DIFFERENT COLLABORATIVE LEARNING SETTINGS TO FOSTER MATHEMATICAL ARGUMENTATION SKILLS

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Since mathematical argumentation skills comprise both, skills genuine to mathematics of differing complexity, and general abilities in argumentation, domain-general as well as domain-specific interventions, can be expected to have a positive effect on its acquisition. In a collaborative learning setting, we combined both interventions, heuristic worked examples vs. problem solving with collaboration script vs. no script support. Results of this experimental study with 119 teacher students indicate that, in collaborative settings, heuristic worked examples are more effective for the acquisition of low-level argumentation skills, whereas solving corresponding problems is more effective for high-level argumentation skills. Structuring the collaboration by a script did not affect the acquisition of domain-specific argumentation skills significantly.

INTRODUCTION

To inquire mathematical conjectures, convince oneself and also the mathematical community about the truth of a conjecture determines the work of mathematicians (Heintz, 2000). Thereby not only deductions by rules of logic play an important role, but also empirical explorations. During the last two decades national and international curricula also focussed on such complex processes of mathematical work (e.g. CCSSI, 2010). As mathematical argumentation tasks require diverse skills and abilities, mathematical argumentation can be considered as an example of a complex skill (Ufer, Heinze, & Reiss, 2008). Several studies showed, that not only students have problems in this field (Reiss, Heinze, Kessler, Rudolph-Albert, & Renkl, 2007), but also prospective and in-service teachers (e.g. Barkai, Tsamir, Tirosh, & Dreyfus, 2002).

Mathematical argumentation skills are understood here as the ability to find and evaluate a mathematical conjecture, generate adequate arguments for or against this conjecture and finally combine these arguments to a proof in an individual or social discursive context. A closer look on this definition shows, that this skill comprises one component which is genuine to mathematics and another component which refers to more general argumentation skills (Kollar, Fischer, & Slotta, 2007). It is an open question, to what extent it is feasible to foster mathematical argumentation skills by using domain-general interventions compared to interventions that aim at domain-specific knowledge and strategies. Hence this contribution investigates the effects of mathematics-specific interventions – heuristic worked example vs. problem solving – together with domain-general interventions – collaboration script vs. no script support – on individual mathematical argumentation skills in collaborative learning settings.

Moreover, under a domain-specific view mathematical argumentation skills comprise facets of diverse complexity. There are low-level demands like schematic argumentation skills based on a routine application of simple rules, proof tasks that require complex problem-solving processes in building up a coherent line of deductive arguments (Reiss et al., 2007), and even more complex facets like the ability to evaluate and prove or disprove mathematical conjectures (conjecturing).

Domain-general interventions: Collaboration script vs. No Collaboration script

According to Kuhn and Udell (2003) learning in collaborative settings can have positive effects on the acquisition of general argumentation skills. But research has also shown that such collaboration is not always effective, especially when no external structure for the collaboration is provided (Mullins, Rummel, & Spada, 2011). One solution is to provide learners with a computer supported collaboration script (Kollar, Fischer, & Hesse, 2006). It assigns the learners of a small group to specific roles or activities in a defined sequence (e.g. A: give an argument, B: give a counterargument, A&B: try a synthesis). A number of authors have studied if and how collaboration scripts facilitate collaborative argumentation. Indeed, these scripts have positive effects on the general quality of constructed arguments and – less frequently – also on domain specific learning outcomes (Weinberger, Ertl, Fischer, & Mandl, 2005).

Domain-specific interventions: Heuristic worked examples vs. Problem solving

Studying heuristic worked examples as well as solving authentic problems are considered as effective means to foster complex skills, like mathematical argumentation skills. But it remains still an open question, whether one of these two learning modes is superior with respect to different facets of argumentation skills.

To distinguish the two learning modes, we use a categorization of instructional information by Schworm and Renkl (2007) originally developed for worked examples. Instructional settings for complex tasks can differ according to the availability of information on three levels: Structural aspects, which are relevant for the solution of the problem and should be learned, belong to the *learning domain level* (e.g. principles of mathematical proof and argumentation). The *exemplifying domain level* contains information about surface features of a task and especially about the context in which the contents of the learning domain are embedded (e.g. a specific number theory argumentation task). Finally the *strategy level* refers to the meta-cognitive aspects of the task, like the choice of heuristic strategies.

Reiss and Renkl (2002) developed the idea of heuristic worked examples, which provide information on all three content levels. These examples do not only explicate the problem formulation and the solution (as a usual worked example would), but also the solution process, heuristic strategies to approach a problem and a process model of the corresponding skills of a more advanced learner or an expert. When studying a heuristic worked example, the learner follows the solution procedure of a fictitious peer, i.e. the solution process is not perfect and can also contain explorative and misleading approaches. Positive effects of those heuristic worked examples can be explained with Cognitive Load Theory (Kalyuga, Ayres, & Sweller, 2011). With an adapted process model of an expert for proving by Boero (1999), heuristic worked examples have been shown to be more effective than typical school lessons for fostering proving skills (Reiss et al., 2007).

Another promising instructional mode for the learning of such complex skills is solving authentic problems. According to Halmos (1980) mathematics and problem solving belong together. Working on mathematical argumentation tasks means also, solving a problem in the sense of Funke and Frensch (2007). This goes beyond the application of well-known rules or algorithmic steps to an unknown solution method for the learner. According to Funke and Frensch (2007) problem solving can be learned by making various experiences in solving problems. A meta-analysis of 43 studies by Dochy, Segers, van den Bossche, and Gijbels (2003) showed that problem solving has positive effects on the acquisition of problem solving skills but not on the acquisition of domain knowledge.

There is ample research for studying traditional worked examples and also problem solving, restricted to individual learning settings (Kalyuga et al., 2011). What remains mostly open so far is the effectiveness of the two learning modes in collaborative learning settings (Kirschner, Paas, Kirschner, & Janssen, 2011). In a first study, Kirschner et al. (2011) compared problem solving with usual worked examples in an unstructured collaborative setting and found problem solving to be superior in this case. Nevertheless, also in line with the results of Dochy et al. (2003), it is an open question how *heuristic* worked examples in collaborative settings influence the acquisition of facets of mathematical argumentation skills with different complexity.

RESEARCH QUESTIONS AND DESIGN OF THE STUDY

The present study is guided by the following questions:

- Is there a positive effect of the availability of instructional support on all three content levels on differing facets of mathematical argumentation skills? Here we compare problem solving to studying heuristic worked examples in *collaborative settings.*
- What impact do collaboration scripts have on the acquisition of these facets of mathematical argumentation skills?
- Are there differential effects of collaboration scripts on mathematical argumentation skills when combined with two different domain-specific interventions (heuristic worked examples vs. problem solving)?

Sample and Design

119 pre-service mathematics teacher students from two German universities took part in our experimental study with pre- and post-test. The different instructional settings

were implemented during a voluntary two-week preparatory course for university mathematics. The participants were assigned randomly to one of four intervention groups, controlling for high vs. low final school qualification grade (see Table 1).

		Collaboration script	
		Without	With
Learning mode	Problem Solving	N=29	N=29
	Heuristic worked example	N=29	N=32

Table 1: Experimental design

On three days, students worked for 45 minutes on one mathematical argumentation task from elementary number theory (e.g. "Choose an odd amount of consecutive numbers, e.g. 3, 5 or 7 consecutive numbers. Sum up these consecutive numbers. Do you notice anything special? Find a conjecture and prove it."). The dyads were homogenous with respect to their final high-school qualification grade and were changed every day. The students worked face to face in a computer supported learning environment, each equipped with a laptop, a graphic tablet and a mouse.

Materials and Instruments

On the right side of the screen (see Fig. 1) the two students working together had a shared work space, which functioned like a (graphical) chat window. At the top right side the students got varying instructions depending on the intervention group. In the condition with collaboration script the students had additionally a range of script Buttons at the bottom of the right side.

The left side of the screen contained the domain-specific instruction. In the *heuristic worked example condition*, illustrated texts, describing how a fictitious peer solved the problem following a 6-phase process model adapted from Boero (1999) were shown. To prevent superficial processing of the heuristic worked example, a self-explanation prompt addressing the strategy level (Schworm & Renkl, 2007) was presented in each phase: After the students were asked to think individually about that question, they were prompted to discuss their thoughts with their partner. The heuristic worked example of the two learning partners differed on the strategy level in every second phase. In the *problem solving condition* students were given the problem formulation and asked to find a solution. They were first prompted to think individually about a possible solution step and afterwards discuss their ideas with their partner.

Thus, individual learning phases and collaborative discussions were systematically alternated. Collaborative discussions were structured in three phases according to the cycle of argumentative discourse of Leitão (2000) in the conditions *with collaboration script*: (1) argument, (2) counterargument and (3) synthesis. Additional support was provided in each step based on Toulmin's (1958) argumentation model.

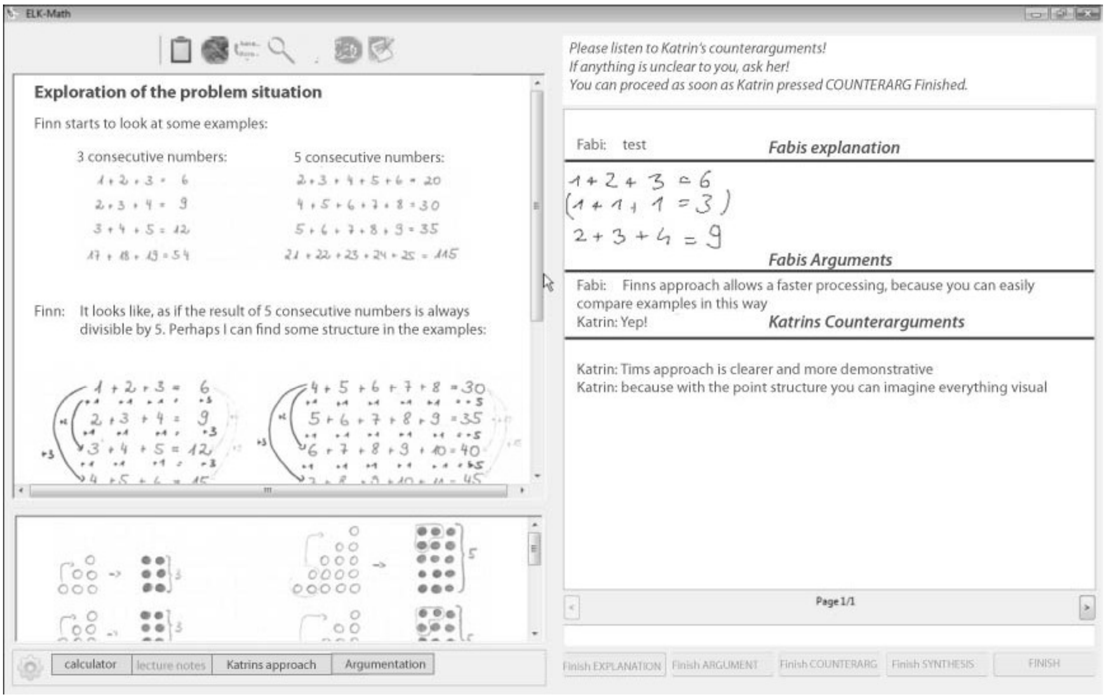

Fig. 1: Screenshot of the computer supported learning environment (heuristic worked example with collaboration script; translated by the authors)

To assess students' progress, parallel pre- and post-tests were designed covering different facets of mathematical argumentation skills. The first part required *schematic argumentation* through divisibility rules (e.g., "Show that for natural numbers, a and b the following statement is true: If 5 divides (a+2b) then 5 divides (4a+3b).") (5 items, Cronbach's α =.66/.67). Students *proof skills* in elementary number theory were measured in the second part (e.g. "Prove the following statement: The sum of a natural number, its square plus one is odd.") (6 items, Cronbach's α =.74/.73) and in the third part, the students had to solve open ended *conjecturing* problems (e.g. "Prove or refute the following statement: The sum of six consecutive numbers is divisible by 6.") (6 items, Cronbach's α =.57/.57). A three-level coding (see also Reiss et al., 2007) was applied to score students' answers. No and irrelevant trials were scored with zero points. For partially correct solutions the students got one point and for a correct solution two points were given. A fourth part of the post-test contained a question on *heuristic strategies* for the mathematical argumentation process ("You should formulate and proof a mathematical conjecture. How would you proceed?"). The students were given one point for each strategy corresponding to the 6-phase model underlying the heuristic worked examples (max. 6 points). All the items were coded by two independent raters and interrater reliability for each part of the pre- and post-test was found to be good (Mean of ICC_{unjust}=.86, SD =.12).

RESULTS

All four learning conditions were appropriate for the learning of mathematical argumentation skills. To get a deeper insight, four ANCOVAs, one for each test part as dependent variable, *learning mode* and *collaboration script* (with/without) as independent variables and pre-test scores as a covariate were conducted.

	Problem solving		Heuristic worked example	
Coll. script	Without	With	Without	With
schematic arg.	.52 (.04)	.58 (.04)	.61 (.04)	.67 (.04)
proof	.50 (.04)	.50 (.03)	.43 (.04)	.46 (.03)
conjecturing	.59 (.03)	.61 (.03)	.54 (.03)	.50 (.03)
heuristic strat.	.30 (.05)	.24 (.05)	.43 (.05)	.35 (.05)

Table 2: Adjusted means (standard deviations indicated in brackets) for mathematical argumentation skills in the four experimental conditions.

For post-test performance, the ANCOVA results show a significant main effect of the learning mode in the first ($F(1,114)=5.93$; $p<.05$; $\eta^2=.05$), third ($F(1,114)=6.63$; $p<.05$; $\eta^2=.06$) and fourth ($F(1,114)=7.66$; $p<.05$; $\eta^2=.06$) part of the test. For *schematic argumentation* (part 1) and *heuristic strategies* (part 4), students who learned in the heuristic worked example condition outperformed those who studied in the problem solving condition (see Table 2). The opposite effect was found for *conjecturing skills* (part 3). Students from the problem solving condition did significantly better when working on open ended conjecturing problems. The main effect for collaboration script and also the interaction effect between collaboration script and learning mode did not reach statistical significance ($F(1,114)<3$, *n.s.*).

DISCUSSION

This study examined the effects of two different domain-specific interventions within a structured resp. unstructured collaborative learning setting on different facets of mathematical argumentation skills.

The main effect of collaboration scripts did not reach statistical significance, but the descriptive results in Table 2 indicate that for most facets of mathematical argumentation skills, students profited from the script. In further analyses, students' behavior in an unstructured collaborative argumentation situation after the intervention will be analysed and we expect to find clearer effects there. We found no interaction effect of the learning mode and the script on the acquisition of mathematical argumentation skills. This indicates that one intervention did not affect the other negatively. Also no synergy effect of combining both interventions was observed.

Regarding the comparison of the two different content-related instructions, our results are at least partly contrary to the results of Kirschner et al. (2011) who found problem solving to be superior to (usual) worked examples in unstructured collaborative settings. Collaborative learning from worked examples led to significantly higher performance in low-level facets of mathematical argumentation skills: Regarding *schematic argumentation skills*, students were required to do transformations of the algebraic expressions, find adequate divisibility rules or use the definition of divisibility to prove the statement. Similarly, knowledge of the *strategies* taught in the heuristic worked example condition can be considered low-level. The reverse relationship was found for *conjecturing skills* and was also indicated for *proof skills*, but failed to reach statistical significance. The *proof skills* test required finding multiple proof steps and for most items a formalization of a verbal statement was conducive. A further demand in the *conjecturing test* was to evaluate the conjecture as true or false. Students' performance on false statements (M=4.18, SD=1.76) was better than on true statements (M=2.52, SD=1.41), since false statements only required counterexamples and no deductive argumentations. This explains higher mean values in the *conjecturing test* compared to the *proof test* (see Table 2). Altogether, the items in the *proof* and *conjecturing tests*, especially the true conjecturing items, can be considered as complex, high-level argumentation tasks.

Our results indicate a first answer on how the availability of solution steps on all content levels influences the acquisition of mathematical argumentation skills. Solution steps on all content levels (heuristic worked examples) proved to be superior for the acquisition of low-level argumentation skills. Problem solving, with no solution steps available, was more effective for the acquisition of high-level argumentation skills in our collaborative setting. Worked examples have repeatedly proved to be effective interventions (Schworm & Renkl, 2007) in individual learning settings. It seems necessary to take the general learning setting as well as the complexity of target skills carefully into account when judging the effectiveness of domain-specific interventions. A noticeable result is that the learning mode influences facets of mathematical argumentation skills differently in our collaborative learning setting. Further research is necessary, modifying these learning modes regarding the availability of solution steps on different levels. Also students' cognitive abilities should be considered (see e.g. Kalyuga et al., 2011)

References

Barkai, R., Tsamir, P., Tirosh, D., & Dreyfus, T. (2002). Proving or refuting arithmetic claims: The case of elementary school teachers. In A. D. Cockburn & E. Nardi (Eds.). *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education*, Vol. 2, 57-64. Norwich, UK: PME.

Boero, P. (1999). Argumentation and mathematical proof: A complex, productive, unavoidable relationship in mathematics and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof, 7*(8).

CCSSI: Common Core State Standards Initiative (2010). *Common Core State Standards for Mathematics,* Retrieved from www.corestandards.org.

Dochy, F., Segers, M., van den Bossche, P., & Gijbels, D. (2003). Effects of problem-based learning: a meta-analysis. *Learning and Instruction, 13*, 533–568.

Funke, J., & Frensch, P. (2007). Complex Problem Solving: The European Perspective-10 Years After. In D. H. Jonassen (Ed.), *Learning to solve complex scientific problems* (pp. 25–47). New York: Lawrence Erlbaum Associates.

Halmos, P. R. (1980). The Heart of Mathematics. *The American Mathematical Monthly, 87*(7), 519-524.

Heintz, B. (2000). *Die Innenwelt der Mathematik: Zur Kultur und Praxis einer beweisenden Disziplin.* Wien; New York: Springer.

Kalyuga, S., Ayres, P., & Sweller, J. (2011). *Cognitive load theory*. New York: Springer.

Kirschner, F., Paas, F., Kirschner, P. A., & Janssen, J. (2011). Differential effects of problem-solving demands on individual and collaborative learning outcomes. *Learning and Instruction, 21,* 587-599.

Kollar, I., Fischer, F., & Hesse, F. W. (2006). Collaboration scripts - a conceptual analysis. *Educational Psychology Review, 18*(2), 159–185.

Kollar, I., Fischer, F., & Slotta, J. D. (2007). Internal and external scripts in computer-supported collaborative learning. *Learning and Instruction, 17*, 708–721.

Kuhn, D., & Udell, W. (2003). The Development of Argument Skills. *Child Development, 74*(5), 1245–1260.

Leitão, S. (2000). The Potential of Argument in Knowledge Building. *Human Development*, *43,* 332–360.

Mullins, D., Rummel, N., & Spada, H. (2011). Are two heads always better than one? Differential effects of collaboration on students' computer-supported learning in mathematics. *Intern Journal of Computer-Supported Collaborative Learning, 6*(3), 421–443.

Reiss, K., & Renkl, A. (2002). Learning to prove: The idea of heuristic examples. *Zentralblatt für Didaktik der Mathematik, 34*(1), 29–35.

Reiss, K., Heinze, A., Kessler, S., Rudolph-Albert, F., & Renkl, A. (2007). Forstering argumentation and proof competencies in the mathematics classroom. In M. Prenzel (Ed.): *Studies on the educational quality of schools: the final report on the DFG priority programme* (pp. 251-264). Münster: Waxmann.

Schworm, S., & Renkl, A. (2007). Learning argumentation skills through the use of prompts for self-explaining examples. *Journal of Educational Psychology, 99*, 285–296.

Toulmin, S. E. (1958). *The uses of argument*. Cambridge: Cambridge University Press.

Ufer, S., Heinze, A., & Reiss, K. (2008). Individual predictors of geometrical proof competence. In O. Figueras, J. L. Cortina, S. Alatorre, T. Rojano, & Sepúlveda (Eds.), *Proceedings of the Joint Meeting of PME 32 and PME-NA XXX*, Vol. 4, 361–368. Mexico: PME.

Weinberger, A., Ertl, B., Fischer, F., & Mandl, H. (2005). Expistemic and social scripts in computer-supported collaborative learning. *Instructional Science, 33*(3), 1–30.