

# Thermodynamically consistent evaluation of equilibrium properties of normal-liquid $^3\text{He}$

Marcus Kollar, Dieter Vollhardt\*

*Theoretische Physik III, Universität Augsburg, D-86135 Augsburg, Germany*

## Abstract

The specific heat  $C_V(T, V)$  together with the molar volume  $V(T = 0.1 \text{ K}, P)$  of normal liquid  $^3\text{He}$  is shown to contain the complete thermodynamic information about this phase. Using the data by Greywall we thus calculate the  $T$ - $P$  dependence of all equilibrium quantities (entropy  $S$ , specific heat at constant pressure  $C_P$ , molar volume  $V$ , compressibility  $\kappa$ , and thermal expansion coefficient  $\alpha$ ) in zero magnetic field. These results are guaranteed to be thermodynamically consistent. We find, for example, that the curves of  $\alpha(T, P)$  versus  $T$  cross near  $T \approx 0.35 \text{ K}$  over a wide temperature range, in contrast to the data presented in some of the literature.

*Keywords:* Compressibility;  $^3\text{He}$  normal liquid; Specific heat; Thermal expansion coefficient

The availability of thermodynamically consistent data is essential for the understanding of any system. This is particularly true in the case of normal liquid  $^3\text{He}$  which plays a fundamental role in the field of strongly correlated Fermi systems. In this communication we show that *all* equilibrium properties of normal liquid  $^3\text{He}$  for temperatures up to 2.5 K (in zero magnetic field) can be obtained from Greywall's [1] high-precision measurements of the specific heat and his formula for the molar volume at  $T_0 = 0.1 \text{ K}$ . Since, our calculations are based on data of a single experimental setup, they are guaranteed to be thermodynamically consistent.

The equation of state is obtained from Greywall's interpolation formulas [1] as follows. His formula for the specific heat at constant volume  $C_V(T, V)$  is integrated to yield the entropy  $S(T, V) = \int_0^T dT' C_V(T', V)/T'$ . From the Maxwell relation  $(\partial P/\partial T)_V = (\partial S/\partial V)_T$  the pressure is then obtained as  $P(T, V) = P(T_0, V) + \int_{T_0}^T dT' (\partial S/\partial V)_{T'}$ . From the formula for  $V(T_0 = 0.1 \text{ K}, P)$  we can calculate the integration constant  $P(T_0, V)$ . From the functions  $P(T, V)$  and  $S(T, V)$  all thermodynamic equilibrium properties can be determined since they de-

termine the free energy  $F$  as a function of its natural variables  $T$  and  $V$ , with  $dF = -SdT - PdV$ .

The available temperature range is  $0.007 \text{ K} \leq T \leq 2.5 \text{ K}$ ; below we will also extrapolate the data to  $T \rightarrow 0$ , disregarding the occurrence of superfluid phases. The range of molar volumes is  $26.16 \text{ cm}^3 \leq V \leq 36.85 \text{ cm}^3$ , leading to range of pressures of  $0 \leq P \leq 28\text{--}29 \text{ bar}$ , depending on temperature.

The molar volume is shown in Fig. 1. At low temperatures the molar volume decreases with temperature. This anomalous behavior is due to the correlation-induced, anomalous pressure dependence of the entropy at low temperatures, where  $(\partial S/\partial P)_T > 0$ , which leads to  $(\partial V/\partial T)_P = -(\partial S/\partial P)_T < 0$ . The change in molar volume over the entire temperature range is less than  $< 4\%$ . We compared our results to those tabulated by Wilks [2], as well as those published by Wheatley [3] for  $T \rightarrow 0$ , finding good agreement.

The isothermal compressibility  $\kappa = -(1/V)(\partial V/\partial P)_T$  is shown in Fig. 2. Due to the small temperature dependence of  $V(T, P)$  the temperature dependence of  $\kappa$  is also small. We find good agreement between our evaluation of  $\kappa(0, P)$  and Wheatley's values [3] readjusted to Greywall's determination of the effective mass  $m^*/m$ .

The isothermal expansion coefficient  $\alpha = (1/V)(\partial V/\partial T)_P$  is shown in Fig. 3. Due to the cross-over from anomalous ( $\alpha < 0$ ) to normal ( $\alpha > 0$ ) behavior  $\alpha$  attains

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\* Corresponding author.

*E-mail address:* vollha@physik.uni-augsburg.de (D. Vollhardt)

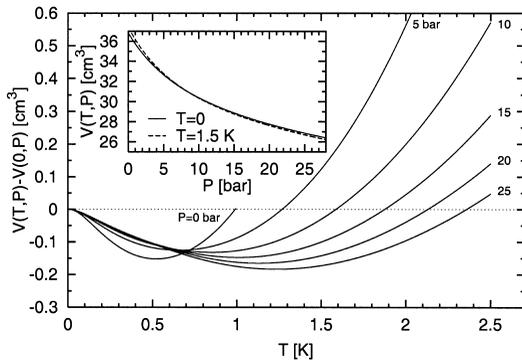


Fig. 1. Molar volume  $V(T, P)$  as a function of temperature for several pressures, relative to its value at  $T = 0$ . The inset shows the pressure dependence of  $V(T, P)$  for  $T = 0, 1.5$  K.

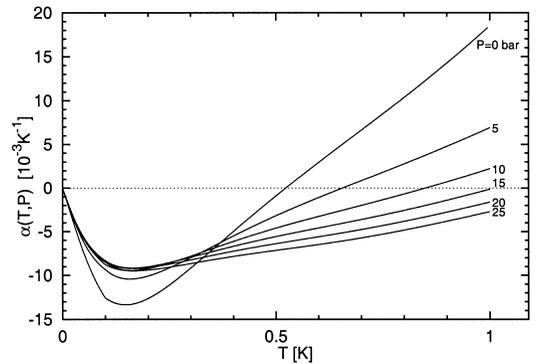


Fig. 3. Thermal expansion coefficient  $\alpha(T, P)$  as a function of temperature for several pressures.

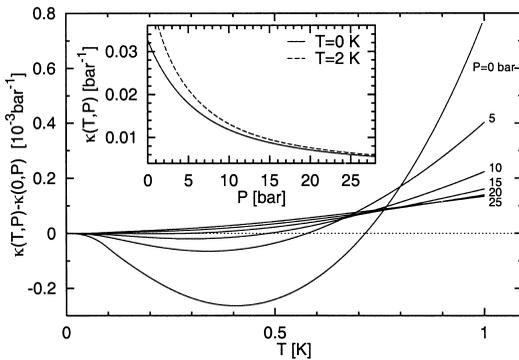


Fig. 2. Compressibility  $\kappa(T, P)$  as a function of temperature for several pressures, relative to its value at  $T = 0$ . The inset shows the pressure dependence of  $\kappa(T, P)$  for  $T = 0, 2$  K.

a minimum at an intermediate temperature. The curves of  $\alpha(T, P)$  versus  $T$  are seen to intersect in a broad temperature range  $T \approx 0.3-0.4$  K. This corrects the figure shown in Ref. [2] where these curves cross at precisely one temperature for all pressures. We find that such a sharp crossing point is inconsistent with other thermodynamic data for  $^3\text{He}$ .

In conclusion, we showed that all equilibrium properties of normal liquid  $^3\text{He}$  can be determined from the data of Greywall [1] in a thermodynamically consistent way. More details of the calculation as well as numerical tables will be presented elsewhere.

## References

- [1] D. Greywall, Phys. Rev. B 27 (1983) 2747.
- [2] J. Wilks, The Properties of Liquid and Solid Helium, Oxford, 1967.
- [3] J.C. Wheatley, Rev. Mod. Phys. 47 (1975) 415.