

THE CASH-IN-ADVANCE CONSTRAINT IN MONETARY GROWTH MODELS WITH LABOR MARKET SEARCH

BURKHARD HEER

University of Augsburg
and
CESifo

ALFRED MAUßNER

University of Augsburg

We study the sensitivity of the inflation–growth trade-off in monetary growth models to the introduction of search frictions in the labor market. We consider three types of endogenous growth models: (1) the AK model, (2) the Lucas supply-side model, and (3) the two-sector model of Jones and Manuelli. We show that the effects depend on the specification of the cash-in-advance constraint and the magnitude of the semi-interest elasticity of the income velocity. For the AK model, economic growth increases with higher inflation. For the other two models, growth declines for the case of the standard cash-in-advance constraint on consumption, whereas it either increases or decreases if money is introduced, as in the cash–credit good economy of Dotsey and Ireland, depending on the semi-interest elasticity of the income velocity. The welfare effects of inflation are shown to be economically significant in the presence of search unemployment.

Keywords: Inflation, Growth, Costly Credit, Search Unemployment

1. INTRODUCTION

Frictions in the labor market are central to the understanding of the monetary facts of the business cycle and were already emphasized in early work, including Keynes (1936) and Phillips (1958). In this tradition, a vast modern literature of New Keynesian models has evolved that relies on search and matching frictions, following Mortenson and Pissarides (1994). Labor market frictions have been found particularly useful in explaining the persistent and hump-shaped response of output to a monetary policy shock, e.g., in Christiano et al. (2005), Walsh (2005),

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Trigari (2009), and Heer and Maußner (2010), and the inflation dynamics, e.g., in Christoffel et al. (2009a, 2009b). Although the short-run business-cycle effects of monetary policy and the inflation dynamics have been studied extensively in these dynamic stochastic general equilibrium models, the long-run effects of inflation on economic growth in the presence of labor market frictions have not received any attention yet. The present article is intended to fill this gap.

Inflation and economic growth are a central subject in the literature, both on growth and on monetary economics. Most empirical cross-country studies support the fact that inflation has a negative effect on growth.¹ However, this evidence is much less clear-cut for countries characterized by low inflation, so that inflation appears to have a nonlinear effect on growth. Barro (2013) and Gillman et al. (2004) point out that the latter effect is not robust if the endogeneity problem is accounted for. In a recent study using cross-country panel data in a dynamic GMM model, López-Villavicencio and Mignon (2011) find that there exist a threshold level of inflation below which higher inflation results in higher economic growth.

There are also a variety of studies that analyze the effects of inflation in models of endogenous growth. In this vein, Jones and Manuelli (1995) review several growth models, including the AK model and the model with human capital accumulation. A role for money is introduced with the help of a cash-in-advance (CIA) constraint on consumption. In their models, they find relative modest effects of inflation on economic growth. Inflation distorts the leisure–consumption choice of the households so that labor decreases. The same mechanism is at work in Gomme (1993) and Wu and Zhang (1998), where higher inflation results in lower employment and, hence, less economic growth. In the two-sector monetary growth model of Maußner (2004), the effect of inflation on the growth rate depends on the value of the intertemporal rate of substitution. A value lower than one implies a negative relation between the growth rate of the money supply and the growth rate of per capita income. Gillman and Kejak (2005) study the sensitivity of two-sector endogenous growth models to the specification of the CIA constraint. They find a robust negative effect of inflation on growth.

In our analysis, we study the sensitivity of this result to the assumption of labor market frictions in three endogenous growth models: (1) the AK model, (2) the Lucas (1990) supply-side model, and (3) the model with human capital accumulation, as in Jones and Manuelli (1995).² All models are calibrated for the U.S. economy and have the property that they are able to replicate the finding that inflation reduces growth when we use the standard CIA constraint on consumption c and assume Walrasian labor markets. The mechanism is well known: In the case of the standard CIA constraint, higher inflation introduces an inflation tax on the labor supply and employment declines. As a consequence, higher inflation has a negative effect on economic growth.

To illustrate this effect, consider the market clearing condition in the AK model with Walrasian labor markets,

$$w = [1 + (r + \pi)]\text{MRS},$$

where w , r , π , and MRS denote the real wage, the real interest rate, the rate of inflation, and the marginal rate of substitution between hours and consumption, respectively. The nominal interest rate factor $1 + r + \pi$ drives a wedge between the real wage and the MRS (the inflation tax). Therefore, a higher rate of inflation reduces employment and, in the end, economic growth.

As an alternative specification of money demand, we will also introduce a cash-credit good as in the Dotsey and Ireland (1996) model, so that the labor market clearing condition becomes

$$w = [1 + (r + \pi)(1 - \zeta)]\text{MRS},$$

where ζ is the endogenously determined number of goods purchased on credit. With increasing inflation, the costs of holding real money balances m increase and the household accumulates less money. The CIA constraint $m \geq (1 - \zeta)c$ becomes more severe. If ζ were fixed, the household would have to reduce consumption. Instead, he can acquire more goods on credit and can thus relax the CIA constraint. Depending on the relative strength of the impact of π on the wedge and the ensuing increase in the share of credit goods, employment may either decline or increase. The size of the response of ζ to an increase of the inflation rate depends on the semi-interest elasticity of income velocity $\epsilon_{v,r+\pi}$.

If we assume search unemployment rather than Walrasian labor markets, we alter the transmission channel from inflation to growth. In search models, wages result from a Nash bargaining process and depend on the reservation wage of the workers. With increasing inflation, households decrease their cash balances and, thus, consumption. As a consequence, the marginal rate of substitution between consumption and working hours decreases, which in turn reduces the reservation wage and, hence, equilibrium wages. Firms find it more profitable to post vacancies. In addition, the households reduce their search effort in order to increase leisure time. Both effects increase the number of vacancies relative to the number of searching agents and, thus, the probability of finding a job, so that employment increases. Whether this effect prevails and ultimately enhances economic growth depends on the specific engine of growth and the modeling of the CIA constraint.

The paper is structured as follows. In Section 2, we present the AK model with two different specifications for the cash-in-advance constraint. In Section 3, we study the balanced growth equilibrium. In Section 4, we analyze two models of human capital accumulation. In the first model, human capital increases with the time spent on learning, as in Lucas (1990). In the second model, we follow Jones and Manuelli (1995), where the households use goods to increase human capital. Section 5 provides a short summary of our results on how inflation affects growth and unemployment. In Section 6, we present a welfare analysis of inflation effects. Section 7 concludes. The results for the models with Walrasian labor markets and the systems of the equilibrium equations for all models are described in more detail in the Technical Appendix that is available from the authors upon request.

2. THE MODEL

The model introduces money demand and search unemployment into the endogenous growth model of Romer (1986). Money is incorporated by assuming that households finance part of their consumption by using cash. The other part of consumption is financed by credit that is either free of transactions costs or costly as in the economy of Dotsey and Ireland (1996). In order to study the effects of inflation on unemployment, we consider frictions in the labor market in the form of search unemployment and wage bargaining, following Shi and Wen (1997, 1999). Unemployment results from time-consuming and costly matching of vacancies with agents who are searching for a job.

The economy consists of four sectors: households, production firms, financial intermediaries, and the monetary authority. The representative household maximizes its expected intertemporal utility subject to its budget constraint and a CIA constraint. Firms produce a consumption–investment good using capital and labor. To hire workers, firms post vacancies. Financial markets provide credit services, whereas the central bank supplies money.

2.1. Households

A single household $h \in [0, 1]$ consists of different members who are either employed, searching for a job, or enjoying leisure. The members pool their income. Let n and s denote the fractions of employed and searching household members, respectively. An employed person loses his job with an exogenously given probability of θ . Searching households will find jobs with probability ξ , so that the share of employed households (both for each $h \in [0, 1]$ and for the unit mass of households) evolves according to

$$\dot{n} = \xi s - \theta n. \tag{1}$$

The share of household members who enjoy leisure is equal to $x = 1 - n - s$.

Households maximize lifetime utility,

$$\int_0^\infty u(c, x) e^{-\rho t} dt, \tag{2}$$

where c and ρ denote consumption and the discount rate of the household, respectively.

Leisure x increases utility, or equally, searching and working, $s + n = 1 - x$, causes disutility to the household. Following Shi and Wen (1999), we parameterize the current-period utility function as follows:

$$u(c, x) = \ln c - \beta \frac{(1 - x)^\eta}{\eta}. \tag{3}$$

Consumption financing. Consumers can purchase consumption with either cash or credit, as in Schreft (1992), Gillman (1993), or Dotsey and Ireland (1996). The specific modeling of the transactions technology follows Heer et al. (2011). The consumption goods are indexed by $i \in [0, 1]$, and the consumption aggregator is given by $c = \inf_i \{c(i)\}$. Therefore, the individuals will consume the same amounts of all goods.

In the first specification of the monetary economy, a fraction $\zeta \in [0, 1)$ of consumption goods is financed on credit, whereas a fraction $1 - \zeta$ of goods is financed with cash:

$$c(1 - \zeta) \leq \frac{M}{P}, \tag{4}$$

where M and P denote nominal money and the price level, respectively. The inflation rate is defined by $\pi \equiv \frac{\dot{P}}{P}$.³ In this case, the credit costs are zero and the fraction ζ is given exogenously.

In the second specification that follows Heer et al. (2011), ζ is determined within the model: In order to buy good i on credit, the household must pay the fee $q(i)$ to a financial intermediary who certifies the seller of the good that the household is worthy of credit. The financial intermediary requires $w\kappa(i)$ units of output to produce this service, where w denotes the real wage. With free market entry in this sector, $q(i) = w\kappa(i)$. The function $\kappa(i)$ is strictly increasing in i and satisfies $\lim_{i \rightarrow 1} \kappa(i) = \infty$. According to the latter assumption, some goods will be purchased with cash, and the demand for money is well defined. In particular, we parameterize the transaction technology as

$$\kappa(i) = \kappa_0 \left(\frac{i}{1-i} \right)^\chi. \tag{5}$$

Budget constraint. In addition to the CIA constraint (4) and the employment dynamics (1), households face a budget constraint. They receive income from capital k , labor n , profits Ω , and real lump-sum transfers τ from the monetary authority. Real assets a consist of capital k and real money balances $m \equiv M/P$ and accumulate according to

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wn + rk + \Omega + \tau - c - \pi m & \text{case 1,} \\ wn + rk + \Omega + \tau - c - \pi m - \int_0^\zeta q(i) di & \text{case 2.} \end{cases} \tag{6}$$

The household maximizes (2) subject to (1), (4)–(6), and given initial values of capital, employment, and real money balances, k_0 , n_0 , and m_0 , respectively.

2.2. The Monetary Authority

The economywide nominal money supply $M = Pm$ grows at the rate μ :

$$\frac{\dot{M}}{M} = \mu. \tag{7}$$

The seignorage obtained from money creation is paid to the households as a lump-sum transfer, implying

$$\tau = \mu m. \tag{8}$$

2.3. Firms

Firms are identical and of measure one. They use labor n and capital k to produce the consumption–investment good y with the technology $f(k, \bar{k}, n)$. The externality in aggregate capital accumulation \bar{k} (which equals k in equilibrium) results in constant returns to capital as in Romer (1986):

$$y = Ak^\alpha n^{1-\alpha} \bar{k}^{1-\alpha}. \tag{9}$$

Workers separate from the representative firm with probability θ . To attract new workers, the firm posts vacancies v at cost ϕwv . The probability that a vacancy is filled is ϑ . Therefore, employment at the firm level (and, at equilibrium, in the entire economy) evolves according to

$$\dot{n} = \vartheta v - \theta n, \tag{10}$$

and profits are given by

$$\Omega = y - wn - (r + \delta)k - \phi wv. \tag{11}$$

The firm maximizes discounted profits,

$$\int_0^\infty \Omega e^{-\int_0^t r(\xi) d\xi} dt, \tag{12}$$

subject to (10), (11), the specification of the production function in (9), and a given initial employment n_0 .

2.4. Matching and Bargaining

At the aggregate level, the mass of successfully filled vacancies M is determined by

$$M = v^\gamma s^{1-\gamma}, \tag{13}$$

where v and s denote the mass of vacancies and the share of searching households. Because households cannot search while being employed,⁴ the unemployment rate u is given by

$$u = \frac{s}{n + s}. \tag{14}$$

The probability for a household of finding a job is

$$\xi = \frac{M}{s} = (v/s)^\gamma, \tag{15}$$

and the probability that an open vacancy is filled follows from

$$\vartheta = \frac{M}{v} = (v/s)^{\gamma-1}. \tag{16}$$

Wages result from decentralized Nash bargaining between the firm and the marginal worker. In particular, the wage w maximizes

$$\max_w [\text{MPL} - w]^{1-\lambda} [w - \text{MRS}]^\lambda, \tag{17}$$

where MPL denotes the marginal product of labor and MRS the marginal rate of substitution between working hours and consumption. The Nash bargaining solution is given by

$$w = \lambda(1 - \alpha)Ak^\alpha n^{-\alpha} \bar{k}^{1-\alpha} + (1 - \lambda)\beta(n + s)^{\eta-1}c. \tag{18}$$

2.5. Balanced Growth Path

The stationary equilibrium or the balanced growth path of the model has the following properties:

1. The stock of capital k , real money balances m , output y , consumption c , and the real wage w at the same constant rate g .
2. The rental rate of capital r , the share of goods purchased with credit ζ , the inflation rate π , the unemployment rate u and thus employment n and the fraction of unemployed households s , and the mass of vacancies v are constant.

The growth rate and the stationary values of the variables are determined by the following system of equations, in which the tilde $\tilde{}$ refers to variables scaled by k :

$$g = An^{1-\alpha} - \delta - \tilde{c} - \tilde{w} \left[\phi v + \kappa_0 \int_0^\zeta \left(\frac{i}{1-i} \right)^x \right], \tag{19a}$$

$$g = r - \rho, \tag{19b}$$

$$g = \mu - \pi, \tag{19c}$$

$$r = \alpha An^{1-\alpha} - \delta, \tag{19d}$$

$$\tilde{w} = \lambda \alpha n^{-\alpha} + (1 - \lambda)\beta(n + s)^{\eta-1} \tilde{c}, \tag{19e}$$

$$(v/s)^\gamma = \theta(n/s), \tag{19f}$$

$$\tilde{c}(r + \pi) = \tilde{w} \kappa_0 \left(\frac{\zeta}{1-\zeta} \right)^x, \tag{19g}$$

$$g = r + \theta - \frac{(v/s)^{\gamma-1}}{\phi} \left[\frac{(1 - \alpha)An^{-\alpha}}{\tilde{w}} - 1 \right], \tag{19h}$$

$$\tilde{w} = [1 + (r + \pi)(1 - \zeta)]\beta(n + s)^{\eta-1} \tilde{c}[(\theta + \rho)(v/s)^{-\gamma} + 1]. \tag{19i}$$

Equation (19a) is the economy's resource constraint, which derives from the household's budget constraint (6), the definition of profits (11), and the zero profit condition for financial intermediation. Equation (19b) derives from the household's Euler equation for capital accumulation. Equation (19c) follows from the CIA constraint (4). Equation (19d) is the firm's first-order condition for the demand of capital. Equation (19e) solves the Nash bargaining problem [see (18)]. Equation (19f) combines (1) for $\dot{n} = 0$ and (15). Equation (19g) is the first-order condition for the optimal share of credit goods. Equation (19h) combines the firm's first-order condition for vacancies and the growth rate of the firm's shadow price of employment. Finally, equation (19i) is implied by the household's shadow price of employment and the first-order condition for the time spent searching.⁵

It is obvious from equation (19d) that in a model with Walrasian labor markets and exogenous labor supply n the growth rate depends only on the four parameters A , α , n , and δ . In this case, money is superneutral. Thus, the effect of money on employment is decisive in understanding the inflation–growth nexus.

2.6. Calibration

The effects of a change in the inflation rate (as resulting from a change in the growth rate of the money supply) cannot be studied analytically but only numerically.⁶ For this reason, the model is calibrated to match characteristics of the U.S. economy. The unit time length corresponds to one quarter. In case 1 (case 2), we have to find the values of 12 (13) parameters. For a subset of 8 parameters, $\{\alpha, \eta, \rho, \delta, \mu, \theta, \gamma, \lambda\}$, we use observations to calibrate them individually. Most of these values, namely α , δ , η , and ρ , are taken from the literature, such as Kydland and Prescott (1982). Their empirical estimates are based on time series evidence covering subsamples of the post–World War II period. For the remaining parameters, i.e., either $\{\beta, A, \zeta, \phi\}$ in case 1 or $\{\beta, A, \kappa_0, \chi, \phi\}$ in case 2, we calibrate them simultaneously so that both a set of empirical observations and the equilibrium conditions (19) hold.

We set the production elasticity of private capital equal to $\alpha = 0.36$ and the quarterly rate of capital depreciation equal to $\delta = 0.025$. The household discounts future utilities at the rate $\rho = 0.01$. $\eta = 3.5$ implies a labor supply elasticity of $\epsilon_{n,w} = 1/(\eta - 1) = 0.4$, and the value of μ follows from our inflation target, which is either 0 or 10% p.a. The parameters $\{\theta, \gamma, \lambda\}$ are set as in Shi and Wen (1999).

The remaining parameters, β , A , ϕ , and (in case 1) ζ , are set so that

- i. the labor force participation rate $n + s$ equals 68%;⁷
- ii. the unemployment rate is equal to 6%;
- iii. the annual growth rate of output $(1 + g)^4 - 1$ (where g is the quarterly rate) is equal to 2%;
- iv. the share of cash goods in total consumption is equal to 82%.⁸

TABLE 1. Calibration of the benchmark model

Preferences	$n + s = 0.68$	$\eta = 3.5$	$\rho = 0.01$
Production	$\alpha = 0.36$	$\delta = 0.025$	
CIA	$\zeta = 0.18$	$\epsilon_{v,r+\pi} = 5.95$	
Labor market	$\theta = 0.05$	$\gamma = 0.6$	$\lambda = 0.6$
	$u = 0.06$		
Output growth	$(1 + g)^4 - 1 = 0.02$		
Money supply	$(1 + \pi)^4 - 1 = 0$		

In case 2, we use yet another observation in addition to (iv) to determine κ_0 and χ simultaneously: the semi-interest elasticity of the income velocity of money $\epsilon_{v,r+\pi}$ implied by the model must equal 5.95, as estimated by Dotsey and Ireland (1996) for the U.S. economy during 1959–1991. The values of the exogenously given parameters are summarized in Table 1.

3. INFLATION AND ECONOMIC GROWTH

In this section, we study the effects of inflation on economic growth in the model of Section 2. We consider annual inflation rates between 0% and 15% p.a.⁹ In this range the rate of growth increases with inflation. This result holds for both specifications of the CIA constraint. In the version of our model with Walrasian labor markets (which we derive in the Technical Appendix), the inflation–growth relation depends on the kind of CIA constraint: if the share of credit goods ζ is given exogenously, inflation depresses growth, whereas it is growth-enhancing if ζ is determined within the model.

In the second part of this section, we study the sensitivity of our result for the economy in case 2 to the magnitude of the semi-interest elasticity of income velocity.¹⁰ We show that the inflation–growth trade-off does not depend on this parameter in the AK model. For the models with human capital accumulation that we consider in the next section, however, this parameter will be crucial.

Moreover, we show that unemployment declines initially with rising inflation irrespective of the form of the cash-in-advance constraint.

3.1. Specification of the Cash-in-Advance Constraint

Figure 1 displays the relation between the annual rate of inflation and various endogenous variables along a balanced growth path. In each of the six panels the annual inflation rate on the abscissa increases from 0% to 15% p.a. The respective left ordinates correspond to solid lines, whereas the right ones refer to the broken lines. The figure depicts three cases:

Case 1 refers to the model with search frictions and an exogenously fixed share of credit goods.

Case 2 refers to the model with search frictions and an endogenously determined share of credit goods.

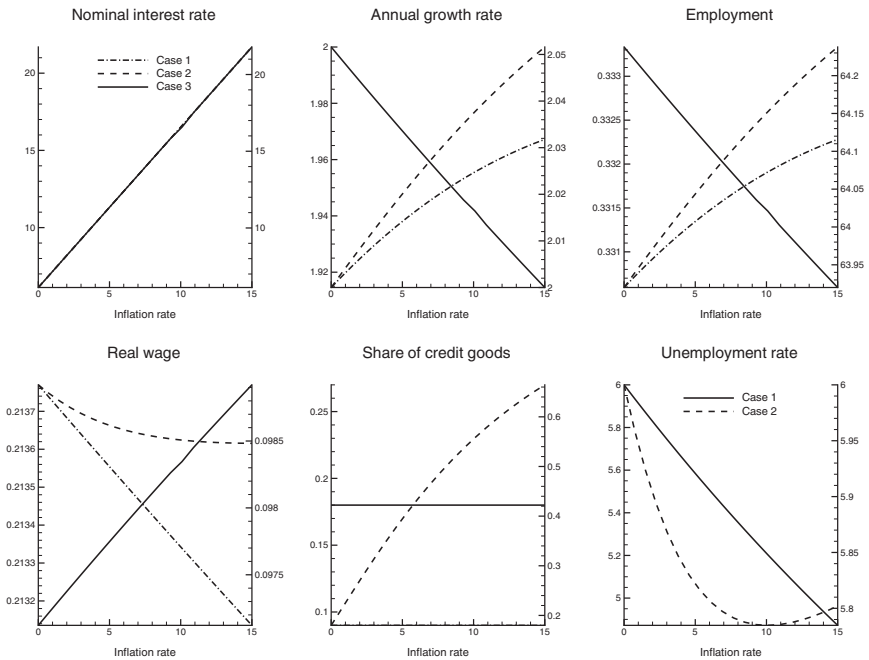


FIGURE 1. Inflation and growth in the AK model.

Case 3 refers to the model with a Walrasian labor market and an exogenously fixed share of credit goods.

The upper right panel of Figure 1 displays the behavior of the nominal interest rate, $r + \pi$. In cases 1 and 2 it increases from 6.12% to 21.79% p.a. In case 3 it reaches 21.69% for an inflation rate of 15%. The nominal interest rate distorts the household’s consumption–leisure choice. With Walrasian labor markets this choice is determined by

$$w = [1 + (r + \pi)(1 - \zeta)]c\beta n^{\eta-1}.$$

Thus, for a given real wage w and a fixed share of credit goods, a higher nominal interest rate requires less consumption and fewer working hours. As shown by the black line in the rightmost upper panel, hours decrease with inflation. It is then obvious from equations (19d) and (19b) (which also hold in the case of Walrasian labor markets) that the rate of growth must decline.

With search frictions, the previous equation is replaced by equation (19i) and two additional effects arise. First, the household may increase leisure by lowering its search effort. Second, the probability of finding a job ξ is a decreasing function of labor market tightness s/v . Inflation—by increasing the costs of money holdings—reduces consumption and thus the household’s reservation wage (see the lower leftmost panel in Figure 1). Firms find it more profitable to post vacancies. This

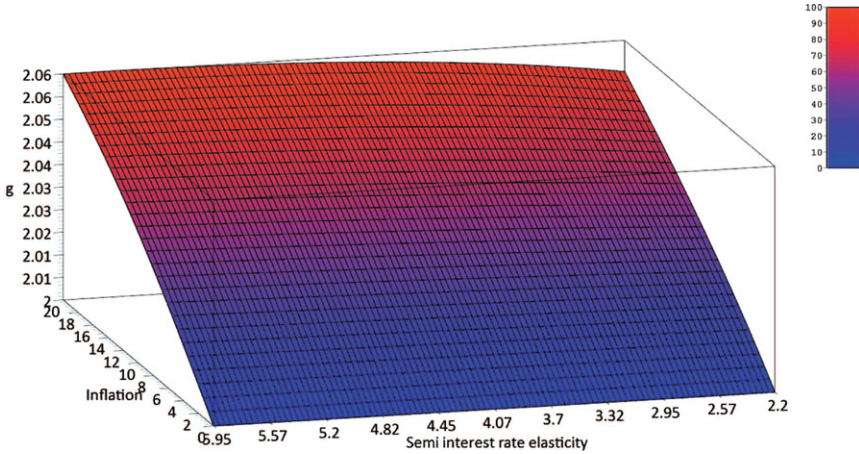


FIGURE 2. Growth and inflation in the AK model: Sensitivity to $\epsilon_{v,r+\pi}$.

effect is reinforced by the lower search effort of households, so that ξ increases (not shown in the figure), raising employment. Obviously, for our calibration, this effect dominates the effect that results from the household’s desire to increase leisure because of increased nominal interest rates. Therefore, employment and growth increase with inflation.

A third effect appears in the case of credit costs. It reinforces the positive effect on employment (compare case 1 and case 2 in the rightmost upper panel of Figure 1) but dampens the effect on unemployment. In its desire to reduce the costs of money holdings, the household buys more goods on credit, so that the wedge $[1 + (r + \pi)(1 - \zeta)]$ in equation (19i) declines. As a consequence, the household is not forced to reduce consumption by as much as in the case of a fixed share of credit goods. For this reason, wages decline less, and beyond a certain threshold for inflation even start to increase. This also explains the U-shaped behavior of the rate of unemployment: Higher wages deter firms from posting vacancies, whereas they encourage households to search for jobs. The labor market becomes tighter, decreasing the probability of finding a job.

3.2. Sensitivity to the Semi-interest Elasticity

In Figure 2, we study the sensitivity of the growth effects of inflation for the benchmark model of Section 2 (case 2) to the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. For the value estimated by Dotsey and Ireland (1996), $\epsilon_{v,r+\pi} = 5.95$, the growth-enhancing effect is more pronounced than for lower values, $\epsilon_{v,r+\pi} = 2.20$.¹¹ If the income velocity is very sensitive to the interest rate (higher values of $\epsilon_{v,r+\pi}$), households reduce their cash holdings more markedly with rising inflation, or, equally, ζ is more sensitive to inflation. As a consequence, the inflation costs are lower and growth is higher for larger $\epsilon_{v,r+\pi}$. For the present

calibration, however, the effect is very small, as can be seen by comparing the inflation–growth trade-off for the values $\epsilon_{v,r+\pi} = 5.95$ and $\epsilon_{v,r+\pi} = 2.20$. The growth rate amounts to 2.063% and 2.054%, respectively, if inflation increases to 15%.

4. ENDOGENOUS GROWTH AND HUMAN CAPITAL

In the following, we examine the sensitivity of our results to the specification of the engine of growth. In Lucas (1990), hours spent on learning increase the stock of human capital, which is used along with raw labor and physical capital to produce consumption and investment goods. Jones and Manuelli (1995) assume that goods rather than time are used to accumulate human capital. In this section, we describe only the essential new features of these endogenous growth models in comparison to the benchmark AK model in Section 2. A full description of the models and their equilibrium conditions is relegated to the Technical Appendix.

4.1. The Lucas Supply-Side Model

To keep the description of the model as brief as possible, we just present the maximization problems of the household and the firm sector, respectively. Everything else remains unchanged.¹²

Households. The representative household can allocate its time endowment 1 to work n , searching s , learning l , or leisure x :

$$1 = n + s + l + x. \tag{20}$$

Human capital of the individual, h , is determined by the time l he allocates to learning:

$$\dot{h} = Dh l^\nu. \tag{21}$$

The individual’s labor income is given by the product of the wage, w , and his effective labor, hn . In case 2, costs per unit of credit service are proportional to the costs of labor given by $w\bar{h}$, where \bar{h} is the stock of human capital of the average worker.¹³ The household’s budget constraint, thus, is

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wnh + rk + \Omega + \tau - c - \pi m & \text{case 1,} \\ wnh + rk + \Omega + \tau - c - \pi m - \int_0^\zeta q(i) di & \text{case 2.} \end{cases} \tag{22}$$

The household maximizes (2) subject to (4), (5), (21), and (22). The first-order and equilibrium conditions of the model are derived in the Technical Appendix.

Firms. Output is produced using capital k and effective labor, hn , according to

$$y = Ak^\alpha (hn)^{1-\alpha}. \tag{23}$$

In the Lucas model, the vacancy costs must rise with the human capital of the workers in the financial sector, implying the profits

$$\Omega = Ak^\alpha(nh)^{1-\alpha} - (r + \delta)k - wnh - \phi whv. \tag{24}$$

Firms maximize

$$\int_0^\infty \Omega e^{-\bar{r}(t)} dt$$

subject to (10) and (24). Otherwise, the production sector is identical to that in the model of Section 2.

Calibration. As compared to the AK model, the Lucas model has two additional parameters, D and ν . We employ the value of $\nu = 0.8$ from Lucas (1990). D is calibrated so that the steady-state annual rate of growth equals 2% for $\pi = 0\%$. In addition, we set $A = 1$.¹⁴

4.2. The Model of Jones and Manuelli

The model. As a second variation of the endogenous growth model with human capital, we assume that human capital accumulation requires goods rather than time:

$$\dot{h} = i_h - \delta_h h, \tag{25}$$

where i_h and δ_h denote investment in human capital and the rate of human capital depreciation, respectively. As a consequence, the household’s budget constraint is modified to

$$\dot{a} = \dot{k} + \dot{m} = \begin{cases} wnh + rk + \Omega + \tau - c - \pi m - i_h & \text{case 1,} \\ wnh + rk + \Omega + \tau - c - \pi m - i_h - \int_0^\zeta q(i) di & \text{case 2.} \end{cases} \tag{26}$$

Households maximize (2) [with the current-period utility function specified by (3)] subject to (4), (25) and (26).

The equilibrium conditions of the firm are the same as in the Lucas model in Section 4.1. The stationary equilibrium conditions are described in more detail in the Technical Appendix. We follow Jones and Manuelli (1995) and set $\delta_h = \delta$. The remaining parameters are set to the values presented in Table 1.

4.3. Results

Effects on unemployment. Figure 3 displays the relation between the rate of inflation, the unemployment rate, and the rate of per capita growth obtained from simulations of our two models for cases 1 and 2 (with $\epsilon_{v,r+\pi} = 5.95$). To distinguish the different models, we use the shorthands L and JM for the models of Sections 4.1 and 4.2, respectively.

In both models, the equilibrium unemployment rate declines with inflation in case 1 and is a U-shaped function of the rate of inflation in case 2. As in

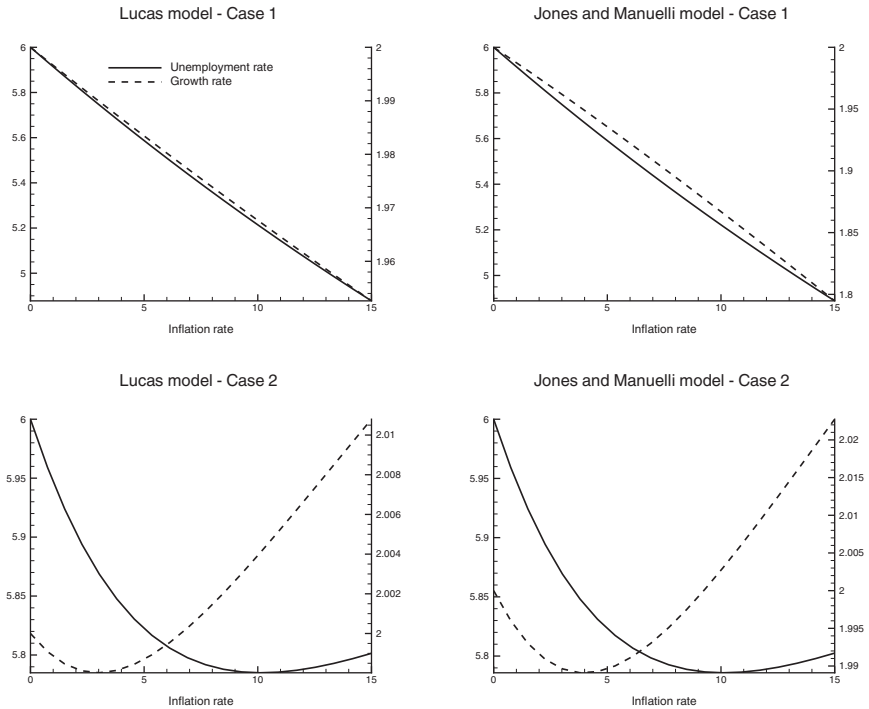


FIGURE 3. Inflation, unemployment, and growth in models with labor market frictions.

the AK model, these results are driven by two effects. First, with increasing inflation, households decrease their cash balances and thus consumption. As a consequence, the marginal rate of substitution between consumption and working hours decreases, which in turn reduces the reservation wage. Firms find it more profitable to post vacancies. In addition, the households reduce their search effort in order to increase leisure time. Both effects decrease labor market tightness s/v so that the probability of finding a job and, in turn, employment increase. The net effect of less searching agents and a larger number of vacancies on employment is negative in the case of the JM model with constant ζ (case 1), and positive in all other cases. Second, in case 2, the households increase their share of credit goods, which mitigates the effect of inflation on consumption and, thus, on the real wage. If inflation exceeds a threshold of about 10%, this effect reverses the first one.

Effects on the rate of growth. Differently from the AK model, the effect on employment does not necessarily mimic the effect on the rate of growth. In the Lucas model, the fraction of the time endowment devoted to human capital accumulation l determines the rate of growth [see equation (21)]. For low rates of inflation, the household demands more leisure, and because the marginal benefit of employment increases, the household substitutes market work for time spent

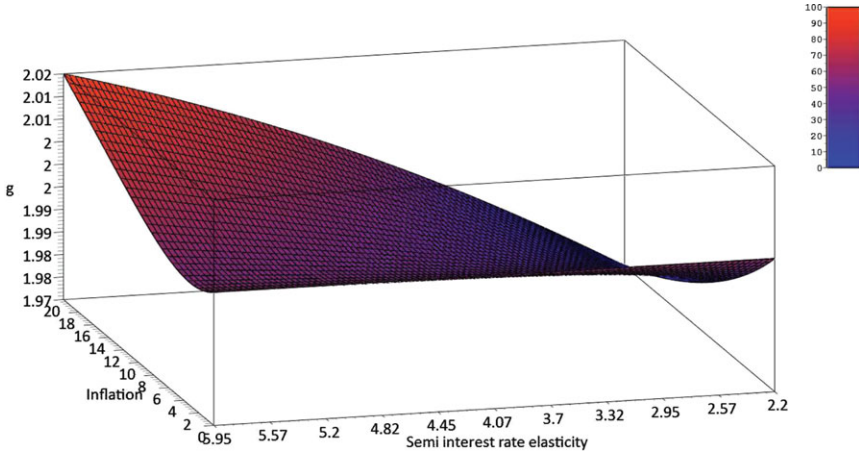


FIGURE 4. Growth and inflation in the Lucas model: Sensitivity to $\epsilon_{v,r+\pi}$.

to increase human capital. For higher rates of inflation, the increasing share of credit goods decreases the demand for leisure, and the household devotes more time to increase its human capital. The threshold level of inflation for this effect to dominate the overall effect is about 3% p.a.

In the Jones and Manuelli model the trade-off between investment in physical, human, and monetary capital determines the effect on the rate of growth. With increasing inflation, the household substitutes physical and human capital for monetary balances. The household invests in human capital until its return $wn - \delta_h$ equals the return on physical capital r .¹⁵ In case 2, there is again a threshold level of inflation of about 4% p.a. at which the decline of wn comes to a halt. Beyond this threshold, the portfolio reallocation increases the rate of growth.

Sensitivity. Figures 4 and 5 show that the U-shaped effects of inflation on the rate of growth are sensitive to the semi-interest elasticity of the income velocity of money $\epsilon_{v,r+\pi}$. Differently from the AK model, there are threshold values of $\epsilon_{v,r+\pi}$ below which inflation reduces the rate of economic growth over the entire range of inflation rates considered. In the Lucas model the threshold is at $\epsilon_{v,r+\pi} = 2.2$, whereas it occurs at $\epsilon_{v,r+\pi} = 3.3$ in the Jones and Manuelli model. The qualitatively different behavior of the growth rate below these thresholds in both models of human capital accumulation as compared to the AK model demonstrates the central role played by the engine of growth rather than the modeling of money demand.

5. SUMMARY OF RESULTS

Our results on the effects of inflation on growth and unemployment are summarized in Tables 2 and 3. The entries display the economic growth rate and the rate of

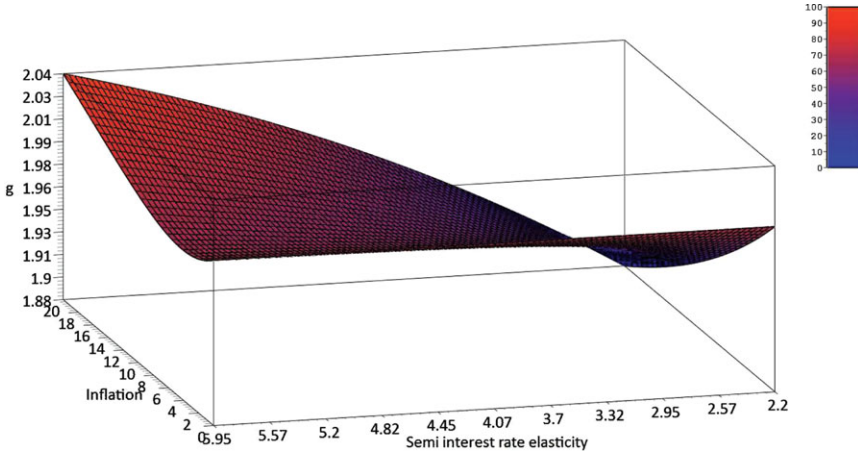


FIGURE 5. Growth and inflation in the Jones and Manuelli model: Sensitivity to $\epsilon_{v,r+\pi}$.

unemployment, respectively, for 10% annual inflation. The benchmark values for zero inflation are 2% and 6%, respectively. In the first entry column, the numbers refer to the models with the standard CIA constraint. Depending on the endogenous growth mechanism, economic growth may increase (in the case of the AK model) or decrease (in the case of the Lucas and Jones/Manuelli models). Notice, however, that the magnitude of the quantitative effect on growth is small. For example, in the AK model, economic growth only increases from 2.00% to 2.025% if inflation rises from 0% to 10%. The effects on the rate of unemployment are much stronger, with a minimum impact of 0.21 and a maximum of 0.79 percentage points.

TABLE 2. Effects of inflation on per capita income growth

Model	CIA	DI	
		$\epsilon_{v,r+\pi}$	
		5.95	2.20
AK	2.025%	2.037%	2.031%
Lucas (1990)	1.968%	2.004%	1.983%
Jones and Manuelli (1995)	1.867%	2.003%	1.928%

Notes: The columns display the growth rate after an increase of the inflation rate π from 0% to 10% in the three prototype endogenous growth models for different specifications of the CIA constraint. The first entry column displays the results for the standard CIA constraint. In the second and third entry columns under the caption DI, the results are derived in the cash-credit economy for two different values of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).

TABLE 3. Effects of inflation on the rate of unemployment

Model	CIA	DI	
		$\epsilon_{v,\pi}$	
		5.95	2.20
AK	5.21%	5.79%	5.45%
Lucas (1990)	5.21%	5.79%	5.45%
Jones and Manuelli (1995)	5.22%	5.79%	5.46%

Notes: The columns display the rate of unemployment after an increase of the inflation rate π from 0% to 10% in the three prototype endogenous growth models for different specifications of the CIA constraint. The first entry column displays the results for the standard CIA constraint. In the second and third entry columns under the caption DI, the results are derived in the cash–credit economy for two different values of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).

The impact of inflation on economic growth and unemployment depends on the specification of the alternative payment means.¹⁶ In the case of the CIA constraint in Dotsey and Ireland (1996), consumers purchase consumption goods with either cash or credit; the latter is subject to transaction costs in the financial market. If inflation increases, households buy a smaller number of goods with cash. In the AK model, this mechanism strengthens the positive effect of inflation on growth and reverses the negative growth effects of inflation in the Lucas and Jones and Manuelli models, if the semi-interest elasticity of income velocity is not too small. This can be seen by comparing the first with the second and third entry columns of Table 2.

6. WELFARE ANALYSIS

In this section, we provide a welfare analysis of the inflation effects.¹⁷ Based on Lucas (1987), we express the welfare change associated with an increase of inflation from 0% to 10% by the change in total consumption (as percentage of output), Δ , that is necessary to make the individual in the old equilibrium indifferent between these two steady states. In addition, following Gomme (1993), we decompose the welfare effects into the three factors affecting steady-state lifetime utility: (1) consumption, (2) leisure, and (3) the growth rate. Our welfare results are summarized in Table 4.

To illustrate the decomposition of the welfare effects, let $\{\tilde{c}, \tilde{y}, x, g\}$ ($\{\tilde{c}^a, \tilde{y}^a, x^a, g^a\}$) denote the steady-state values of consumption, output, leisure, and the growth rate in the steady state with zero annual inflation (10% inflation), where variables with a tilde are expressed as the respective levels relative to capital, e.g. consumption per capita $c = \tilde{c} \cdot k = c \cdot k_0 e^{gt}$ grows at rate g . Given the specification of lifetime utility in (2), the welfare effect Δ can be computed

TABLE 4. Effects of inflation on welfare

Model	Walras			Search		
	CIA	DI $\epsilon_{v,\pi}$		CIA	DI $\epsilon_{v,\pi}$	
		2.2	4.95		2.2	4.95
AK						
Total	-0.926	-0.732	-0.485	0.282	0.133	-0.113
Consumption	-0.230	-0.417	-0.656	-0.455	-0.535	-0.658
Leisure	0.352	0.161	-0.088	0.330	0.157	-0.073
Growth Rate	-1.048	-0.476	0.259	0.407	0.512	0.618
Lucas (1990)						
Total	0.032	-0.297	-0.722	-0.611	-0.693	-0.815
Consumption	-0.097	-0.357	-0.689	-0.656	-0.703	-0.775
Leisure	0.573	0.261	-0.143	0.580	0.288	-0.105
Growth Rate	-0.443	-0.201	0.109	-0.535	-0.278	0.065
Jones and Manuelli (1995)						
Total	0.000	-0.111	-0.243	-0.327	-0.295	-0.282
Consumption	0.000	-0.108	-0.244	-0.064	-0.145	-0.251
Leisure	0.033	0.190	-0.105	0.538	0.281	-0.047
Growth Rate	-0.034	-0.193	0.106	-0.801	-0.430	0.016

Notes: The column entries give the percentage of output that must be given or taken as additional consumption to the representative household in the zero-inflation equilibrium so that it is indifferent between this equilibrium and the one with 10% inflation. The rows labeled consumption, leisure, and growth rate split the total welfare effect into those parts that can be attributed to the change in consumption, leisure, and the growth rate, respectively. CIA refers to the cash-in-advance model, DI to the Dotsey and Ireland (1996) model of money holdings.

from the following equation:¹⁸

$$\begin{aligned}
 W(\tilde{c} + \Delta \tilde{y}, x, g) &= \int_0^\infty \ln(c + \Delta y) - \beta \frac{(1-x)^\eta}{\eta} e^{-\rho t} dt \\
 &= \frac{\ln(\tilde{c} + \Delta \tilde{y})}{\rho} + \frac{\ln k_0}{\rho} + \frac{g}{\rho} + \beta \frac{(1-x)^\eta}{\rho \eta} \\
 &\stackrel{!}{=} W(\tilde{c}^a, \tilde{y}^a, x^a, g^a) = \frac{\ln \tilde{c}^a}{\rho} + \frac{\ln k_0}{\rho} + \frac{g^a}{\rho} + \beta \frac{(1-x^a)^\eta}{\rho \eta}.
 \end{aligned}$$

A Taylor series approximation then implies

$$\Delta = \frac{\tilde{c}}{\tilde{y}} \left\{ [\ln \tilde{c}^a - \ln \tilde{c}] + \frac{\beta}{\eta} [(1-x)^\eta - (1-x^a)^\eta] + \frac{g^a - g}{\rho} \right\}.$$

The entries in Table 4 labeled “Consumption,” “Leisure,” and “Growth rate” correspond to the three additive terms on the right-hand side of this equation.

The following results presented in Table 4 are noteworthy:

1. In the Walras economies, the role of money demand, i.e., the specification of the CIA constraint, is crucial for the sign and the significance of the inflation effect. In the two-sector models of both Lucas (1990) and Jones and Manuelli (1995), the standard CIA constraint implies negligible welfare effects, as in Gomme (1993).¹⁹ However, if we consider the Dotsey–Ireland cash–credit economy with a high semi-interest elasticity, $\epsilon_{v,\pi} = 4.95$, an inflation rate of 10% raises the welfare costs to 0.722% and 0.243% of total output in the two models, respectively.
2. The introduction of search unemployment in the two-sector endogenous growth models increases the magnitude of the welfare effects irrespective of the motive for money demand. For example, in the Lucas model with the cash–credit good and $\epsilon_{v,\pi} = 4.95$, the welfare loss of total consumption due to the higher inflation rate amounts to 0.815% of output rather than 0.722% in the case of Walrasian labor markets.
3. In contrast to the Walras model, we find that the effect of leisure remains significant when we introduce search unemployment. Therefore, we also find the same result as Gomme (1993) that “these results point to the importance of leisure in the calculation of welfare costs.”

Alternative labor market specifications. To study the robustness of our welfare results, we consider two additional forms of matching mechanisms. In particular, we analyze (1) efficient bargaining and (2) wage posting.²⁰ In the first case, we consider bargaining over both the real wage and working hours as in Trigari (2006). In this case, both the wage and the optimal working hours are chosen to maximize the product of the weighted surpluses for the worker and the entrepreneur. In the second case, workers search for a job and decide which job they should apply for. The probability of getting selected depends on the number of applicants. For this specification, we follow Shi and Wen (1999), who employ a matching game based on Peters (1991).

In Table 5, we present the welfare effects resulting from these modifications in the AK model and the Lucas (1990) model for the case of the DI constraint with $\epsilon_{v,\pi} = 4.95$. Notice that the magnitude and the direction of the welfare changes remain unaltered. In the case of the AK model, the total welfare loss even increases and amounts to almost 1% in the case of wage posting. In the case of the Lucas (1990) model, the magnitude of the total effect is reduced, but still economically significant, 0.395% and 0.760% in the cases of efficient bargaining and wage posting, respectively.

7. CONCLUSION

In our models of economic growth, we have emphasized the effects of search unemployment. In comparison to models with Walrasian labor markets, inflation has an additional transmission channel of inflation on economic growth in this case. Higher inflation affects the reservation wage of the households and increases

TABLE 5. Sensitivity of welfare effects to wage setting

Model	Wage setting		
	Wage bargaining	Efficient bargaining	Wage posting
	AK		
Total	-0.113	-0.270	-0.943
Consumption	-0.658	-0.293	-0.791
Leisure	-0.073	-0.091	0.078
Growth rate	0.618	0.114	-0.230
	Lucas (1990)		
Total	-0.815	-0.395	-0.760
Consumption	-0.775	-0.322	-0.690
Leisure	-0.105	-0.133	-0.132
Growth rate	0.065	0.060	0.062

labor market tightness. As a consequence, unemployment initially declines with inflation. Therefore, we find that inflation harms long-run economic growth to a smaller extent in monetary growth models with frictions than in those with Walrasian labor markets. Although the magnitude of the growth rate effects is small, we find economically significant welfare effects of inflation. In the case of the two-sector models with human capital, a rise of inflation from 0% to 10% reduces steady-state lifetime utility by 0.33% and 0.61% of total output in the two model variants.

In addition, we show that the specification of the CIA constraint matters. We compare monetary growth models with a standard CIA constraint to those with a costly credit channel, as in Dotsey and Ireland (1996). In the latter case, agents buy more goods with credit if inflation increases and economic growth is higher than in the case of the standard CIA constraint on consumption. Moreover, if households can choose the financing of consumption by means of cash and credit, the qualitative effect of inflation on economic growth becomes sensitive to the semi-interest elasticity of income, for which the empirical evidence provides a broad range of values.

At this point, let us mention one word of caution. We have focused our analysis and also our discussion on models within a CIA context. Our results may be sensitive to the introduction of other monetary frictions and, in particular, those that become more important for higher rates of inflation or during the process of economic development. For example, with high rates of inflation, transaction costs and employment in the financial sector, as emphasized by Ireland (1994), are likely to result in lower employment and less growth. In a similar vein, inflation may increase transaction costs, and therefore it reduces growth in the models of De Gregorio (1993) and Jha et al. (2002). Furthermore, with higher inflation, the volatility of inflation is likely to increase. We intend to study the sensitivity of our results to other monetary frictions in the future.

NOTES

1. A nonexhaustive list includes Kormendi and Meguire (1985), McCandless and Weber (1995), Chari et al. (1996), Bruno and Easterly (1998), Barro (2001), Gylfason and Herbertsson (2001), Gillman and Kejak (2005), and Walsh (2010, Chap. 1).

2. Our choice of growth models is motivated by the fact that they represent the standard models considered in similar inflation–growth studies with Walrasian labor markets, as in Gomme (1993), Jones and Manuelli (1995), or Gillman and Kejak (2005).

3. Because the analysis only considers the situation where π is larger than the negative real interest rate r , $\pi \geq 0 > -r$, (4) will always hold as an equality at an optimum.

4. We refrain from introducing on-the-job search. The motive to search on the job originates from wage dispersion, and, hence heterogeneity among the firms. To deal with this additional complexity would require an entirely different setup, which we leave as an interesting subject for future research.

5. See the Technical Appendix for details.

6. The GAUSS computer programs are available from the authors upon request.

7. The values both for u and for $n + s$ are taken from Shi and Wen (1999).

8. This value for the U.S. economy was found by Avery et al. (1987).

9. The choice of this range is motivated by the fact that the post-World War II regimes of monetary policy fall within this interval. For example, the average inflation rates during the Volker (1979–1987) and Greenspan (1988–2003) eras amounted to 6.43% and 3.06%, respectively.

10. In the Technical Appendix, we also study the sensitivity of the growth effects on the separation rate θ . However, the rate of growth is not affected by its magnitude.

11. Dotsey and Ireland (1996) estimate $\epsilon_{v,r+\pi}$ from a simple regression of the log of velocity on the nominal interest rate using annual U.S. data from 1959 to 1991. A much smaller value of $\epsilon_{v,r+\pi} = 0.1$ follows from Ball (2001, p. 40), who employs more sophisticated methods and who uses data from 1946 to 1996. Thus, $\epsilon_{v,r+\pi} = 2.2$ lies well within the range of empirically plausible estimates.

12. For a more detailed presentation of the model, see Lucas (1990) and Grüner and Heer (2000).

13. Note that at equilibrium $\bar{h} = h$.

14. The calibration exercise is a bit more involved for the Lucas model than for the AK model and is described in more detail in the Technical Appendix.

15. Note that, in the Jones and Manuelli model, physical capital, human capital, and consumption are perfect substitutes, so there are no relative price effects.

16. For monetary growth models with Walrasian labor markets, this finding has also been pointed out by Gillman and Kejak (2005).

17. We would like to thank an anonymous referee for suggesting this analysis to us.

18. We assume that, under both scenarios for the inflation rate, the economy starts with the same initial capital stock, k_0 , at time 0.

19. Gomme (1993) finds a welfare loss of 0.022% following an increase of the annual money growth rate from 0% to 10%.

20. For a more detailed description of these models with the alternative specifications of the labor market, we refer the interested reader to the Technical Appendix that is available from the authors upon request.

REFERENCES

- Avery, R.B., G.E. Elliehausen, A.B. Kennickell, and P.A. Spindt (1987) Changes in the use of transaction accounts and cash from 1984 to 1986. *Federal Reserve Bulletin* 73, 179–196.
- Ball, L. (2001) Another look at long-run money demand. *Journal of Monetary Economics* 47, 31–44.
- Barro, R.J. (2013) Inflation and economic growth. *Annals of Economics and Finance* 14(1), 121–144.
- Barro, R.J. (2001) Economic Growth in East Asia before and after the Financial Crisis. NBER working paper 8330.

- Bruno, M. and W. Easterly (1998) Inflation crises and long-run growth. *Journal of Monetary Economics* 41, 3–26.
- Chari, V.V., L.E. Jones, and R. Manuelli (1996) Inflation, growth, and financial intermediation. *Federal Reserve Bank of St. Louis Review* 78(3), 41–58.
- Christiano, L.J., M. Eichenbaum, and C.L. Evans (2005) Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy* 113, 1–45.
- Christoffel, K., J. Costain, G. de Walque, K. Kuester, T. Linzert, S. Millard, and O. Pierrard (2009a) Inflation Dynamics with Labour Market Matching. European Central Bank working paper 1053.
- Christoffel, K., K. Kuester, and T. Linzert (2009b) The role of labor markets for euro area monetary policy. *European Economic Review* 53(8), 908–936.
- De Gregorio, J. (1993) Inflation, taxation, and long-run growth. *Journal of Monetary Economics* 31(3), 271–298.
- Dotsey, M. and P. Ireland (1996) The welfare cost of inflation in general equilibrium. *Journal of Monetary Economics* 37, 29–47.
- Gillman, M. (1993) The welfare costs of inflation in a cash-in-advance economy with costly credit. *Journal of Monetary Economics* 31(1), 97–115.
- Gillman, M., M.N. Harris, and L. Matyas (2004) Inflation and growth: Explaining a negative effect. *Empirical Economics* 29(1), 149–167.
- Gillman, M. and M. Kejak (2005) Contrasting models of the effect of inflation on growth. *Journal of Economics Surveys* 19, 113–136.
- Gomme, P. (1993) Money and growth revisited: Measuring the costs of inflation in an endogenous growth model. *Journal of Monetary Economics* 32, 51–77.
- Grüner, H.P. and B. Heer (2000) Optimal flat-rate taxes on capital—A re-examination of Lucas' supply side model. *Oxford Economic Papers* 52(2), 289–315.
- Gylfason, T. and T.T. Herbertsson (2001) Does inflation matter for growth? *Japan and the World Economy* 13, 405–428.
- Heer, B. and A. Maußner (2010) Inflation and output dynamics in a model of labor market search and capital accumulation. *Review of Economic Dynamics* 13, 654–686.
- Heer, B., A. Maußner, and P.D. McNelis (2011) The money-age distribution: Empirical facts and the limits of three monetary models. *Journal of Macroeconomics* 33, 390–405.
- Ireland, P.N. (1994) Money and growth: An alternative approach. *American Economic Review* 84(1), 47–65.
- Jha, S.K., P. Wang, and C.K. Yip (2002) Dynamics in transactions-based monetary growth model. *Journal of Economic Dynamics and Control* 26, 611–635.
- Jones, L.E. and R.E. Manuelli (1995) Growth and the effects of inflation. *Journal of Economic Dynamics and Control* 19, 1405–1428.
- Keynes, J.M. (1936) *The General Theory of Employment, Interest, and Money*. London: Macmillan.
- Kormendi, R. and P. Meguire (1985) Macroeconomic determinants of growth: Cross-country evidence. *Journal of Monetary Economics* 16, 141–163.
- Kydland, F.E. and E.C. Prescott (1982) Time to build and aggregate fluctuations. *Econometrica* 50, 1345–1370.
- López-Villavicencio, A. and V. Mignon (2011) On the impact of inflation on output growth: Does the inflation matter? *Journal of Macroeconomics* 33, 455–464.
- Lucas, R.E. (1987) *Models of Business Cycles*. Oxford, UK: Basil Blackwell.
- Lucas, R.E. (1990) Supply-side economics: An analytical review. *Oxford Economic Papers* 42, 293–316.
- Maußner, A. (2004) Endogenous growth with nominal frictions. *Journal of Economics* 83, 1–46.
- McCandless, G. and W. Weber (1995) Some monetary facts. *Federal Reserve Bank of Minneapolis Quarterly Review* 19(3), 2–11.
- Mortensen, D. and C. Pissarides (1994) Job creation and job destruction in the theory of unemployment. *Review of Economic Studies* 61(3), 397–415.

- Peters, M. (1991) Ex ante price offers in matching games: Non-steady state. *Econometrica* 59, 1425–1454.
- Phillips, A.W. (1958) The relation between unemployment and the rate of change of money wage rates in the United Kingdom, 1861–1957. *Economica* 25, 283–299.
- Romer, P.M. (1986) Increasing returns and long-run growth. *Journal of Political Economy* 94, 1002–1038.
- Schreft, S. (1992) Transaction costs and the use of cash and credit. *Economic Theory* 2, 283–296.
- Shi, S. and Q. Wen (1997) Labor market search and capital accumulation: Some analytical results. *Journal of Economic Dynamics and Control* 21(10), 1747–1776.
- Shi, S. and Q. Wen (1999) Labor market search and the dynamic effects of taxes and subsidies. *Journal of Monetary Economics* 43, 457–495.
- Trigari, A. (2006) The Role of Search Frictions and Bargaining for Inflation Dynamics. IGIER working paper 304.
- Trigari, A. (2009) Equilibrium unemployment, job flows and inflation dynamics. *Journal of Money, Credit and Banking* 41(1), 1–33.
- Walsh, C.E. (2005) Labor market search, sticky prices, and interest rate policies. *Review of Economic Dynamics* 8(4), 829–849.
- Walsh, C.E. (2010) *Monetary Theory and Policy*, 3rd ed. Cambridge, MA: MIT Press.
- Wu, Y. and J. Zhang (1998) Endogenous growth and the welfare costs of inflation: A reconsideration. *Journal of Economic Dynamics and Control* 22, 465–482.

APPENDIX

In Table A.1, the effects of an increase in the inflation rate from 0% to 10% are summarized for the three endogenous growth models AK, Lucas, and JM with Walrasian labor markets. In all three models, with the standard CIA constraint on consumption, growth does not increase with higher inflation, whereas for the cash–credit economy, the direction of the inflation effect on growth depends on the magnitude of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$.

TABLE A.1. Summary: Effects of inflation on per-capita income growth in Walrasian labor markets

	CIA	DI	
		$\epsilon_{v,r+\pi} = 5.95$	$\epsilon_{v,r+\pi} = 2.20$
AK model	1.942%	2.014%	1.974%
Lucas (1990)	1.975%	2.006%	1.989%
Jones and Manuelli (1995)	1.930%	2.017%	1.969%

Notes: The columns display the economic growth rate for an inflation rate π equal to 10% in the three endogenous growth models with Walrasian labor markets for different specifications of the cash-in-advance (CIA) constraint. For an inflation rate of 0%, the economic growth rate is equal to 2.00% in all three cases: AK, L, and JM. The first entry column displays the results for the standard CIA constraint. In the second and third columns, under the caption DI, the results are derived in the cash–credit economy for two different values of the semi-interest elasticity of income velocity, $\epsilon_{v,r+\pi}$. Money holdings in the DI case are motivated as in Dotsey and Ireland (1996).