Comment on "Magnetic Response of Disordered Metallic Rings: Large Contributions of Far Levels"

In a recent Letter [1], Schechter *et al.* reconsidered the average magnetic response of disordered metallic rings, on the basis of a calculation first order in the interaction (assumed to be phonon mediated), for zero temperature, and for vanishing magnetic flux. Their result is

$$\chi(0) = \frac{8\pi\lambda E_{\rm Th}}{\Phi_0^2}\,\mathcal{M}^*, \qquad \mathcal{M}^* = \ln(\frac{E^*}{d}), \qquad (1)$$

where $\chi(\Phi) = dI(\Phi)/d\Phi$ is the susceptibility, $I(\Phi)$ the persistent current per ring, Φ the threading magnetic flux, $\lambda(<0)$ the interaction constant, $E_{\rm Th} = \hbar D/L^2$ the Thouless energy, $\Phi_0 = h/2e$ the flux quantum, d the mean level spacing, and E^* a cutoff energy, given by the minimum of $\hbar\omega_D$ and $\hbar/\tau$. In contrast, earlier results [2] indicate that $E^* \sim E_{\rm Th}$ instead; hence Eq. (1) suggests an "increase" of the susceptibility by a factor of about 4 (for typical parameters).

In view of unresolved questions, see, e.g., [3–5], we agree that further studies of the interaction contribution are important. We doubt, however, that a first-order calculation based on a reduced Hamiltonian can give reliable answers. At least this approach must be contrasted with standard many-body calculations [6], which support the approach in [2], as detailed in [7,8].

Let us recall the expression derived in [2] for the grand potential,

$$\Omega(\Phi) = 2\lambda^* \sum_q T \sum_{\omega>0} \frac{\omega}{\omega + Dq_\Phi^2}, \qquad \lambda^* = N(0)\overline{V}, \quad (2)$$

where $q = 2\pi n/L$, $q_\Phi = q + (2\pi/L)\Phi/\Phi_0$, and ω are the Matsubara (Bose) frequencies. The applicability of Eq. (2) is subject to restrictions, implicit in its derivation, namely, $d \ll \omega$, $Dq^2 \ll 1/\tau$, which implies, e.g., that the temperature T must be larger than d (and larger than the superconducting T_c for the attractive case). The coupling constant λ^* contains Hartree and Fock contributions, averaged over the Fermi surface. The validity of the arguments leading to Eq. (2) persists when the screened Coulomb interaction is replaced by the phonon Green's function. In the latter case, an additional cutoff for the frequency summation is provided by the Debye frequency ω_D. From Eq. (2), the m summation in the expansion $I(\Phi) = \sum_{m=1}^{\infty} I_m \sin(2\pi m\Phi/\Phi_0)$, where $I_m = I_1/m^2$ for $T \ll E_{\rm Th}$, is cutoff at $m^* \sim (E_{\rm Th}/T)^{1/2}$, and we recover Eq. (1), however, with $\mathcal{M}^* = \ln(E_{\rm Th}/T)$.

On the other hand, taking the $q = 0$ term into account only [1], and expanding Eq. (2) for small flux, we obtain $\mathcal{M}^* = \ln(E^*/T)$ with E^* the appropriate cutoff for the frequency sum. The connection with the results of [1] is apparent when T approaches d. Clearly, with such a procedure, the flux periodicity is lost.

We emphasize that the wave vector q, appearing above, is the *sum* of the incoming momenta, but the interaction depends on the momentum *transfer* [2]; hence the relevant scale is set by p_F, and not by ω_D/v_F, as argued in [1]; see [6]. Thus there is no convincing argument which could justify singling out the $q = 0$ contribution in Eq. (2), and we conclude that the reduced BCS Hamiltonian leads to erroneous results in the present case. (Taking $q = 0$ only is valid in a superconductor above but close to T_c, such that the coherence length is larger than the system size [8]—but then terms of infinite order have to be summed.) Nevertheless one can imagine starting with an effective Hamiltonian, in which the interaction $V(\mathbf{p} - \mathbf{p}')$ is replaced by some $\tilde{V}(\mathbf{p} - \mathbf{p}', q)$, and then calculate the grand potential in first order in $\tilde{V}$. This leads to Eq. (2) where, however, λ^* depends on q_Φ, thereby guaranteeing the flux periodicity of the results. Clearly, upon differentiation, ambiguous results are obtained, depending on the choice of $\tilde{V}$. On the other hand, the replacement $\lambda^*(q_\Phi) \to \lambda^*(q)$ at some arbitrary point in the calculation has no foundation either.

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