Decay of a Dissipative Object by Quantum Noise

Ulrich ECKERN and Walter LEHR

Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, Postfach 6980, D-7500 Karlsruhe, Federal Republic of Germany

The decay time, τ, of a dissipative object out of the metastable minimum of a (cubic) potential due to quantum noise is determined by a numerical investigation of the quasiclassical Langevin equation. In the zero temperature limit, and for moderate to large damping ($\gamma \gtrsim \omega_o$), we find that $\sigma = \ln(\omega_o\tau/2\pi)$ can be approximated by $\sigma \simeq \kappa \cdot \gamma\Delta V/\hbar\omega_o^2$, where the constant κ is $\kappa \simeq 3.20$. This is in good agreement with an analytic treatment of the Langevin equation, but about a factor three smaller than the result obtained by Caldeira and Leggett by the instanton method.

1. Introduction

Almost eighty years ago, it was suggested by Langevin [1] that the motion of a classical object, when coupled to an environment which consists of infinitely many degrees of freedom, can be described by a stochastic differential equation of the form

$$m\ddot{x} + \eta\dot{x} + V'(x) = \xi(t) \tag{1}$$

where x is the coordinate of the object, m its mass, $V(x)$ the potential, and $\gamma = \eta/m$ the damping constant. The stochastic force on the rhs of this equation, $\xi(t)$, is a white Gaussian stochastic process such that $< \xi(t)\xi(t') > = D \cdot \delta(t-t')$, where $D = 2\eta kT$ according to the fluctuation-dissipation theorem. Equation (1), or the corresponding Fokker-Planck equation, has been investigated in great detail. In particular, for a potential with a metastable minimum (say, at $x = 0$), it is known [2] that the escape rate due to thermal activation is given by $\tau^{-1} = (\omega_o/2\pi) \cdot a_T \cdot \exp(-b_T)$, where $\omega_o^2 = V''(0)/m$, $b_T = \Delta V/kT$, and ΔV is the barrier height; a_T is the usual prefactor, given by $\omega_o \cdot a_T = (\omega_o^2 + \gamma^2/4)^{1/2} - \gamma/2$.

Recently [3], it was suggested that the Langevin equation can be extended to the quantum regime, by replacing the white $\xi(t)$, eq. (1), by a "blue" stochastic force, with correlations given by $< \xi(t)\xi(t') > = D \cdot K(t-t')$, with

$$K(\omega) = (\hbar\omega/2kT)\coth(\hbar\omega/2kT) \tag{2}$$

where $K(\omega)$ denotes the Fourier transform of $K(t-t')$. We call this stochastic force quantum noise. In fact, it has been confirmed experimentally [3] that this "quasiclassical" Langevin equation describes, at low temperatures, quantitatively the voltage noise in current-driven Josephson junctions in the free-running state, i.e. for an applied current larger than the critical current. In addition, theoretical arguments have been given [4] that the quasiclassical Langevin equation should be valid for not too small damping.

On the other hand, in the past few years, the decay of a dissipative object out of a metastable state by quantum tunneling, has been studied intensively [5-7]. While the experiments seem to be in agreement with the theoretical prediction obtained by quantum-mechanical calculations, one may ask [8] to what extent the decay can be described by "quantum activation", i.e. by the quasiclassical Langevin equation.

2. Numerical Investigations

In order to determine the decay time, we solve the quasiclassical Langevin equation for a large number ($n \sim 1000$) of realizations of the stochastic force, $\xi(t)$, under the initial conditions $x(t=0) = 0$, $\dot{x}(t=0) = 0$. For a given realization, $\xi^{(i)}(t)$, the decay time is determined from the condition $x(\tau_i) = x_a$, i.e. τ_i is identified with the time the object needs to reach the "exit point" x_a, defined by $V(x_a) = V(x=0)$. In order to generate the blue stochastic force, we fix a certain time interval, t_o (typically $t_o \sim 10^4/\omega_o$), and the corresponding discrete frequencies $\nu_k = 2\pi k/t_o$. Then we generate the Fourier coefficients, $\xi(\nu_k)$, and finally compute the stochastic force, $\xi(t)$.

There are different possibilities to discretize the equation of motion, eq. (1). We use the following one:

$$m\Big[(\ddot{x})_i + \gamma(\dot{x})_i\Big] + V'(x_i) = \bar{\xi}_i \tag{3}$$

where i labels the time, $t_i = \Delta t \cdot i$, $x_i = x(t_i)$, and

$$\begin{aligned}(\ddot{x})_i &= (x_{i+1} + x_{i-1} - 2x_i)/(\Delta t)^2 \\ (\dot{x})_i &= (x_{i+1} - x_{i-1})/(2\Delta t)\end{aligned} \tag{4}$$

Furthermore, $\bar{\xi}_i$ is the stochastic force averaged over a time interval Δt; we have checked that the results do not depend on the discretization, or on the special way we average $\xi(t)$, as long as $\Delta t << \omega_o^{-1}$. Typically, we have taken $\Delta t \sim 0.1\omega_o^{-1}$, and $N_o = t_o/\Delta t = 2^{16}$.

As an important point, we note that the ensemble (of n realizations) can be observed only over a finite time interval given by t_o. In the interval $0 \ldots t_o$, however, not all realizations have necessarily shown a decay of the object. Let $m(\leq n)$ be the number of realizations which have shown a decay within the observation time; then the best estimate of the decay time is

$$\tau = \frac{1}{m}\left[\sum_{i=1}^{m}\tau_i + (n-m)\cdot t_o\right] \tag{5}$$

In a first step, we have determined the decay time in the classical limit, and found excellent agreement with the result quoted above, including the corrections to Kramers' result which are known to be important [9] for $\gamma/\omega_o < kT/\Delta V$. Secondly, from the quasiclassical Langevin equation, we find that the decay time saturates for $T \to 0$ to a value which depends on γ and ΔV. Some of our results (for $T = 0$) are shown in Fig. 1. Defining the quantity $\sigma = \ln(\omega_o\tau/2\pi)$, it is quite evident that for $\gamma \gtrsim \omega_o$, the data are close to linear in γ, and can be represented by

$$\sigma \simeq \kappa \cdot (\Delta V/\hbar\omega_o) \cdot (\gamma/\omega_o), \quad \gamma \gtrsim \omega_o \tag{6}$$

where κ is approximately independent of the ratio $\Delta V/\hbar\omega_o$ for the parameters considered here, namely $\Delta V/\hbar\omega_o = 1.83 \ldots 0.5$. The average value of κ is found to be given by $\bar{\kappa} \simeq 3.20$; in fact, κ is largest for curve D, $\kappa_D \simeq 3.4$, while its smallest value is $\kappa_J \simeq 2.95$. For small damping($\gamma = 0.2\omega_o$), σ is found to increase with increasing $\Delta V/\hbar\omega_o$.

3. Discussion

In a brief discussion of our results, we concentrate on two questions:

(i) Can the numerical results be confirmed by an independent calculation within the Langevin equation approach? Very recently [10], path integral methods have been applied to this problem. This technique is, for the present case, based on the fact that [4]

$$W[x(t)] \sim \exp\{-\frac{1}{2D}\int dt dt' \xi(t) K^{-1}(t-t')\xi(t')\} \tag{7}$$

into which eq. (1) is inserted, can be interpreted as the probability density for a path $x(t)$ (see ref. 11 for a discussion in the classical limit). Writing the result for the decay rate in the form $\sigma = b - \ln a$, it is found [10], for $T = 0$ and $\gamma >> \omega_o$, that the "exponent" b is given by

$$b \simeq 4.2 \cdot (\Delta V/\hbar\omega_o) \cdot (\gamma/\omega_o) \tag{8}$$

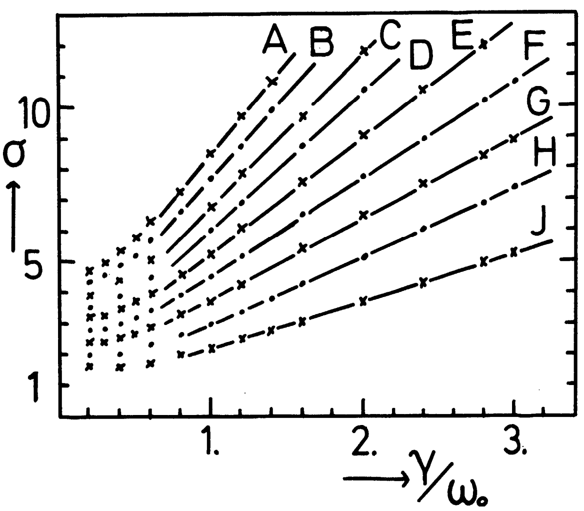

Fig. 1: $\sigma = \ln(\omega_o\tau/2\pi)$ versus γ in the zero temperature limit, for $\Delta V/\hbar\omega_o$ = 1.83(A), 1.67(B), 1.5(C), 1.33(D), 1.17(E), 1.0(F), 0.83(G), 0.67(H), 0.5(J).

Assuming that this equation is reasonably accurate for, say, $\gamma \simeq 3\omega_o$, we find from a comparison with the results of the simulation, that the prefactor is approximately given by $\ln a \sim 2$.

In addition, we have repeated the simulation for a *piecewise parabolic potential*, which is chosen to have the same curvature at the extrema, and the same ΔV, as the cubic potential. For this potential, σ is found to be a few percent less than the one for the cubic potential. Furthermore, the exponent has been calculated analytically [10], with a result similar to eq. (8); however, the numerical factor turns out to be given by π instead, in the large damping limit. In fact, for the piecewise parabolic potential, we find that our data can be fitted rather accurately by assuming that the prefactor is given by its classical value, namely $a \simeq \omega_o/\gamma$.

(ii) In comparison with the results of Caldeira and Leggett [5] and others on quantum tunneling, we quote their result, for $T = 0$ and $\gamma >> \omega_o$, namely

$$b_{CL} = 3\pi \cdot (\Delta V/\hbar\omega_o) \cdot (\gamma/\omega_o) \tag{9}$$

Therefore, we find that the decay rate as calculated from the Langevin equation is much larger than the one calculated, for example, in refs. 5 and 6. Thus, the decay out of a metastable state due to *quantum activation* appears to be much faster than predicted by *quantum tunneling.*

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