

INFLUENCE OF DISSIPATION ON QUANTUM SOLITON PAIR CREATION IN THE SINE-GORDON MODEL

KAZUO HIDA* and ULRICH ECKERN
Institut für Theorie der Kondensierten Materie,
Universität Karlsruhe, D7500 Karlsruhe, FRG.

Abstract The influence of dissipation on the quantum soliton-antisoliton pair creation rate P is calculated for the sine-Gordon model using the path integral formulation. The external field dependence of P is shown to change drastically in the weak field limit. The relation to the electric conductivity of commensurability pinned CDW is discussed.

INTRODUCTION

It was suggested by Maki¹ that the quantum creation of soliton-antisoliton pairs in the sine-Gordon model is related to the electric conduction in commensurate CDW system. In solid state physics, solitons are usually coupled to a heat bath and subject to damping. The purpose of the present work is to investigate the influence of dissipation on the quantum pair creation rate of solitons.

THE MODEL

We consider the one-dimensional quantum sine-Gordon model in an external field whose Lagrangian is given by

$$\mathcal{L} = A \int_{-\infty}^{\infty} dx \left\{ \left(\frac{\partial \varphi}{\partial t} \right)^2 - \left(\frac{\partial \varphi}{\partial x} \right)^2 + 2\omega^2 (:\cos \varphi - 1:) + \varepsilon \varphi \right\} \quad (1)$$

where φ is the bose field, ε the external field, A a positive constant, and ω the frequency of small amplitude oscillations. The notation $::$ denotes the normal product.

When ε is smaller than $\varepsilon_c = 2\omega^2$, this system has an infinite series of metastable states with values of $\varphi = \varphi_0 + 2n\pi$ (n :integer), where φ_0 vanishes if $\varepsilon = 0$. The decay rate P of one of those

states, in the semiclassical approximation, is given by²
 $P = K \cdot \exp(-S_B)$, where K is a determinantal factor, and S_B the
 Euclidean action S , evaluated for a single "bounce"^{1,2,3}.
 Specifically, S is given by

$$S = - \int_{-\infty}^{\infty} \mathcal{L}_E d\tau + S_d, \quad (2)$$

where \mathcal{L}_E is obtained from \mathcal{L} by $t \rightarrow -i\tau$, and the coupling to the heat
 bath is described by³

$$S_d = \frac{\eta_0}{2\pi} \iiint_{-\infty}^{\infty} d\tau d\tau' dx \left\{ \frac{\varphi(x, \tau) - \varphi(x, \tau')}{\tau - \tau'} \right\}^2 \quad (3)$$

In the limit of small ϵ , the bounce is a bubble with $\varphi = 2\pi$ on the
 $x-\tau$ plane in a background with $\varphi = 0$, and the size of the bubble
 is much larger than its wall (thin wall approximation).

RESULTS

For weak damping ($\eta \equiv \eta_0/A \ll \omega$) we find the following results:

$$P \sim \exp(-128\omega^2 A/\epsilon) \quad (4a)$$

$$\text{for } \epsilon_c \gg \epsilon \gtrsim \epsilon_c \exp(-4\omega/\pi\eta)$$

$$P \sim (4\omega\eta/\pi\epsilon)^{\frac{-128\omega\eta A}{\epsilon}} \quad (4b)$$

$$\text{for } \epsilon_c(\eta/\omega)\exp(-4\omega/\pi\eta) \gtrsim \epsilon$$

The expression (4a) coincides with that of Maki¹. In the presence of
 dissipation, however, this does not hold for small ϵ , and the
 crossover to the expression (4b) is found.

For strong damping ($\eta \gg \omega$), we find

$$P \sim (8e\omega^2/\pi^2\epsilon)^{-128\eta\omega A/\epsilon} \quad (5)$$

DISCUSSION

The correspondence between our Lagrangian and that of the
 commensurability pinned CDW is given in Table I^{1,4-6}. In the
 latter system, P is proportional to the electric conductivity

except a prefactor which is weakly dependent on ϵ . In the absence of dissipation, the typical scale of the electric field is given by $\epsilon_1 = 64A\epsilon_c = (16v_F/\pi c_0 N^2)\epsilon_c$. Although $v_F \simeq 10 \dots 10^2 c_0$, quantum tunneling phenomena could be observed before the CDW is depinned classically if commensurability is high enough. On the other hand, our results shows that the typical scale of electric field in the dissipative case is given by $\epsilon_d = 128\eta\omega A$, which is smaller than ϵ_1 for small η . The detailed calculation will be published elsewhere⁷.

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TABLE I Correspondence between the commensurability pinned CDW and the present system. N: commensurability; c_0 : phason velocity; e: electronic charge; E: electric field; v_F : Fermi velocity; τ_2 : backward scattering time due to impurities.

Commensurability pinned CDW	$N \cdot \phi$	$\frac{v_F}{4\pi N^2 c_0}$	$\frac{4eNc_0^2 E}{v_F}$	$\frac{c_0^2}{\tau_2 v_F^2}$
present system	ϕ	A	ϵ	η

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