Meeting Report

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Abstract: This short article is the epilog of the 14 preceding papers in this and the previous issue of CMAM. All are extracted from the 5th Chinese–German Workshop on Computational and Applied Mathematics at Augsburg but submitted as individual papers to the journal.

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The 5th Chinese–German Workshop on Computational and Applied Mathematics was held at the University of Augsburg, Germany, from September 21st to 25th, 2015. The symposium enhanced the mutual understanding of the state of the art of current research on both sides and stimulated future Chinese–German collaborations; it was kindly funded by the Sino-German Center for Research Promotion (grant no. GZ1228). The bilateral workshop followed previous ones in Berlin (2005), Hangzhou (2007), Heidelberg (2009), and Guangzhou (2011) chaired by Carsten Carstensen (Humboldt-Universität zu Berlin, Germany) and Zhong-Ci Shi (Chinese Academy of Science, China). The Augsburg workshop brought together 13 Chinese scientists, 22 participants from German universities and one from a Swiss institution.

In the preceding 14 papers in the journal CMAM, namely ten in the previous issue and four preceding this epilogue in this issue, leading numerical and applied mathematicians of both sides demonstrate the remarkable advances in computational and applied mathematics. The topics range from numerical analysis to scientific computing and include highly current aspects of mathematical modeling, optimization and multi-scale problems.

Bartels and Milicevic compare experimentally the Heron method, the $H^{1/2}$-primal–dual method, and the augmented Lagrangian (splitting) method for total variation regularized minimization problems, where the minimization by numerical methods poses a challenging problem due to the non-differentiability of the BV-seminorm. They investigate the choice of stopping criteria, influence of rough initial data, step sizes as well as mesh sizes, and found that the Heron method outperforms the other two examples in their paper [1].

Chen and Xie propose a weak Galerkin finite element method for two- and three-dimensional linear elasticity problems on conforming or nonconforming polyhedral meshes in [2]. The resulted scheme is in fact

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equivalent to a hybridizable discontinuous Galerkin finite element scheme, which allows for a robust optimal error estimate with respect to the Lamé constant.

Dörfler, Findeisen, and Wieners propose a space–time discretization for linear first-order hyperbolic evolution systems with a discontinuous Galerkin approximation for the space discretization and a Petrov–Galerkin scheme for the time discretization. They show well-posedness and convergence of the discrete system in their paper [3]. Furthermore they introduce an adaptive strategy based on goal-oriented dual-weighted error estimation, and solve the full space–time linear system with a parallel multilevel pre-conditioner.

Feng, Lu, and Xu propose and analyze a hybridizable discontinuous Galerkin (HDG) method for the three-dimensional time-harmonic Maxwell equations coupled with the impedance boundary condition in the case of high wave number. They prove that the HDG method is absolutely stable for all wave numbers $k > 0$ in the sense that no mesh constraint is required for the stability. A wave-number-explicit stability constant is also obtained in [4]. The main ingredients for the analysis consist of a specific penalty parameter $\frac{1}{kh}$ (where h is the meshsize of the mesh) and a PDE duality argument. Utilizing the stability estimate and a non-standard technique, the authors establish the error estimates in the L^2-norm and the energy-norm.

Harbrecht and Schneider investigate the a posteriori error estimation of finite element approximations to the solution of the Poisson equation in their note [5]. By incorporating the discrete residual in terms of the BPX pre-conditioner into the traditional error estimator, they derive a reliable and efficient error estimator. If the finite element approximation is a Galerkin solution, the derived error estimator coincides with the standard element and edge-based a posteriori error estimator. The analysis relies on a hypothetical infinite and dense collection of nested finite element spaces, where the infinite BPX scheme provides a frame in $H^{-1}(\Omega)$.

Ke, Li, and Xiao study the extreme points of a set of multi-stochastic tensors in [6]. They establish two necessary and sufficient conditions for a multi-stochastic tensor to be an extreme point. Those conditions characterize the generators of multi-stochastic tensors, and develop an algorithm to search the convex combination of extreme points for an arbitrary given multi-stochastic tensor. They derive some expression properties for third-order and n-dimensional multi-stochastic tensors, where all extreme points of three-dimensional and four-dimensional triply-stochastic tensors can be produced.

Ling, Marth, Praetorius, and Voigt consider a hydrodynamic multi-phase field problem to model the interaction of deformable objects in [7]. They use an operator splitting approach to discretize in time, and apply the finite element method to discretize in space, where a P_2/P_1 Taylor–Hood element approximates the flow problem and all other quantities are discretized in space with P_2 elements. To improve the efficiency of the numerical approach further, the authors take one phase field variable for each object, where an independent adaptive mesh refinement is allowed for each variable, and the special structure of various terms admits interpolating the solution on one mesh onto another without loss of information. They demonstrate, on an example describing the interaction of red blood cells in an idealized vessel, that the resulting multi-mesh adaptive algorithm shows improvements by a factor of two or higher in computational time, if compared with a classical finite element approach with one adaptively refined mesh.

Schulz and Siebenborn compare numerically the usage of different metrics for the model shape optimization problem of minimizing energy dissipation in Stokes flow around an obstacle in two and three dimensions in [8]. The choice of the surface metric has a major impact in the shape optimization approach based on shape calculus. The paper shows the advantages of the Steklov–Poincaré metric over the classical Laplace–Beltrami metric in terms of convergence properties, overall computational effort, and mesh quality.

Shao, Han, and Hu combine the two-level technique and the bivariate spline method of degree ≥ 5 to propose a new numerical approach within the stream function formulation for Navier–Stokes equations in [9]. A low-order spline approximation is obtained by solving a nonlinear problem, and a fine approximation based on the Newton method taking the aforementioned rough solution as the initial guess in a high-order C^1 piecewise polynomial approximation. The authors prove the optimal convergence of the scheme with respect to the mesh-size and the degree of polynomials.

Su, Chen, Li, and Xu prove the validity of the Ladyzenskaja–Babuska–Brezzi (LBB) condition for the triangular spectral method for the Stokes equations in [10]. The proof uses the equivalence of the LBB condition to the existence of an H_0^1-stable Fortin projection operator. Moreover, an optimal lower bound for the associated constant is shown (in terms of the degree of the basis polynomials) and used to derive an error estimate for the pressure. The estimates are confirmed numerically with a generic example up to large polynomial order. In particular, the results clarify previous speculations on the behavior of the constant.

Gräser, Kahnt and Kornhuber consider a multi-phase extension of the classical Penrose–Fife system derived from a general entropy functional and an associated thin-film approximation variant in [11]. The entropy functional combines a Ginzburg–Landau energy with the thermodynamic entropy and the unknowns of the resulting problem are the phase field and the inverse temperature. The paper focuses on the numerical approximation based on an implicit discretization in time (with explicit treatment of the concave terms) and a discretization by piecewise linear finite elements with adaptive mesh refinement based on hierarchical a posteriori error estimation in space. Numerical experiments are presented to illustrate the properties of the discretization method; simulation results for a liquid phase crystallization process are provided as well.

Huang and Yang propose a class of new tailored finite point methods (TFPM) for the numerical solution of parabolic equations in [12]. Their finite point method has been tailored based on the local exponential basis functions. By the idea of their TFPM, they can recover all the traditional finite difference schemes. They can also construct some new TFPM schemes with better stability condition and accuracy. Furthermore, combining with the Shishkin mesh technique, they construct the uniformly convergent TFPM scheme for the convection dominant convection–diffusion problem.

Kanschat and Lucero review the derivation of weakly penalized discontinuous Galerkin methods for scattering dominated radiation transport and extend the asymptotic analysis to non-isotropic scattering. Their paper [13] focuses on the influence of the penalty parameter on the edges and they derive a new penalty for interior edges and boundary fluxes. Finally, they study how the choice of the penalty parameters influences discretization accuracy and solver speed.

Peterseim and Scheichl present a new approach to the numerical upscaling for elliptic problems with rough diffusion coefficient at high contrast in [14]. It is based on the localizable orthogonal decomposition of H^1 into the image and the kernel of some novel stable quasi-interpolation operators with local contrast-independent L^2-approximation properties. They propose a set of sufficient assumptions on these quasi-interpolation operators that guarantee in principle optimal convergence without pre-asymptotic effects for high-contrast coefficients. They also provide an example of a suitable operator and establish the assumptions for a particular class of examples.

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