Asset Returns, the Business Cycle, and the Labor Market*

Burkhard Heer[a,b], Alfred Maußner[c]

[a] Free University of Bolzano-Bozen, School of Economics and Management, 1 piazza università, I-39100 Bolzano-Bozen, Italy, Burkhard.Heer@unibz.it

[b] CESifo

[c]Corresponding Author, University of Augsburg, Department of Economics, Universitätsstraße 16, 86159 Augsburg, Germany, alfred.maussner@wiwi.uni-augsburg.de

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Abstract:
We review the labor market implications of recent real business cycle and New Keynesian models that successfully replicate the empirical equity premium. We document the fact that all models reviewed in this paper that do not feature either sticky wages or immobile labor between two production sectors as in Boldrin, Christiano, and Fisher (2001) imply a negative correlation of working hours and output that is not observed empirically. Within the class of Neo-Keynesian models, sticky prices alone are demonstrated to be less successful than rigid nominal wages with respect to the modeling of the labor market stylized facts. In addition, monetary shocks in these models are required to be much more volatile than productivity shocks to match statistics from both the asset and labor market.

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1 Introduction

Mehra and Prescott (1985) estimate an equity premium of 6.18% p.a. for the US over the period 1889-1979 and demonstrate that a general equilibrium model of an exchange economy is unable to replicate this fact unless the representative consumer is implausibly risk averse. This puzzle seriously challenges business cycle research that rests on representative agent stochastic dynamic general equilibrium (DSGE) models so that substantial effort has been made to resolve it. Several review articles[2] document this venture. With respect to models with an exogenously given endowment process modifications of the agents preferences and, more recently, the possibility of rare, but severe crises (Barro (2006)) have been proposed. Kocherlakota (1996) argues that generalized expected utility preferences as proposed by Epstein and Zin (1989) do not resolve the equity premium puzzle in the Mehra and Prescott (1985) data set, while some form of habit formation does. Jermann (1998) demonstrates that habit formation alone is not sufficient to resolve the equity premium puzzle in a production economy. In addition to the consumer being eager to smooth consumption, the adjustment of capital must be costly. However, in his model savings in physical capital is the single vehicle to smooth consumption. Once the consumer is allowed to adjust working hours too, there is second channel to cope with productivity shocks and the equity premium disappears. Subsequent research, thus, has focussed on additional frictions hampering the adjustment of labor.

In this paper we consider the ability of these more recent models to resolve the equity premium puzzle while at the same time being consistent with the stylized facts of business cycles. Our main motivation for this venture is the prominent role played by DSGE models in the analysis of monetary policy and our conviction that models suitable for this purpose should be broadly consistent with both asset and labor market stylized facts.

Many studies document that these facts are relatively stable both across time and countries.[3] For this reason it is more or less a matter of (understandable) taste that we will use German data to gauge the models reviewed below. In Appendix B that is available from the authors upon request, we present the results from redoing the

[2]See, among others, Abel (1991), Kocherlakota (1996), Campbell (2003), and Cochrane (2008).

[3]See, among others, Ambler, Clarida, and Zimmermann (2004), Backus and Kehoe (1992), Brandner and Neusser (1992), Basu and Taylor (1999), Hodrick and Prescott (1997), and Maußner (1994), for a survey of these facts.

analysis presented below with parameter values and benchmark business cycle facts related to the US economy to verify this claim.

In our study we consider several ways to introduce frictions in the allocation of labor. Several authors have proposed a habit in leisure which serves as a short-cut to the modeling of either adjustment costs of labor or search frictions in the labor market. Bouakez and Kano (2006) argue that habit formation in leisure fits the US data better with regard to the persistence and propagation of shocks than other standard real-business-cycle models, in particular those allowing for learning-by-doing such as Chang, Gomes, and Schorfheide (2002). Lettau and Uhlig (2000), however, argue that, with habit formation in leisure, labor input is too smooth over the cycle and output and hours are negatively correlated, which is clearly at odds with the stylized facts of the business cycle. Uhlig (2007) combines habits in consumption and leisure with sticky real wages as proposed by Blanchard and Galí (2005). With a considerable degree of real wage stickiness his model is able to produce a sizable equity premium and a positive correlation of hours and output.

In the two sector model of Boldrin, Christiano, and Fisher (2001) (BCF for short), it is not possible to reallocate labor from the consumption goods sector to the investment goods sector after the observation of the shock. Accordingly, the equity premium results from variations in the relative price of the two goods rather than from variations in the firm's value. This model reproduces the positive output-hours correlation found in the data, but fails to predict a positive correlation between the real wage and working hours.

Most studies of the equity premium and asset prices are constrained to the analysis of the real economy that is subject to a technology shock. As one of the very few exceptions, De Paoli, Scott and Weeken (2010) examine the behavior of asset prices in a New Keynesian model with sticky prices. They find that the effect of nominal rigidities on the risk premium depends on the nature of the shock. While the risk premium is reduced, if cycles are driven by technology shocks, it increases in the case of monetary shocks.

In addition to these models we study a model with sticky nominal wages and a model with both sticky nominal prices and wages.

Our results are summarized in Table 1.1. The first column displays the names of the models that we consider in the following sections. The first row presents the

Table 1.1

Summary of Results

	Equity premium	s_Y	s_I/s_Y	s_N/s_Y	s_w/s_Y	r_{YN}	r_{wN}	Score
Data								
	5.18	1.14	2.28	0.69	1.03	0.40	0.27	
Models								
2. Real Business Cycle Models								
Benchmark								
Exogenous labor	5.18	0.90	2.28					
Endogenous labor	0.52	0.51	1.47	1.27	2.08	−0.68	−0.94	26.43
Habit in leisure	5.25	0.65	2.22	0.56	1.53	−0.91	−0.96	3.52
Predetermined hours								
Firms	0.08	0.86	2.19	0.10	1.71	−0.62	−0.25	28.18
Households	5.23	0.78	2.26	0.37	1.23	−0.50	−0.73	1.91
Sticky real wages	5.58	1.36	2.34	0.59	0.61	0.82	0.38	0.54
Two sector models								
Stationary growth	4.77	0.95	2.66	0.13	3.28	0.72	−0.03	5.88
Integrated growth	4.71	0.95	1.55	0.08	3.40	0.73	−0.08	6.99
Adjustment costs	4.58	0.92	2.07	0.07	3.29	0.69	0.00	6.04
3. New Keynesian Models								
Sticky prices	0.43	0.54	1.99	1.06	1.93	−0.76	−0.94	26.38
Sticky wages	5.20	0.98	2.43	1.38	1.14	0.57	−0.69	1.47
Sticky prices and wages	5.05	2.05	2.19	1.40	0.52	0.90	0.66	1.19

Notes: s_x:=Standard deviation of time series x, where $x \in \{Y, I, N, w\}$ and Y, I, and N denote output, investment, hours, and the wage, respectively. Empirical as well as model generated time series were HP-filtered with weight 1600. The empirical moments relate to per capita magnitudes, except for the real wage which was measured as hourly worker compensation. s_x/s_y:=standard deviation of variable x relative to standard deviation of output y. r_{NY}:=Cross-correlation of variable hours with output, r_{wN}:=Cross-correlation of the real wage with hours. The column Score presents the sum of squared differences between the moments from simulations of the model and the moments from the data.

empirical values in Germany that we aim to match.[4] Among the real business cycle models considered in Section 2 the model by Uhlig (2007) comes closest to the empirical moments. This model features slowly adjusting external habits in both consumption

[4]Except for the equity premium, the second moments reported in Table 1.1 are taken from Heer and Maußner (2008), Table 1.2, p. 56. The estimate of the German equity premium during 1900-2002 of 5.18 is from Kyriacou, Madsen, and Mase (2004).

and leisure and sticky real wages. The two-sector models in the spirit of Boldrin, Christiano, and Fisher (2001), where the reallocation of labor between sectors within the current period is impossible, are less successful in this endeavor. In the class of New Keynesian models with nominal frictions our model with sticky prices and wages performs best. Its score is only slightly worse than the score of the Uhlig (2007) model. We find that sticky prices alone are less important than rigid wages for the modeling of the asset and labor market statistics. In addition, we need a sizeable monetary shock in the nominal models in order replicate empirical regularities.

The paper is organized as follows. In the next section we consider real models of the business cycle. We first present the Jermann (1998) model as a benchmark case to which we add one model element after the other. In Subsection 2.2, we show that the equity premium disappears once labor is supplied elastically. In the following subsections we consider habits in consumption and working hours, hours which must be determined before the productivity shock is revealed to either the firm or the household, sticky real wages, and frictions in the allocation of labor between sectors. Section 3 studies models with nominal rigidities. We start with the New Keynesian model of de Paoli, Scott, and Weeken (2010) and show that this model is unable to replicate several labor market statistics. In Sections 3.2 and 3.3, we demonstrate that our model with rigid wages performs much better. All equilibrium conditions and derivations of the individual models are presented in an Appendix that is available from the authors upon request.

2 Real Business Cycle Models

2.1 The Benchmark Model

2.1.1 The Model

The first model that we consider is the asset pricing model of Jermann (1998).[5] We follow the description of this model in Heer and Maußner (2009). Time is discrete and

[5]In Appendix A.3 we consider the time to plan model of Christiano and Todd (1996) as an alternative to the adjustment costs of capital approach employed by Jermann (1998). The model is also able to generate the equity premium observed in the data, if labor supply is exogenous. As in the Jermann model, the equity premium falls close to zero, if labor supply is endogenous. In a separate paper, we will consider extensions of this model similar to those presented for the Jermann model here.

denoted by t.

Households. A representative household supplies labor in a fixed amount of $N_t \equiv N$ at the real wage w_t. Besides labor income he receives dividends d_t per unit of share S_t he holds of the representative firm. The current price of shares in units of the consumption good is v_t. His current period utility function u depends on current and past consumption, C_t and C_{t-1}, respectively. Given his initial stock of shares S_t, the households maximizes

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} \right\}, \quad \eta \geq 0,\ \chi^N \in [0,1),\ \beta \in (0,1)$$

subject to the sequence of budget constraints

$$v_t(S_{t+1} - S_t) \leq w_t N_t + d_t S_t - C_t. \tag{2.1}$$

The operator $\mathbb{E}_t$ denotes mathematical expectations with respect to information as of period t. The first-order conditions of this problem are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta\chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{2.2a}$$

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} R_{t+1}, \tag{2.2b}$$

$$R_t := \frac{d_t + v_t}{v_{t-1}}, \tag{2.2c}$$

where Λ_t is the Lagrange multiplier of the budget constraint.

Firms. The representative firm uses labor N_t and capital K_t to produce output Y_t according to the production function

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \quad \alpha \in (0,1). \tag{2.3}$$

The level of total factor productivity Z_t is governed by the AR(1)-Process

$$\ln Z_t = \rho^Z \ln Z_{t-1} + \epsilon_t^Z, \quad \epsilon_t^Z \sim N\left(0, (\sigma^Z)^2\right). \tag{2.4}$$

The firm finances part of its investment I_t from retained earnings RE_t and issues new shares to cover the remaining part:

$$I_t = v_t(S_{t+1} - S_t) + RE_t. \tag{2.5}$$

It distributes the excess of its profits over retained earnings to the household sector:

$$d_t S_t = Y_t - w_t N_t - RE_t. \tag{2.6}$$

Investment increases the firm's future stock of capital according to:

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \quad \delta \in [0,1], \tag{2.7}$$

where we parameterize the function Φ as

$$\Phi(I_t/K_t) := \frac{a_1}{1-\zeta}\left(\frac{I_t}{K_t}\right)^{1-\zeta} + a_2, \quad \zeta > 0. \tag{2.8}$$

The firm's ex-dividend value at the end of the current period t, V_t, equals the number of outstanding stocks S_{t+1} times the current stock price v_t. This definition implies:

$$\begin{aligned} V_t = v_t S_{t+1} &\overset{(2.5)}{=} I_t + v_t S_t - RE_t \overset{(2.6)}{=} I_t + w_t N_t - Y_t + (v_t + d_t)S_t, \\ &\overset{(2.2c)}{=} I_t + w_t N_t - Y_t + R_t V_{t-1}. \end{aligned}$$

Rearranging and taking expectations as of period t, yields

$$V_t = \mathbb{E}_t \left\{ \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + V_{t+1}}{R_{t+1}} \right\}.$$

Iterating on this equation using the law of iterated expectations and assuming

$$\lim_{s\to\infty} \mathbb{E}_t \left\{ \frac{V_{t+s}}{R_{t+1}R_{t+2}\dots R_{t+s}} \right\} = 0$$

establishes that the end-of-period value of the firm equals the discounted sum of its future cash flows $CF_{t+s} = Y_{t+s} - w_{t+s}N_{t+s} - I_{t+s}$:

$$V_t = \mathbb{E}_t \sum_{s=1}^{\infty} \varrho_{t+s} CF_{t+s}, \quad \varrho_{t+s} = \frac{1}{R_{t+1}R_{t+2}\dots R_{t+s}} \tag{2.9}$$

The firm's objective is to maximize its beginning-of-period value, which equals $V_t^{bop} = V_t + CF_t$. Defining $\varrho_t = 1$ allows us to write

$$V_t^{bop} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} CF_{t+s}. \tag{2.10}$$

The first-order conditions for maximizing (2.10) subject to (2.7) are:

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{2.11a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{2.11b}$$

$$q_t \varrho_t = \mathbb{E}_t \varrho_{t+1} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big] \Big\}. \tag{2.11c}$$

In addition, the transversality condition

$$\lim_{s\to\infty} \mathbb{E}_t \varrho_{t+s} q_{t+s} K_{t+s+1} = 0 \tag{2.11d}$$

must hold.

Market Equilibrium. Using equations (2.5) and (2.6), the household's budget constraint implies the economy's resource restriction:

$$Y_t = C_t + I_t. \tag{2.12}$$

In equilibrium, the labor market clears at the wage w_t so that $N_t = 1$ for all t. Furthermore, using (2.2b), ϱ_{t+1} can be replaced by $\beta\Lambda_{t+1}/\Lambda_t$ so that at any date t the set of equations

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{2.13a}$$

$$Y_t = Z_t K_t^{\alpha}, \tag{2.13b}$$

$$Y_t = C_t + I_t, \tag{2.13c}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta b \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{2.13d}$$

$$q_t = \beta\mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t}\Big\{\alpha Z_{t+1} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta\big]\Big\} \tag{2.13e}$$

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t, \tag{2.13f}$$

determines $(Y_t, C_t, I_t, K_{t+1}, \Lambda_{t+1}, q_{t+1})$ given (K_t, Λ_t, q_t).

Deterministic Stationary Equilibrium. Since our solution strategy rests on a second order approximation of the model, we must consider the stationary equilibrium of the deterministic counterpart of our model that we get, if we put $\sigma^Z = 0$ so that Z_t equals its unconditional expectation $Z = 1$ for all t. In this case we can ignore the expectations operator $\mathbb{E}_t$. Stationarity implies $x_{t+1} = x_t = x$ for any variable in our model. As usual, we specify Φ so that adjustment costs play no role in the stationary equilibrium, i.e., $\Phi(I/K)K = \delta K$ and $q = \Phi'(\delta) = 1$. This requires that we choose

$$a_1 = \delta^{\zeta},$$

$$a_2 = \frac{-\zeta\delta}{1-\zeta}.$$

These assumptions imply via equation (2.13e) the stationary solution for the stock of capital:

$$K = \left(\frac{1 - \beta(1 - \delta)}{\alpha\beta} \right)^{\frac{1}{\alpha - 1}}. \tag{2.14a}$$

Output, investment, consumption, and the stationary solution for Λ are then given by

$$Y = K^{\alpha}, \tag{2.14b}$$

$$I = \delta K, \tag{2.14c}$$

$$C = Y - I, \tag{2.14d}$$

$$\Lambda = C^{-\eta}(1 - \chi^C)^{-\eta}(1 - \chi^C\beta). \tag{2.14e}$$

2.1.2 Calibration and the Equity Premium

Calibration. We calibrate this and the other models considered here in a two-step procedure. In the first step we choose the parameters for which there is direct or (via the models equilibrium conditions) indirect empirical evidence or that are usually set by researchers to some preferred value. In the second step we set the remaining free parameters so that the respective model best fits certain empirical targets. For the first step we employ seasonally adjusted quarterly data for the West German economy over the period 1975.i through 1989.iv. The parameter settings are taken from Heer and Maußner (2009), Section 6.3.4. Table 2.1 displays the respective values.[6] Notice that the wage share in the German data, $1 - \alpha = 0.73$, is larger than the value of 0.64 that is often found in comparable studies relying upon US data,[7] while the depreciation rate, $\delta = 0.011$, is much smaller and amounts to approximately half the US value. In addition, $N = 0.13$ is chosen to match the average quarterly fraction of hours spent on work by the typical German household. Notice that many studies set $N = 1/3$ arguing that the typical worker spends 8 hours per day on the job (see, for example, Hansen (1985)). We consider the typical household to be an average over the total population including children and retired persons rather than consisting of a single worker who is also working on the weekend and does not take any vacation. The discount factor $\beta = 0.994$ yields an annual risk free rate in the simulation of the model of about 1 percent. We choose the unobserved parameters χ^C and ζ to

[6] For future reference it also presents parameters that will be introduced below.

[7] See, for example, King, Plosser, and Rebelo (1988) and Plosser (1989).

match two statistics: the relative volatility of investment expenditures and the equity premium. The former, measured as the standard deviation of the cyclical component of investment expenditures relative to the standard deviation of the cyclical component of GDP, is 2.28 in our data set. The latter equals 5.18 according to a recent study by Kyriacou, Madsen, and Mase (2004) covering the period 1900-2002 (see footnote 4). The solution of this problem is $\chi^C = 0.793$ and $\zeta = 5.53$.

Table 2.1

Benchmark calibration

Preferences	β=0.994	η=2	τ=0.20	N=0.13
Production	α=0.27	δ=0.011		
Stationary Shocks	ρ^Z=0.90	σ^Z=0.0072		
Integrated Shocks	$\ln \bar{z}$=0.006	$\sigma_{\ln z}$=0.0101		

Computation of the Equity Premium. The solution of the model are functions g^i, $i \in \{K, Y, C, I, \Lambda, q\}$, that determine K_{t+1}, Y_t, C_t, I_t, Λ_t, and q_t given the current period state variables K_t, C_{t-1}, and the log of the productivity shock $\ln Z_t$.

In our model the gross risk free rate of return r_t is given by

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}}. \tag{2.15}$$

Since

$$\begin{aligned}
\Lambda_{t+1} &= g^\Lambda(K_{t+1}, C_t, \ln Z_{t+1}) \\
&= g^\Lambda(g^K(K_t, C_{t-1}, \ln Z_t), g^C(K_t, C_{t-1}, \ln Z_t), \rho^Z \ln Z_t + \epsilon^Z_{t+1}) \\
&=: \tilde{g}^\Lambda(K_t, C_{t-1}, \rho^Z \ln Z_t + \epsilon^Z_{t+1},)
\end{aligned}$$

and ϵ^Z_{t+1} is normally distributed, the expected value of the Lagrange multiplier equals

$$\mathbb{E}_t \Lambda_{t+1} = \int_{-\infty}^{\infty} \tilde{g}^\Lambda(K_t, C_{t-1}, \rho^Z \ln Z_t + \epsilon^Z_{t+1},) \frac{1}{\sigma^Z \sqrt{2\pi}} e^{\frac{-(\epsilon^Z_{t+1})^2}{(\sigma^Z)^2}} d\epsilon^Z_{t+1}.$$

We use the quadratic approximation of g^Λ at the stationary equilibrium and the Gauss-Hermite 6-point quadrature formula to approximate the integral on the right-hand-side of this equation.

The labor market equilibrium condition (2.11a) and equation (2.7) imply that the right-hand-side of (2.11c) can be written as

$$\begin{aligned}1 &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \\ &= \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{t+1} + v_{t+1}}{v_t} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} R_{t+1}\end{aligned},$$

where the second equality follows from equations (2.5) and (2.6) and the observation that $q_t K_{t+1} = v_t S_{t+1}$ (see Appendix A.1). Therefore, the gross rate of return on the shares of the representative firm equals[8]

$$R_{t+1} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}. \tag{2.16}$$

We use the quadratic approximations of g^i and a random number generator to compute a long artificial time series for $R_{t+1} - r_t$. The average of this time series is our measure of the ex-post equity premium implied by the model.

We compute the equity premium from a time series of 1,000,000 observations and the second moments of simulated time series from averages over 300 simulations with 80 observations. As our empirical data we pass the artificial time series through the Hodrick-Prescott filter with weight 1600. As noted above, using the parameters in Table 2.1 and a pseudo random number generator, this yields an equity premium of 5.18 and a relative standard deviation of investment of 2.28.[9]

2.2 Endogenous Labor Supply

In this section, we introduce flexible labor in the model of the previous subsection. As a consequence, the equity premium drops from 5.18 to 0.52 percent (see Table 1.1).

The Model. Let

$$\begin{aligned}U_t &\equiv \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} N_{t+s}^{1+\nu_1} \right\}, \\ &\quad \beta \in (0,1),\ \chi^C \in [0,1),\ \eta, \nu_0, \nu_1 \geq 0\end{aligned} \tag{2.17}$$

[8] Note, $\alpha Y_{t+1} = Y_{t+1} - w_{t+1}N_{t+1}$.

[9] The Fortran computer programs are available from Alfred Maußner on request. The solution algorithm is the same as in Heer and Maußner (2009), Chapter 2. The respective code is available from `http://www.wiwi.uni-augsburg.de/vwl/maussner/dgebook/download3.html`.

denote the household's expected life-time utility. Maximizing this expression subject to the budget constraint (2.1) implies the first-order condition:

$$\nu_0 N_t^{\nu_1} = \Lambda_t w_t \tag{2.18}$$

in addition to equations (2.2). The model's dynamics consists of equations (2.18), (2.11a), (2.11b), (2.3), the resource constraint, (2.2a), (2.11c), and (2.7). The equilibrium conditions for this and the following models are summarized in Appendix A.2 that is available from the authors upon request.

We follow Heer and Maußner (2008) and calibrate ν_1 so that the implied Frisch elasticity of labor supply τ equals 0.20.

Equity Premium. In this model the ex post gross return on the firm's shares equals

$$R_{t+1} = \frac{Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \tag{2.19}$$

since

$$Y_{t+1} - w_{t+1}N_{t+1} = \alpha Z_{t+1}(K_{t+1}/N_{t+1})^{\alpha}$$

due to the labor market clearing condition (2.11a).

Using the same sequence of random numbers as in Section 2.1, we find an average risk free rate of return of 2.18 percent p.a. and an equity premium of 0.52 percent p.a. Evidently, the size of the equity premium depends critically on the variability of working hours over the business cycle. Besides the small premium the model has two other deficiencies: hours and output as well as hours and the real wage are negatively correlated (see Table 1.1), which is clearly at odds with the empirical evidence provided in the first row of entries in Table 1.1.

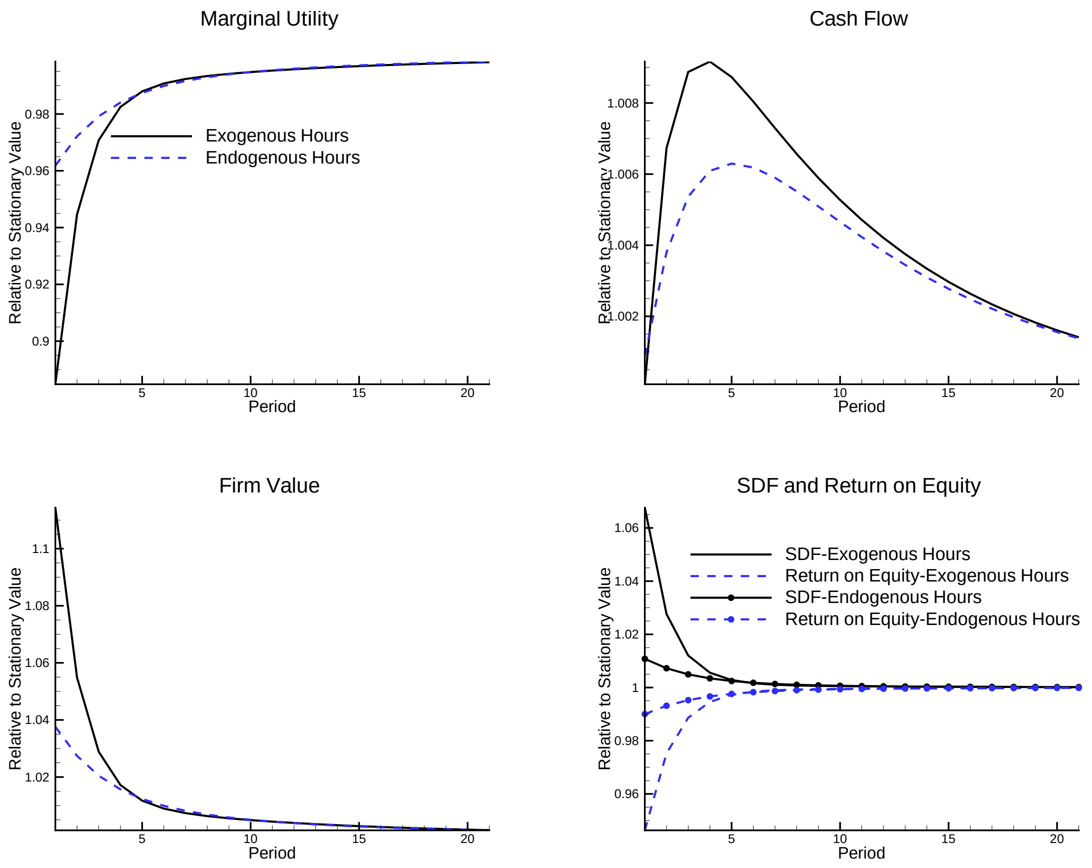
To understand this it will be helpful to recall a well-known asset-pricing formula (see Appendix A.1):

$$\mathbb{E}(R_{t+1}) - r_t^f = -r_t^f \operatorname{cov}(M_{t+1}, R_{t+1}), \tag{2.20}$$

where $M_{t+1} \equiv \beta\Lambda_{t+1}/\Lambda_t$ is the household's stochastic discount factor (SDF). According to this relation, the size of the equity premium on the left-hand side depends on the size of the covariance between the SDF and the equity return R_{t+1}.

Consider a positive, autocorrelated productivity shock. For a while, the firm's business prospects will be better than average as can be seen from the impulse response of the cash-flow in Figure 2.1. Consequently, the firm wants to increase its capital stock. The current price of capital increases and returns slowly to its stationary value. This price effect dominates the effect on the firm's cash flow so that the future return to capital for the firm's shareholders R_{t+1} falls below its average value (see the lower right panel in Figure 2.1). Since the shock raises the household's labor income, current and future consumption increase so that the marginal utility of consumption falls below average and returns slowly to its pre-shock value. Therefore, the stochastic discount factor increases. The opposite movement of the return on equity and the stochastic discount factor generates the negative correlation that accounts for the equity premium according to equation (2.20).

Figure 2.1: Stochastic Discount Factor and Return on Equity



Endogenous labor supply dampens the effect of a productivity shock on both the SDF and the return on equity, and, thus, reduces the equity premium (see the lower right panel of Figure (2.1)). In response to the shock, the household adjusts his income via changes in working hours so that the marginal utility of consumption declines by far less than in the case of a given supply of hours. The effect on hours is the result of two opposing forces: the household wants to supply more labor, since real wages are higher than on average. However, the income effect increases both the demand for consumption and for leisure. The negative effect on hours is reinforced by adjustment costs of investment, which make it expensive to transfer additional labor income into future consumption. Therefore, hours decrease despite higher real wages and dampen the effect of the productivity shock on the firm's cash flow.

2.3 Habit Formation in Leisure

Lettau and Uhlig (2000) introduce habit formation in both consumption and leisure in the standard real business cycle model in order to study the implications for the optimal responses of output, consumption, labor input, and investment to exogenous shocks. Different from our model, they do not allow for capital adjustment costs. Consequently, the equity premium falls close to zero in their model. In the following, we introduce habit in leisure in the above model explicitly allowing for capital adjustment costs. We show that though we are able to produce an equity premium close to the empirical value, the model predicts a strong negative correlation between output and hours and between hours and the real wage.

The Model. With habit in leisure, the household expected life-time utility is given by[10]

$$\begin{aligned} U_t \equiv \mathbb{E}_t \sum_{s=0}^{\infty} \left\{ \frac{(C_{t+s} - \chi^C C_{t+s-1})^{1-\eta} - 1}{1-\eta} - \nu_0 \frac{(N_{t+s} - \chi^N N_{t+s-1})^{1+\nu_1}}{1+\nu_1} \right\}, \\ \eta,\, \nu_0, \nu_1 \geq 0,\, \chi^C, \chi^N \in [0,1). \end{aligned} \tag{2.21}$$

Maximizing (2.21) subject to (2.1) implies the first-order condition

$$\nu_0 (N_t - \chi^N N_{t-1})^{\nu_1} - \beta \nu_0 \chi^N \mathbb{E}_t (N_{t+1} - \chi^N N_t)^{\nu_1} = \Lambda_t w_t \tag{2.22}$$

[10] The exact utility function used by Lettau and Uhlig (2000) differs from ours. They specify the utility as a function of leisure, $1 - N_t$. Bouakez and Kano (2006) use the fraction of labor and the habit stock rather than the first difference.

in addition to equations (2.2). The model's dynamics consists of equations (2.22), (2.11a), (2.11b), (2.3), the resource constraint, (2.2a), (2.11c), and (2.7). The equity premium is computed from (2.19).

Calibration and Results. The model has three unobserved parameters, χ^C, χ^N, and ζ. We searched over a coarse grid with end-points $\chi^C \in [0.1, 0.97]$, $\chi^N \in [0.1, 0.97]$, and $\zeta \in [1.5, 9.0]$, respectively, and selected that combination of parameters that minimized the sum of squared deviations between the empirical moments and those implied by the model. The parameter values obtained from this procedure are $\chi^C = 0.81$, $\chi^N = 0.97$, and $\zeta = 9.0$. The respective moments are presented in Table 1.1. Though the model is able to replicate the equity premium, it performs worse than the model without a habit in labor with regard to the implied correlations between output and hours and between hours and the real wage. Both are negative and (in absolute value) greater than 0.9, and, thus, are strongly at odds with the empirical evidence.

2.4 Predetermined Working Hours

In this subsection we follow Boldrin, Christiano, and Fisher (2001) (BCF) and consider frictions in the allocation of labor. In particular, we assume that working hours are determined before the productivity shock is revealed. We study the question if it makes a difference whether 1) the firm's labor demand or 2) the household's labor supply is predetermined.[11] We show that the distinction mainly concerns the business cycle properties of the real wage, which is much more volatile if firms decide on hours. Therefore, this version fits the facts far less than the model with hours determined by the household.

The Model. In version one, maximizing (2.10) with respect to N_{t+1} yields the first-order condition

$$0 = \mathbb{E}_t \Lambda_{t+1} \left((1-\alpha) Z_{t+1} N_{t+1}^{-\alpha} K_{t+1}^{\alpha} - w_{t+1} \right), \tag{2.23}$$

which replaces (2.11a). Note, however, that equation (2.23) no longer implies

$$R_{t+1} = \frac{d_{t+1} + v_{t+1}}{v_t} = \frac{Y_{t+1} - w_{t+1} N_{t+1} - I_{t+1} + q_{t+1} K_{t+2}}{q_t K_{t+1}},$$

[11] BCF assume that firms must determine labor demand prior to the technology shock.

since $\alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} \neq Y_{t+1} - w_{t+1} K_{t+1}$. Therefore, we assume that the firm uses internal funds only to finance investment. This allows us to employ (2.2c) to compute the return on equity from

$$R_{t+1} = (v_{t+1} + d_{t+1})/v_t. \tag{2.24}$$

In version two, maximizing (2.17) subject to (2.1) with respect to N_{t+1} yields the first-order condition

$$0 = \mathbb{E}_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{t+1} \right\} \tag{2.25}$$

that replaces (2.18), whereas (2.11a) reflects the firm's labor demand schedule. Besides, the model is the same as in Section 2.2.

Table 2.2

Second Moments from the Model with Predetermined Hours

Variable	s_x	s_x/s_Y	r_{xY}	r_{xN}	r_x
Version One: Hours Predetermined by Firms					
Output	0.78	1.00	1.00	−0.50	0.50
Consumption	0.70	0.90	0.94	−0.72	0.75
Investment	1.75	2.26	0.76	0.07	0.00
Real Wage	14.44	18.63	0.69	0.17	−0.06
Hours	0.28	0.37	−0.50	1.00	0.51
Version Two: Hours Predetermined by Households					
Output	0.78	1.00	1.00	−0.50	0.50
Consumption	0.70	0.90	0.94	−0.72	0.75
Investment	1.75	2.26	0.76	0.07	0.00
Real Wage	0.95	1.23	0.97	−0.71	0.69
Hours	0.28	0.37	−0.50	1.00	0.51

Notes: s_x:=Standard deviation of HP-filtered simulated time series x, where x stands for any of the variables in column 1. s_x/s_Y:=standard deviation of variable x relative to standard deviation of output Y. r_{xY}:=Cross-correlation of variable x with output Y. r_{xN}:=Cross-correlation of variable x with hours N , r_x:=First order autocorrelation of variable x.

Calibration and Results. Both versions of the model have two unobserved parameters, χ^C and ζ, which we choose so that the score statistic reported in Table 1.1 is minimized on a grid with endpoints $\chi^C \in [0.1, 0.95]$ and $\zeta \in [0.8, 9]$, respectively. For the model where households predetermine hours, the minimizer is $\chi^C = 0.78$ and $\zeta = 8$ with a score of 1.91 (see Table 1.1).

Table 2.2 presents second moments from simulations of both model versions for the same parameter values.[12] They are averages over 300 simulations of sample size 80. Except for the time series properties of the real wage both models have virtually the same implications for the second moments shown in the table. The annual equity premium is 4.99 in the first version of the model and 5.23 percent in the second version. The real wage is much more volatile in the first version. For this reason, the model fits the facts less well: the moments presented in Table 2.2 imply a score of almost 311. Even the most favorable choice of parameters, $\chi^C = 0.28$ and $\zeta = 0.8$, yields a score of 28 (see Table 1.1 for details). Thus, version two, where households predetermine hours, clearly outperforms version one. However, as in the models considered before, the version implies negative correlations between output and hours and between hours and the real wage.

2.5 Real Wage Stickiness

Uhlig (2007) adds sticky real wages to a model similar to that considered in Subsection 2.3. The main differences concern the slow adjustment of the habits, the non-separability between consumption and leisure in the utility function, and an integrated technology shock.

The Model. The representative household solves

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{[(C_{t+s} - C_{t+s}^h)(A + (1 - N_{t+s} - L_t^h)^\nu]^{1-\eta} - 1}{1-\eta},$$

subject to,

$$V_t(S_{t+1} - S_t) \leq W_t N_t + D_t S_t - C_t,$$

where V_t, S_t, W_t, and D_t denote the price of the firm's shares, the number of shares, the real wage at which the household will supply labor, and dividends per share, respectively. The habits in consumption C_t^h and in leisure L_t^h are exogenous to the household

[12]The parameters other than χ^C and ζ are set to the values given in Table 2.1.

and evolve according to

$$C_t^h = \bar{z}\left[(1-\lambda^C)\chi^C C_{t-1} + \lambda^C C_{t-1}^h\right], \tag{2.26a}$$

$$L_t^h = (1-\lambda^L)\chi^L(1-N_{t-1}) + \lambda^L L_{t-1}^h. \tag{2.26b}$$

The firm's production function allows for stochastic growth in labor-augmenting technical progress and reads

$$Y_t = B(Z_t N_t)^{1-\alpha} K_t^\alpha, \tag{2.27a}$$

$$z_t \equiv \frac{Z_t}{Z_{t-1}}, \tag{2.27b}$$

$$\ln z_t = \ln \bar{z} + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2). \tag{2.27c}$$

Let W_t^f denote the marginal rate of substitution between consumption and hours. Uhlig (2007) assumes that the real wage adjusts according to

$$W_t = (\bar{z} W_{t-1})^\mu (W_t^f)^{1-\mu}. \tag{2.28}$$

Calibration and Results. The model, which we fully describe in Appendix A.5, has six unobserved parameters, χ^C, χ^L, λ^C, λ^L, μ, and ζ. The parameters α, β, δ, and η, are set to the values given in Table 2.1. We use the parameter B to normalize the marginal utility of consumption equal to one in the stationary equilibrium of the deterministic counterpart of the model. The value of ν follows from the Frisch elasticity of labor supply $\tau = 0.2$ (see Table 2.1), and the parameter A in the utility function is a function of other parameters (see Appendix A.5)[13]. We equate $\bar{z}$ with the average growth rate of GDP and compute Z_t from actual data on output, hours, and the capital stock. Our measure of σ is the standard deviation of the growth rate of Z_t. Our estimates are $\bar{z} = 0.006$ and $\sigma = 0.0101$.

With more free parameters, it should come as no surprise that we will get a better score statistic. Indeed, our search over a coarse grid with endpoints $\chi^C \in [0.5, 0.95]$, $\chi^L \in [0.5, 0.96]$, $\lambda^C \in [0.01, 0.90]$, $\lambda^L \in [0.01, 0.9]$, $\zeta \in [0.2, 4.0]$, and $\mu \in [0.01, 0.95]$

[13] Uhlig (2007) employs additional restrictions and also searches over the values of β and η. He uses the log-linearized model to compute second moments and the Sharpe ratio. We, instead, use a second-order approximate solution and compute the equity premium as explained in Appendix A.5. We would like to thank Harald Uhlig for providing us with his MatLab code so that we were able to figure out the differences between his and our computational approach.

produced a score of 0.54. The respective parameters point at strong habits in both consumption and leisure, $\chi^C = \chi^L = 0.90$, with more persistence in the adjustment of the leisure than of the consumption habit, $\lambda^L = 0.70$ and $\lambda^C = 0.45$, and at a very high degree of real-wage stickiness, $\mu = 0.95$. Notably, the correlation between output and hours as well as between hours and the real wage are positive so that the model is in good accordance with both the empirical equity premia (5.58% p.a. from the model versus 5.18% in the data) and the stylized facts of the labor market (see Table 1.1 for the details).

2.6 Two-Sector Models

In this section, we consider the two sector model of Boldrin, Christiano, and Fisher (2001). As a distinctive feature of their model, investment goods are produced in a separate production sector and the mobility of labor between this sector and the sector producing the consumption good is limited. In particular, the household must choose his supply of labor to both sectors before the productivity shock is revealed. Therefore, the price of the investment good is volatile and generates a sizeable equity premium. We study the sensitivity of their model with respect to the assumption on the technology process. In the following, we first consider the case that the (natural) logarithm of total factor productivity $\ln Z_t$ follows the AR(1) given in equation (2.4). Subsequently, we compare our results to the case studied in BCF (2001) where labor augmenting technical progress is driven by a random walk with drift.

2.6.1 Stationary Technology Shocks

The Model. Consumption goods C_t are produced according to the technology

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0,1) \tag{2.29a}$$

where N_{Ct} and K_{Ct} denote labor and capital employed in this sector. The investment goods sector (subscript I) uses the same technology so that

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha} \tag{2.29b}$$

is the amount of investment goods I_t which sell at the relative price p_t. Total labor and capital in the economy equal

$$N_t = N_{Ct} + N_{It}, \tag{2.30a}$$

$$K_t = K_{Ct} + K_{It}. \tag{2.30b}$$

The first-oder conditions with respect to labor demand of both sectors are:

$$w_t = (1-\alpha) Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{2.31a}$$

$$w_t = p_t (1-\alpha) Z_t N_{It}^{-\alpha} K_{It}^{\alpha}. \tag{2.31b}$$

Both sectors rent capital services from the household at the rates r_{Ct} and r_{It}, respectively, so that equilibrium in the respective markets implies:

$$r_{Ct} = \alpha Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{2.32a}$$

$$r_{It} = p_t \alpha Z_t N_{It}^{1-\alpha} K_{It}^{\alpha-1}. \tag{2.32b}$$

The representative household maximizes the same intertemporal utility function (2.17) as in the previous section. Since ex ante the wages in both sectors may differ from each other as do the rental rates of capital, his budget constraint is

$$0 \leq w_{Ct} N_{Ct} + w_{It} N_{It} + r_{Ct} K_{Ct} + r_{It} K_{It} + \Pi_{Ct} + \Pi_{It} - C_t - p_t I_t, \tag{2.33}$$

where w_{Ct} and w_{It} denote the real wage paid in the consumption and the investment goods sector, respectively. Maximizing (2.17) subject to (2.33) and the law of motion for the aggregate capital stock

$$K_{t+1} = I_t + (1-\delta) K_t, \tag{2.34}$$

implies

$$0 = E_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{Ct+1} \right\}, \tag{2.35a}$$

$$0 = E_t \left\{ \nu_0 N_{t+1}^{\nu_1} - \Lambda_{t+1} w_{It+1} \right\}, \tag{2.35b}$$

$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{Ct+1} \right), \tag{2.35c}$$

$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{It+1} \right) \tag{2.35d}$$

in addition to (2.2a) and (2.18).

In equilibrium the budget constraint implies the resource restriction $Y_t = C_t + p_t I_t$. BCF argue that the measure of real output in the national income and product accounts is output at constant prices. They choose the base period price $p = 1$, the relative price of investment goods in the stationary equilibrium of the deterministic version of the model, and compute output as $Y_t = C_t + I_t$. The dynamics of the model is, thus, determined by (2.29)-(2.32), (2.34), (2.35) as well as (2.2a) and (2.18).

Equity Premium. The household's first-order conditions (2.35) imply that the gross rate of return on investment in sector C or I are given by:

$$R_{Ct+1} = \frac{p_{t+1}(1-\delta) + r_{Ct+1}}{p_t} = \frac{p_{t+1}(1-\delta) + \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha}}{p_t}, \tag{2.36a}$$

$$R_{It+1} = \frac{p_{t+1}(1-\delta) + r_{It+1}}{p_t} = \frac{p_{t+1}(1-\delta) + \alpha p_{t+1} Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{\alpha}}{p_t}. \tag{2.36b}$$

BCF compute the average gross rate of return on equity from

$$R_{t+1} = \frac{K_{Ct+1}}{K_{t+1}} R_{Ct+1} + \frac{K_{It+1}}{K_{t+1}} R_{It+1}. \tag{2.36c}$$

The risk free rate of return is the same expression as in the previous models, namely

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}}.$$

We compute the ex-post average equity premium from the time series average of $R_{t+1} - r_t$.

Calibration and Results. The model has just one unobserved parameter χ^C. The search within $\chi^C \in [0.01, 0.95]$ provided $\chi^C = 0.70$ as the minimizer of our score statistic. The model predicts an equity premium of 4.77% p.a., a positive correlation between output and hours of 0.72, which is larger than the empirical value of 0.40, and a negligible negative correlation between hours and the real wage of -0.03 as compared to 0.27 found in the data. The score statistic of 5.88 indicates that this model performs worse than the Uhlig (2007) model considered in the previous subsection. However, as compared to this model it has only one free parameter.

2.6.2 Integrated Technology Shocks

The Model. In the following, we consider the model of the previous paragraph for the case that the technical progress is a difference stationary stochastic process. This is the assumption of the original BCF model. We reformulate the production functions of both sectors accordingly:

$$C_t = (Z_t N_{Ct})^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0,1), \tag{2.37a}$$

$$I_t = (Z_t N_{It})^{1-\alpha} K_{It}^{\alpha}, \tag{2.37b}$$

where the growth factor of Z_t is governed by equations (2.27b) and (2.27c).

Calibration and Results. In the BCF model with stochastic growth and the utility function (2.17) a stationary equilibrium exists only for $\eta = 1$. Since the model has the same technology shock as the Uhlig (2007) model, we employ our estimates of $\bar{z} = 0.006$ and $\sigma = 0.0101$ (see Section 2.5). The value of the habit parameter that minimizes the model's score is $\chi^C = 0.80$. With a score of 6.99 this version fits the empirical facts slightly worse than the model with a stationary technology shock.

2.6.3 A Two Sector Adjustment Cost Model

The equity premium in the model of the previous two subsections is the outcome of variations of the relative price of two goods. In order to study the equity premium that results from variations in the firm value we introduce adjustment costs in the BCF model.

The Model. The representative household holds stocks S_{Xt} of both industries, where, as before, the index $X = C$ denotes consumption goods and $X = I$ refers to the investment goods sector. He chooses his labor supply before the period t productivity shock is realized. The budget constraint is:

$$v_{Ct}(S_{Ct+1} - S_{Ct}) + v_{It}(S_{It+1} - S_{It}) \leq w_{Ct}N_{Ct} + w_{It}N_{It} + d_{Ct}S_{Ct} + d_{It}S_{It} - C_t. \quad (2.38)$$

The representative firm in sector X maximizes

$$V_{Xt} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} \left[X_{t+s} - w_{Xt+s}N_{Xt+s} - p_{t+s}I_{Xt+s}\right] \quad (2.39)$$

subject to

$$X_t = Z_t N_{Xt}^{1-\alpha} K_{Xt}^{\alpha}, \quad \alpha \in (0,1), \quad (2.40a)$$

$$K_{Xt+1} = \Phi(I_{Xt}/K_{Xt})K_{Xt} + (1-\delta)K_{Xt}, \quad \delta \in (0,1]. \quad (2.40b)$$

In equilibrium, the stochastic discount factor equals

$$\varrho_{t+s} = \beta^s \frac{\Lambda_{t+1}}{\Lambda_t}.$$

Firms in each sector transfer their profits less retained earnings as dividends to the household sector. Appendix A.8 presents the model in full detail.

Equity Premium. We compute the equity premium of each sector in the same way as in the one sector model of Section 2.4, i.e.,

$$R_{Ct+1} = \frac{C_{t+1} - w_{Ct+1}N_{Ct+1} - p_{t+1}I_{Ct+1} + q_{Ct+1}K_{Ct+2}}{q_{Ct}K_{Ct+1}}, \tag{2.41a}$$

$$R_{It+1} = \frac{p_{t+1}I_{t+1} - w_{It+1}N_{It+1} - p_{t+1}I_{It+1} + q_{It+1}K_{It+2}}{q_{It}K_{It+1}}, \tag{2.41b}$$

are the gross rates of return on equity in the consumption goods and the investment goods sector, respectively. The average gross rate of return is the weighted average of these rates with the respective shares of capital employed in each sector as weights.

Calibration and Results. The model has two free parameters, χ^C and ζ. The values that minimize the score statistic on a grid with endpoints $\chi^C \in [0.1, 0.95]$ and $\zeta \in [0.001, 6.0]$ are $\chi^C = 0.70$ and $\zeta = 0.005$. Thus, the model that comes closest to the facts is one with negligible adjustment costs. It generates an equity premium and labor market statistics that depart only slightly from those of the BCF model with stationary technology shock.

3 New Keynesian Business Cycle Models

In the following three subsections, we will study monetary models with nominal rigidities. We begin with frictions in the form of price staggering.

3.1 Sticky Prices

In this subsection, we consider a slightly simplified version of the model of De Paoli, Scott, and Weeken (2010). They build on the model described in Section 2.3 and introduce money via the household's utility function. Money prices do not adjust perfectly due to convex costs of price adjustment. However, these costs are modeled as intangible, i.e., they appear in the firms objective function but do not reduce the firm's output.

Households. Households enter the current period t with a given amount of firm shares S_t and given stocks of nominal Bonds B_t.[14] The current price level is P_t. Bonds

[14]The original model also considers the stock of money. However, since monetary policy is modeled via a Taylor rule and since real money balances enter the current period utility function additively,

pay a predetermined nominal rate of interest $Q_t - 1$. The real share price is v_t and real dividend payments per share are d_t. Firms pay the real wage w_t per unit of working hours N_t. Thus,

$$v_t(S_{t+1} - S_t) + \frac{B_{t+1} - B_t}{P_t} \leq w_t N_t + (Q_t - 1)\frac{B_t}{P_t} + d_t S_t - C_t \tag{3.1}$$

is the household's budget constraint. Households maximize (2.17) subject to (3.1) and given initial values of S_t and B_t.

De Paoli, Scott, and Weeken (2010) assume that the consumption and labor habits are exogenous to the household. Thus, different from equations (2.2a) and (2.22), the first order conditions are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{3.2a}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \tag{3.2b}$$

$$v_t \Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1} + d_{t+1}), \tag{3.2c}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \tag{3.2d}$$

where Λ_t is the Lagrange multiplier of the time t budget constraint.

Firms. Final output Y_t is produced from differentiated inputs $Y_t(j)$ distributed on the unit interval according to the function

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_y - 1}{\epsilon_y}} dj \right)^{\frac{\epsilon_y}{\epsilon_y - 1}}, \quad \epsilon_y > 1. \tag{3.3}$$

The zero-profit condition

$$P_t Y_t = \int_0^1 P_t(j) Y_t(j) dj$$

implies the usual demand function for the intermediate product $Y_t(j)$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t} \right)^{-\epsilon_y} Y_t, \tag{3.4}$$

and the price index

$$P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_y} dj \right)^{\frac{1}{1-\epsilon_y}}. \tag{3.5}$$

the time path of money holdings does not interfere with the rest of the model. Therefore, we strip down the presentation of the model. The full version is considered in Appendix A.9.

Consider an arbitrary producer of intermediate product $j \in [0,1]$. His production function is

$$Y_t(j) = Z_t N_t(j)^{1-\alpha} K_t(j)^{\alpha}, \quad \alpha \in (0,1), \tag{3.6}$$

where total factor productivity Z_t is common to all producers and evolves as stated in equation (2.4). The producer finances investment $I_t(j)$ out of retained earnings and distributes the remaining surplus as dividends:

$$D_t(j) = Y_t(j) - w_t N_t(j) - I_t(j). \tag{3.7}$$

Capital accumulation is subject to adjustment costs so that

$$K_{t+1}(j) = (1-\delta)K_t(j) + \Phi\left(\frac{I_t(j)}{K_t(j)}\right) K_t(j), \tag{3.8}$$

with $\Phi(\cdot)$ specified in equation (2.8). Producer j determines his nominal price $P_t(j)$, demand for labor $N_t(j)$, and investment expenditures $I_t(j)$ to maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t}\left[D_{t+s}(j) - \frac{\psi}{2}\left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1\right)^2 Y_{t+s}\right]$$

subject to (3.4), (3.6)-(3.8), and a given initial stock of capital $K_t(j)$. In this expression π denotes the inflation factor in a stationary environment without exogenous shocks. Also note, that the convex cost function in this expression indicates intangible costs, since it appears in the objective function of the producer but does not reduce his profits.

Let Γ_t denote the Lagrange multiplier in minimizing production costs subject to the production function.[15] The first-order conditions are given by:

$$w_t = (1-\alpha)\Gamma_t Z_t N_t(j)^{-\alpha} K_t(j)^{\alpha}, \tag{3.9a}$$

$$q_t = \frac{1}{\Phi'(I_t(j)/K_t(j))}, \tag{3.9b}$$

$$\begin{aligned} q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\Bigg[& \alpha \Gamma_{t+1} Z_{t+1} N_{t+1}(j)^{1-\alpha} K_{t+1}(j)^{\alpha-1} - \frac{I_{t+1}(j)}{K_{t+1}(j)} \\ & + q_{t+1}\left(1 - \delta + \Phi(I_{t+s}(j)/K_{t+1}(j))\right)\Bigg], \end{aligned} \tag{3.9c}$$

$$\begin{aligned} 0 = {} & (1-\epsilon_y)\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_y}\frac{Y_t}{P_t} - \psi\left(\frac{P_t(j)}{\pi P_{t-1}(j)} - 1\right)\frac{Y_t}{\pi P_{t-1}(j)} \\ & + \epsilon_y \Gamma_t \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_y - 1}\frac{Y_t}{P_t} + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\psi\left(\frac{P_{t+1}(j)}{\pi P_t(j)} - 1\right)\frac{P_{t+1}(j) Y_{t+1}}{\pi P_t(j)^2}. \end{aligned} \tag{3.9d}$$

[15] This multiplier is independent of the firm index j, since all firms face the same wages and rental prices for capital and since the production function is linear homogenous.

Monetary Policy. The central bank sets the nominal interest rate Q_{t+1} according to the Taylor rule

$$Q_{t+1} = Q_t^{\delta_1} \left(\frac{\pi}{\beta}\right)^{1-\delta_1} \left(\frac{\pi_t}{\pi}\right)^{\delta_2} e^{\epsilon_t^Q}, \quad \delta_1 \in [0,1), \quad \epsilon_t^Q \sim N(0, \sigma^Q). \tag{3.10}$$

The elasticity of Q_{t+1} with respect to the deviation of the inflation factor π_t from its steady state value π will be chosen so that the equilibrium is determinate. Usually, this requires $\delta_2 > 1$.

Calibration and Results. The model has several additional parameters. We assume an inflation target of zero and set the price elasticity equal to $\epsilon_y = 6.0$ according to markups estimated by Linnemann (1999). We choose $\delta_2 = 1.5$ so that the equilibrium of the model is determinate in all our simulations. The parameters χ^C, χ^N, ζ, ψ, δ_1, and σ^Q are chosen in order to minimize the model's score. We used a coarse grid over the intervals $\chi^C \in [0.5, 0.95]$, $\chi^N \in [0.1, 0.95]$, $\zeta \in [1.5, 5.5]$, $\psi \in [0.01, 120]$, $\delta_1 \in [0.01, 0.90]$, and $\sigma^Q \in [0.5\sigma, 3\sigma]$ (where $\sigma = 0.0072$ is the standard deviation of the innovations of the productivity shock).

The minimizer features a strong consumption habit, $\chi^C = 0.85$, and a moderate labor habit, $\chi^N = 0.40$. The Taylor rule shows no persistence, $\delta_1 = 0.01$, but monetary shocks are more important than productivity shocks, $\sigma^Q/\sigma = 2.5$. Importantly, with $\psi = 0.01$, money prices are almost perfectly flexible.

The model is neither able to come close to the German equity premium (0.43 instead of 5.18) nor is it able to generate the positive correlations between output and hours and between hours and the real wage. According to its score of 26.38 it performs worse than most of the real business cycle models considered in Section 2.

3.2 Sticky Wages

As our second model with nominal frictions, we set up a model with wage staggering as introduced by Erceg, Henderson, and Levin (2000). From the previous section we borrow the modeling of the government sector. The production sector is the same as in Section (2.1) with one exception to which we turn next.

Labor Demand. Labor input N_t in production $Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}$ is an index of the different types of labor $N_t(h)$ supplied by the members $h \in [0,1]$ of the representative household:

$$N_t = \left[\int_0^1 N_t(h)^{\frac{\epsilon_w - 1}{\epsilon_w}} dh \right]^{\frac{\epsilon_w}{\epsilon_w - 1}}, \quad \epsilon_w > 1. \tag{3.11}$$

Let W_t denote the nominal wage rate at date t and $W_t(h)$ the wage paid to labor of type h. Minimizing the wage bill

$$W_t N_t = \int_0^1 W_t(h) N_t(h) dh$$

subject to (3.11) yields the demand function for labor and the wage index:

$$N_t(h) = \left(\frac{W_t(h)}{W_t} \right)^{-\epsilon_w} N_t, \tag{3.12}$$

$$W_t = \left[\int_0^1 W_t(h)^{1-\epsilon_w} \right]^{\frac{1}{1-\epsilon_w}}. \tag{3.13}$$

Since everything else is unchanged, conditions (2.11) continue to describe the firm's optimal decisions with respect to capital accumulation and aggregate labor demand N_t, where $w_t = W_t/P_t$ on the left-hand side of (2.11a). As in the previous section P_t denotes the money price of output Y_t.

Wage Setting. The preferences of household member $h \in [0,1]$ are:[16]

$$\begin{gathered} u(C_t(h), C_{t-1}(h), N_t(h)) \equiv \frac{(C_t(h) - \chi^C C_{t-1}(h))^{1-\eta} - 1}{1-\eta} - \frac{\nu_0}{1+\nu_1} N_t(h)^{1+\nu_1}, \\ \eta, \nu_0, \nu_1 > 0, \chi^C \in [0,1), \end{gathered} \tag{3.14}$$

where $C_t(h)$ denote consumption of household member h.

In each period a fraction φ_w of households updates their wage rate according to the steady state inflation factor π:

$$W_t(h) = \pi W_{t-1}(h). \tag{3.15}$$

The fraction $1 - \varphi_w$ of the households can choose their wage rate $W_t(h)$ optimally. These households maximize

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta \varphi_w)^s u(C_{t+s}(h), C_{t+s-1}(h), N_{t+s}(h)) \tag{3.16}$$

[16] As in the previous section we do not model the demand for money.

subject to the series of budget constraints

$$\frac{W_{t+s}(h)}{P_{t+s}}N_{t+s}(h) + S_{t+s}d_{t+s}(h) + (Q_{t+s}-1)\frac{B_{t+s}(h)}{P_{t+s}} - C_{t+s}(h)$$
$$\geq \frac{B_{t+s+1}(h) - B_{t+s}(h)}{P_{t+s}} + v_{t+s}(S_{t+s+1}(h) - S_{t+s}(h)), \tag{3.17}$$

and the demand function (3.12). As before, d_t are dividends per share S_t with price v_t and B_t are bonds in money units that earn the nominal interest rate $Q_t - 1$. The maximand (3.16) is the expected life time utility assuming that the household were never able to readjust its wage after period t. We assume that there is a sufficiently rich set of contingent security markets so that a representative agent exists. Therefore, all wage setters will opt for the same relative wage $w_{At} \equiv W_t(h)/W_t$. In Appendix A.10 we show that this wage is determined by the set of equations:

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\Gamma_{1t}}{\Gamma_{2t}}, \tag{3.18a}$$

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)}N_t^{1+\nu_1} + \beta\varphi_w\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1}w_{At+1}}\right)^{-\epsilon_w}\Gamma_{1t+1}, \tag{3.18b}$$

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w}N_t + \beta\varphi_w\left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w}\left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w}\Gamma_{2t+1}, \tag{3.18c}$$

$$w_t = \frac{W_t}{P_t} \equiv \frac{\omega_t}{\pi_t}w_{t-1}, \tag{3.18d}$$

$$1 = (1-\varphi_w)w_{At}^{1-\epsilon_w} + \varphi_w(\pi/\omega_t)^{1-\epsilon_w}, \tag{3.18e}$$

$$\omega_t = \frac{W_t}{W_{t+1}}. \tag{3.18f}$$

As can be seen from equation (3.18d), the partial adjustment of nominal wages entails sticky real wages, though in a different manner as introduced in Section 2.5.

Consumption and Portfolio Choice. The pooling assumption allows us to derive the demand for consumption, bonds, and stocks from maximizing

$$\mathbb{E}_t\sum_{s=0}^{\infty}\beta^s u(C_{t+s}, C_{t+s-1}, N_{t+s})$$

subject to the sequence of budget constraints

$$w_{t+s}N_{t+s} + S_t d_{t+s} + (Q_{t+s}-1)\frac{B_{t+s}}{P_{t+s}} - C_{t+s} \geq \frac{B_{t+s+1} - B_{t+s}}{P_{t+s}} + v_{t+s}(S_{t+s+1} - S_{t+s}).$$

The respective first-order conditions coincide with (3.2a), (3.2c), and (3.2d).

Calibration and Results. The equity premium implied by this model can be computed as in Section (2.2) from equation (2.19). The model has two new parameters, the wage markup implied by ϵ_w and the degree of wage stickiness determined by ϕ_w. We set ϵ_w equal to 6 so that the wage markup is 20 percent. The remaining free parameters, χ^C, ζ, ϕ_w, δ_1, and σ^Q are set in order to imply the best possible fit with the data.

The minimizer, found on a coarse grid, implies a very strong consumption habit, $\chi^C = 0.95$, significant adjustment costs, $\zeta = 5.4$, rigid wages, $\phi_w = 0.84$, a high degree of persistence in the Taylor rule, $\delta_1 = 0.91$, and a predominance of monetary as compared to productivity shocks, $\sigma^Q/\sigma = 3$. Except for the correlation between hours and the real wage, which is 0.27 in the data and -0.69 in the simulated time series, the model is in good accordance with empirical evidence, documented by a score statistic of 1.47 (see Table 1.1 for details).

3.3 Sticky Prices and Wages

As our last model we merge the models from the previous two subsections so that both the nominal prices and the nominal wages are sticky. The model is presented in Appendix A.11. We set both the price elasticity of the demand for goods and for labor equal to $\epsilon_y = \epsilon_w = 6.0$. The free parameters χ^C, ζ, ψ, ϕ_w, δ_1 and σ^Q are chosen optimally. The result, $\chi^C = 0.88$, $\zeta = 3.0$, $\psi = 275$, $\phi_w = 0.55$, $\delta_1 = 0.90$, and $\sigma^Q/\sigma = 4.6$, demonstrates that price rigidity enhances the model's performance, especially with respect to the correlation between hours and the real wage. The model's score of 1.19 is close to that of the sticky wage model of Section 2.5 (see Table 1.1).

4 Conclusion

We have evaluated the current-state of the art business-cycle models that try to replicate the empirically observed equity premium with regard to their labor market behavior. In addition to the current studies, we also analyzed a model of the equity premium with sticky wages and with both sticky prices and sticky wages.

In the class of real business cycle models the Uhlig (2007) model of sticky real wages predicts an equity premium and labor market statistics that come very close to the empirical evidence provided in Table 1.1. Less successful are various variants of the Boldrin, Christiano, and Fisher (2001) two-sector model with frictions in the allocation of labor within the current period. Among the New Keynesian models with nominal rigidities the model with sticky prices and wages is favored by the data. The score of this model is only slightly worse than that of the Uhlig (2007) model. The sticky price model of de Paoli, Scott and Weeken (2010) is neither able to generate a sizeable equity premium nor the correct correlations between hours and output and between hours and the real wage.

In future work we will explore along which dimensions more complicated models, like the one introduced by Gourio (2012) with Epstein-Zin preferences and a small risk of economic disaster, improve upon the simple, but successful models presented in this paper.

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Appendix A (Not for Publication)

This Appendix covers technical details of the models considered in the body of the paper and presents models that we have also studied but decided not to include in our review.

A.1 Computation of the Equity Return

In Section 2.1 we demonstrate that the gross return to stockholders of the representative firm equals

$$R_{t+1} = \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}} \equiv \frac{v_{t+1} + d_{t+1}}{v_t}. \tag{A.1.1}$$

The first term on the right-hand side allows us to compute the equity return without any assumption about the firm's dividend policy. The second term includes dividends per share d_{t+1}, and we cannot employ this formula without statement about the amount of self-financing. If we assume pure self-financing so that the number of outstanding shares is constant, both formulas (differences due to the numeric approximation of the respective functions aside) will provide the same rate of return.

The equivalence (A.1.1) rests on the relation $v_t S_{t+1} = q_t K_{t+1}$, which we prove in the following. Note that (2.13e) can be written as:

$$q_t K_{t+1} = \beta \frac{\Lambda_{t+1}}{\Lambda_t} \left[CF_{t+1} + q_{t+1} K_{t+2} \right],$$

since $\alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{-\alpha} = (Y_{t+1} - w_{t+1}N_{t+1})/K_{t+1}$ and $CF_{t+1} = Y_{t+1} - w_{t+1}N_{t+1} - I_{t+1}$. Iterating on this equation yields

$$q_t K_{t+1} = \sum_{s=1}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} CF_{t+s},$$

since the term $\beta^s(\Lambda_{t+s}/\Lambda_t)K_{t+s+1}$ vanishes due to the transversality condition (2.11d) (where $\varrho_{t+s} = \beta^s \Lambda_{t+s}/\Lambda_t$). The right-hand side of this expression is equal to $V_t = v_t S_{t+1}$ (see equation (2.9) in the body of the paper).

Let

$$M_{t+1} \equiv \frac{\beta \Lambda_{t+1}}{\Lambda_t}$$

denote the stochastic discount factor that appears in equation (2.2b) and equation (2.15). Both conditions hold for any t and thus for any information set used in forming expectations $\mathbb{E}_t$. Therefore, they also hold unconditionally.[17] Thus, condition (2.15) implies

$$r_t^f = \frac{1}{\mathbb{E}(M_{t+1})}$$

and equation (2.2b) can be written as

$$1 = \mathbb{E}_t(M_{t+1}R_{t+1}).$$

Using $\mathrm{cov}(x, y) = \mathbb{E}(xy) - \mathbb{E}(x)\mathbb{E}(y)$ as well as the previous equation implies

$$\mathbb{E}(R_{t+1}) - r_t^f = -r_t^f \,\mathrm{cov}(M_{t+1}, R_{t+1}).$$

A.2 Endogenous Labor Supply

The model with endogenous labor supply in Section 2.2 is described by the following equilibrium conditions:

$$\nu_0 N_t^{\nu_1} = \Lambda_t w_t, \tag{A.2.1a}$$

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.2.1b}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.2.1c}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.2.1d}$$

$$Y_t = C_t + I_t, \tag{A.2.1e}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta b \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.2.1f}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta \big] \Big\}, \tag{A.2.1g}$$

$$K_{t+1} = \Phi(I_t/K_t) K_t + (1-\delta) K_t. \tag{A.2.1h}$$

In the stationary equilibrium, equation (A.2.1f) reduces to

$$\frac{K}{N} = \left(\frac{1 - \beta(1-\delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}}. \tag{A.2.2}$$

For $N = 0.13$, equations (A.2.2) allows us to infer K, and we can compute the stationary values of the remaining variables in the same way is in the model of the previous section. Finally, equation (A.2.1a) allows us to fix the value of ν_0.

[17] Our derivation follows Grossman and Shiller (1991).

A.3 Time-to-Plan Model

In this subsection we consider yet another way to explain the equity premium. We embed a consumption habit in the model of Heer and Maußner (2009), Section 2.6.2. This model is a stripped down version of the Kydland and Prescott (1982) model of economic fluctuations. The parameterization of the investment equation follows Beaubrun-Diant (2005), who employs the time-to-plan model of Christano and Todd (1996) to investigate the equity premium puzzle.

The model differs from the model considered in the previous subsection only with respect to the modeling of the firm.

Firms. The firm maximizes its beginning-of-period current value V_t^{bop}. By using $\varrho_{t+s} = \beta^s(\Lambda_{t+s}/\Lambda_t)$, this can be written as:

$$V_t^{bop} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \left[Z_{t+s}(A_{t+s}N_{t+s})^{1-\alpha} K_{t+s}^{\alpha} - w_{t+s}A_{t+s}N_{t+s} - I_{t+s} \right] \tag{A.3.1}$$

subject to

$$I_t = \sum_{i=1}^{4} \omega_i X_{it}, \quad \sum_{i=1}^{4} \omega_i = 1, \tag{A.3.2a}$$

$$K_{t+4} = (1-\delta)K_{t+3} + X_{4t}, \tag{A.3.2b}$$

$$X_{1t+1} = X_{2t}, \tag{A.3.2c}$$

$$X_{2t+1} = X_{3t}, \tag{A.3.2d}$$

$$X_{3t+1} = X_{4t}, \tag{A.3.2e}$$

$$A_{t+1} = aA_t, \quad a \geq 1. \tag{A.3.2f}$$

The time-to-build model assumes that the resource costs are equally spread over the construction period so that $\omega_i = 0.25\ \forall i = 1, 2, 3, 4$. The time-to-plan model instead assumes that in the start-up phase little resources are required. Thus, $\omega_4 = 0.01$ and $\omega_i = 0.33\ \forall i = 2, 3, 4$. This is the parameterization which we will employ here.

The first-order conditions of the firm's problem are:

$$w_t A_t = (1-\alpha) Z_t A_t^{1-\alpha} N_t^{-\alpha} K_t^{\alpha}, \tag{A.3.3a}$$

$$q_t = \omega_4 + \beta\omega_3 \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} + \beta^2 \omega_2 \mathbb{E}_t \frac{\Lambda_{t+2}}{\Lambda_t} + \beta^3 \omega_1 \mathbb{E}_t \frac{\Lambda_{t+3}}{\Lambda_t}, \tag{A.3.3b}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} q_{t+1}(1-\delta) + \beta^4 \mathbb{E}_t \frac{\Lambda_{t+4}}{\Lambda_t} \alpha Z_{t+4} (A_{t+4} N_{t+4})^{1-\alpha} K_{t+4}^{\alpha-1}, \tag{A.3.3c}$$

where q_t denotes the Lagrange multiplier of the constraint (A.3.2b). Equations (A.3.3b) and (A.3.3c) can be condensed to

$$\begin{aligned} 0 = \mathbb{E}_t \Bigg\{ & \omega_4 \left[\beta(1-\delta)\Lambda_{t+1} - \Lambda_t\right] + \omega_3 \beta \left[\beta(1-\delta)\Lambda_{t+2} - \Lambda_{t+1}\right] \\ & + \omega_2 \beta^2 \left[\beta(1-\delta)\Lambda_{t+3} - \Lambda_{t+2}\right] + \omega_1 \beta^3 \left[\beta(1-\delta)\Lambda_{t+4} - \Lambda_{t+3}\right], \\ & + \alpha \beta^4 \Lambda_{t+4} Z_{t+4} (A_{t+4} N_{t+4})^{1-\alpha} K_{t+4}^{\alpha-1} \Bigg\}. \end{aligned} \tag{A.3.3d}$$

Households. Since the model allows for deterministic growth of labor augmenting technical progress A_t, we modify the household's budget constraint in equation (2.1) to read

$$v_t A_t (S_{t+1} - S_t) \leq w_t A_t N_t + d_t A_t S_t - C_t \tag{A.3.4}$$

so that v_t, w_t, and d_t now represent the share price, the real wage and dividends per unit of A_t. Accordingly, the first-order conditions for maximizing (2.17) subject to (A.3.4) are

$$\Lambda_t = \left(C_t - \chi^C C_{t-1}\right)^{-\eta} - \beta \chi^C \mathbb{E}_t \left(C_{t+1} - \chi^C C_t\right)^{-\eta}, \tag{A.3.5a}$$

$$\Lambda_t w_t A_t = \nu_0 N_t^{\nu 1}, \tag{A.3.5b}$$

$$v_t A_t \Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} A_{t+1} (v_{t+1} + d_{t+1}). \tag{A.3.5c}$$

Temporary Equilibrium in Stationary Variables. In this model, a stationary equilibrium exists

i. if labor supply is exogenously fixed so that condition (A.3.5b) does not apply,

ii. if there is no growth, i.e., $a = 1$,

iii. if there is growth, $a > 1$, and $\eta = 1$.

These restriction should be kept in mind, when considering the transformed equations to which we turn next.

We use lower case letters for all variables without a trend. Variables that grow are scaled according to $x_t \equiv X_t/A_t$ except for $\lambda_t \equiv \Lambda_t A^\eta$.

A temporary equilibrium is defined by equations (A.3.5a)-(A.3.5c), (A.3.2a)-(A.3.2e), (A.3.3a), (A.3.3d), the production function $Y_t = Z_t(A_t N_t)^{1-\alpha} K_t^\alpha$, and the economy's resource constraint $Y_t = C_t + I_t$, which follows from the household's budget constraint and the definition of dividends. To put the system into the canonical form of equations (2.51) of Heer and Maußner (2009), we include a set of auxiliary variables v_{it}, $i = 1, 2, \ldots, 10$. The final system reads:

$$\nu_0 N_t^{\nu_1} = \lambda_t w_t, \tag{A.3.6a}$$

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} k_t^\alpha, \tag{A.3.6b}$$

$$y_t = Z_t N_t^{1-\alpha} k_t^\alpha, \tag{A.3.6c}$$

$$y_t = c_t + i_t, \tag{A.3.6d}$$

$$i_t = \sum_{i=1}^{4} \omega_i x_{it}, \tag{A.3.6e}$$

$$d_t = y_t - w_t N_t - i_t, \tag{A.3.6f}$$

$$a x_{1t+1} = x_{2t}, \tag{A.3.6g}$$

$$a x_{2t+1} = x_{3t}, \tag{A.3.6h}$$

$$a x_{3t+1} = x_{4t}, \tag{A.3.6i}$$

$$a k_{t+1} = (1-\delta) k_t + x_{1t}, \tag{A.3.6j}$$

$$\lambda_t = \left(c_t - (\chi^C/a) v_{1t}\right)^{-1} - \mathbb{E}_t \chi^C \beta \left(a c_{t+1} - \chi^C c_t\right)^{-1}, \tag{A.3.6k}$$

$$1 = \beta a^{1-\eta} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left[\frac{v_{t+1} + d_{t+1}}{v_t}\right], \tag{A.3.6l}$$

$$\begin{aligned} 0 = \mathbb{E}_t \Bigg\{ & \omega_4 \left[\left(\beta a^{-\eta}\right)(1-\delta)\lambda_{t+1} - \lambda_t\right] \\ & + \left(\beta a^{-\eta}\right) \omega_3 \left[\left(\beta a^{-\eta}\right)(1-\delta) v_{2t+1} - v_{2t}\right] \\ & + \left(\beta a^{-\eta}\right)^2 \omega_2 \left[\left(\beta a^{-\eta}\right)(1-\delta) v_{3t+1} - v_{3t}\right] \\ & + \left(\beta a^{-\eta}\right)^3 \omega_1 \left[\left(\beta a^{-\eta}\right)(1-\delta) v_{4t+1} - v_{4t}\right] \\ & + \alpha \left(\beta a^{-\eta}\right)^4 v_{4t+1} (Z_{t+1})^{\left(\rho^Z\right)^3} v_{7t+1}^{1-\alpha} v_{10t+1}^{\alpha-1} \Bigg\}, \end{aligned} \tag{A.3.6m}$$

$$v_{1t} = c_{t-1}, \tag{A.3.6n}$$

$$v_{2t} = \lambda_{t+1}, \tag{A.3.6o}$$

$$v_{3t} = v_{2t+1} = \lambda_{t+2}, \tag{A.3.6p}$$

$$v_{4t} = v_{3t+1} = \lambda_{t+3}, \tag{A.3.6q}$$

$$v_{5t} = N_{t+1}, \tag{A.3.6r}$$

$$v_{6t} = v_{5t+1} = N_{t+2}, \tag{A.3.6s}$$

$$v_{7t} = v_{6t+1} = N_{t+3}, \tag{A.3.6t}$$

$$v_{8t} = k_{t+1}, \tag{A.3.6u}$$

$$v_{9t} = v_{8t+1} = k_{t+2}, \tag{A.3.6v}$$

$$v_{10t} = v_{9t+1} = k_{t+3}. \tag{A.3.6w}$$

Equity Premium. The gross risk free rate is

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}} = \frac{\lambda_t}{\beta a^{-\eta} \mathbb{E}_t \lambda_{t+1}}$$

so that the household's Euler equation (A.3.6l) implies that the return on equity equals

$$R_{t+1} = a \frac{d_{t+1} + v_{t+1}}{v_t}.$$

We assume that the firm finances its investment entirely from retained earnings so that equation (A.3.6f) defines the dividend payments to the household sector.

Calibration and Results. Besides the weights ω_i, which implement the time to plan assumption, the model has just on free parameter, χ^C. In the version with exogenous labor supply of $N = 1$ and no technical progress, $a = 1$, we find that $\chi^C = 0.6768$ implies an equity premium of 5.18% p.a. and a standard deviation of investment expenditures relative to output of $s_i/s_y = 2.35$. Thus, the model fits the data almost as perfect as the Jermann (1998) model.

However, if working hours are endogenous, the equity premium disappears as it does in the model of section 2.2. It drops to 0.42% p.a. The relative standard deviation of investment increases to $s_i/s_y = 4.79$. Except for the negative correlation between hours and the real wage of $r_{wN} = -0.55$ the other labor market statistics are empirically reasonable: $s_N/s_y = 0.85$, $s_w/s_y = 1.17$, $r_{yN} = 0.21$.

A.4 Habit in Leisure

The model with habit in leisure in Section 2.3 is described by the following equilibrium conditions:

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.4.1a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.4.1b}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.4.1c}$$

$$Y_t = C_t + I_t, \tag{A.4.1d}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1} - \beta \nu_0 \chi^N (N_{t+1} - \chi^N N_t)^{\nu_1}, \tag{A.4.1e}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.4.1f}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda t} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta \big] \Big\}, \tag{A.4.1g}$$

$$K_{t+1} = \Phi(I_t/K_t) K_t + (1-\delta) K_t. \tag{A.4.1h}$$

In the stationary equilibrium, equations (A.4.1c) and (A.4.1g) imply

$$\frac{K}{N} = \left(\frac{1 - \beta(1-\delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}}. \tag{A.4.2}$$

Equation (A.4.2) allows us to infer K with the help of $N = 0.13$, and we can compute the stationary values of the remaining variables in the same way as in the model of the previous section. Finally, equation (A.4.1e) allows us to fix the value of ν_0 for given value of d.

A.5 Real Wage Rigidity

In this subsection we consider real wage rigidity. We depart from our set up in the previous subsection and consider the model of Uhlig (2007).

Firm. The production function of the representative firm is modified to allow for stochastic growth in labor augmenting technical progress Z_t. In particular, we assume

$$Y_t = B(Z_t N_t)^{1-\alpha} K_t^{\alpha}, \tag{A.5.1a}$$

$$z_t \equiv \frac{Z_t}{Z_{t-1}}, \tag{A.5.1b}$$

$$\ln z_t = \ln \bar{z} + \epsilon_t^z, \quad \epsilon_t^z \sim N(0, \sigma_z^2). \tag{A.5.1c}$$

The firm maximizes

$$V_t^{bop} = \sum_{s=0}^{\infty} \varrho_{t+s} \left[Y_{t+s} - W_{t+s} N_{t+s} - I_{t+s}\right] \tag{A.5.2}$$

subject to

$$K_{t+1} = \Phi(I_t/K_t)K_t + (1-\delta)K_t. \tag{A.5.3}$$

The first-order conditions are

$$W_t = (1-\alpha)BZ_t^{1-\alpha}N_t^{-\alpha}K_t^{\alpha}, \tag{A.5.4a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.5.4b}$$

$$q_t\varrho_t = \beta\mathbb{E}_t\varrho_{t+1}\Big\{\alpha B(Z_{t+1}N_{t+1})^{1-\alpha}K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1}\big[1-\delta+\Phi(I_{t+1}/K_{t+1})\big]\Big\}. \tag{A.5.4c}$$

Household. The representative household solves

$$\max \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{[(C_{t+s} - C_{t+s}^h)(A + (1 - N_{t+s} - L_t^h)^{\nu}]^{1-\eta} - 1}{1-\eta},$$

subject to,

$$V_t(S_{t+1} - S_t) \leq W_t N_t + D_t S_t - C_t,$$

where V_t, S_t, W_t^f, and D_t denote the price of the firm's shares, the number of shares, the real wage at which the household will supply labor, and dividends per share, respectively.[18] The habits in consumption C_t^h and in leisure L_t^h are exogenous to the household. They evolve according to

$$C_t^h = \bar{z}\left[(1-\lambda^C)\chi^C C_{t-1} + \lambda^C C_{t-1}^h\right], \tag{A.5.5a}$$

$$L_t^h = (1-\lambda^L)\chi^L(1-N_{t-1}) + \lambda^L L_{t-1}^h. \tag{A.5.5b}$$

The first-order conditions of the household's problem are:

$$\Lambda_t = (C_t - C_t^h)^{-\eta}\left(A - (1 - N_t - L_t^h)^{\nu}\right)^{1-\eta}, \tag{A.5.6a}$$

$$\Lambda_t W_t^f = \nu(C_t - C_t^h)^{1-\eta}\left(A - (1 - N_t - L_t^h)^{\nu}\right)^{-\eta}(1 - N_t - L_t^h)^{\nu-1}, \tag{A.5.6b}$$

$$1 = \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}\frac{V_{t+1} + D_{t+1}}{V_t} \equiv \beta\mathbb{E}_t\frac{\Lambda_{t+1}}{\Lambda_t}R_{t+1}. \tag{A.5.6c}$$

[18] The reason why we are using upper case variables to denote prices and dividends will become obvious in a moment.

Labor Market. Real wages do not adjust instantaneously to clear the labor market. Instead, near the balanced growth path labor supply will be demand-constrained. Thus, given the real wage W_t, equation (A.5.4a) determines working hours (given the current period capital stock K_t). Between periods the real wage changes according to

$$W_t = (\bar{z}W_{t-1})^{\mu}(W_t^f)^{1-\mu}. \tag{A.5.7}$$

Dynamics. In equilibrium $V_t(S_{t+1} - S_t) = I_t - RE_t$, and $D_tS_t = Y_t - W_tN_t - RE_t$ so that the household's budget constraint implies

$$Y_t = C_t + I_t. \tag{A.5.8}$$

Furthermore, $\varrho_{t+1}/\varrho_t = \beta\Lambda_{t+1}/\Lambda_t$ from (A.5.6c) and the definition of ϱ_t in (2.9) (in the body of the paper).

To study the model's dynamics we have to formulate it in stationary variables. Towards this purpose let

$$x_t \equiv \frac{X_t}{Z_{t-1}}, \tag{A.5.9a}$$

$$\lambda_t \equiv \Lambda_t Z_{t-1}^{\eta}. \tag{A.5.9b}$$

Thus, from (A.5.1a), (A.5.4a), (A.5.8), (A.5.4b), (A.5.6a), (A.5.6b), (A.5.7):

$$y_t = Bz_t^{1-\alpha}N_t^{1-\alpha}k_t^{\alpha}, \tag{A.5.10a}$$

$$w_t = (1-\alpha)\frac{y_t}{N_t}, \tag{A.5.10b}$$

$$y_t = c_t + i_t, \tag{A.5.10c}$$

$$q_t = \frac{1}{\Phi'(i_t/k_t)}, \tag{A.5.10d}$$

$$\lambda_t = \left(c_t - c_t^h\right)^{-\eta}\left(A + (1 - N_t - L_t^h)^{\nu}\right)^{1-\eta}, \tag{A.5.10e}$$

$$\lambda_t w_t^f = \nu\left(c_t - c_t^h\right)^{1-\eta}\left(A + (1 - N_t - L_t^h)^{\nu}\right)^{-\eta}(1 - N_t - L_t^h)^{\nu-1}, \tag{A.5.10f}$$

$$w_t = \left(\frac{\bar{z}}{z_{t-1}}w_{t-1}\right)^{\mu}\left(w_t^f\right)^{1-\mu} \tag{A.5.10g}$$

These seven equations determine y_t, c_t, i_t, N_t, w_t and w_t^f given the state variables k_t, z_t, z_{t-1}, c_t^h, L_t^h, w_{t-1}, and the costate λ_t. The dynamics of the state and costate variables follows from (A.5.3), (A.5.5a), (A.5.5b), and (A.5.4c)

$$z_t k_{t+1} = \Phi(i_t/k_t)k_t + (1-\delta)k_t, \tag{A.5.11a}$$

$$c_t^h = \frac{\bar{z}}{z_{t-1}} \left[(1-\lambda^C)\chi^C c_{t-1} + \lambda^C c_{t-1}^h)\right], \tag{A.5.11b}$$

$$L_t^h = (1-\lambda^L)\chi^L(1-N_{t-1}) + \lambda^L L_{t-1}^h, \tag{A.5.11c}$$

$$q_t = \beta z_t^{-\eta} \mathbb{E}_t \frac{\lambda_{t+1}}{\lambda_t} \left\{\alpha(y_{t+1}/k_{t+1}) - (i_{t+1}/k_{t+1}) + q_{t+1}(1-\delta+\Phi(i_{t+1}/k_{t+1})\right\}. \tag{A.5.11d}$$

Balanced Growth Path. The balanced growth path follows from the set of equations (A.5.10) and (A.5.11) when we ignore the shock to technology and assume that the ensuing deterministic dynamics has converged to stationary variables (denoted without a time index). In order to guarantee that adjustment costs play no role on the balanced growth path we assume

$$\Phi'(i/k) = 1, \tag{A.5.12a}$$

$$\Phi(i/k) = \bar{z} - 1 + \delta. \tag{A.5.12b}$$

For the parameterization of the function Φ used throughout this paper (see (2.8)) this implies

$$a_1 = (\bar{z} - 1 + \delta)^{\zeta}, \tag{A.5.13a}$$

$$a_2 = (\bar{z} - 1 + \delta)\left(\frac{\zeta}{\zeta - 1}\right). \tag{A.5.13b}$$

Given these assumptions, equation (A.5.11d) implies

$$\frac{y}{k} = \frac{\bar{z}^{\eta} - \beta(1-\delta)}{\alpha\beta}. \tag{A.5.14a}$$

Via the resource constraint (A.5.10c) we derive

$$\frac{c}{y} = 1 - \frac{\bar{z} - 1 + \delta}{y/k}, \tag{A.5.14b}$$

$$\frac{c}{k} = \frac{y}{k} - (\bar{z} - 1 + \delta). \tag{A.5.14c}$$

According to (A.5.11b) and (A.5.11c) the stationary values of the consumption habit and the leisure habit equal

$$c^h = \chi^C c, \tag{A.5.14d}$$

$$L^h = \chi^L(1 - N), \tag{A.5.14e}$$

respectively. In our calibration we set N to some given number and determine the shift parameter B in the production function (A.5.1a) so that $\lambda = 1$.[19] Equations (A.5.10e), (A.5.10f), and (A.5.10b) imply

$$\nu = 1 - (1-\chi^L)\left[\frac{1-N}{\tau N} - \left(2 - \frac{1}{\eta}\right)\frac{c/y}{1-\alpha}\frac{N}{1-N}\frac{1}{1-\chi^C}\right], \tag{A.5.14f}$$

where τ is the Frisch elasticity of labor supply with respect to the real wage.[20] Together with the solutions obtained so far, this equation determines the parameter ν from a given value of τ. In the next step we can solve (A.5.4a), (A.5.6a), and (A.5.6b) for the parameter A.[21] This provides:

$$A = \nu(1-\chi^C)(c/y)(N/(1-\alpha))\left[(1-\chi^L)(1-N)\right]^{\nu-1} - \left[(1-\chi^L)(1-N)\right]^{\nu}. \tag{A.5.14g}$$

Together with $\lambda = 1$ (A.5.10e) implies

$$c = \frac{1}{1-\chi^C}\left[A + \left((1-\chi^L)(1-N)\right)^{\nu}\right]^{\frac{1-\eta}{\eta}}, \tag{A.5.14h}$$

so that we can derive the stationary level of stock of capital from $k = c/(c/k)$ and the stationary level of output from $y = (y/k)k$. Given N, k, and $\bar{z}$ (A.5.1a) implies

$$B = \frac{y}{k}\left(\frac{k}{\bar{z}N}\right)^{1-\alpha}. \tag{A.5.14i}$$

Equity Premium. The gross risk-free rate of return in this model equals

$$r_t = \frac{\Lambda_t}{\beta\mathbb{E}_t\Lambda_{t+1}} = \frac{\Lambda_t Z_{t-1}^{\eta}}{\beta\mathbb{E}_t\Lambda_{t+1}(Z_t/z_t)^{\eta}} = \frac{z_t^{\eta}\lambda_t}{\beta\mathbb{E}_t\lambda_{t+1}}.$$

The gross equity return R_{t+1} derives from the household's first-order condition with respect to S_{t+1}, which equals (2.2c), and the definition of the firm's dividends and retained earnings in (2.6) and (2.5), respectively:

$$\begin{aligned} R_{t+1} &\overset{(2.2c)}{=} \frac{V_{t+1}+D_{t+1}}{V_t} = \frac{(V_{t+1}+D_{t+1})S_{t+1}}{V_t S_{t+1}}, \\ &\overset{(2.6)}{=} \frac{V_{t+1}S_{t+1} + Y_{t+1} - W_{t+1}N_{t+1} - RE_{t+1}}{V_t S_{t+1}}, \\ &\overset{(2.5)}{=} \frac{Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + V_{t+1}S_{t+2}}{V_t S_{t+1}}. \end{aligned}$$

[19]Without this normalization of λ the numeric computation of first and second derivatives is very inaccurate since λ would be a very large number.

[20]Differentiate (A.5.10e) and (A.5.10f) with respect to c_t, N_t, w_t and λ_t, evaluate the respective derivatives on the balanced growth path, and solve for $(dN/dw)(w/N)_{d\lambda=0}$.

[21]Note that (A.5.7) implies $w = w^f$ on the balanced growth path.

Using $V_t S_{t+1} = q_t K_{t+1}$ (see Section A.1), this can be written as:

$$\begin{aligned}
R_{t+1} &= \frac{Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}K_{t+2}}{q_t K_{t+1}}, \\
&\overset{(\text{A.5.3})}{=} \frac{Y_{t+1} - W_{t+1}N_{t+1} - I_{t+1} + q_{t+1}\left(\Phi(I_{t+1}/K_{t+1})K_{t+1} + (1-\delta)K_{t+1}\right)}{q_t K_{t+1}}, \\
&\overset{(\text{A.5.4a})}{=} \frac{\alpha Y_{t+1} - I_{t+1} + q_{t+1}\left(\Phi(I_{t+1}/K_{t+1})K_{t+1} + (1-\delta)K_{t+1}\right)}{q_t K_{t+1}}, \\
&= \frac{\alpha(y_{t+1}/k_{t+1}) - (i_{t+1}/k_{t+1}) + q_{t+1}\left(\Phi(i_{t+1}/k_{t+1}) + (1-\delta)\right)}{q_t}, \\
&\overset{(\text{A.5.11a})}{=} \frac{\alpha y_{t+1} - i_{t+1} + q_{t+1}z_{t+1}k_{t+2}}{q_t k_{t+1}}.
\end{aligned}$$

Computation of Second Moments. The solution of the model delivers policy functions for the stationary variables x_t. The period t level of a variable, thus, equals

$$X_t = Z_{t-1}^{\xi} x_t, \quad \xi \in \{1, \eta\}.$$

We assume $Z_0 \equiv 1$. Given a random sequence $\{\hat{z}_i\}_{i=1}^{t}$, $\hat{z}_i = \ln(z_i/z)$,

$$Z_{t-1} = \prod_{i=1}^{t-1} e^{\hat{z}_i} z.$$

The second moments are computed from logged and HP-filtered simulated time series of X_t.

A.6 Two-Sector Model with Predetermined Working Hours by the Households

The entire two-sector model where households decide upon their labor supply prior to the observation of the technology shock consists of the equations:

$$w_{Ct} = (1-\alpha)Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{A.6.1a}$$

$$w_{It} = p_t(1-\alpha)Z_t N_{It}^{-\alpha} K_{It}^{\alpha}, \tag{A.6.1b}$$

$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.6.1c}$$

$$r_{Ct} = \alpha Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{A.6.1d}$$

$$r_{It} = p_t \alpha Z_t N_{It}^{1-\alpha} K_{It}^{\alpha-1}, \tag{A.6.1e}$$

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \tag{A.6.1f}$$

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \tag{A.6.1g}$$

$$Y_t = C_t + I_t, \tag{A.6.1h}$$

$$N_t = N_{Ct} + N_{It}, \tag{A.6.1i}$$

$$K_t = K_{Ct} + K_{It}, \tag{A.6.1j}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{Ct+1}, \tag{A.6.1k}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{It+1}, \tag{A.6.1l}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.6.1m}$$

$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{Ct+1} \right), \tag{A.6.1n}$$

$$p_t \Lambda_t = \beta \mathbb{E}_t \lambda_{t+1} \left(p_{t+1}(1-\delta) + r_{It+1} \right), \tag{A.6.1o}$$

$$K_{t+1} = I_t + (1-\delta) K_t. \tag{A.6.1p}$$

A.7 Two-Sector Model with Stochastic Trend

Equilibrium Dynamics. In order to compute linear or quadratic approximate solutions of the model, we must transform it into a model in stationary variables. However, this requires $\eta = 1$, the assumption used by Boldrin, Christiano, and Fisher (2001). It will be convenient to put

$$\begin{aligned} x_t &:= \frac{X_t}{Z_{t-1}}, \quad X_t \in \{K_t, K_{Ct}, K_{It}, Y_t, C_t, I_t, w_{Ct}, w_{It}\}, \\ \lambda_t &:= \Lambda_t Z_{t-1}. \end{aligned} \tag{A.7.1}$$

This allows us to transform equations (A.6.1) into the following system:[22]

$$w_{Ct} = (1-\alpha) z_t^{1-\alpha} N_{Ct}^{-\alpha} k_{Ct}^{\alpha}, \tag{A.7.2a}$$

$$w_{It} = p_t (1-\alpha) z_t^{1-\alpha} N_{It}^{-\alpha} k_{It}^{\alpha}, \tag{A.7.2b}$$

$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.7.2c}$$

$$r_{Ct} = \alpha z_t^{1-\alpha} N_{Ct}^{1-\alpha} K_{Ct}^{\alpha-1}, \tag{A.7.2d}$$

$$r_{It} = p_t \alpha z_t^{1-\alpha} N_{It}^{1-\alpha} K_{It}^{\alpha-1}, \tag{A.7.2e}$$

$$c_t = z_t^{1-\alpha} N_{Ct}^{1-\alpha} k_{Ct}^{\alpha}, \tag{A.7.2f}$$

$$i_t = z_t^{1-\alpha} N_{It}^{1-\alpha} k_{It}^{\alpha}, \tag{A.7.2g}$$

$$y_t = c_t + i_t, \tag{A.7.2h}$$

[22] Note, that (A.7.1) implies that we redefine wages without using new symbols.

$$N_t = N_{Ct} + N_{It}, \tag{A.7.2i}$$

$$k_t = k_{Ct} + k_{It}, \tag{A.7.2j}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \lambda_{t+1} w_{Ct+1}, \tag{A.7.2k}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \lambda_{t+1} w_{It+1}, \tag{A.7.2l}$$

$$\lambda_t = (c_t - \chi^C (c_{t-1}/z_{t-1}))^{-\eta} - \beta \chi^C \mathbb{E}_t (z_t c_{t+1} - \chi^C c_t)^{-\eta}, \tag{A.7.2m}$$

$$p_t \lambda_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{z_t} \left(p_{t+1}(1-\delta) + r_{Ct+1} \right), \tag{A.7.2n}$$

$$p_t \lambda_t = \beta \mathbb{E}_t \frac{\lambda_{t+1}}{z_t} \left(p_{t+1}(1-\delta) + r_{It+1} \right), \tag{A.7.2o}$$

$$z_t k_{t+1} = i_t + (1-\delta) k_t. \tag{A.7.2p}$$

Balanced Growth Path. The balanced growth path is obtained by assuming $z_t \equiv z$ and $x_t = x_{t-1}$ for all variables x_t of the model. Using these assumption, equations (A.7.2n) and (A.7.2o) imply $r = r_C = r_I$. Together with the stationary versions of (A.7.2a) and (A.7.2b) this implies k_C/N_c=k_I/N_I=k/N and $p = 1$. This allows one to compute k/N from (A.7.2d) as:

$$\frac{k}{N} = \left(\frac{z - \beta(1-\delta)}{\alpha \beta z^{1-\alpha}} \right)^{1/(\alpha-1)}. \tag{A.7.3a}$$

Given the stationary value of average hours N, this equation also implies k. We can then use (A.7.2p) to find

$$i = (z - 1 + \delta) k. \tag{A.7.3b}$$

From $i = z^{1-\alpha} N_I (k/N)^\alpha$ and $i = z^{1-\alpha} k_I (k/N)^{\alpha-1}$, we get the stationary values of N_I and k_I, and, therefore $N_C = N - N_I$ and $k_C = k - k_I$. Given these solutions, $c = z^{1-\alpha} N_C^{1-\alpha} k_C^\alpha$ so that (A.7.2m) yields

$$\lambda = \frac{z - \beta b}{c(z-b)}. \tag{A.7.3c}$$

Equity Premium. The risk free rate of return is given by

$$r_t = \frac{\Lambda_t}{\beta \mathbb{E}_t \Lambda_{t+1}} = \frac{\lambda_t z_t}{\beta \mathbb{E}_t \lambda_{t+1}}$$

and can be computed via Gauss-Hermite quadrature from the policy function for λ_t as explained in Section 2.2. The rates of return on equity from both sectors as defined in (2.36a) involve stationary values only so that no change in the computation of the equity return is required.

As explained in Section A.5, we compute second moments from logged and HP-filtered levels of the variables.

A.8 Two-Sector Model with Capital Adjustment Costs

Households. The household maximizes (2.17) subject to (2.38) with respect to consumption C_t, labor supply N_{Ct+1}, N_{It+1}, S_{Ct+1}, and S_{It+1}. This yields the first-order conditions (2.2a), (2.35a), (2.35b), and

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{Ct+1} + v_{Ct+1}}{v_{Ct}}, \tag{A.8.1a}$$

$$1 = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \frac{d_{It+1} + v_{It+1}}{v_{It}}, \tag{A.8.1b}$$

which determine his portfolio allocation.

Firms. The representative firm in the consumption goods sector maximizes

$$V_{Ct} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} \left[C_{t+s} - w_{Ct+s} N_{Ct+s} - p_{t+s} I_{t+s} \right] \tag{A.8.2}$$

subject to

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \quad \alpha \in (0,1), \tag{A.8.3a}$$

$$K_{Ct+1} = \Phi(I_{Ct}/K_{Ct}) K_{Ct} + (1-\delta) K_{Ct}, \quad \delta \in (0,1], \tag{A.8.3b}$$

where in equilibrium $\varrho_{t+1} = \beta^s \frac{\Lambda_{t+1}}{\Lambda_t}$. The first-order conditions for the optimal choice of N_{Ct}, I_{It}, and K_{Ct+1} are:

$$w_{Ct} = (1-\alpha) Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{A.8.4a}$$

$$q_{Ct} = \frac{p_t}{\Phi'(I_{Ct}/K_{Ct})}, \tag{A.8.4b}$$

$$q_{Ct} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \left\{ \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha-1} - \frac{p_{t+1} I_{Ct+1}}{K_{Ct+1}} + q_{Ct+1} \left(\Phi(I_{Ct+1}/K_{Ct+1}) + 1 - \delta \right) \right\}, \tag{A.8.4c}$$

where q_{Ct} (Tobin's q) is the Lagrange multiplier on the equation governing capital accumulation. In addition, the transversality condition

$$\lim_{s \to \infty} \mathbb{E}_t \varrho_{t+s} q_{Ct+s} K_{Ct+s+1} = 0$$

must hold. As in the one-sector model of A.1, it can be shown that $V_{Ct} = q_{Ct}K_{Ct+1}$.

Analogously, the representative firm in the investment goods sector maximizes

$$V_{It} = \mathbb{E}_t \sum_{s=0}^{\infty} \varrho_{t+s} \left[p_{t+s} I_{t+s} - w_{It+s} N_{It+s} - p_{t+s} I_{It+s} \right] \tag{A.8.5}$$

subject to

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \quad \alpha \in (0,1), \tag{A.8.6a}$$

$$K_{It+1} = \Phi(I_{It}/K_{It})K_{It} + (1-\delta)K_{It}, \quad \delta \in (0,1]. \tag{A.8.6b}$$

The respective first-order conditions are:

$$w_{It} = (1-\alpha) Z_t N_{It}^{-\alpha} K_{It}^{\alpha}, \tag{A.8.7a}$$

$$q_{It} = \frac{p_t}{\Phi'(I_{It}/K_{It})}, \tag{A.8.7b}$$

$$q_{It} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ p_{t+1} \alpha Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{\alpha-1} - \frac{p_{t+1} I_{It+1}}{K_{It+1}} + q_{It+1} \left(\Phi(I_{It+1}/K_{It+1}) + 1 - \delta \right) \Bigg\}. \tag{A.8.7c}$$

Firms from both sectors transfer their profits less retained earnings as dividends to the household sector

$$d_{Ct}S_{Ct} = C_t - w_{Ct}N_{Ct} - RE_{Ct},$$
$$d_{It}S_{It} = p_t I_t - w_{It}N_{It} - RE_{It},$$

and finance the remaining investment expenditures by issuing new equity $v_{Xt}(S_{Xt+1} - S_{Xt}) = p_t I_{Xt} - RE_{Xt}$. Thus, in equilibrium, the budget constraint of the household implies the definition of GDP, $Y_t = C_t + p_t I_t$.

Equilibrium Conditions. The full model is described by 18 equations:

$$w_{Ct} = (1-\alpha) Z_t N_{Ct}^{-\alpha} K_{Ct}^{\alpha}, \tag{A.8.8a}$$

$$w_{It} = (1-\alpha) Z_t N_{It}^{-\alpha} K_{It}^{\alpha}, \tag{A.8.8b}$$

$$q_{Ct} = \frac{p_t}{\Phi'(I_{Ct}/K_{Ct})}, \tag{A.8.8c}$$

$$q_{It} = \frac{p_t}{\Phi'(I_{It}/K_{It})}, \tag{A.8.8d}$$

$$N_t = N_{Ct} + N_{It}, \tag{A.8.8e}$$

$$K_t = K_{Ct} + K_{It}, \tag{A.8.8f}$$

$$w_t = \frac{N_{Ct}}{N_t} w_{Ct} + \frac{N_{It}}{N_t} w_{It}, \tag{A.8.8g}$$

$$C_t = Z_t N_{Ct}^{1-\alpha} K_{Ct}^{\alpha}, \tag{A.8.8h}$$

$$I_t = Z_t N_{It}^{1-\alpha} K_{It}^{\alpha}, \tag{A.8.8i}$$

$$Y_t = C_t + I_t, \tag{A.8.8j}$$

$$I_t = I_{Ct} + I_{It}, \tag{A.8.8k}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.8.8l}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{Ct+1}, \tag{A.8.8m}$$

$$\nu_0 N_{t+1}^{\nu_1} = \mathbb{E}_t \Lambda_{t+1} w_{It+1}, \tag{A.8.8n}$$

$$\begin{aligned} q_{Ct} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ & \alpha Z_{t+1} N_{Ct+1}^{1-\alpha} K_{Ct+1}^{\alpha-1} - \frac{p_{t+1} I_{Ct+1}}{K_{Ct+1}} \\ & + q_{Ct+1} \left(\Phi(I_{Ct+1}/K_{Ct+1}) + 1 - \delta \right) \Bigg\}, \end{aligned} \tag{A.8.8o}$$

$$\begin{aligned} q_{It} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Bigg\{ & p_{t+1} \alpha Z_{t+1} N_{It+1}^{1-\alpha} K_{It+1}^{1-\alpha} - \frac{p_{t+1} I_{It+1}}{K_{It+1}} \\ & + q_{It+1} \left(\Phi(I_{It+1}/K_{It+1}) + 1 - \delta \right) \Bigg\}, \end{aligned} \tag{A.8.8p}$$

$$K_{Ct+1} = \Phi(I_{Ct}/K_{Ct}) K_{Ct} + (1-\delta) K_{Ct}, \tag{A.8.8q}$$

$$K_{It+1} = \Phi(I_{It}/K_{It}) K_{It} + (1-\delta) K_{It}. \tag{A.8.8r}$$

We employ the assumptions about the function Φ from Section 2.2. Therefore, the model has the same stationary solution as the two sector model in the previous subsection.

A.9 New-Keynesian Model with Sticky Prices

Households. Households enter the current period t with a given amount of firm shares S_t and given stocks of nominal money balances M_t and nominal Bonds B_t. The current price level is P_t. Bonds pay a predetermined nominal rate of interest $Q_t - 1$. The real share price is v_t and real dividend payments per share are d_t.[23] Firms pay the

[23] De Paoli, Scott, and Weeken (2010) distinguish between real and nominal bonds and consider different maturities of nominal bonds. They also assume that share prices and dividends are denoted

real wage w_t per unit of working hours N_t. Government transfers to the households are T_t in units of money. Thus,

$$v_t(S_{t+1} - S_t) + \frac{M_{t+1} - M_t}{P_t} + \frac{B_{t+1} - B_t}{P_t} \leq w_t N_t + (Q_t - 1)\frac{B_t}{P_t} + d_t S_t + \frac{T_t}{P_t} - C_t \quad \text{(A.9.1)}$$

is the household's budget constraint.

The current period utility function is

$$\begin{aligned} u(C_t, C_{t-1}, N_t, N_{t-1}, M_{t+1}/P_t) = & \frac{(C_t - \chi^C C_{t-1})^{1-\eta} - 1}{1-\eta} \\ & - \nu_0 \frac{(N_t - \chi^N N_{t-1})^{1+\nu_1} - 1}{1+\nu_1} \\ & + \theta^M \frac{\left(\frac{M_{t+1}}{P_t}\right)^{1-\gamma} - 1}{1-\gamma}. \end{aligned} \quad \text{(A.9.2)}$$

Households maximize

$$U_t = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s u(C_{t+s}, C_{t+s-1}, N_{t+s}, N_{t-1+s}, M_{t+1+s}/P_{t+s})$$

subject to (A.9.1) and given initial values of S_t, M_t, and B_t. De Paoli, Scott, and Weeken (2010) assume that the household treats previous consumption C_{t-1} and previous working hours N_{t-1} as given, when he decides on current consumption and working hours. Thus, different from equations (2.2a) and (2.22), the first order conditions are:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \quad \text{(A.9.3a)}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \quad \text{(A.9.3b)}$$

$$v_t = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1} + d_{t+1}), \quad \text{(A.9.3c)}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \quad \text{(A.9.3d)}$$

$$\Lambda_t = \beta \mathbb{E}_t \left(\theta^M m_{t+1}^{-\gamma} + \frac{\Lambda_{t+1}}{\pi_{t+1}} \right), \quad m_{t+1} = \frac{M_{t+1}}{P_t}, \quad \text{(A.9.3e)}$$

where Λ_t is the Lagrange multiplier of the time t budget constraint.

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in units of money and not in units of goods. However, since the equilibrium conditions of the model boil down to conditions in real share prices and real dividends, we can assume this right away. Furthermore, since our focus is on the cross-correlations of output, hours, and the real wage, we restrict the spectrum of financial assets to a one-period nominal bond, money, and stocks.

Firms. The Lagrange function of the firm's maximization problem can be written as:

$$\begin{aligned}\mathscr{L} = \mathbb{E}_t \sum_{s=0}^{\infty} \beta^s \frac{\Lambda_{t+s}}{\Lambda_t} \Bigg\{ & \left(\frac{P_{t+s}(j)}{P_{t+s}}\right)^{-\epsilon_y} Y_{t+s} - w_{t+s} N_{t+s}(j) - I_{t+s}(j) \\ & - \frac{\psi}{2}\left(\frac{P_{t+s}(j)}{\pi P_{t+s-1}(j)} - 1\right)^2 Y_{t+s} \\ & + q_{t+s}\left[(1-\delta)K_{t+s}(j) + \Phi\left(\frac{I_{t+s}(j)}{K_{t+s}(j)}\right) K_{t+s}(j) - K_{t+s+1}(j)\right] \\ & + \Gamma_{t+s}\left[Z_{t+s} N_{t+s}(j)^{1-\alpha} K_{t+s}(j)^{\alpha} - \left(\frac{P_{t+s}(j)}{P_{t+s}}\right)^{-\epsilon_y} Y_{t+s}\right]\Bigg\}.\end{aligned}$$

Differentiating this expression with respect to $N_t(j)$, $I_t(j)$, $K_{t+1}(j)$, and $P_t(j)$ and setting the ensuing results equal to zero yields the first-order conditions stated in (3.9).

Money Supply. The central bank satisfies the money demand that originates from the Taylor rule (3.10). This implies the money growth factor μ_t:

$$\mu_t = \frac{M_{t+1}}{M_t}. \tag{A.9.4}$$

The government transfers the seignorage lump sum to the households so that

$$\frac{T_t}{P_t} = \frac{M_{t+1} - M_t}{P_t}. \tag{A.9.5}$$

Temporary Equilibrium. In equilibrium the supply of bonds is zero, $B_t = 0$, the supply of shares is constant, and all intermediate producers choose the same nominal price $P_t(j)$ so that – via the definition of the price index (3.5) – the relative price of each producer equals unity, and individual prices $P_t(j)$, output $Y_t(j)$, working hours $N_t(j)$, capital services $K_t(j)$, investment expenditures $I_t(j)$, and dividend payments $D_t(j)$ equal the respective aggregate quantities. Therefore, the budget constraint (A.9.1) simplifies to the economy's resource constraint $Y_t = C_t + I_t$, and (3.6) implies the aggregate production function $Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}$. The dynamics of the model is governed by equations (A.9.3), the simplified equations (3.7), (3.8), (3.9), the resource constraint, the production function, the Taylor rule (3.10), and (A.9.4). For convenience, we repeat this set of equations, yet in a different ordering with the static equations appearing first.

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{A.9.6a}$$

$$\Lambda_t w_t = \nu_0 (N_t - \chi^N N_{t-1})^{\nu_1}, \tag{A.9.6b}$$

$$w_t = (1-\alpha)\Gamma_t Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.9.6c}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.9.6d}$$

$$Y_t = C_t + I_t, \tag{A.9.6e}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.9.6f}$$

$$d_t = Y_t - w_t N_t - I_t, \tag{A.9.6g}$$

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t)K_t, \tag{A.9.6h}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}\Bigg[\alpha \Gamma_{t+1} Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - \frac{I_{t+1}}{K_{t+1}} + q_{t+1}\left(1-\delta+\Phi(I_{t+1}/K_{t+1})\right)\Bigg], \tag{A.9.6i}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \tag{A.9.6j}$$

$$\Lambda_t = \mathbb{E}_t \left(\theta^M m_{t+1}^{-\gamma} + \beta \frac{\Lambda_{t+1}}{\pi_{t+1}}\right), \tag{A.9.6k}$$

$$v_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t}(v_{t+1} + d_{t+1}), \tag{A.9.6l}$$

$$m_{t+1} = \frac{\mu_t m_t}{\pi_t}, \tag{A.9.6m}$$

$$Q_{t+1} = Q_t^{\rho^Q} \left(\frac{\pi}{\beta}\right)^{1-\rho^Q} \left(\frac{\pi_t}{\pi}\right)^{\varphi(1-\rho^Q)} e^{\epsilon_t^Q}, \tag{A.9.6n}$$

$$0 = (1-\epsilon_y)Y_t + \epsilon_y \Gamma_t Y_t - \psi\left(\frac{\pi_t}{\pi} - 1\right)\frac{\pi_t}{\pi} Y_t + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{\pi_{t+1}}{\pi} - 1\right)\frac{\pi_{t+1}}{\pi} Y_{t+1}. \tag{A.9.6o}$$

Note that equation (A.9.6g) derives from equation (3.7) if we normalize the outstanding shares to unity. Equation (A.9.6m) is just another way to write the definition of end-of-period real money balances $m_t = M_{t+1}/P_t$ given the definition of the money growth factor μ_t and the inflation factor π_t. Since the nominal interest rate Q_{t+1} is determined in period t, it is non-stochastic with respect to the conditional expectations operator

$\mathbb{E}_t$. Thus, condition (3.2d) can be written as

$$\frac{\Lambda_t}{Q_{t+1}} = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\pi_{t+1}},$$

which allows one to reduce the first-order condition (A.9.6k) to a static equation by using the definition (A.9.6m) and the Taylor rule (A.9.6n).[24] Considering the maximization problem of the firm, equation (A.9.6l) recursively defines the end-of-period value of the firm, if investment is entirely financed from internal funds.

Stationary Equilibrium. As usual, the stationary equilibrium is defined by setting the shocks equal to their unconditional means and by assuming $x_{t+1} = x_t = x$ for all variables x of the model. In this case, equation (A.9.6o) simplifies to

$$\Gamma = \frac{\epsilon_y - 1}{\epsilon_y}, \tag{A.9.7a}$$

and equation (A.9.6i) reduces to

$$\frac{Y}{K} = \frac{1 - \beta(1-\delta)}{\alpha\beta\Gamma}, \tag{A.9.7b}$$

so that for given N the stationary stock of capital equals

$$K = N(Y/K)^{\frac{1}{\alpha-1}}. \tag{A.9.7c}$$

and output Y is determined by (A.9.6d). Given the properties of the adjustment cost function Φ (see Section 2.2), equation (A.9.6h) implies

$$I = \delta K, \tag{A.9.7d}$$

and we get the stationary value of consumption from the resource constraint (A.9.6e). Given the solution for C we can compute the solution for Λ from (A.9.6a). The stationary real wage follows from equation (A.9.6c). This allows us to determine the parameter ν_0:

$$\nu_0 = \Lambda w \left(N - \chi^N N\right)^{-\nu_1}. \tag{A.9.7e}$$

Dividends d follow from equation (A.9.6g). The stationary share price derives from (A.9.6l):

$$v = \frac{\beta}{1-\beta} d. \tag{A.9.7f}$$

[24] Otherwise, the model must be solved by using the generalized Schur factorization.

In the stationary equilibrium, the Taylor rule (A.9.6n) fixes the nominal interest rate factor Q for a given inflation target π:

$$Q = \frac{\pi}{\beta}, \tag{A.9.7g}$$

and (A.9.6m) implies $\mu = \pi$. Finally, given θ^M, equation (A.9.6k) can be used to determine the stationary end-of-period level of real money balances m:

$$m = \left(\frac{\Lambda(1 - (\beta/\pi))}{\theta^M}\right)^{-1/\gamma}. \tag{A.9.7h}$$

A.10 New Keynesian Model with Sticky Wages

The Optimal Relative Wage. Substituting from (3.12) in (3.16) and (A.9.1) yields the Lagrangian for choosing the optimal wage:

$$\begin{aligned}
\mathscr{L} = \mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \Bigg\{ \Bigg[& \frac{(C_{t+s}(h) - \chi^C C_{t+s-1}(h))^{1-\eta} - 1}{1-\eta} \\
& - \frac{\nu_0}{1+\nu_1} \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+s}^{1+\nu_1} + \theta \frac{\left(\frac{M_{t+s+1}(h)}{P_{t+s}}\right)^{1-\gamma} - 1}{1-\gamma} \Bigg] \\
& + \Lambda_{t+s}(h) \Bigg[\frac{\pi^s W_t(h)}{P_{t+s}} \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w} N_{t+s} + S_{t+s}(h) d_{t+s} \\
& + (Q_{t+s} - 1) \frac{B_{t+s}(h)}{P_{t+s}} - \frac{T_t(h)}{P_{t+s}} - C_{t+s}(h) \\
& - \frac{M_{t+s+1}(h) - M_{t+s}(h) + B_{t+s+1}(h) - B_{t+s}(h)}{P_{t+s}} \\
& - v_{t+s}(S_{t+s+1}(h) - S_{t+s}(h)) \Bigg] \Bigg\}.
\end{aligned}$$

Differentiating with respect to $W_t(h)$ and setting the ensuing expression equal to zero delivers

$$\mathbb{E}_t \sum_{s=0}^{\infty} (\beta\varphi_w)^s \left\{ N_{t+s}(h) \left[\nu_0 N_{t+s}(h)^{\nu_1} - \frac{\epsilon_w - 1}{\epsilon_w} \Lambda_{t+s}(h) \frac{\pi^s W_t(h)}{P_{t+s}} \right] \right\}.$$

We assume that there is a sufficiently rich set of contingent security markets so that a representative agent exists. Thus, $\Lambda_{t+s}(h) = \Lambda_{t+s}$ and all wage setters will opt for the same relative wage $w_{At} \equiv \frac{W_t(h)}{W_t}$. Therefore, the preceding condition can be stated as:

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\Gamma_{1t}}{\Gamma_{2t}}, \tag{A.10.1a}$$

$$\Gamma_{1t} = \mathbb{E}_t \sum_{s=0}^{\infty}(\beta\varphi_w)^s \nu_0 \left(\frac{\pi^s W_t(h)}{W_{t+s}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+s}^{1+\nu_1}, \tag{A.10.1b}$$

$$\Gamma_{2t} = \mathbb{E}_t \sum_{s=0}^{\infty}(\beta\varphi_w)^s \Lambda_{t+s}\frac{\pi^s W_t}{P_{t+s}} \left(\frac{\pi^s W_t(h)}{W_{t+s}}^{-\epsilon_w}\right) N_{t+s}. \tag{A.10.1c}$$

The auxiliary variables Γ_{1t} and Γ_{2t} have a recursive definition. Consider (3.18b):

$$\begin{aligned}\Gamma_{1t} = \mathbb{E}_t \Bigg\{ &\nu_0 \left(\frac{W_t(h)}{W_t}\right)^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + (\beta\phi_w)\nu_0 \left(\frac{\pi W_t(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} \\ &+ (\beta\phi_w)^2\nu_0 \left(\frac{\pi^2 W_t(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} + \dots \Bigg\}\end{aligned} \tag{A.10.2}$$

Therefore,

$$\begin{aligned}\Gamma_{1t+1} = \mathbb{E}_{t+1} \Bigg\{ &\nu_0 \left(\frac{W_{t+1}(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} + (\beta\phi_w)\nu_0 \left(\frac{\pi W_{t+1}(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} \\ &+ (\beta\phi_w)^2\nu_0 \left(\frac{\pi^2 W_{t+1}(h)}{W_{t+3}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+3}^{1+\nu_1} + \dots \Bigg\}\end{aligned}$$

From the perspective of period $t+1$ the variables $W_t(h)$, $W_{t+1}(h)$, and W_{t+1} are non-random. Thus, multiplying the previous equation on both sides by

$$(\beta\phi_w)\left(\pi \frac{(W_t(h)/W_t)}{W_{t+1}(h)/W_{t+1}}\frac{W_t}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} \equiv (\beta\phi_w)\left(\frac{\pi w_{At}}{w_{At+1}}\frac{1}{\omega_{t+1}}\right)^{-\epsilon_w(1+\nu_1)}$$

and taking expectations as of period t yields (since $\mathbb{E}_t\mathbb{E}_{t+1}\{\cdot\} = \mathbb{E}_t\{\cdot\}$ by the law of iterated expecations)

$$\begin{aligned}&(\beta\phi_w)\mathbb{E}_t\left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}}\right)^{-\epsilon_w(1+\nu_1)} \Gamma_{1t+1} \\ &= \mathbb{E}_t \Bigg\{ (\beta\phi_w)\nu_0 \left(\frac{\pi W_t(h)}{W_{t+1}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+1}^{1+\nu_1} \\ &\qquad + (\beta\phi_w)^2\nu_0 \left(\frac{\pi^2 W_t(h)}{W_{t+2}}\right)^{-\epsilon_w(1+\nu_1)} N_{t+2}^{1+\nu_1} + \dots \Bigg\}.\end{aligned}$$

Together with (A.10.2) this establishes:

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + \beta\varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}}\right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.10.3a}$$

Analogously, the recursive definition of the auxiliary variable Γ_{2t},

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta\varphi_w \left(\frac{\pi}{\omega_{t+1}}\right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}}\right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.10.3b}$$

can be derived, where

$$w_t = \frac{W_t}{P_t}, \tag{A.10.3c}$$

$$\omega_t = \frac{W_t}{W_{t-1}}. \tag{A.10.3d}$$

Finally, note that $W_{t-1}(h) = W_{t-1}$ for those that cannot adjust their wage optimally. Thus, equation (3.13) implies:

$$W_t^{1-\epsilon_w} = (1-\varphi_w) W_{At}^{1-\epsilon_w} + \varphi_w (\pi W_{t-1})^{1-\epsilon_w}$$

or

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w (\pi/\omega_t)^{1-\epsilon_w}. \tag{A.10.4}$$

Equilibrium Conditions. The equilibrium conditions of the model consist of the firm's optimality conditions stated in (2.11), the production function (2.3), the capital accumulation equation (2.7), the economy's resource constraint implied by the household's budget constraint, the wage setting equations (3.18a)-(3.18d), the household's optimality conditions (2.2a), (A.9.3c)-(A.9.3e), and the Taylor rule (3.10). We disregard the solution for the stock of real balances so that the following 14 equations determine the time path of Y_t, C_t, I_t, N_t, K_t, w_t, w_{At}, ω_t, Q_t, π_t, q_t, Λ_t, Γ_{1t}, and Γ_{2t}.

$$w_t = (1-\alpha) Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.10.5a}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.10.5b}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.10.5c}$$

$$Y_t = C_t + I_t, \tag{A.10.5d}$$

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\Gamma_{1t}}{\Gamma_2 t}, \tag{A.10.5e}$$

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w (\pi/\omega_t)^{1-\epsilon_w}, \tag{A.10.5f}$$

$$w_t = \frac{\omega_t}{\pi_t} w_{t-1}, \tag{A.10.5g}$$

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t)K_t, \tag{A.10.5h}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Big\{ \alpha Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta \big] \Big\} \tag{A.10.5i}$$

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta} - \beta \chi^C \mathbb{E}_t (C_{t+1} - \chi^C C_t)^{-\eta}, \tag{A.10.5j}$$

$$\Lambda_t = \beta \mathbb{E}_t \Lambda_{t+1} \frac{Q_{t+1}}{\pi_{t+1}}, \tag{A.10.5k}$$

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + \beta \varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}} \right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.10.5l}$$

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta \varphi_w \left(\frac{\pi}{\omega_{t+1}} \right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}} \right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.10.5m}$$

$$Q_{t+1} = Q_t^{\rho^Q} \left(\frac{\pi}{\beta} \right)^{1-\rho^Q} \left(\frac{\pi_t}{\pi} \right)^{\varphi(1-\rho^Q)} e^{\epsilon_t^Q}. \tag{A.10.5n}$$

Stationary Solution. In the stationary equilibrium of the deterministic counterpart of the model, equation (A.10.5i) implies

$$\frac{Y}{K} = \frac{1-\beta(1-\delta)}{\alpha\beta}.$$

Given the stationary value of hours N, (A.10.5e) yields the stationary stock of capital

$$K = N \left(\frac{1-\beta(1-\delta)}{\alpha\beta} \right)^{\frac{1}{\alpha-1}}.$$

Given the assumptions with respect to $\Phi(I/K)$ investment equals $I = \delta K$ so that consumption follows from (A.10.5d). Given C the stationary version of (A.10.5j) yields Λ.

In equilibrium, wage inflation ω must equal price inflation π – the target of the monetary authority. Equation (A.10.5f), thus, implies $w_A = 1$. Therefore, equations (A.10.5e), (A.10.5l), and (A.10.5m) reduce to

$$1 = \frac{\epsilon_w}{\epsilon_w - 1} \frac{\nu_0 N^{1+\nu_1}}{\Lambda w N}.$$

We use this equation to fix the unknown parameter ν_0 yielding:

$$\nu_0 = (1-\alpha) \frac{\epsilon_w - 1}{\epsilon_w} \Lambda K^\alpha N^{-(\alpha+\nu_1)}.$$

A.11 Sticky Prices and Sticky Wages

This model merges the models of Section 3.1 and 3.2. As in the body of the paper we neglect real money balances since they do not interact with the rest of the model.

Households. The preferences of a member h of the household sector are the same as in (3.14), where N_t is defined in (3.11). The budget constraint of the representative household is

$$v_t(S_{t+1} - S_t) + \frac{B_{t+1} - B_t}{P_t} \leq w_t N_t + (Q_t - 1)\frac{B_t}{P_t} + d_t S_t - C_t.$$

Therefore, the household's decisions with respect to consumption, portfolio allocation, and nominal wages satisfy equations (3.2a), (3.2c), (3.2d), and (3.18a)-(3.18f).

Firms. The production sector is the same as depicted in Section 3.1 so that equations (3.9a)-(3.9d) describe the optimal decisions of producer j.

Equilibrium Dynamics. The full model, thus, is described by the following set of equations:

$$\Lambda_t = (C_t - \chi^C C_{t-1})^{-\eta}, \tag{A.11.1a}$$

$$w_t = (1-\alpha)\Gamma_t Z_t N_t^{-\alpha} K_t^{\alpha}, \tag{A.11.1b}$$

$$Y_t = Z_t N_t^{1-\alpha} K_t^{\alpha}, \tag{A.11.1c}$$

$$Y_t = C_t + I_t, \tag{A.11.1d}$$

$$q_t = \frac{1}{\Phi'(I_t/K_t)}, \tag{A.11.1e}$$

$$d_t = Y_t - w_t N_t - I_t, \tag{A.11.1f}$$

$$w_{At} = \frac{\epsilon_w}{\epsilon_w - 1}\frac{\Gamma_{1t}}{\Gamma_2 t}, \tag{A.11.1g}$$

$$1 = (1-\varphi_w) w_{At}^{1-\epsilon_w} + \varphi_w (\pi/\omega_t)^{1-\epsilon_w}, \tag{A.11.1h}$$

$$w_t = \frac{\omega_t}{\pi_t} w_{t-1}, \tag{A.11.1i}$$

$$K_{t+1} = (1-\delta)K_t + \Phi(I_t/K_t)K_t, \tag{A.11.1j}$$

$$v_t = \beta \mathbb{E}_t \Lambda_{t+1}(v_{t+1} + d_{t+1}), \tag{A.11.1k}$$

$$\Lambda_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1} Q_{t+1}}{\pi_{t+1}}, \quad \pi_{t+1} = \frac{P_{t+1}}{P_t}, \tag{A.11.1l}$$

$$q_t = \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \Big\{ \alpha \Gamma_{t+1} Z_{t+1} N_{t+1}^{1-\alpha} K_{t+1}^{\alpha-1} - (I_{t+1}/K_{t+1}) + q_{t+1} \big[\Phi(I_{t+1}/K_{t+1}) + 1 - \delta \big] \Big\} \tag{A.11.1m}$$

$$\Gamma_{1t} = \nu_0 w_{At}^{-\epsilon_w(1+\nu_1)} N_t^{1+\nu_1} + \beta \varphi_w \mathbb{E}_t \left(\frac{\pi w_{At}}{\omega_{t+1} w_{At+1}} \right)^{-\epsilon_w} \Gamma_{1t+1}, \tag{A.11.1n}$$

$$\Gamma_{2t} = \Lambda_t w_t w_{At}^{-\epsilon_w} N_t + \beta \varphi_w \left(\frac{\pi}{\omega_{t+1}} \right)^{1-\epsilon_w} \left(\frac{w_{At}}{w_{At+1}} \right)^{-\epsilon_w} \Gamma_{2t+1}, \tag{A.11.1o}$$

$$\begin{aligned} 0 &= (1-\epsilon_y) Y_t + \epsilon_y \Gamma_t Y_t - \psi \left(\frac{\pi_t}{\pi} - 1 \right) \frac{\pi_t}{\pi} Y_t \qquad \text{(A.11.1p)} \\ &\quad + \beta \mathbb{E}_t \frac{\Lambda_{t+1}}{\Lambda_t} \psi \left(\frac{\pi_{t+1}}{\pi} - 1 \right) \frac{\pi_{t+1}}{\pi} Y_{t+1}. \end{aligned}$$

$$Q_{t+1} = Q_t^{\rho^Q} \left(\frac{\pi}{\beta} \right)^{1-\rho^Q} \left(\frac{\pi_t}{\pi} \right)^{\varphi(1-\rho^Q)} e^{\epsilon_t^Q}. \tag{A.11.1q}$$

Appendix B

B.1 Fixed Parameters and Targets

This appendix presents the results of our study with respect to the US economy. We take the parameters that remain the same in all models from Heer and Maußner (2008). They are displayed in Table B.1. The sources of our empirical targets (displayed in the first row of Table B.2) are Mehra and Prescott (1985) for the equity premium of 6.18% p.a., Cooley and Prescott (1995) for the relative volatilities of investment $s_I/s_Y = 2.97$, hours $s_N/s_Y = 0.98$, the real wage $s_w/s_Y = 0.44$ and the correlation between output and hours $r_{YN} = 0.78$, and Galí and van Rens (2010) for the correlation between hours and the reals wage $r_{wN} = 0.21$.

Table B.1

US Calibration

Preferences	β=0.99	η=1	τ=0.3	N=0.33
Production	α=0.36	δ=0.025		
Stationary Shocks	ρ^Z=0.95	σ^Z=0.00712		
Integrated Shocks	$\ln \bar{z}$=0.004	$\sigma_{\ln z}$=0.018		

B.2 Results

Table B.2 presents the results obtained from stochastic simulations of the models considered in the body of the paper.

We set the free parameters in the benchmark model of Jermann (1998), χ^C and ζ, so that the model reproduces the equity premium and the relative volatility of investment expenditures. Since it is common in studies of the US economy to assume log-linear preferences in consumption, $\eta = 1$, we need a stronger habit than in the German calibration ($\chi^C = 0.951$ instead of $\chi^C = 0.793$) to achieve this goal. The adjustment cost parameter $\zeta = 5.345$ is not very different from its value of $\zeta = 5.53$ in the German calibration.[25]

[25] Jermann (1998) employs $\eta = 5$, $\chi^C = 0.82$, and $\zeta = 4.35$.

Table B.2

Summary of Results - US Calibration

	Equity premium	s_Y	s_I/s_Y	s_N/s_Y	s_w/s_Y	r_{YN}	r_{wN}	Score
Data								
	6.18	1.72	2.97	0.98	0.44	0.78	0.21	
Models								
2. Real Business Cycle Models								
Benchmark								
exogenous labor	6.18	0.92	2.97					
endogenous labor	0.48	0.33	2.37	2.96	3.78	−0.77	−0.99	51.71
Habit in leisure	5.58	0.42	1.80	1.87	2.82	−0.92	−0.99	12.52
Predetermined hours								
Firms	0.01	0.93	2.87	0.05	0.94	0.67	0.68	39.47
Households	6.12	0.69	2.78	0.94	1.58	−0.32	−0.80	3.56
Sticky real wages	5.96	1.88	2.49	0.74	0.79	0.63	−0.13	0.60
Two sector models								
Stationary growth	4.26	0.96	1.79	0.18	4.18	0.75	−0.13	19.83
Integrated growth	5.80	1.52	1.43	0.13	3.09	0.75	−0.02	10.30
Adjustment costs	5.08	0.96	1.71	0.17	4.55	0.75	−0.15	20.49
3. New Keynesian Models								
Sticky prices	0.24	0.63	2.27	0.88	1.77	−0.65	−0.78	40.51
Sticky wages	6.19	2.38	2.38	1.41	0.61	0.92	−0.77	1.62
Sticky prices and wages	6.13	2.72	2.95	1.49	0.14	0.94	0.13	0.38

Notes: s_x:=Standard deviation of time series x, where $x \in \{Y, I, N, w\}$ and Y, I, and N denote output, investment, hours, and the wage, respectively. s_x/s_y:=standard deviation of variable x relative to standard deviation of output y. r_{NY}:=Cross-correlation of variable hours with output, r_{wN}:=Cross-correlation of the real wage with hours. The column Score presents the sum of squared differences between the moments from simulations of the model and the moments from the data.

As in the German case, the model with endogenous labor fails to generate a sizeable equity premium and predicts counterfactual negative correlations between output and hours and between hours and the real wage.

The model with habits in both consumption and hours has three free parameters. Our search on a coarse grid for the minimizing values of $\chi^C \in [0.1, 0.95]$, $\chi^N \in [0.1, 0.95]$, and $\zeta \in [0.2, 20]$ found $\chi^C = 0.90$, $\chi^N = 0.95$, and $\zeta = 17.5$. Though the model is able to predict a sizeable equity premium of 5.58% p.a. it implies that hours and the real wage are much more volatile than in the data. Furthermore, it is also unable to produce the right signs of the correlations between output and hours and between hours and the real wage.

Among the models with predetermined hours, the model where firms employ workers before the realization of the productivity shock is clearly inferior to the model where the household predetermines the supply of hours. In the first version of the model, the search over the free parameters yielded $\chi^C = 0.70$ and $\zeta = 0.2$ with a score of 39.47. In the second version, $\chi^0.91$ and $\zeta = 7.8$ imply a score of 3.56. While the first version produces the correct signs for the labor market correlations it badly fails with respect to the equity premium. The second version is able to predict the equity premium but fails with respect to the signs of the labor market correlations.

The search over the six free parameters of the sticky real wage model found $\chi^C = 0.935$, $\chi^L = 0.96$, $\lambda^C = 0.68$, $\lambda^L = 0.01$, $\zeta = 2.7$, and $\mu = 0.95$ implying a model score of 0.60. Thus, as in the German case, it requires strong habits and a high degree of real wage rigidity to explain both the equity premium and the labor market facts.

The two sector model of Boldrin, Christiano, and Eichenbaum (2001) has just one free parameter. In the version with a stationary productivity shock, $\chi^C = 0.79$ minimized our score statistic. Though the model is able to predict a sizeable equity premium, it implies an excessive volatility of the real wage. Overall, it fits the US data quite less well than does the German calibration with respect to our German target statistics. The version with an integrated technology shock and $\chi^C = 0.76$ performs much better with a score of 10.30, though in this model, too, wages are too volatile. In the adjustment cost version of the model $\chi^C = 0.80$ and $\zeta = 0.001$ yield a score of 20.49. Thus, as in the German calibration, adjustment costs do not yield a better model.

The results reported for the sticky price model employ $\epsilon_y = 6$ and $\delta_2 = 1.5$. The five free parameters are χ^C, χ^N, ζ, ψ, δ_1, σ^Q. We searched over a coarse grid covering the intervals $\chi^C \in [0.5, 0.95]$, $\chi^N \in [0.2, 0.95]$, $\zeta \in [1.2, 4.25]$, $\psi \in [0.01, 140]$, $\delta_1 \in [0.01, 0.95]$ and $\sigma^Q \in [0.5\sigma^Z, 5\sigma^Z]$. The minimizer is the set of parameters $\chi^C = 0.92$, $\chi^N = 0.20$, $\zeta = 1.8$, $\psi = 1.0$, $\delta_1 = 0.01$, and $\sigma^Q = 5\sigma^Z$. As in the German calibration, the model is neither able to predict a sizeable equity premium nor the correct sign of the labor market correlations. With a score of 40.54 it is only slightly better that the real model with endogenous labor supply. Remarkably, price rigidity does not play any role in this model, since with $\psi = 1$ prices are almost perfectly flexible.

Our simulations of the sticky wage model use $\epsilon_w = 4$ from Erceg, Henderson, and Levin (2000). The best choice (on a coarse grid) of the free parameters is $\chi^C = 0.95$, $\zeta = 2.5$, $\phi_w = 0.93$, $\delta_1 = 88$, and $\sigma^Q = 4.8\sigma^Z$. The model is in good accordance with the US-targets, except for the negative correlation between hours and the real wage. With a score of 1.62 it fits the fact much better than the sticky price model. Note that $\phi_w = 0.93$ indicates a high degree of wage stickiness and that it requires a predominance of monetary shocks over productivity shocks $\sigma^Q/\sigma^Z = 4.8$ to achieve this result.

In the model with both sticky nominal prices and wages we set $\epsilon_y = 6.0$ and $\epsilon_w = 4.0$. The score statistic for this model is 0.38. It is achieved for $\chi^C = 0.95$, $\zeta = 1.8$, $\psi = 170$, $\phi_w = 0.85$, $\delta_1 = 0.90$, and $\sigma^Q = 4.5\sigma^Z$. As compared to the sticky real wage model of Uhlig (2007) the real wage stickiness that results from a combination of nominal price and wage stickiness comes slightly closer to the US targets.

Summarizing, the overall picture is similar to the results obtained with respect to the German calibration. Among the one-sector real business cycle models the sticky real wage model has the smallest score statistic. The model with hours predetermined by the household comes closer to the facts than the two sector models. In the US calibration this result is more pronounced. Different from the German calibration the two-sector model with integrated technology shocks dominates the model with stationary shocks. With respect to the three New Keynesian models sticky wages are in much better accordance with the empirical facts than the sticky prices model. For US data, adding sticky prices to this model even improves the fit with the data.

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