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# Diamagnetic response of mesoscopic superconducting rings above $T_c$

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Various small effects that might appear to be obstacles in the way of measuring the persistent current in a mesoscopic ring of a superconductor above but close to $T_c$ are investigated, among them the effect of the magnetic field penetrating into the metal, the Zeeman splitting, and the inductance of the ring. Analogies and differences with a (single-junction) superconducting quantum interference device (SQUID) are pointed out, the differences being due to the high harmonic content of the current flux relation. Typical parameter values make flux trapping virtually impossible.

## I. INTRODUCTION

The observation[1] of persistent currents periodic in applied magnetic flux in an ensemble of mesoscopic copper rings at low temperature has stimulated much theoretical activity in the past year. There have been studies of the interaction-induced currents,[2–5] large-scale numerical simulations of a tight-binding model of noninteracting electrons,[6] and analytical theories of the single-electron effect in an ensemble of rings.[5,7,8] As of this writing, it seems fair to say that the physics discussed in Refs. 2 and 3 (see also, Ref. 5), which has previously appeared in other guises,[9,10] comes closest to explaining the phenomenon. In particular, it reproduces the observed temperature dependence perfectly.[11] Questions remain, however, about the calculated magnitude and, more importantly, about the sign of the observed effect.

In this context we investigated[4] a case in which the sign is certainly diamagnetic, and the interaction between electrons certainly dominant, namely, rings of superconductors above the transition temperature $T_c$. As one subcase, we considered materials of ordinary $T_c$, say, of a few K, in the region $T-T_c \simeq T_1 \equiv \hbar D/L_\parallel^2$, where $D$ and $L_\parallel$ are, respectively, the electron diffusion constant and the perimeter of the ring. (In Ref. 1, $T_1 \simeq 30$ mK.) In this regime, results follow straightforwardly from Ginzburg-Landau theory. They can be interpreted as due to "classical" Gaussian fluctuations of the superconducting order parameter.[9] Furthermore, there is no ambiguity about the magnitude of the current as $T \to T_c^+$: the empirical $T_c$ that occurs in the final formulas is determined by the extent to which the delicate, and difficult to calculate, balance between Coulomb repulsion and phonon-induced attraction is won by the latter.

The purpose of this paper is to examine various corrections that could, in principle, complicate the testing of simple predictions of the theory alluded to in the last paragraph. In particular, we study the role of coherence destroying processes, including the penetration of the magnetic field into the ring material, spin-orbit scattering, and Zeeman splitting. We also examine inductive effects and answer in the negative the question of whether SQUID-like metastable behavior is observable above $T_c$ in such a mesoscopic ring. These two topics are discussed in Secs. II and III. A very brief recapitulation comprises Sec. IV.

## II. FINITE FIELD AND OTHER CORRECTIONS

In the regime we are discussing here, i.e., $T-T_c \simeq T_1 \ll T_c$, extreme case $B$ in the classification of Ref. 4, the flux-dependent part of the grand potential was obtained from the expression

$$\Delta\Omega = T_c \sum_q \ln \mathcal{E}_0(q) \ , \tag{1}$$

where $\mathcal{E}_0(q)$, equal to $\mathcal{E}(\omega=0,q)$ in our earlier notation,[4] is the static eigenvalue of the pair propagator

$$\mathcal{E}_0(q) = \epsilon + \pi\hbar D q^2/8T_c \ . \tag{2}$$

Here $\epsilon = \ln(T/T_c) \ll 1$, $q=(2\pi/L_\parallel)(n-\Phi/\Phi_0)$, $n=0,\pm1,\ldots$, for the ring geometry, $\Phi$ is the threading flux, and $\Phi_0 = h/2e$. (Transverse wave vectors are frozen out since $L_\perp \ll L_\parallel$, typically $L_\perp \sim 10^{-2} L_\parallel$). With these substitutions, Eq. (1), apart from a flux-independent term, becomes

$$\Delta\Omega = T_c \ln[\cosh(L_\parallel/\xi) - \cos(2\pi\Phi/\Phi_0)] \ , \tag{3}$$

where the symbol is $\xi$, the dirty-limit coherence length; $\xi^2$ is given by $\pi\hbar D/8\epsilon T_c$. Equation (3) is plotted in Fig. 1 for $L_\parallel/\xi = 0.5$; $\Delta\Omega$ is periodic in $\Phi$ with period $\Phi_0$, but it is by no means a simple harmonic. The equilibrium current, $I$, is then obtained from Eq. (3) via the general formula

$$I = -\frac{\partial\Omega}{\partial\Phi} \ . \tag{4}$$

Near $T_c$, where $\xi \to \infty$, the current due to fluctuations

 

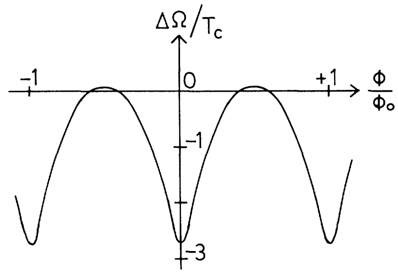


FIG. 1. Thermodynamic potential given by Eq. (3) for $L_\parallel/\xi=0.5$ plotted against flux $\Phi$ in units of $\Phi_0=h/2e$.

in the Cooper channel, studied here, will be the dominant effect, both for a single ring and an ensemble of rings. In the latter case, however, an implicit assumption, which could well be violated, is that the $T_c$'s of the different rings are identical. Experiments on a single ring may thus be needed to observe the pairing fluctuations unambiguously.

Turning now to corrections, phase breaking (e.g., due to inelastic scattering), the penetration of the magnetic field into the ring material, and the Zeeman splitting are taken into account by generalizing (1) to[12,10,3]

$$\Delta\Omega=\tfrac{1}{2}T_c\sum_q \ln[\mathcal{E}_+(q)\mathcal{E}_-(q)] , \tag{5}$$

where

$$\mathcal{E}_\pm=\mathcal{E}_0+\frac{\pi\hbar}{8T_c}\left[\frac{1}{\tau_\varphi}+\frac{1}{\tau_H}\pm i\omega_s\right] . \tag{6}$$

Here $1/\tau_\varphi$ is the inelastic-scattering rate, and (assuming a circular ring of square crosssection)

$$\frac{\hbar}{\tau_H}\simeq\frac{\hbar}{3}\left[\frac{eHL_\perp}{\hbar}\right]^2 D=\frac{16\pi^4}{3}\left[\frac{\Phi}{\Phi_0}\right]^2\left[\frac{L_\perp}{L_\parallel}\right]^2 T_1 . \tag{7}$$

The Zeeman splitting is

$$\hbar\omega_s=2\mu_B H=\frac{12\pi^2}{k_F l}\frac{\Phi}{\Phi_0}T_1\equiv a_s\frac{2\pi\Phi}{\Phi_0}T_1 , \tag{8}$$

where $l$, the elastic mean free path, is typically $\simeq 10^2/k_F$. Thus, $\hbar\omega_s\simeq T_1$ for $\Phi\simeq\Phi_0$. Note that (6) only holds for $\hbar/\tau_\varphi$, $\hbar/\tau_H$, $\hbar\omega_s \ll T_c$.

Spin-orbit scattering leads to a negligible correction as long as $\hbar/\tau_{SO}\ll T$, but reduces the marked effect of the Zeeman splitting as given in (6) in the regime $\hbar/\tau_{SO}\gg T$: In the latter case, one finds[12,3] that $\pm i\omega_s$ in (6) is replaced by $1/\tau_z=3\omega_s^2\tau_{SO}/4$, thus leading to a contribution similar to $1/\tau_H$ (proportional to $\Phi^2$), but typically even smaller.

The pair of equations (5) and (6) yield the following generalization of Eq. (3):

$$\Delta\Omega=T_c\mathrm{Re}\ln[\cosh\gamma_+-\cos(2\pi\Phi/\Phi_0)] , \tag{9}$$

where

$$\gamma_+=\left[\left[\frac{L_\parallel}{\xi}\right]^2+\frac{\hbar}{T_1}\left[\frac{1}{\tau_\varphi}+\frac{1}{\tau_H}+i\omega_s\right]\right]^{1/2} \tag{10}$$

and the square root has a positive real part.

The interesting physical question is then the following: By how much does the current predicted by (9), (10), and (4) differ from that obtained from (3) and (4)? That the differences are small is brought about by the following three observations. (i) Phase-breaking scattering, described by $\tau_\varphi$, simply lowers $T_c$ by the amount $\delta T_c=-\pi\hbar/8\tau_\varphi$ and needs no further consideration. (ii) Field penetration, $\tau_H$, leads to a small field-dependent shift in $T_c$, given in order of magnitude by $\delta T_c\sim -0.02T_1$ for $\Phi\sim\Phi_0$, assuming $L_\perp=10^{-2}L_\parallel$. (A similar shift occurs for strong spin-orbit scattering, as discussed above.) The correction due to $\tau_H$ also affects the current at small flux, which is given by

$$I=-\frac{4\pi T_c}{\Phi_0}\frac{\varphi}{\varphi^2+(L_\parallel/\xi)^2} , \quad \varphi=\frac{2\pi\Phi}{\Phi_0} \tag{11}$$

through the minor modification

$$\left[\frac{L_\parallel}{\xi}\right]^2\to\left[\frac{L_\parallel}{\xi}\right]^2\left[1+\frac{4\pi^2}{3}\left[\frac{L_\perp}{L_\parallel}\right]^2\right]^{-1} . \tag{12}$$

(iii) In order to study the effect of the Zeeman splitting, we consider the regime $L_\parallel\ll\xi$, $\hbar\omega_s\ll T_1$ for small flux. An explicit calculation of (9) and (10) in this limit gives

$$\Delta\Omega=\tfrac{1}{2}T_c\ln\left\{\left[\frac{\hbar\omega_s}{T_1}\right]^2+\left[\varphi^2+\left[\frac{L_\parallel}{\xi}\right]^2\right]^2\right\} . \tag{13}$$

Assuming, in addition, $\varphi\ll L_\parallel/\xi$, the current obtained from (13), (8), and (4) is

$$I=-\frac{2\pi T_c}{\Phi_0}\frac{[a_s^2+2(L_\parallel/\xi)^2]\varphi}{a_s^2\varphi^2+(L_\parallel/\xi)^4} , \tag{14}$$

which, for dominant Zeeman splitting (where $I\simeq -T_c/\Phi$), is a factor of 2 smaller than that obtained from (11) as $T\to T_c$. Since, typically, $2\pi a_s\simeq 1$, Zeeman splitting dominates the small-flux regime close to $T_c$ (except for very strong spin-orbit scattering, see above).

## III. INDUCTANCE AND SQUID ANALOGY

A thermodynamic potential periodic in the enclosed flux, such as we encounter here, is also characteristic of a superconducting ring containing a Josephson junction, i.e., the essential element of a SQUID.[13] A natural question thus arises:[14] Is flux trapping, characteristic of SQUID's, also possible in mesoscopic rings? The answer is that, even in the most favorable case, both the energy barriers and the inductances are too small to make this a realistic possibility. The assumption that the threading flux is equal to the external flux, implicit in the literature on rings, is very well justified.

To examine these questions one must consider the total potential

$$U(\Phi)=\frac{1}{2L}(\Phi-\Phi_x)^2+\Delta\Omega(\Phi)\ , \tag{15}$$

where $L$ [$\simeq 2L_{\parallel}\ln(L_{\parallel}/L_{\perp})$] is the inductance of the ring, $\Phi_x$ is the external flux, and $\Delta\Omega$, neglecting the corrections of Sec. III, is given by Eq. (3). The characteristic scale of the energy barriers is

$$\Delta\Omega(\Phi_0/2)-\Delta\Omega(0)\simeq 2T_c\ln(2\xi/L_{\parallel}) \tag{16}$$

in the most favorable limit $\xi \gg L_{\parallel}$. This is only logarithmically larger than the thermal energy. By comparison, the energy barrier for a Josephson junction is $\hbar I_J/2e$, where $I_J=\pi\Delta/2eR_N$ at low temperatures. [Here $\Delta$ is the energy gap ($\sim T_c$) and $R_N$ the normal-state resistance.] The ratio of the two energies essentially is given by $\sim e^2R_N/h$. Now, the metastable state of a SQUID is robust against thermal fluctuations because $R_N$ is typically smaller than the "quantum of resistance" $h/e^2$ ($\sim 25.8k\Omega$). No such factor is present here. Furthermore, the energy barriers in (15) are reduced from (16) because of the inductance $L$. A natural measure of the inductive energy is $T_L=\Phi_0^2/4\pi^2L$. What is the condition for the appearance of a secondary minimum in (15)? (Take $\Phi_x=0$ for simplicity.) In the most favorable case $\xi \gg L_{\parallel}$, minima occur near $\Phi=\pm\Phi_0$ when

$$2\pi T_L<(\xi/L_{\parallel})T_c\ . \tag{17}$$

Since the inductance of a single ring is typically $10^{-3}$ cm, $T_L\simeq 5\times 10^3$ K, and (17) can only be realized closer to $T_c$ than any experiment we would dare advocate. Note also that for an ensemble of $N$ rings, arranged as described in Ref. 1, the inductance per ring decreases as $N^{-1/2}$, i.e., $T_L$ increases as $N^{1/2}$.

Concentrating thus on the regime of small flux, we consider first the linear response regime $\Phi$, $\Phi_x\to 0$. In this limit, $\Phi$ is reduced compared to $\Phi_x$ according to the relation

$$\Phi=\Phi_x\left[1+\frac{2T_c}{T_L}\frac{\xi^2}{L_{\parallel}^2}\right]^{-1} \tag{18}$$

and the "susceptibility" is given by

$$\Phi_x\to 0,\quad \frac{I}{\Phi_x}=-\frac{8\pi^2T_c}{\Phi_0^2}\left[\frac{2T_c}{T_L}+\left[\frac{L_{\parallel}}{\xi}\right]^2\right]^{-1}\ , \tag{19}$$

which means a smearing out of the temperature dependence ($\sim\epsilon^{-1}$ for $T_L\to\infty$) for $T_c/T_L>(L_{\parallel}/\xi)^2$, i.e., $\epsilon<T_1/T_L$.

We note, finally, that a metastable point of (15) exists for $\Phi_x\sim 3\Phi\sim\Phi_0\ (L_{\parallel}/\xi)$ but only when $\epsilon<T_1/T_L$. As above, we conclude that this is an unobservably small region in temperature.

## IV. CONCLUSIONS

The results of this study can be stated briefly. Various real-world corrections to the equilibrium flux-dependent current in a mesoscopic ring of an ordinary superconductor above $T_c$ have been considered and found small, except for the Zeeman splitting. The latter, for not too strong spin-orbit scattering and typical parameter values, dominates the small-flux regime close to the transition temperature, but nevertheless leads only to minor modifications of the current flux relation. The possibility that such a ring might trap flux has been examined and found to occur in an unobservably small temperature region. Experiments on such rings thus should be able to uncover interesting physics.

*Note added in proof:* Although not explicitly stated in the above, our results are also applicable in the region $T_c(\phi)<T<T_c$, where $T_c(\phi)$ denotes the actual flux-dependent critical temperature, and $\phi\equiv 2\pi\Phi/\Phi_0$. For the simple case described by Eqs. (1) and (2),

$$T_c(\phi)=T_c-(\pi T_1\phi^2/8)$$

for $-\pi<\phi<\pi$, and periodic extensions thereof, as obtained in Ref. 15. The susceptibility derived from (1) diverges as $[T_c(\phi)-T]^{-1}$. We thank E. Riedel for suggesting that we make this explicit.

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[1]L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990); L. P. Levy, Physica B **169**, 245 (1991).

[2]V. Ambegaokar and U. Eckern, Phys. Rev. Lett. **65**, 381 (1990).

[3]U. Eckern, Z. Phys. B **82**, 393 (1991).

[4]V. Ambegaokar and U. Eckern, Europhys. Lett. **13**, 733 (1990).

[5]A. Schmid, Phys. Rev. Lett. **66**, 80 (1991); **66**, 1379 (1991).

[6]H. Bouchiat and G. Montambaux, J. Phys. (Paris) **50**, 2695 (1989); G. Montambaux, H. Bouchiat, D. Sigeti, and R. Friesner, Phys. Rev. B **42**, 7647 (1990); H. Bouchiat, G. Montambaux, and D. Sigeti (unpublished).

[7]F. von Oppen and E. K. Riedel, Phys. Rev. Lett. **66**, 84 (1991).

[8]B. L. Alshuler, Y. Gefen, and Y. Imry, Phys. Rev. Lett. **66**, 88 (1991).

[9]P. A. Lee and M. G. Payne, Phys. Rev. B **5**, 923 (1972); J. Kurkijärvi, V. Ambegaokar, and G. Eilenberger, *ibid.* **5**, 868 (1972).

[10]B. L. Altshuler and A. G. Aronov, in *Electron-Electron Interactions in Disordered Systems,* edited by A. L. Efros and M. Pollak (North-Holland, Amsterdam, 1985), p. 1, and references therein.

[11]V. Ambegaokar and U. Eckern (unpublished).

[12]P. Fulde and K. Maki, Phys. Rev. **141**, 275 (1966); **147**, 414 (1966); K. Maki, in *Superconductivity,* edited by R. Parks (Marcel Dekker, New York, 1969), Vol. II, p. 1035.

[13]M. Tinkham, *Introduction to Superconductivity* (Krieger, Malabar, 1980), Chap. 6.

[14]D. Wohlleben, M. Esser, P. Freche, E. Zipper, and M. Szopa, Phys. Rev. Lett. **66**, 3191 (1991).

[15]W. A. Little and R. D. Parks, Phys. Rev. Lett. **9**, 9 (1962); R. P. Groff and R. D. Parks, Phys. Rev. **176**, 567 (1968).