Righi–Leduc effect in Y- and Bi-based high-$T_c$ superconductors

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Abstract

Measurements of the transverse ($k_{xy}$) and longitudinal ($k_{xx}$) thermal conductivity in the superconducting state of single crystals of YBa$_2$Cu$_4$O$_{7-\delta}$ (YBCO) and Bi$_2$Sr$_2$CaCu$_2$O$_{8-\delta}$ (BSCCO) are presented in applied magnetic fields up to 14 T. We separate the electronic thermal conductivity ($k_{xx}^{el}$) of the CuO$_2$-planes from the phononic thermal conductivity ($k_{xx}^{ph}$) using $k_{xx}$ and the magnetic field dependence of $k_{xx}$. Our main results are: (1) In YBCO $k_{xx}^{el}(T)$ shows a pronounced maximum. We attribute this to a rapid increase of the quasiparticle scattering time $\tau$ below $T_c$. The maximum of $k_{xx}^{el}(T)$ in BSCCO is much smaller due to stronger impurity scattering. The maximum of $k_{xx}$ is strongly suppressed by a magnetic field, presumably because of scattering of QPs on vortices. (2) Our data analysis reveals that below $T_c$ a transport ($\tau$) and Hall ($\tau_{HI}$) relaxation time must be distinguished as in the normal state. Whereas $\tau$ is strongly enhanced below $T_c$, $\tau_{HI}$ displays the same temperature dependence as above $T_c$.

Keywords: Righi–Leduc effect; Thermal conductivity; Single crystal

1. Introduction

A characteristic feature of the high-$T_c$ superconductors (HTSC) is a pronounced maximum of their thermal conductivity $k_{xx}$ in the superconducting state [1–4]. On the one hand, this maximum has been attributed to the quasiparticle (QP) contribution to the thermal conductivity, resulting from a strong suppression of the QP scattering rate below $T_c$, which overcompensates the decrease of the number of QPs [3,5,6]. Alternatively, the maximum has been attributed to the phononic contribution $k_{xx}^{ph}$ to the thermal conductivity [1,2]. It is well known from conventional superconductors that the scattering of phonons on electrons is reduced in the superconducting state, when electrons condense into the superfluid [1]. In order to settle this issue, a separation of the QP and phonon heat currents is required. This is up to now an open problem.

It has been pointed out in Refs. [7–9] that the transverse thermal conductivity $k_{xy}$, also called the Righi–Leduc or thermal Hall effect, has no phonon contribution, i.e., $k_{xy}$ is purely electronic. One there-
fore expects to gain direct information on the QP relaxation time from the measurement of $k_{xy}$. However, in the cuprates additional complications arise when effects transverse in a magnetic field such as the Hall and Righi–Leduc effect are involved. The study of the normal state electrical transport phenomena clearly showed that a consistent interpretation of the experimental data requires a distinction of two relaxation times [10–12]: whereas a longitudinal (transport) relaxation time $\tau$ enters the dc conductivity, $\sigma_{xx} \propto \tau$, a transverse Hall relaxation time $\tau_{H}$ is necessary to describe the Hall conductivity $\sigma_{xy} \propto \tau \tau_{H}$ and thus the Hall angle $\tan \alpha_{H} = \sigma_{xy} / \sigma_{xx} \propto \tau_{H}^{-1}$. In this paper, we present a detailed study of $k_{xx}$ and $k_{xy}$ of single crystals of YBa$_2$Cu$_3$O$_{7-\delta}$ and Bi$_2$Sr$_2$CaCu$_2$O$_8+\delta$. Our data analysis shows that a distinction of two relaxation times is also necessary for the description of thermal transport properties both, in the normal and in the superconducting state. In particular, $\tau$ is strongly enhanced below $T_c$ and becomes magnetic field dependent, whereas $\tau_{H}$ shows the same magnetic field and temperature dependence as in the normal state. We also separate the QP and phononic contribution to the heat current using $k_{xy}$ and the magnetic field dependence of $k_{xx}$.

2. The Righi–Leduc effect

The thermal conductivity tensor $k$ is defined via the heat current density:

$$j_h = -k \nabla T.$$  \hspace{0.5cm} (1)

Here, $k$ is the sum of an electronic and a phononic part:

$$k = k_{el} + k_{ph}.$$  \hspace{0.5cm} (2)

It is natural to assume that $k_{ph}$ remains diagonal even for $B \neq 0$, i.e., $k_{xy} = 0$. In this case the transverse components of $k$ are purely electronic. The transverse thermal conductivity is the thermal analogue of the Hall effect and is measured as follows: in a magnetic field $B = (0,0,B)$ a temperature gradient $\nabla_y T$ is applied in the $x$-direction. Under the condition $j_{h,x} = 0$ a transverse temperature gradient $\nabla_y T$ is found in the $y$-direction. Using the Onsager relations we find in this situation

$$j_{h,y} = k_{xy} \nabla_y T - k_{xx} \nabla_x T = 0$$  \hspace{0.5cm} (3)

with $k_{xy} = k_{yx}$ for twinned crystals without in-plane anisotropy. Hence, $k_{xy}$ can be determined experimentally by measuring $\nabla_x T$, $\nabla_y T$, and the total longitudinal thermal conductivity $k_{xx}$. According to standard transport theory $k_{el}$ is related to the conductivity tensor $\sigma$ via the Wiedemann–Franz law (see, for example, Ref. [13])

$$k_{el} = LT \sigma.$$  \hspace{0.5cm} (4)

where $L$ is the Lorenz number. This relation implies that the thermal and electrical Hall angles, $\tan \alpha_R = k_{xy} / k_{xx}$ and $\tan \alpha_H = \sigma_{xy} / \sigma_{xx}$ are equal.

3. Experimental

Our measurements were carried out at constant temperatures with the magnetic field applied perpendicular to the CuO$_2$-planes and all temperature gradients within the CuO$_2$-planes. Typically, temperature gradients $\nabla_y T$ of the order of 0.5 K/mm were applied using a small manganin heater mounted on top of the samples (Fig. 1). The resulting transverse temperature gradients $\nabla_y T$ of about $10^{-3}$ K/mm were measured with AuFe–Chromel thermocouples in magnetic fields up to 14 T. The thermocouples were calibrated in the same field range (1) with an $\alpha$-quartz crystal and (2) against a thermocouple placed in zero field, but kept at the same temperature difference (for details see Ref. [14,15]). To eliminate offset voltages due to misalignment of the thermocouple $\nabla_y T$ was measured for both field directions $\pm B$, since the Righi–Leduc component of $\nabla_y T$ must be antisymmetric with respect to field reversal. We have measured in two different modes: Either $B$ was reversed at fixed temperature or the sample was heated up to temperatures above $T_c$ before the field was reversed. In order to avoid hysteresis as observed in Ref. [16], e.g., due to vortex pinning effects this latter mode was used for all low temperature measurements. We have tested our method by measurements on an insulator ($k_{xy} = 0$) and on sim-
ple metals. Details of our experimental setup will be described elsewhere [14,15].

The results presented here have been obtained on a high quality twinned single crystal of YBa$_2$Cu$_3$O$_{7-\delta}$ (YBCO) ($a = b \approx 2$ mm; $c = 0.4$ mm) with a superconducting transition at $T_c = 90.5$ K and on a single crystal of Bi$_2$Sr$_2$CaCu$_2$O$_{8+\delta}$ (BSCCO) ($a = b = 1$ mm; $c = 0.05$ mm) with $T_c = 80$ K.

4. Results

The thermal conductivity $k_{xx}$ of the YBCO sample in zero magnetic field as a function of temperature is shown in Fig. 2. The absolute value of $k_{xx}$ at $T_c$ is of the order of 10 W/K m. $k_{xx}$ increases drastically below $T_c$ and reaches a maximum at $\approx 40$ K. The normal state thermal conductivity is independent of the magnetic field within our experimental resolution. In contrast, $k_{xx}$ is strongly suppressed by a magnetic field below $T_c$. The magnetic field dependence of $k_{xx}$ is shown in Fig. 4, where we plot $\Delta k_{xx} = k_{xx}(B) - k_{xx}(B = 0)$ as a function of the magnetic field at fixed temperatures. $k_{xx}$ varies non-linearly with $B$. Moreover, the magnetic field dependence changes with temperature: at the lowest temperatures measured, $k_{xx}$ has a tendency to become magnetic field independent at high fields. This is reminiscent of the behavior reported recently by Krishana et al. [17] for BSCCO.

In Fig. 3, $k_{xy}$ is plotted vs. temperature. $k_{xy}$ is positive and of the order $10^{-2}$ (W/K m) for $B = 1$ T. It exhibits a pronounced maximum below $T_c$. The position of this maximum shifts slightly to higher temperatures with increasing magnetic field from about 40 K at $B = 3$ T to nearly 50 K at $B = 14$ T. Comparing $k_{xy}$ to $k_{xx}$ one notes that the maximum...
in $k_{xy}(T)$ occurs at a higher temperature and that the relative increase in $k_{xy}(T)$ below $T_c$ is much larger. Comparing the absolute value of $k_{xy}(T_c)$ to $k_{xx}(T_c)$

\[ \approx 10 \text{ W/K m} \] reveals that the thermal Hall angle at 1 T is less than $10^{-3}$. A quantitative determination is not possible at this point, since $k_{xx}$ is dominated by the phonon contribution.

The magnetic field dependence of $k_{xy}$ is also shown in Fig. 4, where we show $k_{xy}/B$. We emphasize that the magnetic field dependencies of $k_{xy}$ and of $k_{xy}/B$ are identical within our experimental accuracy, which implies that

\[ \frac{\partial}{\partial B} \left( \frac{k_{xy}}{B} \right) \propto \frac{\partial k_{xx}}{\partial B}. \] (5)

At the lowest temperatures used in our experiments $k_{xy}$ has the tendency to become independent of $B$ at high magnetic fields above 3 T. We note that the data of Fig. 4 are similar to the results of a previous study [8]. On the other hand, we do not find a decrease of $k_{xy}$ with increasing $B$ at high magnetic fields, in contrast to what is reported in Ref. [18].
The thermal conductivity of BSCCO is shown in the upper panel of Fig. 5. \( k_{xx} \) is of order 3 W/K m near \( T_c \). This value is significantly smaller than that found for YBCO. Below \( T_c \) in zero magnetic field a weak upturn in \( k_{xx}(T) \) is observed with a maximum around 70 K. This maximum is almost completely suppressed when applying a magnetic field of 14 T.

\( k_{xy} \) measured on this sample is shown in the lower panel of Fig. 5. The behavior of \( k_{xy} \) is qualitatively similar to that found in YBCO. In particular, although the maximum of \( k_{xy}(T) \) clearly shows a pronounced maximum at about 50 K. However, note that in BSCCO the absolute magnitude of \( k_{xy} \) is smaller by about a factor of 4 in the normal state compared to that in YBCO, see, e.g., Refs. 14, 15.

We note that vortex motion contributes to the heat current below \( T_c \). This contribution is, however, small compared to the heat carried by the QPs (see, e.g., Refs. [14,15]).

5. Data analysis and discussion

For our analysis, we assume that \( k_{xx} \) is the sum of three contributions

\[
k_{xx} = k^{el}_{xx} + k^{ch}_{xx} + k^{ph}_{xx} = k^{el}_{xx} + k^{ch}_{xx}.
\]

(6)

Here, \( k^{el}_{xx} \) is the electronic thermal conductivity of the CuO\(_2\)-planes (in fact, bilayers in YBCO and BSCCO) and \( k^{ph}_{xx} \) is the phononic thermal conductivity. \( k^{ch}_{xx} \) must be included in the data analysis only if an additional electronic channel of heat conduction is present with a magnetic field or temperature dependence different from that of \( k^{el}_{xx} \). Such a situation is most probably realized in YBCO, where in addition to the CuO\(_2\)-planes (pl) common to all HTSCs, CuO-chains (ch) are present along the \( b \)-direction of the orthorhombic crystal structure. In untwinned crystals, these chains lead to significant \( a-b \) anisotropy of the electronic properties. Note that a contribution from the chains to the longitudinal transport properties is present even for twinned samples, where an in-plane average of \( k^{ch}_{xx} \) contributes. Since the CuO-chains form a one-dimensional channel of conduction the magnetic field and temperature dependence of \( k^{ch}_{xx} \) may be different from that of \( k^{el}_{xx} \). In particular, one expects no magnetic field dependence. Moreover, the CuO-chains should not contribute to the transverse effects.

5.1. Thermal Hall angle

We relate the transverse and longitudinal electronic thermal conductivity (of the CuO\(_2\)-planes) according to

\[
k_{xy} = k^{el}_{xy} \tan \alpha_R = k^{el}_{xy} \omega_c \tau_R.
\]

(7)

This equation defines a ‘relaxation time’ \( \tau_R(T,B) \), which is used to parametrize the temperature and magnetic field dependence of the thermal Hall angle. The distinction between \( \tau \) and \( \tau_R \) is of course motivated by the distinction between \( \tau \) and \( \tau_H \) required for the description of the electrical transport properties in the normal state. Nevertheless, at this point we leave open, whether \( \tau_R \) must be identified with the transport or the Hall relaxation time in order to demonstrate that the subsequent analysis is independent of this.

The key experimental observation underlying our analysis is that \( \Delta k_{xx} = k_{xx}(B) - k_{xx}(B = 0) \) and \( k_{xy}/B \) have the same magnetic field dependence, as expressed by Eq. (5). To understand the implication of this result, we note that Eqs. (7) and (6) yield

\[
\frac{m}{e} \frac{\partial}{\partial B} \left( k_{xy} \right) = \tau_R \frac{\partial k^{el}_{xy}}{\partial B} + k^{el}_{xy} \frac{\partial \tau_R}{\partial B} = \tau_R \frac{\partial k_{xy}}{\partial B} + \left[ k^{el}_{xy} \frac{\partial \tau_R}{\partial B} - \tau_R \frac{\partial k^{ph}_{xy}}{\partial B} - \tau_R \frac{\partial k^{ch}_{xy}}{\partial B} \right].
\]

(8)

Apparently, our experimental results, i.e., Eq. (5), imply that the term in brackets is zero. Since the three terms in brackets refer to three different channels of heat conduction, this requires that \( \tau_R \), \( k^{ph}_{xy} \), and \( k^{ch}_{xy} \) are separately field independent.

With these results and using Eqs. (8) and (7) \( \tau_R \) can be determined according to

\[
\tau_R = \frac{\partial}{\partial B} \left( \frac{k_{xy}}{B} \right) = \frac{\Delta k_{xy}/B}{\Delta k_{xx}}.
\]

(9)

The result of this analysis is shown in Fig. 6 for YBCO.
As a check of our result for \( \tau_R \), we have also determined \( \varepsilon \tau_{HH}/m = \sigma_{xy}/B \sigma_{xx} \) for the same sample from measurements of \( \sigma_{xy} \) and \( \sigma_{xx} \) in the normal state \([14,15]\). These data as well as their extrapolation \(^2\) to temperatures below \( T_c \) are also shown in Fig. 6. Comparing the extrapolated values for \( \tau_{HH}^{-1} \) to \( \tau_R^{-1} \), we find that the temperature dependence is similar, approximately a \( T^2 \)-dependence. However, \( \tau_R \) appears to be systematically smaller by about a factor of 2. Although we do not regard this factor of 2 as important, we note that it can be explained in a straightforward way: we first note that \( \tau_R \) as extracted from the thermal transport data is clearly unaffected by the CuO-chains and that this is also true for \( \sigma_{xy} \). In contrast, \( \sigma_{xx} \) has a contribution from the CuO-chains, i.e., \( \sigma_{xx} = \sigma_{xx}^\text{el} + \langle \sigma_{xx}^\text{ch} \rangle \), where \( \sigma_{xx}^\text{el} \) is the electrical conductivity of the CuO\(_2\)-planes and \( \langle \sigma_{xx}^\text{ch} \rangle \) is an average of the chain contribution appropriate for a twinned crystal. With \( \sigma_{xx}^\text{el} = \langle \sigma_{xx}^\text{ch} \rangle \) \([12]\), we conclude that \( \varepsilon \tau_{HH}/m = \sigma_{xy}/B \sigma_{xx} \) is underestimated by a factor of 2. Correcting the normal state data for this factor, we find excellent agreement between \( \tau_{HH} \) and \( \tau_R \), i.e., our data suggest \( \tau_R = \tau_{HH} \).

We thus arrive at an important result of our analysis: our data show clearly that \( \tau_R = \tau_{HH} \) and that — anticipating our results for \( \tau - \tau_{HH} \) and \( \tau \) behave differently also below \( T_c \) in the same way as in the normal state. In particular, \( \tau_{HH} \) is unaffected by the superconducting transition and shows the same temperature dependence as above \( T_c \), whereas \( \tau \) is strongly suppressed below \( T_c \). This finding should provide important information for the theoretical understanding of transport phenomena in the cuprates.

### 5.2. Electronic thermal conductivity

Once \( \tau_R \) is known, the remainder of our analysis is straightforward: \( k_{xx}^{el} (B \neq 0) \) follows from Eq. (7) using the data for \( k_{xy}(B) \). Subsequently, \( k_{xx}^{rest} \) is obtained from Eq. (6) for each field strength. As a test of consistency, we have verified that \( k_{xx}^{rest} \) is indeed field independent. Finally, knowing \( k_{xx}^{el} \) the zero field electronic thermal conductivity \( k_{xx}^{el} (B = 0) \) follows from Eq. (6) using the zero field data for \( k_{xy} \). We have also determined \( k_{xx}^{el} (B = 0) \) directly from Eq. (7) using an extrapolation of \( B/k_{xy} \) to \( B = 0 \) in good agreement with the results obtained from using \( k_{xx}^{rest} \) and Eq. (6).

In the lower panel of Fig. 7, we show \( k_{xx}^{el} (B) \) as obtained from our data analysis for YBCO. \( k_{xx}^{el} \) represents the electronic thermal conductivity of the CuO\(_2\)-planes in YBCO. Our results confirm explicitly that \( k_{xx}^{el} \) is strongly enhanced below \( T_c \). Since \( k_{xx}^{el} \propto \text{Tr} \tau_{QP} \) this implies that \( \tau \) is strongly enhanced below \( T_c \) overcompensating the decrease of the QP number density \( n_{QP}(T) \) with decreasing temperature. This confirms the results obtained for the QP relaxation time from the microwave conductivity \([5,6]\). In the upper panel, \( k_{xx}^{pl} \) for BSCCO is shown. Since the field dependence of \( k_{xx}^{pl} \) is much weaker in BSCCO,
we have in this case assumed that $\tau_R$ is field independent below $T_c$ in order to calculate $k_{\text{el}}^{(1)}$ from $k_{\text{el}}^{(1)}$. The much weaker increase of $k_{\text{el}}^{(1)}$ below $T_c$ in BSCCO should be attributed to stronger impurity scattering in this compound. However, the inelastic, temperature dependent part of the scattering rate must increase also in this compound in order to explain the increase of $k_{\text{el}}^{(1)}$. Thus, the collapse of the inelastic scattering rate below $T_c$ appears to be a generic property of the cuprates.

We observe that $k_{\text{el}}^{(1)}$ is very sensitive to magnetic fields below $T_c$. This is in contrast to what is found in the normal state, where the total thermal conductivity and thus $k_{\text{el}}^{(1)}$ is field independent. This field dependence can be attributed to scattering of QPs on vortices [19,20], with a scattering rate $\tau_e^{-1}$ proportional to the density of vortices, i.e., $\tau_e^{-1} \propto B$ [9].

$k_{\text{el}}^{(1)}$ as obtained from our data analysis for YBCO is shown in Fig. 2. Remarkably, $k_{\text{el}}^{(1)}$ shows a pronounced maximum below $T_c$, too. This maximum may be due to the phononic contribution which is expected to have a maximum at low temperatures [1]. However, it may also arise from the chain contribution. The latter has previously been determined experimentally from the $a$-$b$-anisotropy of $k_{xx}$ in detwinned single crystals of YBCO [3,21]. It was found that $k_{xx}^{\text{ph}}$ shows a pronounced maximum below $T_c$ with an overall temperature dependence similar to that found here for $k_{xx}^{\text{el}}$ [21]. Furthermore, the field independence of $k_{xx}^{\text{ph}}$ implies that the phononic contribution $k_{xx}^{\text{ph}}$ is independent of $B$. Such a conclusion has recently been drawn also on the basis of low temperature results for the thermal conductivity in Bi-based HTSC [17].

6. Summary

We have presented a separation of the QP and phononic contributions to the thermal conductivity below $T_c$ in $\text{YBa}_2\text{Cu}_3\text{O}_y - \delta$ and $\text{Bi}_2\text{Sr}_2\text{Cu}_2\text{O}_{8+y}$ based on measurements of the longitudinal and transverse thermal conductivity in high magnetic fields. Our data analysis shows explicitly that the QP contribution to $k_{xx}$ is strongly enhanced below $T_c$ and that it is the QP contribution to the heat current which is responsible for the magnetic field dependence of $k_{xx}$. We find that two relaxation times must be distinguished not only in the normal state but also below $T_c$: whereas the QP relaxation time $\tau$ is strongly enhanced below $T_c$ and is magnetic field dependent, the Hall relaxation time $\tau_H$ remains independent of $B$ below $T_c$ and has the same temperature dependence as above $T_c$.

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