firm by *Broll* and various co-authors in a large number of papers for different hedging instruments (see e.g. *Broll, 1992, Broll and Wahl, 1992a, 1992b, Broll and Eckwert, 1996*). The two main results are that under certain assumptions hedging permits output decisions independent of exchange rate expectations, uncertainty and attitude towards risk (separation), and that a firm will prefer to fully hedge its risky foreign exchange position (full hedge).

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enables us to write c_2 and ec_2 for these marginal costs. It is also implicitly assumed that the same realization of e holds for the payments for inputs and the revenue from output.

In order to examine whether these insights carry over to a quantity-setting or price-setting duopoly, denote by e_f the forward exchange rate - as opposed to the spot rate e - at which firm 2 can exchange its revenue earned in country 1 against a certain amount of country 2's currency. Let h be the number of forward contracts firm 2 decides to hold where each contract pays e_f in exchange for one unit of country 1's currency. Duopolist 2's profit function is now

$$\pi_2 = eTR_2 - c_2x_2 + h(e_f - e), \tag{52}$$

which leads to the objective function

$$u(\pi_2) = \mu_e TR_2 - c_2x_2 + h(e_f - \mu_e) - \frac{\rho_2}{2}(TR_2 - h)^2\sigma_e^2. \tag{53}$$

Maximization with respect to x_2 and h yields the first-order conditions

$$\mu_e MR_2 - c_2 - \rho_2(TR_2 - h)MR_2\sigma_e^2 = 0, \tag{54}$$

$$e_f - \mu_e + \rho_2(TR_2 - h)\sigma_e^2 = 0. \tag{55}$$

Multiplying (55) by MR_2 and plugging the resulting term for $\rho_2(TR_2 - h)MR_2\sigma_e^2$ into (54) gives

$$\mu_e MR_2 - c_2 + (e_f - \mu_e)MR_2 = 0 \quad \Leftrightarrow \quad e_f MR_2 - c_2 = 0. \tag{56}$$

μ_e, σ_e^2 or ρ_2 are not included in (56) which therefore can be solved for the optimal output response x_2 to a given x_1 without any reference to the mean or variance of the uncertain exchange rate or to the risk preferences of duopolist 2. The separation result also holds in the duopoly framework: If there is the possibility to sell the revenue earned in currency 1 forward at a fixed forward rate e_f, exchange rate uncertainty and risk do not affect output or volume of trade which can be set independently from the hedging decision. Firm 2's reaction curve which - as we saw earlier on - was shifted downward by the existence of exchange rate uncertainty - moves upward again to the location where it would have been under a fixed exchange rate of e_f. We can therefore use our previous comparative-static results to predict the effects of the hedging opportunity on the duopoly equilibrium. In particular, x_2 and - given some substitutability - x_1 will increase compared to the situation without hedging, thereby improving market performance.

A second look at (55) immediately leads to the conclusion on the extent of hedging. If the forward market is unbiased in the sense of $e_f = \mu_e$, firm 2 will choose a full hedge, i.e., $h = TR_2$. If, on the other hand, transaction costs or some other inefficiency implies $e_f < \mu_e$, the foreign duopolist will prefer to get less than a full hedge.

If we perform the same exercise for a price-setting firm 2, we arrive at $e_f MR_2 + c_2 = 0$ which is again the separation result. As a consequence of hedging there is a downward shift of duopolist 2's reaction curve towards the certainty location. From the first-order condition with respect to h one immediately concludes that the full hedge property holds again for an unbiased forward market.

All this shifting and turning of reaction curves raises the question whether hedging could be used as a strategic device by duopolist 2 in order to alter the duopolistic interaction in its own interest. To model such a strategic move, we have to analyze the duopoly as a two-stage game where in stage 1 firm 2 decides on a hedging level h and in stage 2 both firms choose quantities or prices.

Consider the case of quantity-setting. Intuition suggests that firm 2 will have an incentive to „over-hedge“ because this would shift its reaction curve even above the one for the certainty, i.e., full hedge, situation and commit it to a more aggressive behavior. Solving the game backwards to ensure subgame perfection, we first look at the maximization of $\pi_1 = TR_1 - c_1 x_1$ and (53) with respect to x_1 and x_2, respectively, in stage 2. Assume existence of an equilibrium $\left(x_1(h), x_2(h)\right)$. Firm 2's maximization of (53) in stage 1 then leads to a first-order condition

$$\begin{aligned} &\mu_e\left(\frac{\partial TR_2}{\partial x_1}\frac{dx_1}{dh} + \frac{\partial TR_2}{\partial x_2}\frac{dx_2}{dh}\right) - c_2\frac{dx_2}{dh} + e_f - \mu_e \\ &\qquad - \rho_2\left(TR_2 - h\right)\left(\frac{\partial TR_2}{\partial x_1}\frac{dx_1}{dh} + \frac{\partial TR_2}{\partial x_2}\frac{dx_2}{dh} - 1\right)\sigma_e^2 = 0 \end{aligned} \tag{57}$$

Substituting from firm 2's first-order condition of stage 2 and simplifying the expression yields

$$TR_2 - h = \frac{1}{\rho_2 \sigma_e^2}\left(\mu_e - e_f\left(1 - \frac{\partial TR_2}{\partial x_1}\frac{dx_1}{dh}\right)^{-1}\right). \tag{58}$$

For our intuition to be confirmed, i.e., for $TR_2 - h < 0$, the expression in the inner bracket has to be between 0 and 1 for an unbiased forward market. It is below 1 because both $\partial TR_2 / \partial x_1$ and dx_1 / dh are negative. It is also plausible to assume a value above zero, since then the effect of a marginal increase in h through x_1 on TR_2 is less than 1. There is indeed an incentive for strategic „over-hedging“ which only vanishes in the limiting case of $k = 0$ where $\partial TR_2 / \partial x_1 = 0$. If there is no substitutability between the domestic and the foreign product, there is no point in using a hedge as a strategic device. The only role for hedging is then to get rid of the currency risk.

If we perform the same exercise for a price-setting firm, we arrive at an expression analogous to (58):

$$TR_2 - h = \frac{1}{\rho_2 \sigma_e^2}\left(\mu_e - e_f\left(1 - \frac{\partial TR_2}{\partial p_1}\frac{dp_1}{dh}\right)^{-1} + c_2 k \frac{dp_1}{dh}\left(1 - \frac{\partial TR_2}{\partial p_1}\frac{dp_1}{dh}\right)^{-1}\right). \quad (59)$$

$\partial TR_2/\partial p_1$ is positive whereas dp_1/dh. Taken together this implies that $TR_2 - h$ is positive if firms choose prices. There is no strategic incentive for „over-hedging". In fact, duopolist 2 hedges less than in the one-stage game where the hedging decision cannot be used as a strategic move influencing duopolistic interaction. These results are in line with basic insights from the literature on strategic trade policy which tells us that under price competition a firm should be made less aggressive, and under quantity competition it should be made more aggressive.

While the use of hedging as a strategic device was shown to be theoretically feasible, we still have to ask whether this would work in practice. The main caveat against the previous argument concerns the issue of credibility. A strategic move „influences the other person's choice, in a manner favorable to one's self, by affecting the other person's expectations on how one's self will behave" (*Schelling, 1960, p. 160*). For duopolist 1 to really believe that duopolist 2 will behave differently in stage 2 due to a hedging decision in stage 1, the fact and the extent of hedging have to be known and to be irreversible. In reality, however, duopolist 2 could rather easily offset part or all of its initial hedge by an opposite forward contract, i.e. by selling forward currency 2 against currency 1. In fact, only transaction costs would somewhat prevent firm 2 from doing the reverse hedge and going back to the full-hedge decision which was shown to be optimal in the one-stage game.

Given the credibility problems attached to a duopolist's own hedging decision, we can then ask whether another agent could provide a hedge which credibly includes the strategic element. Clearly enough, government in a broad sense is the prime candidate. Consider therefore a risk-neutral government of country 2 which in stage one decides on the extent h of hedging it will offer to firm 2, e.g., through a government agency for export promotion. Social welfare in this partial equilibrium framework can be written as $w_2 = \pi_2 + h(e - e_f)$ where we neglect like most of the strategic trade policy literature the social cost of raising public funds. Government's objective is to maximize the expected value of welfare

$$E(w_2) = E(\pi_2) + h(\mu_e - e_f) \quad (60)$$

with respect to h. As before, we assume for the case of quantity-setting existence of an equilibrium $(x_1(h), x_2(h))$ from the duopolists' stage 2 maximization of $\pi_1 = TR_1 - c_1 x_1$ and (53) with respect to x_1 and x_2, respectively. Government 2's first-order condition can then be written as

$$\mu_e \left(\frac{\partial TR_2}{\partial x_1} \frac{dx_1}{dh} + \frac{\partial TR_2}{\partial x_2} \frac{dx_2}{dh} \right) - c_2 \frac{dx_2}{dh} = 0 . \tag{61}$$

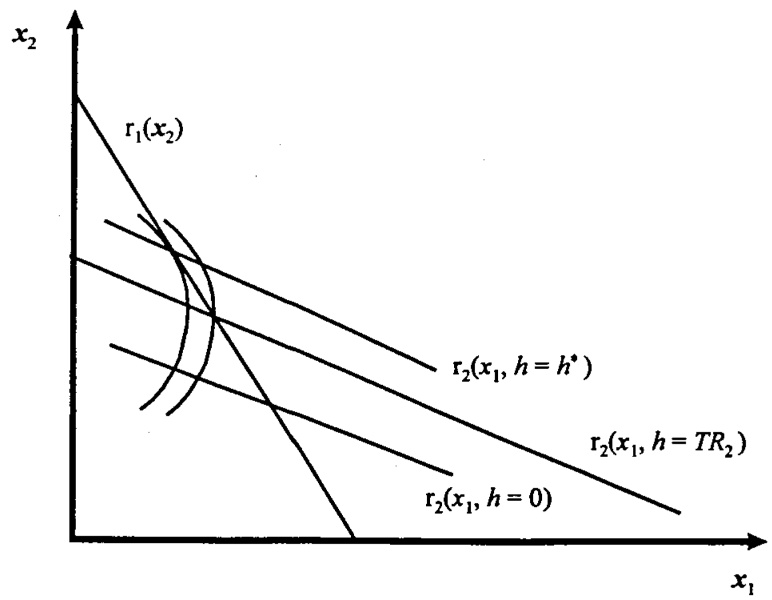
If we substitute for c_2 from duopolist 2's stage 1 first-order condition (54), we arrive at

$$TR_2 - h = \frac{-\mu_2}{\rho_2 \sigma_e^2} \left(\frac{\partial TR_2}{\partial x_1} \Big/ \frac{\partial TR_2}{\partial x_2} \right) \left(\frac{dx_1}{dh} \Big/ \frac{dx_2}{dh} \right) \tag{62}$$

The sign of this expression is negative, i.e., it is in the welfare maximizing government's interest not only to provide a full hedge but to shift firm 2 to a position of „over-hedging" where it chooses its output quantity more aggressively than under certainty. Again, for $k = 0$ the strategic incentive is no longer present and the government would only like to provide a full hedge. Optimal government intervention can be depicted in the standard diagram from the strategic trade policy literature:

Figure 5

Optimal hedge provided by government under quantity competition



If the government provided a full hedge ($h = TR_2$), firm 2's reaction curve would shift to the position that would hold for a certain exchange rate μ_e. The iso-profit contours drawn in the figure are those for a duopolist 2 facing μ_e with certainty. They also represent iso-welfare contours for a risk-neutral government 2 with the objective function defined above. As can be seen, it is in the interest of a welfare-maximizing government to shift firm 2's reaction curve further up even more in order to reach optimum welfare given firm 1's behavior.

In the case of price-setting the analysis yields

$$TR_2 - h = \frac{-\mu_e}{\rho_2 \sigma_e^2}\left(k(a - p_2 + p_1)\frac{dp_1}{dh}\right)\left(\left(\frac{dp_2}{dh} - k\frac{dp_1}{dh}\right)MR_2\right)^{-1}. \tag{63}$$

$x_2 = a - p_2 + kp_1 > 0$ implies that $a - p_2 + p_1 > 0$. Due to stability considerations we have $|dp_2/dh| > |dp_1/dh|$ which together with $MR_2 < 0$ leads to a positive sign of the denominator and a positive sign of $TR_2 - h$. The risk-neutral government 2 has no incentive to provide a full hedge except in the limiting case $k = 0$.

The previous analysis of government incentives to provide a currency hedge for an exporting duopolist suffers from many or all the well-known criticisms against strategic trade policy. It should therefore rather be interpreted as an examination of the effects government policies can have on an international duopoly. First of all, there is a role for government, if private capital markets do not offer hedging opportunities or if only indirect (cross) hedging is possible (for cross hedging see *Broll, Wahl and Zilcha, 1995*). Secondly, real-world governments would probably offer a partial or a full hedge. From our analysis we know, thirdly, that a partial hedge would implicitly amount to stra'egic trade policy, if firms compete in prices, whereas it would fall behind even providing exchange rate certainty, if firms compete in quantities.

4 Conclusions

In this paper a simple model of an international oligopoly or duopoly with product differentiation was used to work out some basics of the effects of exchange rate movements, and exchange rate uncertainty in particular, on market conduct and market performance. For a non-stochastic shift of the exchange rate it was shown that less than perfect pass-through is a property independent of the nature of oligopolistic interaction, i.e., it was found to exist under both under quantity-setting and under price-setting. If the domestic and the foreign variety of the good are substitutable, there will be an effect on the price of the domestic good, too. As to exchange rate uncertainty we identified its unfavorable effects on market performance. Both for the game in quantities and for the game in prices we found exchange rate uncertainty to increase prices and reduce the volume of imports. This result was confirmed in the appendix for a specification of the utility function more general than the one used in the main text. Three ways of hedging against exchange rate uncertainty were finally examined. In all cases we found an incentive to strategically „over-hedge" in the case of quantity-setting and to „under-hedge" in the case of price-setting.

Clearly enough, other interesting issues related to international oligopolies and exchange rates are worth examining. One is the question of how exchange rate uncertainty affects the feasibility of open or tacit collusion. For the case of tacit collusion *Meckl (1995, 1996)* recently showed in a supergame setting that under flexible as opposed to fixed exchange rates duopolistic firms find it more difficult to sustain the collusive outcome, and if they succeed in full collusion, industry output will be higher under flexible exchange rates. *Meckl's* analysis was based on a price-setting game with a homogeneous good. It will be interesting to extend this analysis to a more general class of duopoly models.

5 Appendix

Asplund (1995) recently suggested a fairly general way of dealing with oligopolistic interaction under uncertainty. Denote by $u_i(\pi_i)$ a *von Neumann-Morgenstern* utility function with $\partial u_i/\partial\pi_i > 0$ and $\partial^2 u_i/\partial\pi_i^2 \leq 0$, i.e., firms or their owners are assumed to be risk-averse or risk-neutral. Let s_i, $i = 1,2$, be firm i's strategy which in our analysis could be quantity x_i or price p_i. A firm $i \neq j$ then maximizes expected utility $\mathrm{E}\left(u_i(\pi_i(s_i, s_j))\right)$. The first-order conditions are

$$\mathrm{E}\left(\frac{\partial u_i}{\partial \pi_i}\frac{\partial \pi_i}{\partial s_i}\right) = 0. \tag{A-1}$$

By definition of the covariance

$$\mathrm{Cov}\left(\frac{\partial u_i}{\partial \pi_i}, \frac{\partial \pi_i}{\partial s_i}\right) = \mathrm{E}\left(\frac{\partial u_i}{\partial \pi_i}\frac{\partial \pi_i}{\partial s_i}\right) - \mathrm{E}\left(\frac{\partial u_i}{\partial \pi_i}\right)\mathrm{E}\left(\frac{\partial \pi_i}{\partial s_i}\right) \tag{A-2}$$

we can write the first-order conditions as

$$\mathrm{E}\left(\frac{\partial \pi_i}{\partial s_i}\right) + \frac{\mathrm{Cov}\left(\frac{\partial u_i}{\partial \pi_i}, \frac{\partial \pi_i}{\partial s_i}\right)}{\mathrm{E}\left(\frac{\partial u_i}{\partial \pi_i}\right)} = 0. \tag{A-3}$$

Suppose now, π_i and $\partial\pi_i/\partial s_i$ are bivariate normally distributed with a correlation coefficient $\mathrm{Cov}\left(\pi_i, \partial\pi_i/\partial s_i\right)/\left(\mathrm{Var}(\pi_i)\mathrm{Var}(\partial\pi_i/\partial s_i)\right)$. The simplifying assumption unfortunately includes the possibility of either infinitely positive or infinitely negative profits with a non-zero probability. We nevertheless follow *Asplund (1995)* in using the bivariate normal and in considering it a reasonable approximation of the true distribution over a relevant range of profits and marginal profits.

Using *Stein's Lemma*

$$\mathrm{Cov}\left(\frac{\partial u_i}{\partial \pi_i}, \frac{\partial \pi_i}{\partial s_i}\right) = \mathrm{E}\left(\frac{\partial^2 u_i}{\partial \pi_i^2}\right)\mathrm{Cov}\left(\pi_i, \frac{\partial \pi_i}{\partial s_i}\right) \tag{A-4}$$

and the definition of the *Arrow-Pratt* measure of (global) absolute risk aversion

$$\rho_i = -\frac{\mathrm{E}\left(\partial^2 u_i / \partial \pi_i^2\right)}{\mathrm{E}\left(\partial u_i / \partial \pi_i\right)}, \tag{A-5}$$

we can now write a duopolist's first-order condition as

$$\frac{\partial V_i}{\partial s_i} = \mathrm{E}\left(\frac{\partial \pi_i(s_i, s_j)}{\partial s_i}\right) - \rho_i\left(\pi_i(s_i, s_j)\right)\mathrm{Cov}\left(\pi_i(s_i, s_j), \frac{\partial \pi_i(s_i, s_j)}{\partial s_i}\right) = 0. \tag{A-6}$$

Given our assumption on the second derivative of u_i, ρ_i is positive or zero. Under risk-neutrality ($\rho_i = 0$) the second term in the first-order condition vanishes, and we are back to maximization of expected profit. With $\rho_i > 0$ the sign of the covariance between profits and marginal profits matters. If it is positive (negative), the expected value of profit has to be greater (less) than zero in producer i 's optimum.

From the first-order conditions $\partial V_i / \partial s_i = 0$ we can define reaction functions $s_i = \mathrm{r}_i(s_j)$. Implicit differentiation of the first-order conditions yields

$$\frac{ds_i}{ds_j} = \mathrm{r}_i' = -\frac{\partial^2 V_i / (\partial s_i \partial s_j)}{\partial^2 V_i / \partial s_i^2}. \tag{A-7}$$

From the second-order conditions we know that $\partial^2 V_i / \partial s_i^2 < 0$. Furthermore,

$$\frac{\partial^2 V_i}{\partial s_i \partial s_j} = \mathrm{E}\left(\frac{\partial^2 \pi_i}{\partial s_i \partial s_j}\right) - \frac{d\rho_i}{d\pi_i}\frac{\partial \pi_i}{\partial s_j}\mathrm{Cov}\left(\pi_i, \partial \pi_i / \partial s_i\right) - \rho_i \frac{\partial \mathrm{Cov}\left(\pi_i, \partial \pi_i / \partial s_i\right)}{\partial \pi_j}. \tag{A-8}$$

In the absence of uncertainty this second derivative consists only of the effect of s_j on firm i 's marginal profit. If this is negative (positive) we call s_i and s_j strategic substitutes (complements), which corresponds to quantity (price) competition analyzed above. Following *Asplund (1995)* we extend this terminology to the uncertainty case and call the choice variables strategic substitutes (complements), if $\partial^2 V_i / \partial s_i \partial s_j < 0$ (> 0). Notice that this inequality is now influenced, among other things, by the covariance between profit and marginal profit. It is therefore important to determine the sign of $\mathrm{Cov}\left(\pi_i, \partial \pi_i / \partial s_i\right)$. In his paper *Asplund (1995)* considers uncertainty with respect to parameters of the linear demand functions - in our case a, k - and to marginal cost c_i.

Examine now exchange rate uncertainty as faced by the duopolistic firms in the simple product differentiation model used in the main text. Notice first that both firms are affected by e: firm 2 directly, and both firm 1 and firm 2 through the effect of e on the equilibrium choice of s_1 and s_2, respectively. However, only the direct uncertainty faced by firm 2 has to be taken into account explicitly. Firm 1 acts like a „normal" duopolist choosing s_1 and knowing that its opponent rationally chooses s_2.

The sign of the critical covariance $\text{Cov}(\pi_2, \partial\pi_2/\partial s_2)$ can be determined from the profit functions defined above. If firms compete in quantities, we get

$$\text{Cov}(\pi_2, \partial\pi_2/\partial x_2) = \text{Cov}\left[\left(e\left(a - x_2 - kx_1\right) - c_2\right)x_2, e\left(a - 2x_2 - kx_1\right) - c_2\right]. \qquad \text{(A-9)}$$

This is equivalent to

$$\text{Cov}(\pi_2, \partial\pi_2/\partial x_2) = \underbrace{x_2}_{>0}\underbrace{\left(a - x_2 - kx_1\right)}_{p_2>0}\underbrace{\left(a - 2x_2 - kx_1\right)}_{MR_2>0}\underbrace{\sigma_e^2}_{>0} > 0. \qquad \text{(A-10)}$$

The reasoning for the positive sign of MR_2 was outlined in the main text. Intuitively speaking, a risk-averse duopolist 2 requires a (perceived) marginal revenue even above the value of c_2/e reached under certainty.

If, on the other hand, firms compete in prices, we have to examine

$$\text{Cov}(\pi_2, \partial\pi_2/\partial p_2) = \text{Cov}\left(\left(ep_2 - c_2\right)\left(a - p_2 + kp_1\right), e\left(a - 2p_2 + kp_1\right)\right). \qquad \text{(A-11)}$$

This yields

$$\text{Cov}(\pi_2, \partial\pi_2/\partial p_2) = \underbrace{p_2}_{>0}\underbrace{\left(a - p_2 + kp_1\right)}_{x_2>0}\underbrace{\left(a - 2p_2 + kp_1\right)}_{MR_2<0}\underbrace{\sigma_e^2}_{>0} < 0. \qquad \text{(A-12)}$$

Again, the reasoning for the negative sign of MR_2 can be found in the main text. Since MR_2 is now defined with respect to a price variable, the same intuition as above applies. Given that we are able to determine the sign of the critical covariance, we arrive at the following result: A risk-averse duopolistic firm selling in a market where it faces an uncertain exchange rate supplies a smaller quantity under quantity competition and sets a higher price under price competition compared to a risk-neutral firm. This induces its competitor located in this market to produce a higher quantity under quantity competition and to sell at a higher price under price competition.

This is simply a special case of proposition 1 in *Asplund (1995)* (a similar insight can be found in *Hviid, 1989*). For a proof denote by Θ a parameter which shifts firm 2's risk aversion and assume $\partial\rho_2/\partial\Theta > 0$. The system of first-order conditions

$$\frac{\partial V_1}{\partial s_1} = 0 = \frac{\partial \pi_1}{\partial s_1} \tag{A-13}$$

$$\frac{\partial V_2}{\partial s_2} = 0 = \mathrm{E}\left(\frac{\partial \pi_2}{\partial s_2}\right) - \rho_2 \cdot \mathrm{Cov}\left(\pi_2, \frac{\partial \pi_2}{\partial s_2}\right) \tag{A-14}$$

can be differentiated to get

$$\begin{pmatrix} ds_1/d\Theta \\ ds_2/d\Theta \end{pmatrix} = \frac{-1}{\det} \underbrace{\begin{pmatrix} \partial^2 V_2/\partial s_2^2 & -\partial^2 V_1/\partial s_1 \partial s_2 \\ -\partial^2 V_2/\partial s_2 \partial s_1 & \partial^2 V_1/\partial s_1^2 \end{pmatrix}}_{\mathrm{M}} \begin{pmatrix} \partial^2 V_1/\partial s_1 \partial \Theta \\ \partial^2 V_2/\partial s_2 \partial \Theta \end{pmatrix} \tag{A-15}$$

where

$$\det = \frac{\partial^2 V_1}{\partial s_1^2} \cdot \frac{\partial^2 V_2}{\partial s_2^2} - \frac{\partial^2 V_1}{\partial s_1 \partial s_2} \cdot \frac{\partial^2 V_2}{\partial s_2 \partial s_1} > 0 \tag{A-16}$$

is required by the usual stability condition (cf. *Dixit, 1986*). Since firm 1 is only affected by strategic uncertainty but not by exchange rate uncertainty, we know that $\partial^2 V_1/\partial s_1 \partial \Theta = 0$. For firm 2, however, we get

$$\frac{\partial^2 V_2}{\partial s_2 \partial \Theta} = -\frac{\partial \rho_2}{\partial \Theta} \mathrm{Cov}\left(\pi_2, \partial \pi_2/\partial s_2\right), \tag{A-17}$$

which is negative under quantity competition and positive under price competition due to the sign of the covariance. We then find

$$\frac{ds_1}{d\Theta} = \frac{\partial^2 V_1/\partial s_1 \partial s_2 \cdot \partial^2 V_2/\partial s_2 \partial \Theta}{\det} \geq 0 \tag{A-18}$$

$$\frac{ds_2}{d\Theta} = -\frac{\partial^2 V_1/\partial s_1^2 \cdot \partial^2 V_2/\partial s_2 \partial \Theta}{\det} \lessgtr 0 \tag{A-19}$$

Under quantity competition $\partial^2 V_1/\partial x_1 \partial x_2 = -k \leq 0$ which implies $ds_1/d\Theta \geq 0$, whereas under price competition we have $\partial^2 V_1/\partial p_1 \partial p_2 = k \geq 0$ which also leads to $ds_1/d\Theta \geq 0$.

For the intuition of this result notice e.g. for the case of quantity-setting that for a given s_1 expected marginal profit of a risk-averse firm 2 is positive at its best response $r_2(s_1)$. This implies a lower s_2 compared to the case of risk-neutrality. Firm 2's reaction function is shifted downward by an increasing risk aversion ρ_2. A risk-averse firm puts relatively more weight on unfavorable realisations of the uncertain exchange exchange rate e. If - as is the case in our model - profit and marginal profit are positively correlated, low quantities and prices are optimal in those bad realisations.

Next we examine the effect of an increase in the variability of the exchange rate on the market equilibrium. A rise in σ_e^2 affects firm 2's first-order condition through its covariance of profits and marginal profits. Denote by Φ the marginal effect of σ_e^2 on this covariance. Comparative-static analysis yields

$$\begin{pmatrix} ds_1/d\sigma_e^2 \\ ds_2/d\sigma_e^2 \end{pmatrix} = \frac{-1}{\det} M \begin{pmatrix} 0 \\ -\left(\rho_2 + \partial\rho_2/\partial \text{Cov}(\cdot)\cdot \text{Cov}(\cdot)\right)\Phi \end{pmatrix}, \tag{A-20}$$

where $\text{Cov}(\cdot)$ stands for $\text{Cov}(\pi_2, \partial\pi_2/\partial s_2)$. This implies the following result: If absolute risk aversion is non-increasing and convex, a risk-averse duopolist selling to a market where it faces an uncertain exchange rate reacts to an increase in exchange rate uncertainty by supplying a smaller quantity under quantity competition and by setting a higher price under price competition compared to a risk-neutral firm. This induces its competitor located in the market to produce a higher quantity under quantity competition and to sell at a higher price under price competition.

For a proof notice from the definition of $\text{Cov}(\pi_2, \partial\pi_2/\partial s_2)$ that the sign of Φ equals the sign of this covariance and is therefore positive. As for $\rho_2 + \partial\rho_2/\partial \text{Cov}(\cdot)\cdot \text{Cov}(\cdot)$, we know that it is positive due to the assumption of a non-increasing and convex measure of risk aversion.[11] For $\text{Cov}(\cdot) > 0$, an increase in $\text{Cov}(\cdot)$ means an increase in risk by moving mass away from the mean which increases ρ_2. If $\text{Cov}(\cdot) < 0$, an increase in $\text{Cov}(\cdot)$ would be a reduction in risk, reducing ρ_2 and leading to the same sign. Using $\det$ and M as defined before, the results stated above follow immediately, where for firm 1 we again have to distinguish between $\partial^2 V_1/\partial x_1 \partial x_2 = -k \le 0$ under quantity competition and $\partial^2 V_1/\partial p_1 \partial p_2 = k \ge 0$ under price competition. An increase in exchange rate fluctuation therefore affects the equilibrium in the same qualitative way as an increase in risk aversion.

6 References

Asplund, M. (1995), Oligopoly and Risk Aversion, Working Paper No. 69, Stockholm School of Economics

Baldwin, R. (1988), Hysteresis in Import Prices: The Beachhead Effect, American Economic Review, vol. 78, 773-785

Baldwin, R., Krugman, P.R. (1989), Persistent Trade Effects of Large Exchange Rate Shocks, Quarterly Journal of Economics, vol. 104, 635-654

11 This holds in particular for the utility functions of the HARA class which are widely used in finance (see e.g. *Ingersoll, 1987, pp. 39-40*). The utility function with constant risk aversion used in the main text belongs to this class.

Baniak, A., Phlips, L. (1995), La Pléiade and Exchange Rate Pass-through, International Journal of Industrial Organization, vol. 13, 195-211

Baron, D.P. (1976), Flexible Exchange Rates, Forward Markets, and the Level of Trade, American Economic Review, vol. 66, 253-266

Broll, U. (1992), International Capital Market, Currency Forward Contract and the Export Decision, Schweizerische Zeitschrift für Volkswirtschaft und Statistik, vol. 128, 125-131

Broll, U., Eckwert, B. (1996), Cross Hedging of Exchange Rate Risk, Review of International Economics, vol. 4, 282-286

Broll, U., Wahl, J.E. (1992a), Hedging with Synthetics, Foreign Exchange Forwards and the Export Decision, Journal of Futures Markets, vol. 12, 511-517

Broll, U., Wahl, J.E. (1992b), Exports under Exchange Rate Uncertainty and Hedging Markets, Journal of Institutional and Theoretical Economics, vol. 148, 577-587

Broll, U., Wahl, J.E., Zilcha, I, (1995), Indirect Hedging of Exchange Rate Risk, Journal of International Money and Finance, vol. 14, 667-78

Caselli, P. (1991), Exchange Rate and Pricing Strategies in a Model of International Duopoly, Giornale degli Economisti e Annali di Economia, vol. 50, 103-123

Dixit, A.K. (1989), Hysteresis, Import Penetration and Exchange Rate Pass-Through, Quarterly Journal of Economics, vol.104, 205-228

Dixit, A.K. (1986), Comparative Statics for Oligopoly, International Economic Review, vol. 27, 107-122

Dornbusch, R. (1987a), Exchange Rates and Prices, American Economic Review, vol. 77, 93-106

Dornbusch, R. (1987b), Exchange Rate Economics: 1986, Economic Journal, vol. 97, 1-18

Froot, K.A., Klemperer, P.D. (1989), Exchange Rate Pass-Through when Market Share Matters, American Economic Review, vol. 79, 637-653

Hooper, P., Mann, C. (1989), Exchange Rate Pass-through in the 1980s: The Case of U.S. Imports of Manufactures, Brookings Papers on Economic Activity 1, 297-329

Hviid, M. (1989), Risk-Averse Duopolists and Voluntary Information Transmission, Journal of Industrial Economics, vol. 38, 49-64

Ingersoll, J.E. (1987), Theory of Financial Decision Making, Totowa, NJ: Rowman and Littlefield

Kirman, A., Phlips, L. (1996), Exchange-rate Pass-through and Market Structure, Journal of Economics (Zeitschrift für Nationalökonomie), vol. 64, 129-154

Knetter, M. (1989), Exchange Rate Fluctuations and Price Discrimination by U.S. and German Exporters, American Economic Review, vol. 79, 198-210

Leland, H.E. (1972), Theory of the Firm Facing Uncertain Demand, American Economic Review, vol. 62, 278-291

Mann, C.L. (1986), Prices, Profit Margins, and Exchange Rates, Federal Reserve Bulletin, vol. 72, 366-379

Meckl, J. (1995), Market Power of Firms and Exchange Rate Fluctuations, Discussion Paper No. 278, Sonderforschungsbereich 178 „Internationalisierung der Wirtschaft", Universität Konstanz

Meckl, J. (1996), Market Power of Firms and Exchange-Rate Fluctuations, Journal of Economics (Zeitschrift für Nationalökonomie), vol. 63, 57-77

Menon, J. (1995), Exchange Rate Pass-through, Journal of Economic Surveys, vol. 9, 197-231

Schelling, T.C. (1960), The Strategy of Conflict, Cambridge, MA: Harvard University Press

Singh, N., Vives, X. (1984), Price and Quantity Competition in a Differentiated Duopoly, Rand Journal of Economics, vol. 15, 546-554

Tivig, T. (1996), Exchange Rate Pass-through in Two-period Duopoly, International Journal of Industrial Organization, vol. 14, 631-645

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