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Josephson effect and quantum fluctuations

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The supercurrent $I = I_c \sin(\varphi)$ through a Josephson junction at zero voltage bias is determined by the phase difference φ across the junction and limited by the critical current I_c. This ideal picture is modified by thermal or quantum fluctuations of the phase. While thermal fluctuations [1] and quantum fluctuations at not too low temperatures [2] are discussed in the literature, we restrict ourselves here to the case of quantum fluctuations at zero temperature.

In the following we consider the minimal model shown in Fig. 1. The junction is characterized by the Josephson coupling energy $E_J = \hbar I_c/2e$ and the charging energy $E_c = 2e^2/C$ where C is the junction capacitance. Unless $E_J \gg E_c$, the capacitance will lead to fluctuations of the phase which destroy the supercurrent in the absence of an external voltage. Therefore, we have to consider the case of a finite applied voltage V and need to include a resistance $R = \rho(h/4e^2)$ which allows the tunneling Cooper pairs to loose their excess energy 2 eV. The voltages of interest will be much smaller than the gap voltage and we may therefore neglect quasiparticle excitations.

The dynamics of a Josephson junction is related to the motion of a Brownian particle in a periodic potential whereby the coordinate of the particle corresponds to the phase difference across the junction. Using exact results for the mobility of the particle in the limit of strictly Ohmic damping [3], one can show [4] that for low voltages, $V \ll \hbar/(eRC)$, the current–voltage characteristics can be obtained from two complementary expansions

$$I = \frac{V}{R}\sum_{n=1}^{\infty} c_n(\rho)\left(\frac{V}{V_0}\right)^{2(\rho-1)n} \tag{1}$$

and

$$I = \frac{V}{R}\left(1 - \sum_{n=1}^{\infty} c_n(1/\rho)\left(\frac{V}{V_0}\right)^{2(1/\rho-1)n}\right) \tag{2}$$

with the coefficients

$$c_n(\rho) = (-1)^{n-1}\frac{\Gamma(1+\rho n)\Gamma(3/2)}{\Gamma(1+n)\Gamma(3/2+(\rho-1)n)} \tag{3}$$

and the voltage scale

$$V_0 = \frac{\pi E_J}{e}\left[\Gamma(\rho)\left(\frac{e^{\gamma}}{\pi^2\rho}\frac{E_c}{E_J}\right)^{\rho}\right]^{1/(\rho-1)}, \tag{4}$$

where γ is the Euler constant.

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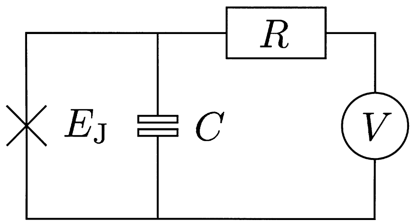

Fig. 1. A Josephson junction characterized by its Josephson coupling energy E_J and its capacitance C is coupled to an ideal voltage source via a resistance R.

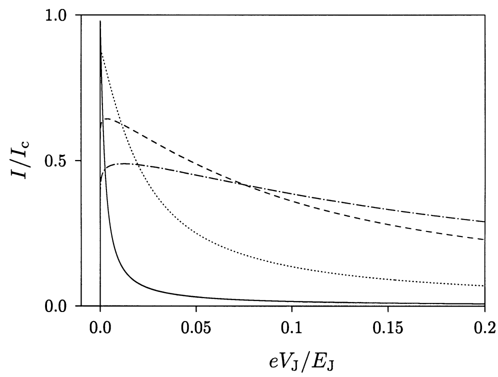

Fig. 2. Cooper pair current–voltage characteristics for $(e^\gamma/\pi^2)(E_c/E_J) = 1$ and $\rho = 0.001, 0.01, 0.05$, and 0.1 shown as full, dotted, dashed, and dashed–dotted line, respectively. The characteristics are shown as a function of the junction voltage $V_J = V - RI$.

The two expansions have a finite radius of convergence given by

$$V_c = V_0(|1-\rho|\rho^{\rho/(1-\rho)})^{1/2} \tag{5}$$

in such a way that one of the two expansions is a low-voltage expansion while the other is a high-voltage expansion. In any case, there exists a convergent expansion for all voltages within the range of validity of the theory.

For $\rho \to 0$, expansions (1) and (2) reduce to the result of classical phase diffusion [1]. The expansion (1) displays a zero bias anomaly $I \sim V^{2\rho-1}$ in agreement with the prediction of Coulomb blockade theory [5,6]. For $\rho < 1$, this leads to a diverging conductance at zero bias. In fact, in this regime (1) represents a high-voltage expansion and the divergence is unphysical.

The complementary expansion (2) reduces for small voltages to the result of macroscopic quantum tunneling [7]. The two expansions (1) and (2) thus provide a connection between the regimes of classical phase diffusion, Coulomb blockade and macroscopic quantum tunneling. This bridging as well as the formation of the zero bias supercurrent with decreasing resistance ρ is shown in Fig. 2. For these values of ρ, the current–voltage characteristics are associated with macroscopic quantum tunneling on the left and Coulomb blockade on the right. On the basis of recent measurements on small voltage-biased Josephson junctions [8] one may expect that the effect of quantum fluctuations discussed here should be observable experimentally.

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