A New Methodology to Derive a Bank's Maturity Structure Using Accounting-Based Time Series Information

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1 Introduction

While over the past few years both banking supervisors and researchers have focussed their attention on banks' credit risk, the spotlight is now being turned again on interest rate risk. One reason for this is its character as a kind of systemic risk: there is evidence that a rise in interest rates affects most banks negatively. An historical example of a banking crisis caused by high interest rates is the 'Savings and Loan Crisis' which occurred in the USA during the 1980s.³ Between 1980 and 1988, 563 of the approximately 4,000 savings and loan institutions failed, while further failures were prevented by 333 supervisory mergers. The total costs of the crisis are estimated at USD160 billion. Recently, the Basel Committee on Banking Supervision published principles for the management and supervision of interest rate risk that go far beyond current practice.⁴ However, few data are available concerning banks' interest rate risk exposure.

Whereas the Deutsche Bundesbank has exclusive information for some sub-samples of banks that is based on bank-internal risk management systems, most methods proposed in literature rely on accounting-based information. Accounting-based information on the banks' maturity structure usually contains information on the amount of positions within certain time bands: the total outstanding amount of a given position is distributed among a number of time bands according to initial maturity and/or remaining time to maturity. Approaches proposed in literature use

³ See [4] for a detailed analysis.
⁴ See [1].
one-point-in-time data, typically the most recent report. In order to derive a cash flow structure and its interest rate sensitivity, a certain distribution within the time bands is assumed. The Economic Value Model (EVM) of the Federal Reserve assumes a concentration in the middle of a time band. A similar approach is applied by [7] who quantify the interest rate risk of the Indian banking system. In contrast, [2] and [8] assume a uniform distribution. The main shortcoming of these approaches is the omission of further information about the distribution within a time band, which can either be taken from time series information or additional data sources. We present here a methodology to systematically use different accounting-based data sources and time series information to derive the maturity structure of banks' assets and liabilities. In a simulation approach we compare our new methodology with models proposed in literature and show that time series can contain important information for inferring the banks' maturity structure.

The model presented here was developed within a joint research project of the Deutsche Bundesbank and the Catholic University of Eichstaett-Ingolstadt. An extended model was applied to quantify the interest rate risk exposure of all German banks. For further details and applications see [3].

2 Model

Let time be discrete: \( t \in T = \mathbb{N}_0 \). In each \( t_{beg} \in T \) the bank invests the amount \( X_{pos,t_{beg},t_{end}} \) in a position \( pos \) (such as loans on the asset side or deposits on the liability side) that matures in \( t_{end} > t_{beg} \). We will refer to these variables as 'business items'. To keep things simple, we omit possible provisions and premature redemptions here. Under this assumption, the outstanding amount in \( t \) that is due in \( s > t \) is given by

\[
OA(pos, t, s) \sum_{i=0}^{t} X_{pos,i,s}.
\]

The knowledge of the outstanding amounts \( OA(pos, t, s) \) for each \( s \) and \( pos \) is the basis for deriving the maturity and cash flow structure of the bank in \( t \). The cash flow structure in turn determines the bank's on-balance-sheet interest rate risk exposure and makes it possible to apply risk measures like duration or value at risk. Unlike the bank itself, external analysts do not have detailed information on the maturity structure. However, at certain dates the bank reveals some information on its maturity structure by reporting the amount of a position broken down into a certain number of time bands. These contain for each \( pos \) the sum of all business within specified ranges of initial maturity or remaining time to maturity, respectively. We assume that in each \( t^i_{obs} \in T^i_{obs} \subset T \) there are \( N \) report items characterized by the business' initial maturity: \( ITM^i_t \) with \( n \in \{1,\ldots,N\} \) reports the amount of \( pos^n \) in \( t \) with an initial maturity within the time band \( t^i_{lower} < t_{end} - t_{beg} \leq t^i_{upper} \). Analogously, in each \( t^i_{obs} \in T^i_{obs} \subset T \) there are \( M \) items characterized by the business' remaining

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5 See [6] for an advanced approach using detailed information on the banks' assets and liabilities.

6 See [5] and [9]. For an evaluation of the model see [11], [10] and [9].

7 Accounting-based data can here include both publicly available and non-publicly available data like confidential reports to the banking supervisors.
time to maturity: $RTM_t^m$ with $m \in \{1, \ldots, M\}$ reports the amount of $pos^m$ in $t$ with a remaining time to maturity within the time band $t_{lower}^m < t_{end} - t \leq t_{upper}^m$. Since the amount within a certain time band contains the sum of the relevant business items, the former can be expressed as a linear function of the latter. Thus we obtain one function (restriction) for each amount in a time band and each reporting date, leading to a system of linear equations:

$$
\sum_{i=0}^{t_{lower}^n} \sum_{j=t_{lower}^n+1}^{t_{upper}^n} X^{pos^n, t-i, t-i+j} + \sum_{i=t_{lower}^n+1}^{t_{upper}^n} \sum_{j=i+1}^{t_{upper}^n} X^{pos^n, t-i, t-i+j} = ITM_t^n \quad \forall t \in T_{obs}^n, n \in \{1, \ldots, N\}
$$

$$
\sum_{i=0}^{t} \sum_{j=t_{lower}^m+1}^{t_{upper}^m} X^{pos^m, t+j} = RTM_t^m \quad \forall t \in T_{obs}^m, m \in \{1, \ldots, M\}.
$$

All variables are assumed to be nonnegative:

$$
X^{pos, t_{beg}, t_{end}} \geq 0 \quad \forall pos, t_{beg}, t_{end}.
$$

Equation (2) represents the restrictions due to reports on the initial maturity and (3) those on the remaining time to maturity. The respective right-hand side denotes the reported items (which are given to the analyst) whereas the left-hand side represents the sum of all relevant business items.

The system of equations given by (2), (3) and (4) restricts the values of the business items $X^{pos, t_{beg}, t_{end}}$ and hence via (1) the bank’s maturity structure. The system of equations describes the bank over time. To calculate the bank’s maturity structure on a certain date $t_{ref}$ (‘reference date’), it can be useful to restrict the sample period and thus the included reports. In general, the system of equations obtained is not uniquely determined, i.e. it is not possible to infer a unique maturity structure using accounting-based data. Thus, further assumptions are necessary. These can be formulated in terms of the optimization of a function (‘objective function’) on the space of solutions. Hence, the problem is to optimize a function subject to linear restrictions, and the model can be stated as:

$$
\text{optimize } \quad F(X) \\
\text{subject to } \quad (2),(3),(4),
$$

where $F$ denotes the objective function and $X$ the set of all business items $X^{pos, t_{beg}, t_{end}}$.

### 3 Simulation Results

To analyze whether the integration of time series information improves the ability to infer a bank’s maturity structure, we here compare the results of the presented approach with an approach using only one-point-in-time data on a sample of synthetic banks. For the latter we assume a uniform distribution within the time bands.

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8 Here we assume that business due in $t$ is not included in the reports in $t$.

9 See Sect. 3 for a possible objective function.
according to [2].
We restrict our sample period to 72 months. For the simulation, we only consider one position (interbank loans), as the remaining positions are modeled analogously. We take the information from the Monthly balance sheets statistics ('Monatliche Bilanzstatistik') and the Data schedule pursuant to the auditor's report ('Sonderdatenkatalog').\textsuperscript{10} The Monthly balance sheets statistics contain, on a monthly basis, the assets and liabilities in time bands according to initial maturity. The position interbank loans is broken down into 4 time bands: i) daily maturing, ii) up to 1 year but not daily maturing, iii) more than 1 year and up to 5 years, and iv) more than 5 years. The Data schedule pursuant to the auditor's report contains, on a yearly basis, a breakdown according to the remaining time to maturity. There are 4 time bands available: i) up to 3 months, ii) more than 3 months and up to 1 year, iii) more than 1 year and up to 5 years, and iv) more than 5 years.
The analysis is performed as follows. We first simulate synthetic banks by specifying 'typical' parameter constellations for the variables $X_{t, t_{ref}, t_{ref} + i}^{pos, t_{ref}, t_{ref} + j}$ and create synthetic reports that the banks would have reported to the Deutsche Bundesbank. We then apply our model as presented in Sect. 2 to derive an implied maturity structure. As an objective function we assume a possibly constant relative distribution of new business on the different maturities over time, specified as follows:

$$\text{minimize} \quad \sum_{t \in T_{obs}} \sum_{i \in \mathbb{N}} d_{t,i}^2$$

subject to

$$\frac{X_{t, t_{ref}, t_{ref} + i}^{pos, t_{ref}, t_{ref} + j}}{\sum_{j \in \mathbb{N}} X_{t, t_{ref}, t_{ref} + j}^{pos, t_{ref}, t_{ref} + j}} = d_{t,i} \quad \forall t \in T_{obs}, i \in \mathbb{N},$$

where $t_{ref}$ denotes the reference date at which the maturity structure is to be estimated and $T_{obs}$ the set of all reporting dates: $T_{obs} = T_{obs}^{1m} \cup T_{obs}^{2m}$. Since the objective function is quadratic, we obtain a quadratic programming problem.
Here we present the results of 4 sample banks. Bank 1a tracks a constant business strategy. In each month it contracts a constant amount of new business that is uniformly (discretely) distributed among all maturities between 1 month and 10 years. Bank 2a, by contrast, contracts only business with an initial maturity of 2, 4, 6, 8, and 10 years. Additionally, the amount of new business increases linearly over time. Banks 1b and 2b are analogous, but contain a stochastic component. Figure 1 shows the results of the 4 sample banks. The solid line gives the true maturity structure, whereas the other two lines represent the estimated ones. The simulation results show that the new approach is able to infer the maturity structure quite well both in a deterministic and a stochastic setting. The approach using only of one-point-in-time data yields a stepwise constant maturity structure. The jumps correspond to the limits of the time bands according to the Data schedule pursuant to the auditor’s report. The new methodology, in contrast, yields a smoothed function that fits the true maturity structure quite well, in particular for the banks 1a and 1b that track a constant maturity structure over time.

Thus time series can contain important information on the banks' maturity structure.

\textsuperscript{10} For further information on these reports, see \url{www.bundesbank.de}. 
Fig. 1. Simulation results

and should be taken into account when quantifying the interest rate risk exposure of banks using accounting-based data.

4 Concluding Remarks

We presented a methodology to quantify the maturity structure of a bank's assets and liabilities using accounting-based information. In contrast to approaches proposed in literature, the new methodology is able to integrate both the information of different data sources and time series information. In an analysis based on a simulation we showed that the integration of this information yields a better inference of the banks’ maturity structure than approaches proposed in literature. In addition to what we presented in this paper, we quantified the interest rate risk exposure of all German banks with an extended model using the data available to the Deutsche Bundesbank.\footnote{See [3].} We evaluated the results on a sub-sample of banks for which bank-internal data concerning the interest rate risk is available. The empirical results confirm that our approach is able to quantify the interest rate risk exposure of banks more accurately than approaches proposed in literature.

References