Charge Transfer between Weakly Coupled Normal Metals and Superconductors at Low Temperatures

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Summary: We give a review of recent advances in the theoretical description of ultrasmall tunnel junctions, i.e. weakly coupled normal metals and superconductors, at very low temperatures. The description, which is based on the microscopic theory, accounts for the quantum effects associated with Cooper pair tunneling, and for the dissipation due to single electron tunneling (SET) or normal current flow. The former is described by the familiar periodic potential which, in the quantum regime, leads to energy bands, and to coherent Bloch oscillations in response to an external current. The stochastic SET, which may be blocked by the Coulomb interaction, yields voltage oscillations even in normal junctions, and also modifies the Bloch oscillations in Josephson junctions. The theory is extended to networks and granular superconducting films as well.

1 Introduction

During the past ten years, a great deal of theoretical as well as experimental efforts have been devoted to what is now commonly called Quantum Mechanics of Macroscopic Variables [1]. The experimental system which has been studied extensively is a Josephson junction, i.e. a system of two weakly coupled superconductors. It is well known that the relevant variable for the description of the dynamics of a Josephson junction is the difference φ of the phases of the superconducting order parameters in the two electrodes. If the capacitance of the junction is large enough, the dynamics of the phase is determined by the following classical equations:

$$\hbar\dot{\varphi} = 2eV \; ; \quad C\dot{V} + V/R + I_c \sin\varphi = I_x \tag{1}$$

where the first is Josephson's equation, and the second expresses the balance of currents; V, I_x, C, R, and I_c denote the voltage, the external current, the capacitance, the resistance, and the critical current, respectively. Obviously, Eq. (1) describes the damped motion of a particle in a tilted periodic potential $U(\varphi)$, given by

$$U(\varphi) = -E_J \cos\varphi - \hbar I_x \cdot \varphi/2e \tag{2}$$

backward across the barrier, at random times $\{t_i^+\}$ and $\{t_j^-\}$. Since the time for tunneling is negligible, the current is given by a sum of delta-functions as follows:

$$I(t) = e\sum_i \delta(t - t_i^+) - e\sum_j \delta(t - t_j^-) \tag{4}$$

The statistics governing these processes is Poissonian, characterized by the rates ν^+ and ν^-. Thus the probability density for n particles to tunnel forward at times $0 < t_1^+, \ldots, t_n^+ < t_f$ is given by

$$P_n^+(t_1^+, \ldots, t_n^+) = e^{-\nu^+ t_f} \cdot (\nu^+)^n/n! \tag{5}$$

and similar for the backward tunneling. Detailed balance determines the ratio $\nu^+/\nu^- = \exp(-\epsilon/kT)$, where ϵ is the energy difference needed for the forward tunneling process. The average current and the current fluctuations are expressed by ν^+ and ν^- as follows:

$$< I(t) >= e(\nu^+ - \nu^-) \tag{6}$$

$$< \delta I(t)\delta I(t') >=< I(t)I(t') > - < I >^2= e^2\delta(t - t') \cdot (\nu^+ + \nu^-) \ . \tag{7}$$

Let us assume that for a fixed voltage, which means fixed energy difference $\epsilon^\pm = \pm eV$, the tunneling current voltage characteristic $I_t(V)$ is known (in simple cases it is given by $I_t(V) = V/R_t$). This provides a relation for the average current, $I_t(V) = e(\nu^+ - \nu^-)$, and combined with the detailed balance requirement it fixes the rates to

$$\nu^\pm = \frac{1}{e} I_t(\frac{\epsilon^\pm}{e}) \left[\exp(\frac{\epsilon^\pm}{kT}) - 1\right]^{-1} \ . \tag{8}$$

Thus the fluctuations of the voltage biased junction are given by

$$< \delta I(t)\delta I(0) >= e\delta(t) I_t(V) \coth(\frac{eV}{2kT}) \tag{9}$$

which obviously interpolates between Schottky's result and the standard Nyquist formula for thermal noise ($eV >> kT$ and $eV << kT$, respectively). After generalization to the quantum mechanical form, the power spectrum is found to be given by [12]

$$S_I(\omega) = \frac{e}{4\pi} \sum_\pm I_t(V \pm \hbar\omega/e) \coth(\frac{eV \pm \hbar\omega}{2kT}) \ . \tag{10}$$

In more general situations, the energy is not fixed by the external voltage but depends on the state of the system. In particular in junctions with very small capacitance, the energy is dominated by the charging energy $E(Q) = Q^2/2C$, which changes in the tunneling process since $Q \to Q \pm e$. The

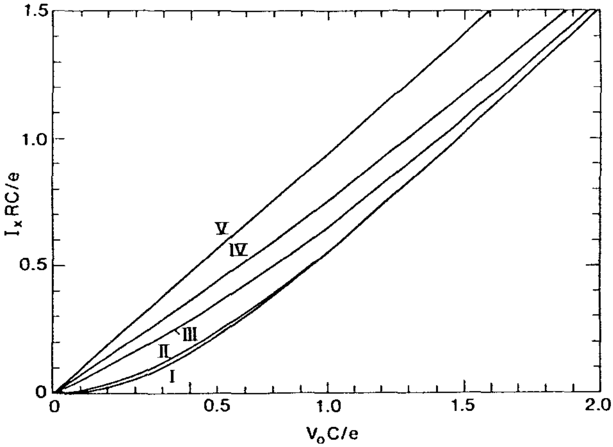


Fig. 1 The current voltage characteristic of a normal junction driven by a dc current I_x is shown for different temperatures. (I) to (V): $kT/E_C = 0.01, 0.1, 0.5, 1$, and 5. At low temperature a pronounced deviation from an Ohmic form is found.

It extrapolates to a nonzero dc voltage $V_g = e/2C$ at $I_x = 0$ which is called the *Coulomb gap.* The Coulomb gap and other manifestations of the discrete nature of single electron tunneling have been observed by now in several experiments on small capacitance junctions [14] and granular materials.

If the junction in addition is shunted by a parallel Ohmic resistor R_s, the junction also discharges continuously, which can be accounted by the following equation:

$$dQ/dt = I_x(t) + \dot{Q}|_{tunneling} - Q/R_sC + \delta\tilde{I}(t) \ . \tag{18}$$

Here $\delta\tilde{I}(t)$ denotes the Gaussian current noise of the shunt resistor, whose power spectrum is given by

$$S_I(\omega) = (2\pi)^{-1} < \delta\tilde{I}(t)\delta\tilde{I}(t') >_\omega = kT/\pi R_s \ . \tag{19}$$

A weak shunt mostly modifies the low voltage part of the I-V-characteristics. The noise associated with it destroys the long-time correlations in the charge correlation function $< Q(t)Q(0) >$.

3 Microscopic Theory of Weakly Coupled Superconductors

The derivation of an effective action for Josephson junctions [4] from microscopic theory has been described in detail in the literature, and will only briefly be summarized here. The starting point is the Hamiltonian

$$\mathcal{H} = \mathcal{H}_L + \mathcal{H}_R + \mathcal{H}_Q + \mathcal{H}_t \tag{20}$$

result, we arrive at a description in terms of the relevant variable (namely the phase difference φ) alone, and we obtain the following representation:

$$Z = \int \mathcal{D}\varphi \; e^{-S[\varphi]/\hbar} \ . \tag{25}$$

Here $S = S_0 + S_1 + S_2$, with

$$S_0 = \int_0^{\hbar\beta} d\tau \Big[\frac{\hbar^2 C}{8e^2}\dot{\varphi}^2 + U(\varphi)\Big] \tag{26}$$

where the potential is given in Eq. (2), and

$$S_1 = \hbar \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha(\tau - \tau')\Big[1 - \cos\frac{\varphi(\tau) - \varphi(\tau')}{2}\Big] \ . \tag{27}$$

The contribution S_2, which is classically related to the "cosφ" term in the equation of motion (which has been omitted in Eq. (1)), has (up to now) not led to any important consequences in the quantum behavior, and thus will not be discussed here. The contribution S_1 represents the single electron tunneling, which becomes evident by noting that the Fourier transform, $\alpha(\omega_j)$, where $\omega_j = 2\pi j/\hbar\beta$ denotes the Matsubara frequency, is related to the normal current, $I_n(V)$, as follows:

$$\alpha(\omega_j) = \frac{1}{e}\int \frac{d\nu}{2\pi}\frac{\nu}{\nu^2 + \omega_j^2} I_n(\frac{\hbar\nu}{e}) \ . \tag{28}$$

Thus, considering identical electrodes for simplicity, we find for ideal junctions in the zero temperature limit (in which case $I_n(V) = 0$ for $V < 2|\Delta|/e$), that $\alpha(\tau)$ decreases exponentially $\sim \exp(-2|\Delta|\tau/\hbar)$ for large times; this clearly reflects the gap in the single particle excitation spectrum. On the other hand, for short times such that $|\Delta|\tau << \hbar$, we obtain $\alpha(\tau) = \alpha/(\pi\tau)^2$, which is the characteristic behavior also found [5] in the case of an Ohmic shunt. Note that Eq. (28) covers both cases and in addition more general ones, provided we consider $I_n(V)$ an experimentally determined input for the theory. We mention that, after the fact, other "microscopic" models can be devised [15,16] which reduce to the effective action given above. As an important point, these models involve coupling $\sin(\varphi/2)$ and $\cos(\varphi/2)$ to a suitably chosen environment, in contrast to φ in the phenomenological treatment [5], the latter nevertheless being adequate in cases where a shunt resistor is present.

We emphasize that the appearance of the trigonometric functions in Eqs. (26) and (27) is intimately related to the discreteness of charge transfer, which is also directly connected with the boundary conditions in the path integral (25), and with the allowed eigenvalues of the charge operator [15]. As an illustration, consider S_0 only, and the simplest choice (i): $\varphi(0) = \varphi(\hbar\beta)$; consequently, Eq. (25) can be interpreted as the partition function of a potential. Typical parameters for which quantum tunneling dominates over thermal activation are $C \leq 10^{-12}F$ and $T \leq 0.1K$; also, for $E_C << E_J$ the current has to be chosen close to I_c in order for quantum tunneling to occur at an observable rate. Eq. (29) has been confirmed quantitatively in experiments [6].

4.2 Energy Levels

Another immediate consequence of quantum mechanics is (for weak dissipation) the appearance of discrete energy levels. Though in the tilted washboard potential of a current biased junction these states are not true eigenstates, they are still well defined since the tunneling rate is small. The levels show up as resonances if the frequency f_x of an external perturbation matches an energy difference, i.e. $hf_x = E_{n+1} - E_n$. On resonance, the probability for transitions out of a metastable minimum increases [7].

4.3 Bloch Oscillations

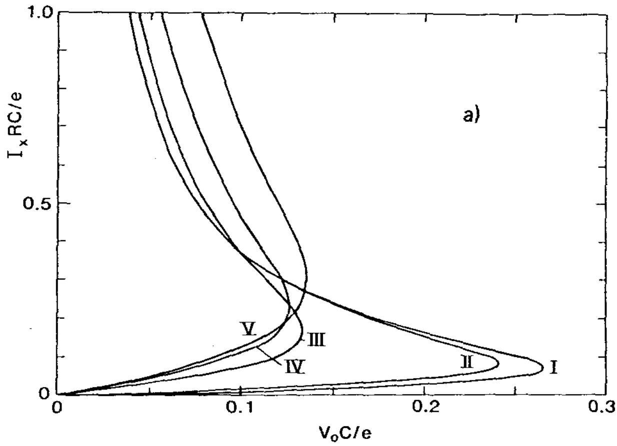
In an unbiased junction, we can realize a perfectly periodic potential (see Eq. (2)). If, in addition, the capacitance is in the range $C < 10^{-15}F$, i.e. smaller than typically in the MQT and energy level experiments, the wavefunction is not longer sharply localized in a single minimum. Hence, the appropriate boundary conditions (see Ch. 3) become crucial. Formally, the eigenstates of the Hamiltonian (3) (with $I_x = 0$) are Bloch states depending on the parameter Q_x, which we denote as "quasi-charge" in analogy to the quasi-momentum of a Bloch state in a periodic potential:

$$\psi_n(\varphi + 2\pi) = e^{i2\pi Q_x/2e}\psi_n(\varphi) \ . \tag{30}$$

The energy levels thus develop into bands [13], $E_n(Q_x)$, and the equivalent of the first Brillouin zone extends between $-e < Q_x < e$. The detailed form of $E_n(Q_x)$ depends on the ratio E_C/E_J. In the limit $E_J << E_C$, corresponding to the "nearly free electron" limit, the lowest energy band for small Q_x is $E_0(Q_x) = Q_x^2/2C$, and the bandsplitting at $|Q_x| = e$ is given by $E_1(e) - E_0(e) = E_J$.

The bands have a transparent physical interpretation. The quasi-charge is the charge provided in a continuous way by the external circuit, just as the external charge Q_x in a normal junction (see Eq. (14)). For Q_x close to $\pm e$, Cooper pair tunneling mixes the states with $Q_x = +e$ and $Q_x = -e$, leading to a reduction of the energy at the zone boundary. In addition, if Q_x is increased adiabatically beyond e, Cooper pairs tunnel (*Umklapp processes*) such that the actual charge remains small, $|Q_x| < e$, i.e. $E_0(Q_x) < E_C$. Considering the case where the junction is driven by a weak external current $I_x = \dot{Q}_x$, we conclude that the energy and the voltage oscillate with the fundamental frequency [13]

$$f_{\text{Bloch}} = I_x/2e \ . \tag{31}$$



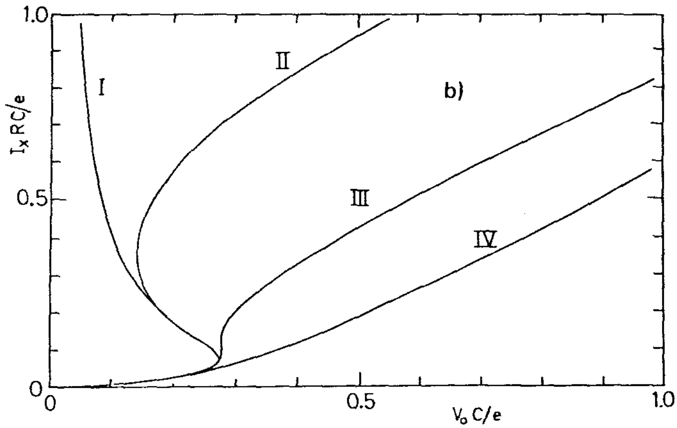


Fig. 2 The dc current voltage characteristic of a superconducting tunnel junction for $E_J = 0.2E_C$. (a) $\alpha_t << 1$, and different temperatures, (I) to (V): $kT/E_C = 10^{-3}$, 0.1, 0.5, 1, 5. (b) Same as (a), but $kT/E_C = 10^{-3}$ fixed, and (I) to (IV): $\alpha_t = 10^{-3}$, 0.01, 0.05, 0.5. The Zener tunneling increases with increasing α_t, suppressing the nose structure.

characteristic frequencies I_x/e and $I_x/2e$, respectively, show up when the junction is driven by a dc plus ac current.

Finally, we remark that Zener tunneling between neighboring bands leads to an additional modification, which can be taken into account for $E_J << E_C$ by the standard formula for the tunneling probability. As a result, it is found that Zener tunneling is only negligible for very small dissipation, namely $\alpha_t << (E_J/E_C)^2$. In the opposite case, one observes a transition from the nose-like I-V-characteristic, typical for a system where bandgaps

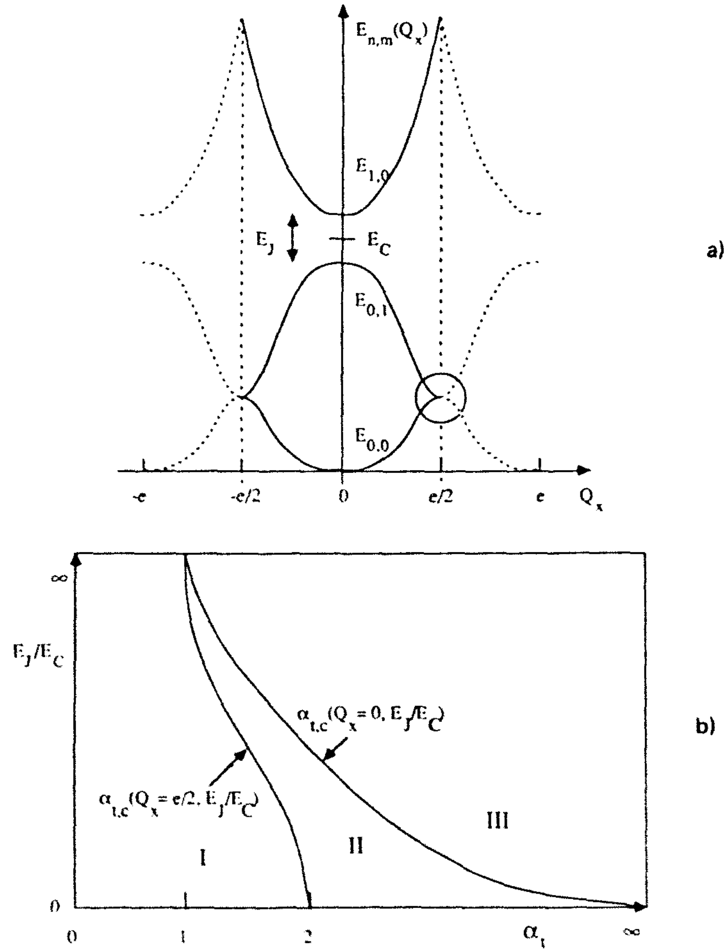


Fig. 3 (a) Overall picture of the energy bands in the "nearly free electron" limit $E_J << E_C$. (b) Phase diagram for strong quasiparticle tunneling. In region (I), the two lowest bands are degenerate only at $|Q_x| = e/2$; in (II), they are degenerate in a finite interval near the zone boundary; in (III), the bands are degenerate for all Q_x.

2(b)). Only at larger values, $1 < \alpha_t < \alpha_{t,c}$, the modifications of the energy bands begin to become important. The band gap increases, and hence the probability of Zener tunneling decreases, leading to a reappearance of the nose (but on a smaller current and voltage scale). Above the phase transition, the two lowest bands are degenerate. Eventually, for very large α_t, the classical superconducting response (i.e. zero voltage) is recovered [15, 11].

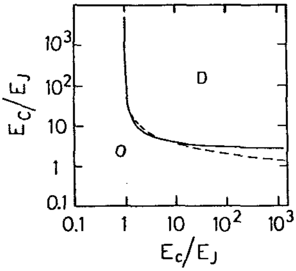


Fig. 4 Zero temperature phase diagram based on the mean-field Hamiltonian Eq. (38). D and O indicate the disordered and the ordered phase, respectively. A deviation from the result of [22] (dashed line) appears only for very small c.

decreasing the temperature, with a critical temperature given (implicitly) by $kT \sim E_J$. On the other hand, for $T = 0$, the model is in the same universality class as the 3D-XY model, and shows the standard second order transition with the well-known exponents, upon decreasing E_c/E_J, at a critical value ~ 1. In order to gain further insight into the phase diagram, e.g. the reentrant behavior found by the numerical investigation [20], it seems appropriate to consider the dual description in terms of vortices and vortex loops [21].

The more interesting case, however, is $C >> c$ as discussed above. Employing path integral methods, it is possible to derive an effective Ginzburg-Landau-Wilson functional [22], confirming that at $T = 0$ the transition is of the 3D-XY type. Generally, it is found that the ordered region of the phase diagram is increased upon increasing the nearest neighbor capacitance C. In fact, a mean-field analysis based on the ansatz [23]

$$\mathcal{H}_{mf} = \frac{1}{2}\sum_{i,j} \hat{Q}_i (C^{-1})_{ij} \hat{Q}_j - \sum_i h_i^{mf} \cos\hat{\varphi}_i \tag{38}$$

leads to the critical value $E_C = 2E_J$ for $c = 0$. Thus, inserting the renormalized capacitance Eq. (36) in the limit $C_0 << C_{qp}$, and the standard relation $E_J = \alpha|\Delta|/2$, one finds a phase transition [22] at $\alpha \sim 1$, consistent with experiment. More precisely, mean-field theory predicts a transition at $\alpha_c = (8/3)^{1/2}$, though the $c = 0$ limit is approached only very slowly. The zero temperature phase diagram [23], based on the optimal choice (in the usual sense) of h_i^{mf} in Eq. (38), is shown in Fig. 4. However, we point out that for ideal junctions, the response of the system necessarily involves frequencies of the order $\hbar\omega \sim 2|\Delta|$, which is beyond the adiabatic limit Hamiltonian Eq. (37); thus it becomes crucial to consider the full effective action, in order to include correctly the process of quasiparticle creation [21]. The origin of dissipation at zero temperature, and especially the strong dependence of the resistance on the normal state resistance, require further investigations.

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