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Wave Scattering by a Circular Elastic Plate in Water of Finite Depth: A Closed Form Solution

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ABSTRACT

We present a solution for a circular thin plate of shallow draft on water of finite depth subject to linear wave forcing of a single frequency. The solution, which is given in a closed form, is based on decomposing the solution into angular eigenfunctions. The coefficients in the expansion are then found by matching the potential and its derivative at the plate edge and imposing the free edge conditions for the plate. The matching is accomplished by taking the inner product with respect to the vertical eigenfunctions for the free surface. The equations that are derived are transformed so that the final system of equations involves only the unknowns under the plate. Solutions are presented and compared to the results of Meylan (2002), who presented a solution for a plate of arbitrary geometry.

INTRODUCTION

The problem of a linear, floating thin plate of shallow draft subject to wave forcing is a standard problem in hydroelasticity which can be used to model a range of physical systems. In 2 dimensions, many different solution methods exist. For shallow water, a solution was presented by Stoker (1958); for finite depth, solutions have been presented by Meylan and Squire (1994) and Newman (1994), amongst others. In 3 dimensions, a solution for a circular plate on shallow water was presented by Zilman and Miloh (2000), and general solution methods have been presented by Meylan and Squire (1994), Meylan (2002) and Kashiwagi (1998), amongst others. However, with the exception of Zilman and Miloh (2000), these solutions were based on the free-surface Green's function and were highly numerical. Even the solution presented by Meylan and Squire (1996) for a circular plate only exploited the circular geometry to calculate the modes of vibration of the free plate.

The only 3-dimensional solution for a thin plate of shallow draft that is not based on a highly numerical method was presented by Zilman and Miloh (2000). The exact same solution method was also independently derived by Tsubogo (2001). Their solution was for the case of shallow water and a circular plate. The circular geometry allows separation of variables in the angular direction, so that the solution may be found by decoupling the solutions for each angular eigenfunction. Once this has been accomplished, the solution for each angular direction can be found by solving a linear system of 4 equations. These were derived by matching the potential and its derivative, and by imposing the 2 boundary conditions at the edge of the plate.

We present here an extension of the method of Zilman and Miloh (2000) to the case where the water depth is finite. In this case we can still solve for each angular eigenfunction separately, and we match the potential and its derivative and impose the boundary conditions at the plate edge. However, we must match the potential not at a point but throughout the water depth. This matching is accomplished by taking the inner product with respect to the vertical eigenfunctions which satisfy the free surface condition. We present results for the method, which are compared to the results of Meylan (2002).

GOVERNING EQUATIONS

We begin with the equations for the plate-water system in nondimensional form as the problem is so well known. The derivation and nondimensionalisation are discussed in detail in Meylan (2002). We nondimensionalise the spatial variables with respect to a length parameter L (for example, L may be derived from the area of the plate, or L may be the characteristic length $(D/\rho g)^{1/4}$, where D is the rigidity constant of the plate, ρ the density of the water, and g the gravitational constant) and the time variables with respect to $\sqrt{L/g}$. We assume that all motions are time-harmonic with radian frequency $\omega = \sqrt{\alpha}$, so that the velocity potential of the water, $\bar{\phi}(x, t)$, can be expressed as the real part of a complex quantity ϕ:

$$\bar{\phi}(\mathbf{x}, t) = \mathrm{Re}\left\{\phi(\mathbf{x})\mathrm{e}^{-\mathrm{i}\sqrt{\alpha}t}\right\} \tag{1}$$

We will use a cylindrical coordinate system, $\mathbf{x} = (r, \theta, z)$, assumed to have its origin at the centre of the circular plate with radius a. The water is assumed to have constant finite depth H, and the z-direction points vertically upward with the water surface at $z = 0$ and the seafloor at $z = -H$. The boundary value problem can thus

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be expressed as:

$$\left.\begin{array}{ll}\Delta\phi = 0, & -H < z < 0,\\ \phi_z = 0, & z = -H,\\ \phi_z = \alpha\phi, & z = 0,\ r > a,\\ (\beta\Delta^2 + 1 - \alpha\gamma)\phi_z = \alpha\phi, & z = 0,\ r < a\end{array}\right\} \tag{2}$$

where the constants β and γ are given by:

$$\beta = \frac{D}{\rho L^4 g}, \quad \gamma = \frac{\rho_i h}{\rho L} \tag{3}$$

and ρ_i is the density of the plate. We must also apply the edge conditions for the plate and the radiation condition as $r \to \infty$. The subscript z denotes the derivative in the z-direction.

SOLUTION METHOD

Separation of Variables

We now separate variables, noting that, since the problem has circular symmetry, we can write the potential as:

$$\phi(r,\theta,z) = \zeta(z)\sum_{n=-\infty}^{\infty}\rho_n(r)\mathrm{e}^{in\theta} \tag{4}$$

Applying Laplace's equation we obtain:

$$\zeta_{zz} + \mu^2\zeta = 0 \tag{5}$$

so that:

$$\zeta = \cos\mu(z+H) \tag{6}$$

where the separation constant μ^2 must satisfy the standard dispersion equations:

$$k\tan(kH) = -\alpha, \quad r > a \tag{7}$$

$$\kappa\tan(\kappa H) = \frac{-\alpha}{\beta\kappa^4 + 1 - \alpha\gamma}, \quad r < a \tag{8}$$

Note that we have set $\mu = k$ under the free surface and $\mu = \kappa$ under the plate. The dispersion equations are discussed in detail in Fox and Squire (1994). We denote the positive imaginary solution of Eq. 7 by k_0 and the positive real solutions by k_m, $m \geq 1$. The solutions of Eq. 8 will be denoted by κ_m, $m \geq -2$. The fully complex solutions with a positive imaginary part are κ_{-2} and κ_{-1} (where $\kappa_{-1} = \overline{\kappa_{-2}}$), the negative imaginary solution is κ_0, and the positive real solutions are κ_m, $m \geq 1$. We define:

$$\phi_m(z) = \frac{\cos k_m(z+H)}{\cos k_m H}, \quad m \geq 0 \tag{9}$$

as the vertical eigenfunction of the potential in the open water region and:

$$\psi_m(z) = \frac{\cos\kappa_m(z+H)}{\cos\kappa_m H}, \quad m \geq -2 \tag{10}$$

as the vertical eigenfunction of the potential in the plate-covered region. For later reference, we note that:

$$\int_{-H}^{0}\phi_m(z)\phi_n(z)\,\mathrm{d}z = A_m\delta_{mn} \tag{11}$$

where δ_{mn} is the Kronecker delta and:

$$A_m = \frac{1}{2}\left(\frac{\cos k_m H\sin k_m H + k_m H}{k_m\cos^2 k_m H}\right) \tag{12}$$

and:

$$\int_{-H}^{0}\phi_n(z)\psi_m(z)\,\mathrm{d}z = B_{mn} \tag{13}$$

where:

$$B_{mn} = \frac{k_n\sin k_n H\cos\kappa_m H - \kappa_m\cos k_n H\sin\kappa_m H}{(\cos k_n H\cos\kappa_m H)\,(k_n^2 - \kappa_m^2)} \tag{14}$$

We now solve for the function $\rho_n(r)$. Using Laplace's equation in polar coordinates we obtain:

$$\frac{\mathrm{d}^2\rho_n}{\mathrm{d}r^2} + \frac{1}{r}\frac{\mathrm{d}\rho_n}{\mathrm{d}r} - \left(\frac{n^2}{r^2} + \mu^2\right)\rho_n = 0 \tag{15}$$

where μ is k_m or κ_m, depending on whether r is greater or less than a. We can convert this equation to the standard form by substituting $y = \mu r$ to obtain:

$$y^2\frac{\mathrm{d}^2\rho_n}{\mathrm{d}y^2} + y\frac{\mathrm{d}\rho_n}{\mathrm{d}y} - (n^2 + y^2)\rho_n = 0 \tag{16}$$

The solution of this equation is a linear combination of the modified Bessel functions of order n, $I_n(y)$ and $K_n(y)$. (See Abramowitz and Stegun, 1964.) Since the solution must be bounded, we know that under the plate the solution will be a linear combination of $I_n(y)$, while outside the plate the solution will be a linear combination of $K_n(y)$. Thus the potential can be expanded as:

$$\phi(r,\theta,z) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty}a_{mn}K_n(k_m r)\mathrm{e}^{in\theta}\phi_m(z), \quad r > a \tag{17}$$

$$\phi(r,\theta,z) = \sum_{n=-\infty}^{\infty}\sum_{m=-2}^{\infty}b_{mn}I_n(\kappa_m r)\mathrm{e}^{in\theta}\psi_m(z), \quad r < a \tag{18}$$

where a_{mn} and b_{mn} are the coefficients of the potential in the open water and the plate-covered region, respectively.

Incident Potential

The incident potential is a wave of amplitude A in displacement travelling in the positive x-direction. Following Zilman and Miloh (2000), the incident potential can then be written as:

$$\phi^{\mathrm{I}} = \frac{A}{\mathrm{i}\sqrt{\alpha}}e^{k_0 x}\phi_0(z) = \sum_{n=-\infty}^{\infty}e_n I_n(k_0 r)\phi_0(z)\,\mathrm{e}^{\mathrm{i}n\theta} \tag{19}$$

where $e_n = A/\left(\mathrm{i}\sqrt{\alpha}\right)$. (We retain the dependence on n for situations where the incident potential might take another form.)

Boundary Conditions

The boundary conditions for the plate also have to be considered. The vertical force and bending moment must vanish, which, following Zilman and Miloh (2000), can be written as:

$$\left[\bar{\Delta} - \frac{1-\nu}{r}\left(\frac{\partial}{\partial r} + \frac{1}{r}\frac{\partial^2}{\partial\theta^2}\right)\right]w = 0 \tag{20}$$

and:

$$\left[\frac{\partial}{\partial r}\bar{\Delta} - \frac{1-\nu}{r^2}\left(\frac{\partial}{\partial r} + \frac{1}{r}\right)\frac{\partial^2}{\partial \theta^2}\right] w = 0 \tag{21}$$

where w is the time-independent surface displacement, ν is Poisson's ratio, and $\bar{\Delta}$ is the polar coordinate Laplacian:

$$\bar{\Delta} = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} \tag{22}$$

Displacement of Plate

The surface displacement and the water velocity potential at the water surface are linked through the kinematic boundary condition:

$$\phi_z = -\mathrm{i}\sqrt{\alpha} w \quad \text{at } z = 0 \tag{23}$$

From Eq. 2, the potential and the surface displacement are thus related by:

$$w = \mathrm{i}\sqrt{\alpha}\phi, \quad r > a \tag{24}$$

$$(\beta\bar{\Delta}^2 + 1 - \alpha\gamma) w = \mathrm{i}\sqrt{\alpha}\phi, \quad r < a \tag{25}$$

The surface displacement can also be expanded in eigenfunctions as:

$$w(r,\theta) = \sum_{n=-\infty}^{\infty}\sum_{m=0}^{\infty} \mathrm{i}\sqrt{\alpha} a_{mn} K_n(k_m r) \mathrm{e}^{\mathrm{i}n\theta}, \quad r > a \tag{26}$$

$$w(r,\theta) = \sum_{n=-\infty}^{\infty}\sum_{m=-2}^{\infty} \mathrm{i}\sqrt{\alpha}(\beta\kappa_m^4 + 1 - \alpha\gamma)^{-1} b_{mn} I_n(\kappa_m r) \mathrm{e}^{\mathrm{i}n\theta}, \quad r < a \tag{27}$$

using the fact that:

$$\bar{\Delta}\left(I_n(\kappa_m r)\mathrm{e}^{\mathrm{i}n\theta}\right) = \kappa_m^2 I_n(\kappa_m r) \mathrm{e}^{\mathrm{i}n\theta} \tag{28}$$

Infinite Dimensional System of Equations

The boundary conditions (Eqs. 20 and 21) can be expressed in terms of the potential using Eq. 28. Since the angular modes are uncoupled, the conditions apply to each mode, giving:

$$\sum_{m=-2}^{\infty} (\beta\kappa_m^4 + 1 - \alpha\gamma)^{-1} b_{mn}\left(\kappa_m^2 I_n(\kappa_m a) - \frac{1-\nu}{a}\left(\kappa_m I_n'(\kappa_m a) - \frac{n^2}{a} I_n(\kappa_m a)\right)\right) = 0 \tag{29}$$

$$\sum_{m=-2}^{\infty} (\beta\kappa_m^4 + 1 - \alpha\gamma)^{-1} b_{mn}\left(\kappa_m^3 I_n'(\kappa_m a) + n^2\frac{1-\nu}{a^2}\left(\kappa_m I_n'(\kappa_m a) + \frac{1}{a} I_n(\kappa_m a)\right)\right) = 0 \tag{30}$$

The potential and its derivative must be continuous across the transition from open water to the plate-covered region. Thus, the potentials and their derivatives at $r = a$ have to be equal. Again, we know that this must be true for each angular mode, and we obtain:

$$e_n I_n(k_0 a)\phi_0(z) + \sum_{m=0}^{\infty} a_{mn} K_n(k_m a)\phi_m(z) = \sum_{m=-2}^{\infty} b_{mn} I_n(\kappa_m a)\psi_m(z) \tag{31}$$

$$e_n k_0 I_n'(k_0 a)\phi_0(z) + \sum_{m=0}^{\infty} a_{mn} k_m K_n'(k_m a)\phi_m(z) = \sum_{m=-2}^{\infty} b_{mn}\kappa_m I_n'(\kappa_m a)\psi_m(z) \tag{32}$$

for each n. We solve these equations by multiplying both equations by $\phi_l(z)$ and integrating from $-H$ to 0 to obtain:

$$e_n I_n(k_0 a) A_0 \delta_{0l} + a_{ln} K_n(k_l a) A_l = \sum_{m=-2}^{\infty} b_{mn} I_n(\kappa_m a) B_{ml} \tag{33}$$

$$e_n k_0 I_n'(k_0 a) A_0 \delta_{0l} + a_{ln} k_l K_n'(k_l a) A_l = \sum_{m=-2}^{\infty} b_{mn}\kappa_m I_n'(\kappa_m a) B_{ml} \tag{34}$$

Eq. 33 can be solved for the open-water coefficients a_{mn}:

$$a_{ln} = -e_n \frac{I_n(k_0 a)}{K_n(k_0 a)}\delta_{0l} + \sum_{m=-2}^{\infty} b_{mn}\frac{I_n(\kappa_m a) B_{ml}}{K_n(k_l a) A_l} \tag{35}$$

which can then be substituted into Eq. 34 to give us:

$$\left(k_0 I_n'(k_0 a) - k_0\frac{K_n'(k_0 a)}{K_n(k_0 a)} I_n(k_0 a)\right) e_n A_0 \delta_{0l} = \sum_{m=-2}^{\infty}\left(\kappa_m I_n'(\kappa_m a) - k_l \frac{K_n'(k_l a)}{K_n(k_l a)} I_n(\kappa_m a)\right) B_{ml} b_{mn} \tag{36}$$

for each n. Together with Eqs. 29 and 30, Eq. 36 gives the required equations to solve for the coefficients of the water velocity potential in the plate-covered region.

NUMERICAL SOLUTION

To solve the system of Eq. 36 together with the boundary conditions, we set the upper limit of m as M. We also set the angular expansion to be from $n = -N$ to N. This gives us:

$$\phi(r,\theta,z) = \sum_{n=-N}^{N}\sum_{m=0}^{M} a_{mn} K_n(k_m r)\mathrm{e}^{\mathrm{i}n\theta}\phi_m(z), \quad r > a \tag{37}$$

$$\phi(r,\theta,z) = \sum_{n=-N}^{N}\sum_{m=-2}^{M} b_{mn} I_n(\kappa_m r)\mathrm{e}^{\mathrm{i}n\theta}\psi_m(z), \quad r < a \tag{38}$$

Since l is an integer with $0 \le l \le M$, this leads to a system of $M+1$ equations. The number of unknowns is $M+3$, and the 2 extra equations are obtained from the boundary conditions for the free plate (Eqs. 29 and 30). The equations to be solved for each

n are:

$$\left(k_0 I_n'(k_0 a) - k_0 \frac{K_n'(k_0 a)}{K_n(k_0 a)} I_n(k_0 a)\right) e_n A_0 \delta_{0l} = \sum_{m=-2}^{M} \left(\kappa_m I_n'(\kappa_m a) - k_l \frac{K_n'(k_l a)}{K_n(k_l a)} I_n(\kappa_m a)\right) B_{ml} b_{mn} \tag{39}$$

$$\sum_{m=-2}^{M} (\beta\kappa_m^4 + 1 - \alpha\gamma)^{-1} b_{mn} \left(\kappa_m^2 I_n(\kappa_m a) - \frac{1-\nu}{a}\left(\kappa_m I_n'(\kappa_m a) - \frac{n^2}{a} I_n(\kappa_m a)\right)\right) = 0 \tag{40}$$

$$\sum_{m=-2}^{M} (\beta\kappa_m^4 + 1 - \alpha\gamma)^{-1} b_{mn} \left(\kappa_m^3 I_n'(\kappa_m a) + n^2 \frac{1-\nu}{a^2}\left(\kappa_m I_n'(\kappa_m a) + \frac{1}{a} I_n(\kappa_m a)\right)\right) = 0 \tag{41}$$

It should be noted that the solutions for positive and negative n are identical, so that they do not both need to be calculated. Some minor simplifications, which are a consequence of this, are discussed in more detail in Zilman and Miloh (2000).

SHALLOW-WATER THEORY OF ZILMAN AND MILOH

The shallow-water theory of Zilman and Miloh (2000) can be recovered by simply setting the depth shallow enough so that the shallow-water theory is valid and the setting is $M = 0$. If the shallow-water theory is valid, then the first 3 roots of the dispersion equation for the ice will be exactly the same roots found in the shallow-water theory by solving the polynomial equation. The system of equations has 4 unknowns (3 under the plate and 1 in the open water), exactly as for the theory of Zilman and Miloh (2000). We do not present any comparison with the results of Zilman and Miloh (2000), because they applied their theory in a situation (wavelength 200, water depth 20) where the shallow-water approximation is not valid. It should be noted, however, that our equations become identical to those of Zilman and Miloh (2000) when $M = 0$ and the water depth is chosen so that the shallow-water approximation is valid.

NUMERICAL RESULTS

We present solutions for a plate of radius $a = 100$. The wavelength is $\lambda = 50$ (recall that $\alpha = 2\pi/\lambda \tanh(2\pi H/\lambda)$), $\beta = 10^5$ and $\gamma = 0$. The incident wave is of unit amplitude. We begin with some convergence results, first of all fixing the number of roots of the dispersion equation $M = 8$ and varying the number of terms in the angular expansion N. Fig. 1 shows the real part of the displacement. The number of points in the angular expansion is $N = 2$ (a), 4 (b), 8 (c) and 16 (d). The depth is $H = 25$. For this situation it follows that we require only $N = 8$ for an accurate solution, which means we only need solve 9 systems of equations. Now we fix the number of points in the angular expansion as $N = 16$ and vary the number of roots of the dispersion equation M. Fig. 2 shows the real part of the displacement. The number of roots of the dispersion equation is $M = 0$ (a), 2 (b), 4 (c) and 8 (d). The depth is $H = 25$. It follows that we require only $M = 2$ for an accurate solution, which means that we only need solve a 5×5 system of equations. This shows how efficient this closed form solution is.

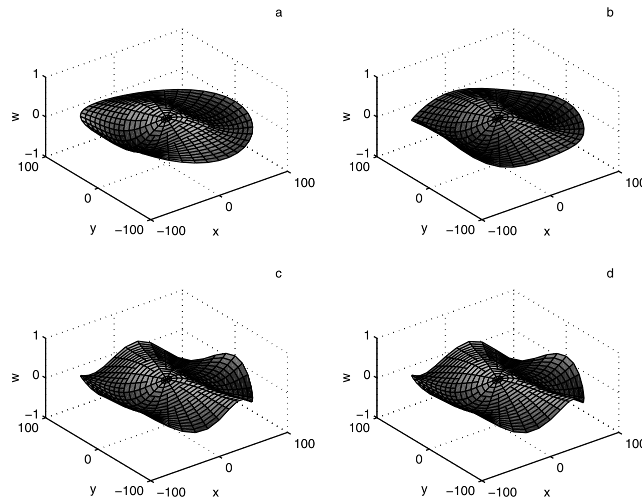

Fig. 1 Real part of displacement for a plane wave of unit amplitude incident from negative x-direction. Parameters chosen as $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 25$. M fixed as 8 while N chosen as $N = 2$ (a), 4 (b), 8 (c) and 16 (d).

We compare with the results presented in Meylan (2002) for an arbitrary shaped plate modified to compute the solution for finite depth. The circle is represented in this scheme by square panels which are arranged to form a circular shape, as nearly as possible. Figs. 3 and 4 show the real part (a and c) and imaginary part (b and d) of the displacement for depth $H = 25$ and $H = 1$, respectively. The number of points in the angular expansion is $N = 16$. The number of roots of the dispersion equation is $M = 8$. Plots (a) and (b) are calculated using the circular plate method described here. Plots (c) and (d) are calculated using an arbitrary shaped plate method, with the panels shown being the actual panels used in the calculation. We see the expected agreement between the 2 methods.

Finally, Tables 1 and 2 show the values of the coefficients b_{mn} for the case $H = 25$. The very rapid decay of the higher evanescent modes is apparent, which explains the rapid convergence in M shown in Fig. 2.

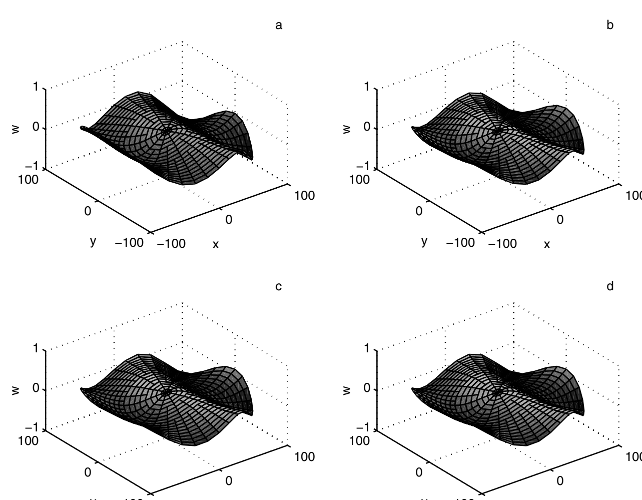

Fig. 2 Real part of displacement for a plane wave of unit amplitude incident from negative x-direction. Parameters chosen as $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 25$. N fixed as 16 while M chosen as $M = 0$ (a), 2 (b), 4 (c) and 8 (d).

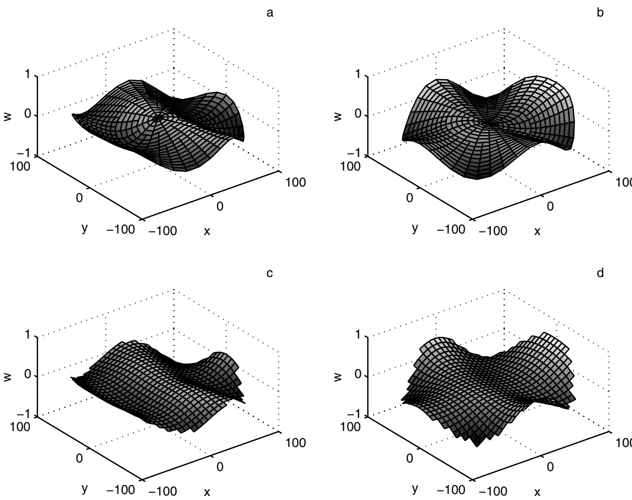

Fig. 3 Real part (a and c) and imaginary part (b and d) of displacement for a plane wave of unit amplitude incident from negative x-direction. Parameters chosen as $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 25$. Number of points in angular expansion: $N = 16$ and $M = 8$. Plots (c) and (d) calculated using arbitrary shaped plate method.

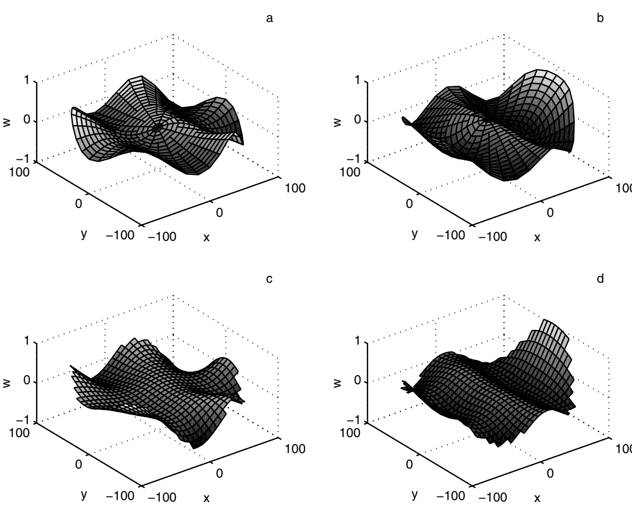

Fig. 4 Real part (a and c) and imaginary part (b and d) of displacement for a plane wave of unit amplitude incident from negative x-direction. Parameters chosen as $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 1$. Number of points in angular expansion: $N = 16$ and $M = 8$. Plots (c) and (d) calculated using arbitrary shaped plate method.

SUMMARY

We have presented a closed form solution for a circular thin plate of shallow draft floating on the water surface subject to linear wave forcing. The solution was based on decomposing the solution into angular eigenfunctions that are uncoupled. A matched eigenfunction expansion is then used in the 2 regions, inside and outside the plate-covered region. The number of roots of the dispersion equation in the regions is always chosen such that there are 2 extra roots under the plate. The conditions that the potential and its derivative must match under the plate edge

b_{mn}	$n = 0$	$n = 1$
$m = -2$	$1.32 \cdot 10^{-1} - 9.71 \cdot 10^{-1}\mathrm{i}$	$6.85 \cdot 10^{-1} - 6.37 \cdot 10^{-1}\mathrm{i}$
$m = -1$	$-6.38 \cdot 10^{-5} + 1.47 \cdot 10^{-3}\mathrm{i}$	$-3.92 \cdot 10^{-3} + 3.99 \cdot 10^{-3}\mathrm{i}$
$m = 0$	$-3.29 \cdot 10^{-4} + 1.43 \cdot 10^{-3}\mathrm{i}$	$4.26 \cdot 10^{-3} - 3.62 \cdot 10^{-3}\mathrm{i}$
$m = 1$	$4.31 \cdot 10^{-7} - 3.18 \cdot 10^{-6}\mathrm{i}$	$-6.64 \cdot 10^{-6} - 7.14 \cdot 10^{-6}\mathrm{i}$
$m = 2$	$6.79 \cdot 10^{-13} - 5.01 \cdot 10^{-12}\mathrm{i}$	$-5.78 \cdot 10^{-12} - 6.21 \cdot 10^{-12}\mathrm{i}$
$m = 3$	$1.35 \cdot 10^{-18} - 9.95 \cdot 10^{-18}\mathrm{i}$	$-9.69 \cdot 10^{-18} - 1.04 \cdot 10^{-17}\mathrm{i}$

Table 1 Coefficients b_{mn} for parameters $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 25$

b_{mn}	$n = 2$	$n = 3$
$m = -2$	$2.95 \cdot 10^{-1} - 1.12 \cdot 10^{0}\mathrm{i}$	$6.09 \cdot 10^{-1} - 4.95 \cdot 10^{-1}\mathrm{i}$
$m = -1$	$1.41 \cdot 10^{-3} + 2.82 \cdot 10^{-3}\mathrm{i}$	$-4.28 \cdot 10^{-3} + 3.89 \cdot 10^{-3}\mathrm{i}$
$m = 0$	$-2.62 \cdot 10^{-3} + 1.76 \cdot 10^{-3}\mathrm{i}$	$4.68 \cdot 10^{-3} - 3.39 \cdot 10^{-3}\mathrm{i}$
$m = 1$	$2.07 \cdot 10^{-7} - 7.89 \cdot 10^{-7}\mathrm{i}$	$-6.30 \cdot 10^{-6} - 7.74 \cdot 10^{-6}\mathrm{i}$
$m = 2$	$8.87 \cdot 10^{-13} - 3.38 \cdot 10^{-12}\mathrm{i}$	$-5.54 \cdot 10^{-12} - 6.81 \cdot 10^{-12}\mathrm{i}$
$m = 3$	$1.94 \cdot 10^{-18} - 7.39 \cdot 10^{-18}\mathrm{i}$	$-9.37 \cdot 10^{-18} - 1.15 \cdot 10^{-17}\mathrm{i}$

Table 2 Coefficients b_{mn} for parameters $\lambda = 50$, $a = 100$, $\beta = 10^5$, $\gamma = 0$ and $H = 25$

are converted to a system of equations by taking an inner product with respect to the vertical eigenfunctions for the free surface. Two further equations are found by imposing the edge conditions of the plate. We have shown that the solutions agree with those found using Meylan's method (2002) for an arbitrary shaped plate.

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