

Thermodynamically consistent evaluation of equilibrium properties of normal-liquid ^3He

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Abstract

The specific heat $C_V(T, V)$ together with the molar volume $V(T = 0.1 \text{ K}, P)$ of normal liquid ^3He is shown to contain the complete thermodynamic information about this phase. Using the data by Greywall we thus calculate the T - P dependence of all equilibrium quantities (entropy S , specific heat at constant pressure C_P , molar volume V , compressibility κ , and thermal expansion coefficient α) in zero magnetic field. These results are guaranteed to be thermodynamically consistent. We find, for example, that the curves of $\alpha(T, P)$ versus T cross near $T \approx 0.35 \text{ K}$ over a wide temperature range, in contrast to the data presented in some of the literature.

Keywords: Compressibility; ^3He normal liquid; Specific heat; Thermal expansion coefficient

The availability of thermodynamically consistent data is essential for the understanding of any system. This is particularly true in the case of normal liquid ^3He which plays a fundamental role in the field of strongly correlated Fermi systems. In this communication we show that *all* equilibrium properties of normal liquid ^3He for temperatures up to 2.5 K (in zero magnetic field) can be obtained from Greywall's [1] high-precision measurements of the specific heat and his formula for the molar volume at $T_0 = 0.1 \text{ K}$. Since, our calculations are based on data of a single experimental setup, they are guaranteed to be thermodynamically consistent.

The equation of state is obtained from Greywall's interpolation formulas [1] as follows. His formula for the specific heat at constant volume $C_V(T, V)$ is integrated to yield the entropy $S(T, V) = \int_0^T dT' C_V(T', V)/T'$. From the Maxwell relation $(\partial P/\partial T)_V = (\partial S/\partial V)_T$ the pressure is then obtained as $P(T, V) = P(T_0, V) + \int_{T_0}^T dT' (\partial S/\partial V)_{T'}$. From the formula for $V(T_0 = 0.1 \text{ K}, P)$ we can calculate the integration constant $P(T_0, V)$. From the functions $P(T, V)$ and $S(T, V)$ all thermodynamic equilibrium properties can be determined since they de-

termine the free energy F as a function of its natural variables T and V , with $dF = -SdT - PdV$.

The available temperature range is $0.007 \text{ K} \leq T \leq 2.5 \text{ K}$; below we will also extrapolate the data to $T \rightarrow 0$, disregarding the occurrence of superfluid phases. The range of molar volumes is $26.16 \text{ cm}^3 \leq V \leq 36.85 \text{ cm}^3$, leading to range of pressures of $0 \leq P \leq 28\text{--}29 \text{ bar}$, depending on temperature.

The molar volume is shown in Fig. 1. At low temperatures the molar volume decreases with temperature. This anomalous behavior is due to the correlation-induced, anomalous pressure dependence of the entropy at low temperatures, where $(\partial S/\partial P)_T > 0$, which leads to $(\partial V/\partial T)_P = -(\partial S/\partial P)_T < 0$. The change in molar volume over the entire temperature range is less than $< 4\%$. We compared our results to those tabulated by Wilks [2], as well as those published by Wheatley [3] for $T \rightarrow 0$, finding good agreement.

The isothermal compressibility $\kappa = -(1/V)(\partial V/\partial P)_T$ is shown in Fig. 2. Due to the small temperature dependence of $V(T, P)$ the temperature dependence of κ is also small. We find good agreement between our evaluation of $\kappa(0, P)$ and Wheatley's values [3] readjusted to Greywall's determination of the effective mass m^*/m .

The isothermal expansion coefficient $\alpha = (1/V)(\partial V/\partial T)_P$ is shown in Fig. 3. Due to the cross-over from anomalous ($\alpha < 0$) to normal ($\alpha > 0$) behavior α attains

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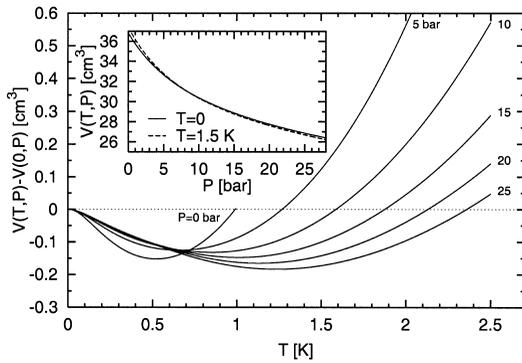


Fig. 1. Molar volume $V(T, P)$ as a function of temperature for several pressures, relative to its value at $T = 0$. The inset shows the pressure dependence of $V(T, P)$ for $T = 0, 1.5$ K.

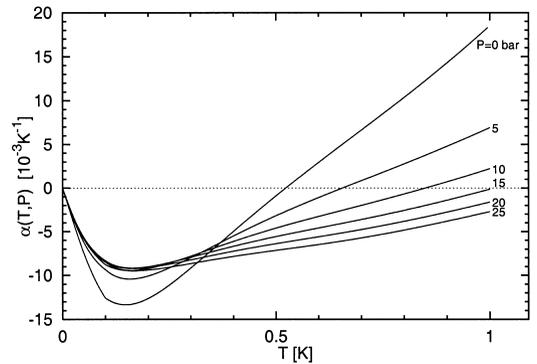


Fig. 3. Thermal expansion coefficient $\alpha(T, P)$ as a function of temperature for several pressures.

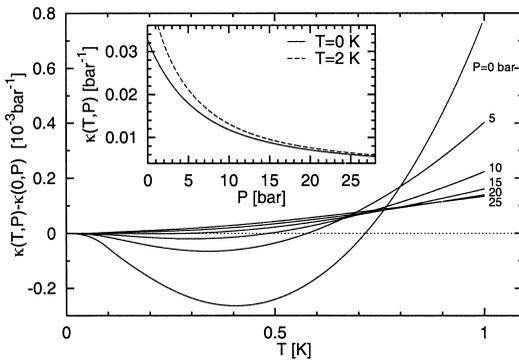


Fig. 2. Compressibility $\kappa(T, P)$ as a function of temperature for several pressures, relative to its value at $T = 0$. The inset shows the pressure dependence of $\kappa(T, P)$ for $T = 0, 2$ K.

a minimum at an intermediate temperature. The curves of $\alpha(T, P)$ versus T are seen to intersect in a broad temperature range $T \approx 0.3-0.4$ K. This corrects the figure shown in Ref. [2] where these curves cross at precisely one temperature for all pressures. We find that such a sharp crossing point is inconsistent with other thermodynamic data for ^3He .

In conclusion, we showed that all equilibrium properties of normal liquid ^3He can be determined from the data of Greywall [1] in a thermodynamically consistent way. More details of the calculation as well as numerical tables will be presented elsewhere.

References

- [1] D. Greywall, Phys. Rev. B 27 (1983) 2747.
- [2] J. Wilks, The Properties of Liquid and Solid Helium, Oxford, 1967.
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