Guest editor's introduction: Special issue on Relation Algebra and Kleene Algebra

Relation Algebra was originally introduced for compacting first-order logic formulas, by eliminating certain quantifiers, into a form that is amenable to simple algebraic manipulation, i.e., (in)equational reasoning. While initially only a fragment of predicate logic was covered, full expressiveness was achieved by the extension to Fork Algebra. In parallel, Kleene Algebra, originating in formal language theory, developed into a general algebraic system for calculating with sequential composition, choice and finite iteration. More recently it has also been extended to deal with infinite iteration and modal operators. Relation Algebra and Kleene Algebra fit well together, since the former is an enrichment of the latter by the operation of conversion and additional axioms.

In the recent years, applications of both approaches to a wide variety of problems in practical and theoretical computer science have emerged. The present special issue is intended to show a number of substantial samples of this. They were selected from the submissions in a strict and thorough refereeing process.

First, R. Backhouse uses the particular class of Regular Algebras, also known as Standard Kleene Algebras or Quantales, to characterize the functions on context-free languages that can be defined homomorphically along their grammars and to give a systematic method for evaluating such functions.

Next, R. Berghammer employs Relation Algebra to derive algorithms for computing all fixed points of certain functions on powersets. The corresponding algebraic expressions allow a direct and efficient evaluation in the RelView system. This is illustrated by some examples from different problem domains.

Third, J. Desharnais, B. Möller and F. Tchier use Modal Kleene Algebra to give an abstract algebraic semantics for imperative loop programs with demonic non-deterministic choice and explicit forms of the least and greatest fixed points of the corresponding semantic function.

Following that, M. Frías and C. López Pombo show how to embed the first-order temporal logics LTL and TL into Fork Algebra and prove a corresponding interpretability result. This has important applications in the relational specification of properties of systems within the authors' ARGENTUM tool.

Then H. Leiss considers Kleene Modules, i.e., left semiring modules over Kleene Algebras. He shows that the linear context-free languages form a Kleene Module over a

Kleene Algebra of binary regular word relations and gives a purely algebraic proof that the simultaneous linear fixed-point operator on languages can be reduced to finite iteration. Next, S. Li and Y. Li demonstrate the application of Relation Algebra in temporal and spatial reasoning. They study the algebras generated by the "part of" and "connection" relations, in particular, the complemented disk algebra. They provide a general representation method for it in the Region Connection Calculus and show that connected regions bounded by Jordan curves and their complements form such a representation.

Following that, G. Schmidt again uses Relation Algebra to develop a particular theory of partial semantic information in the setting of non-strict and parallel evaluation along the lines of denotational semantics. This prepares the ground for a successor paper on universal characterisations of parallel products and correctness rules for parallel programs.

Finally, G. Struth employs Kleene Algebra extended by infinite iteration as a natural formal semantics for abstract rewriting diagrams. This is used in purely algebraic proofs of general Church?Rosser related theorems and reduction and transformation theorems for termination issues that depend on abstract commutation, cooperation or simulation properties.

Hopefully, the papers show that Relation Algebra and Kleene Algebra and their relatives are useful and convenient formal tools that help reasoning in quite a number of ways, not least by admitting suitable abstractions from irrelevant details. In using them we really have just scratched the surface; it is to be hoped that other researchers will be enticed to try them out and use them to good effect for their own work.

I am most grateful to J. Bergstra for inviting me to guest-edit this special issue. Next I want to thank the authors for their interesting and profound papers and the referees for their careful and constructively critical evaluation of the submissions. Finally I gratefully acknowledge the help of I. Bethke, P. Höfner and G. Struth in the technicalities involved in preparing the issue.

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Spring 2006