Quasiclassical theory of superconducting multi-layers

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Abstract

We study the density of states of a thin normal metal layer in proximity to a bulk superconductor in the clean as well as in the dirty limit. We find that for increasing impurity scattering a mini gap is established in the normal metal. Furthermore, we consider a spin-active surface of the normal metal (e.g. due to an additional ferromagnetic layer) and observe a suppression of the mini gap.

Keywords: Superconductivity; Quasiclassical theory; Multi-layers

1. Introduction

We study a system consisting of a thin normal metal layer on top of a bulk superconductor as shown in Fig. 1 \((d_n \approx \xi_0, \xi_0\) : superconducting coherence length). We calculate the density of states (DOS) in the normal metal layer using the quasiclassical theory of superconductivity. We generalize the earlier work of Belzig et al. [1] by considering arbitrary impurity concentrations and reduced transparency between the normal metal and the superconductor. Furthermore, we examine a spin-active surface which can be realized by an insulating ferromagnetic layer when in contact with the normal metal.

2. Quasiclassical theory

In thermal equilibrium, the quasiclassical Green’s function \(\hat{g}(E, p_F, \mathbf{r})\) is determined by the Eilenberger equation and the normalization condition:

\[
[i \hat{\tau}_3 E + i \hat{\Delta}(p_F, \mathbf{r}) + i \hat{\sigma}(\mathbf{r}), \hat{g}] \hat{\nu}_F \cdot \nabla \hat{g} = 0, \quad (1)
\]

\[
\hat{g}^2 = \mathbb{1}. \quad (2)
\]

The hats denote the \(4 \times 4\) matrix structure, which combines spin and particle-hole space (\(\hat{\tau}_3, \hat{\sigma}\) are the Pauli matrices in particle-hole/spin space); for simplicity we assume an identical spherical Fermi surface in the normal metal and the superconductor. The s-wave order parameter \(\Delta\) and the impurity self-energy \(\hat{\sigma}\) (in Born approximation) must be determined self-consistently:

\[
\hat{\Delta}(\mathbf{r}) = i \frac{\delta \hat{2} n T N_0 V}{d_n} \sum_{|E_n| < E_F} \frac{1}{2} \Gamma_{\alpha \beta} \hat{g}_{\alpha \beta}(iE_n; \mathbf{r}), \quad (3)
\]

\[
\hat{\sigma}(E; \mathbf{r}) = \frac{1}{2} \hat{\gamma}_i(\mathbf{r}). \quad (4)
\]

Here \(N_0\) is the DOS of the superconductor in the normal state, \(V\) is the pairing interaction, and \(\hat{\gamma}_i\) is the s-wave part of the Green’s function; \(\Gamma_{\alpha \beta}\) is the trace in spin space. This approach allows us to study the cross-over from the clean \((1/2\tau = 0)\) to the dirty \((1/2\tau > T_c)\) limit.

The quasiclassical theory is not directly applicable at interfaces and the Eilenberger equation must be supplemented by appropriate boundary conditions. The NS interface characterized by the transparency \(T_0\) is

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig1.png}
\caption{F-N-S-structure studied in this work.}
\end{figure}

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incorporated with the help of Zaitsev's boundary conditions [2]. The spin-flip processes at the ferromagnetic layer are taken into account by the boundary conditions of Tokuyasu et al. [3] where a phenomenological parameter $\theta$ describes the degree of spin rotation. In this paper, we use the boundary condition in terms of the Maki–Schopohl parameterization of the Green’s function [4,5].

3. Results

We calculated the DOS in the normal metal at $x = 0$ for $d_x = 1.1\zeta_0$ for ideal ($T_0 = 1$, see Fig. 2) and reduced transparency ($T_0 = 0.8$, see Fig. 3) of the NS contact. We compared the case of a spin-active surface ($\theta = \pi/4$) with the spin-conserving case ($\theta = 0$) for various impurity concentrations. The order parameter was determined self-consistently at $T = 0.1T_c$. In the dirty limit with a spin-conserving surface a mini gap can be observed; the DOS is almost unaffected when varying the transparency. Otherwise in the clean limit a clear dependence on the transparency occurs. Note that at a spin-active surface the mini gap is suppressed.

The sub-gap structure of the DOS in the normal metal stems from bound states due to Andreev reflection at the superconductor. In the clean case their energies can be calculated for a constant order parameter in the superconductor and ideal transparency at the NS contact with the result

$$E^2 = \frac{|A|^2}{2} \left[ 1 + \cos \theta \cos \frac{2EL}{v_F} \pm \sin \theta \sin \frac{2EL}{v_F} \right]$$

(5)

with $L = 2d_x p_F / |p_F|$, the signs $\pm$ describe the bound states in the two spin channels. In the non-magnetic case the low energy contribution to the DOS comes from trajectories with small angles of incidence ($L \gg d_x$). These states are strongly affected by impurities in the normal metal. Their spectral weight is shifted to higher energies which results in a mini gap. For a magnetic surface the energies of these states are shifted up or down depending on the spin channel; this means that the spectral weight at low energies increases, and hence the mini gap is suppressed.

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References