Editorial
Relations and Kleene Algebras in Computer Science

Relation algebras, Kleene algebras and related algebraic approaches are concerned with the abstraction and compaction of frequently occurring patterns of certain sets of formulas into a form amenable to simple algebraic manipulation, that is, (in)equational reasoning, and hence also to mechanised theorem proving. Among others, they cover the fundamental computer science concepts of choice, sequential composition and (in)finite iteration. These structures have received increasing interest over the last decade, notably through many joint conferences on Relational Methods in Computer Science (RelMiCS) and Applications of Kleene Algebra (AKA). Two special issues of JLAP devoted to these topics appeared in 2006 and 2008. This series is now continued with the present issue. Its seven research contributions underwent a thorough two round refereeing process. In the first round, a programme committee chose 28 technical contributions for the joint proceedings of the *Tenth International Conference on Relational Methods in Computer Science* and the *Fifth International Conference on Applications of Kleene Algebra* in 2008. These have been published as Volume 4988 of Springer's Lecture Notes in Computer Science. In the second round, selected and substantially revised papers were reviewed again for this journal by new referees. The papers cover both theory and application and reflect an interesting cross-section of current work in the field.

In *Boolean logics with relations*, Philippe Balbiani and Tinko Tinchev introduce a Boolean language to which relation symbols have been added. This is useful for describing relational and algebraic structures. The paper introduces the concepts of Kripkean and Boolean semantics for that language and addresses the traditional issues of decidability and complexity; moreover it defines the new concepts of weak and strong canonicity.

The paper *Relation-algebraic specification and solution of special university timetabling problems* by Rudolf Berghammer and Britta Kehden tackles a concrete practical problem. It was posed by a university administration and concerns the construction of a timetable with an even distribution of certain mandatory courses. Two relation-algebraic models and corresponding algorithmic solutions are developed; one of them is directly implementable in the Kiel RelView tool.

Another paper on the foundational side is *Abstract representation theorems for demonic refinement algebras* by Jean-Lou De Carufel and Jules Desharnais. Its main result is that every demonic refinement algebra with enabledness and termination, as defined in earlier RelMiCS/AKA papers, is isomorphic to an algebra of ordered pairs of elements of a Kleene algebra with domain and with a divergence operator satisfying a mild condition. In addition, it is shown that every demonic refinement algebra with enabledness is also a demonic refinement algebra with termination.

In *Imperative abstractions for functional actions* Walter Guttmann studies an application of algebraic concepts to the semantics of programming languages. It elaborates on an earlier relational model of non-strict, imperative computations by the same author. The resulting theory also supports infinite data structures and covers concepts such as procedures, parameters, partial application, algebraic data types, pattern matching and list comprehension. Moreover, a relational treatment of programming patterns ?– such as fold, unfold and divide-and-conquer –? is given, including proofs of functional programming laws like fold-map fusion. The approach is validated by a number of examples.

The paper *Algebraic notions of nontermination: omega and divergence in idempotent semirings* by Peter Höfner and Georg Struth is again on the foundational side. It compares two notions of nontermination in the setting of idempotent semirings. Their behaviour in various computational models is determined, and conditions for their existence and their coincidence are given. It confirms and deepens earlier results that the approach using divergence yields a simple and natural way of modelling infinite behaviours of programs and discrete systems, whereas the omega operator exhibits some anomalies.

Next, *Determinisation of relational substitutions in ordered categories with domain* by Wolfram Kahl applies algebraic methods within the realm of term rewriting and unification. Rather than describing unification in terms of functional substitutions (or terms), it uses a single relational substitution which becomes a "determiniser" arrow in a locally ordered category with domain. This generalises an earlier characterisation of determinacy by Desharnais and Möller in Kleene algebras with domain. It is shown how "most general determinisers" can be viewed as a generalisation of quotient projections of partial equivalence relations. Moreover, a characterisation via restricted residuals avoids the use of converse or symmetry.

The issue concludes with another foundational paper, *Cardinality functions in allegories*, by Yasuo Kawahara and

Michael Winter. It investigates three ways of generalising the cardinality of relations in the context of allegories, which roughly are heterogeneous relation algebras without complements. Two of the three axiom systems presented are motivated by the existence of injective and surjective functions, respectively. Finally, a third definition of cardinality unifies those possibly differing notions. For all three variants a canonical cardinality function is shown to be initial in the category of all cardinality functions over the given allegory.

We are indebted to all the authors for submitting their papers and to the referees for their careful scrutiny. We are most grateful to Jan Bergstra and John Tucker for making such a special issue once again possible. We would like to thank the Steering Committee of the RelMiCS/AKA conference series for preselecting papers. Last, but not least, special thanks go to Inge Bethke and Peter Höfner for helping us with the technicalities of preparing this special issue.

Rudolf Berghammer
Institut für Informatik
Universität Kiel
Olshausenstraße 40, 24098 Kiel, Germany
E-mail address: rub@informatik.uni-kiel.de

Bernhard Möller
Institut für Informatik
Universität Augsburg
D-86135 Augsburg, Germany
E-mail address: moeller@informatik.uni-augsburg.de

Georg Struth
Department of Computer Science
The University of Sheffield
Regent Court, 211 Portobello Street
Sheffield S1 4DP, UK
E-mail address: g.struth@dcs.shef.ac.uk

Spring 2010