

## Relations and Kleene algebras in computer science

Rudolf Berghammer, Ali Mohamed Jaoua, Bernhard Möller

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## Editorial

### Relations and Kleene Algebras in Computer Science

Algebraic techniques are being applied to more and more areas in computer science. Their main advantage over purely logical formulations is the economy in notation and the emphasis on (in)equational reasoning, largely in a pointfree way. In particular the latter aspect makes them amenable to fully automated reasoning using off-the-shelf theorem provers. These techniques have been fostered and propagated through a, by now well established, series of conferences on Relational Methods in Computer Science (RelMiCS) and Applications of Kleene Algebra (AKA). Three special issues of JLAP devoted to these topics appeared in 2006, 2008 and 2010. This series is now continued with the present issue. Its six research contributions underwent a thorough two round refereeing process. In the first round, a programme committee chose 22 technical contributions for the joint proceedings of the Eleventh International Conference on Relational Methods in Computer Science and the Sixth International Conference on Applications of Kleene Algebra in 2009. These have been published as Volume 5827 of Springer's Lecture Notes in Computer Science. In the second round, selected and substantially revised papers were reviewed again for the journal issue by new referees. The papers are on the foundational side, showing, however, also strong links to applications. They cover a wide variety of topics and provide a good view of some main issues in the area of relational and algebraic methods. In Algebraic Separation Logic, Han-Hing Dang, Peter Hfner and Bernhard Mller give a characterisation of various classes of assertions of separation logic and prove abstract laws fully algebraically. Further, they present a relational semantics of a simple programming language associated with separation logic. This is used to give a general version of the central modular reasoning instrument of separation logic, the frame rule, and to prove it in a pointfree way. This includes an algebraic formulation of the requirement that a command preserves certain variables. Moreover, the treatment is parametric in the operator of separating conjunction, of which two particular variants are discussed. The paper Fixpoints for General Correctness by Walter Guttmann uses various fixpoint operators for a semiring-based general correctness semantics of programs. This involves combinations of least/greatest (pre/post)fixpoints with respect to the refinement ordering. The paper also presents another view of the semantics based on distributive lattices rather than semirings. These can be augmented in a natural way with a second order similar to the Egli-Milner order, to which the fixpoint operators are then extended. Finally, the combination of semirings and lattices is used to derive further results about general correctness. In Concurrent Kleene Algebra and its Foundations Tony Hoare, Bernhard Mller, Georg Struth and Ian Wehrman develop a new class of semantic algebras. Next to choice, these offer the operators of sequential and concurrent composition, related by a weak version of an exchange law. They are exemplified in a concrete trace model of program semantics and defined in terms of a primitive independence relation between traces. The basic abstract algebra is enriched to build a series of richer algebras, culminating in one that validates a proof calculus for programs similar to Jones-style rely/guarantee reasoning. Finally it is shown how the original trace model can be algebraically reconstructed using the lattice-theoretic notion of atoms. The paper Collagories: Relation-Algebraic Reasoning for Gluing Constructions by Wolfram Kahl provides an abstract algebraic view of generalised graph structure transformation. It simplifies and extends an earlier approach using complete distributive allegories by defining collagories, i.e., essentially distributive allegories without zero morphisms, and also a variant of Kleene star to obtain difunctional closures. A number of typical collagories are obtained from basic ones via nestable constructions. Particular emphasis is laid on co-tabulations, the core of the original relation-algebraic gluing concept. Finally, the paper discusses how van Kampen squares can be included in collagories; these squares are the central ingredient of adhesive categories, a recent important foundation for algebraic graph transformation. In Contact, Closure, Topology, and the Linking of Row and Column Types of Relations Gunther Schmidt and Rudolf Berghammer investigate closures of subset of a given base set in a relational setting. One application of this fundamental concept, the contact relation, was introduced by Georg Aumann in 1970. The paper generalises Aumann's notion from powersets to general membership relations and their induced partial orders. It also investigates the relationship between contacts and closures in this general setting and presents some applications. Particular cases are the connections between contacts, closures and topologies and a one-to-one correspondence between the column and row intersection spaces of relations.

The issue concludes with the paper Normal Forms in Total Correctness for While Programs and Action Systems by Kim Solin. It uses demonic refinement algebra to prove a classical normal-form theorem for while-programs in a more abstract setting. Contrary to Kozen's partial-correctness proof in Kleene algebra with tests, the derivation establishes the theorem for the case of total correctness. The techniques are extended to discuss a normal form for action systems as used for reasoning about concurrent systems, and a proof that every action system can be brought

into normal form is outlined. We are grateful to all the authors for submitting their papers and to the referees for their careful scrutiny. We are most grateful to Jan Bergstra and John Tucker for making such a special issue once again possible. We would like to thank the Steering Committee of the ReLMiCS/AKA conference series for preselecting the papers and Peter Jipsen for his help in the refereeing process. Last, but not least, we would like to thank the sponsors of the conferences ReLMiCS11 and AKA 6 from which the papers were selected; these are Qatar University (QU), the Supreme Education Council of Qatar (SEC) and Qatar National Research Funds (QNRF).

Rudolf Berghammer  
Institut für Informatik  
Universität Kiel  
Olshausenstraße 40, 24098 Kiel, Germany  
E-mail address: [rub@informatik.uni-kiel.de](mailto:rub@informatik.uni-kiel.de)

Ali Mohamed Jaoua  
Qatar University  
College of Engineering  
CSE 209, P.O. Box 2713  
Doha, Qatar

E-mail address: [jaoua@qu.edu.qa](mailto:jaoua@qu.edu.qa) Bernhard Möller  
Institut für Informatik  
Universität Augsburg  
D-86135 Augsburg, Germany  
E-mail address: [moeller@informatik.uni-augsburg.de](mailto:moeller@informatik.uni-augsburg.de)

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