POPULATION, PENSIONS, AND ENDOGENOUS ECONOMIC GROWTH*

Burkhard Heer[1]

University of Augsburg and CESifo, Munich

Andreas Irmen[2]

University of Luxembourg and CESifo, Munich

Abstract: We study the effect of a declining labor force on the incentives to engage in labor-saving technical change and ask how this effect is influenced by institutional characteristics of the pension scheme. When labor is scarcer it becomes more expensive and innovation investments that increase labor productivity are more profitable. We incorporate this channel in a new dynamic general equilibrium model with endogenous economic growth and heterogeneous overlapping generations. We calibrate the model for the US economy and obtain the following results. First, the effect of a decline in population growth on labor productivity growth is positive and quantitatively significant. In our benchmark, it is predicted to increase from an average annual growth rate of 1.74% over 1990-2000 to 2.41% in 2100. Second, institutional characteristics of the pension

---

*We would like to thank Dirk Krueger, Alexander Ludwig, Klaus Wälde, and Ulrich Woitek for the provision of their respective data sets. Moreover, we are grateful to an anonymous referee and an associate editor for their comments.

[1]University of Augsburg, Universitaetsstrasse 16, D-86316 Augsburg, Germany, Burkhard.Heer@wiwi.uni-augsburg.de.

[2]University of Luxembourg, CREA, Faculty of Law, Economics and Finance, 162a, avenue de la Faïencerie L-1511 Luxembourg, Luxembourg, airmen@uni.lu.

system matter both for the growth performance and for individual welfare. Third, the assessment of pension reform proposals may depend on whether economic growth is endogenous or exogenous.

Keywords: Growth, Demographic Transition, Capital Accumulation, Pension Reform.

JEL-Classification: O41, C68, O11, D91.

This Version: June 15, 2014 (final version submitted to JEDC).

1 Introduction

Population aging is one of the major economic challenges for today's industrialized societies. An increasing life expectancy in conjunction with declining birth rates tends to reduce the part of the population in working age and raises the part of the economically dependent old. For instance, the United Nations predicts for the US an increase in the old-age dependency ratio, i. e., the ratio of the population aged 65 years or over to the population aged 15-64, from 18% in 2005 to 36% in 2050 (United Nations (2013), medium variant). To meet this challenge it is necessary to understand the economic consequences of such a demographic change.

This paper studies the economic consequences of the link between demographic changes, pensions, and the incentives of firms to engage in innovation investments that affect total factor productivity (TFP). With a focus on the US economy and the time span between 1950 and 2200, we argue that this link may substantially modify the predicted *positive* and *normative* implications of actual policy reform proposals of the pension scheme. We obtain these implications in a new dynamic general equilibrium model with endogenous economic growth and heterogeneous overlapping generations in the spirit of Auerbach and Kotlikoff (1987).

We calibrate our model for the US economy and establish the following major results. First, we find that the effect of a decline in population growth on labor productivity growth is positive and quantitatively significant.[1] For instance, under the current pension system it is predicted to increase from an average annual growth rate of 1.74% over 1990-2000 to approximately 2.41% in 2100. Second, we show that the pension system indeed matters both for the growth performance of the economy and for the welfare of each generation. Finally, we show that the assessment of pension reform proposals differs in an endogenous growth framework as opposed to the standard framework where TFP growth is exogenous and costless.

In times of pronounced population aging, these findings suggest to policymakers that the pressure on a pay-as-you-go pension scheme may be somewhat less severe. Conventional wisdom has it that the receipts of such a system must decline since the number of contributors falls with the number of workers. However, the main message of the present paper indicates that this tendency is at least weakened since labor productivity, wages and, hence, the contributions per worker

[1]This runs counter to an often held view that reduced population growth leads to lower growth rates of per-capita income (see, e. g., Bös and von Weizsäcker (1989) for a discussion).

grow faster. The reason is that firms will respond to the decline in the work force with more labor-saving investments. In other words, there will be faster endogenous growth. However, our analysis also shows that the extent to which this channel will materialize hinges crucially on the institutional characteristics of the pension scheme and, hence, on the way policymakers shape and adapt it during a demographic transition.

Our analytical framework has a supply side that satisfies two requirements which prove particularly useful when it comes to the identification of the role of endogenous growth for our positive and normative results. First, endogenous growth is obtained in a competitive neoclassical economy where TFP growth is the result of labor-saving innovation investments of profit-maximizing firms and capital accumulation is subject to diminishing returns. Since this economy may readily be turned into the standard neoclassical growth model of Solow (1956) with exogenous and costless technical change there is a natural benchmark. Second, there is a single channel linking the demographic trend to innovation activities. This parsimony allows for clear-cut intuitions when we compare policy reform proposals under endogenous and exogenous growth.[2]

In a broad sense, our research methodology follows the lead of modern macroeconomics and takes the famous Lucas Critique seriously. This requires an assessment of policy proposals that accounts for the optimal responses of households *and* firms. The novel feature of our approach is that firms are allowed to respond with labor-saving investments to the demographic trend of population aging.

More precisely, a mechanism often attributed to Hicks (1932) provides the link between the demographic trend and TFP growth. According to Hicks' argument, a demographic transition that reduces the labor force may render labor scarcer relative to capital. Accordingly, the relative price of labor increases and so does the profitability of innovation investments that raise labor productivity and TFP.[3]

[2]We do not want to dispute the possible presence of other channels that may strengthen or weaken the mechanism expounded here. However, the analytical and the quantitative analysis of such interactions is quite involved. We leave this question for future research and shall get back to this point in the concluding Section 7.

[3]Evidence supporting Hicks' intuition is provided by Cutler, Poterba, Sheiner, and Summers (1990). These authors' cross-country regressions establish a negative and significant partial correlation between productivity growth and labor force growth. In a similar vein, several cross-country growth regressions find a negative and significant partial correlation between population growth and growth of per-capita GDP (see, e. g., Barro and Sala-í-Martin (2004), Kelley and Schmidt (1995), and Kormendi and Meguire (1985)). Romer (1987) shows a strong inverse relationship between

However, from a general equilibrium point of view, the relevance of Hicks' argument hinges on the economy's ability to accumulate capital, i. e., on the propensity of a declining population in working age to save. Hence, to account for the Hicks effect we have to allow firms to adjust their innovation investments and households to adjust their savings decision to a changing demographic environment. To provide for such an analytical framework we adapt the production sector of Irmen (2005), which complies with the aforementioned attributes, and combine it with a household sector comprising heterogeneous overlapping generations of Auerbach and Kotlikoff (1987). Since the pension system is a major determinant of an economy's propensity to save, this framework is well suited for our *comparative institutional analysis* that asks how institutional characteristics of the pension scheme affect the incentive to engage in labor-saving technical change during a demographic transition.

Our research builds on and contributes to a recent and growing literature that extends the seminal framework of Auerbach and Kotlikoff (1987) to analyze the economic consequences of population aging in different settings. To the best of our knowledge, the existing literature does neither incorporate the endogenous response of firms to the demographic transition through the Hicks effect, nor does it provide an analysis of the consequences of policy reforms in endogenous growth economies.[4] The present study is a first step to fill this gap.

Important existing contributions study the consequences of policy reform proposals of the social security system in closed economies. For instance, Huang, İmrohoroğlu, and Sargent (1997), Conesa and Krüger (1999), and Fuster, İmrohoroğlu, and İmrohoroğlu (2007) deal with the transition from an unfunded to a more funded pension scheme, where the latter two emphasize political economy aspects of such a switch. Closer to our analysis are the studies of İmrohoroğlu,

---

average annual growth rates of the labor force and of output per hour worked in the US for successive twenty year intervals covering the 20th century. In a famous study, Habakkuk (1962) uses ideas similar to the Hicksian argument to explain why technological progress in the 19th century was faster in the US than in Britain.

[4]Heijdra and Romp (2009a) as well as a previous version of Ludwig, Schelkle, and Vogel (2012) include a calibration analysis of endogenous growth through endogenous human capital investments of the households and a growth mechanism along the lines of Lucas (1988). The latter growth mechanism is also at the heart of a theoretical literature on the link between demographics, pensions, and growth. It includes, e. g., Zhang (1995) emphasizing the role of social security on fertility, Kemnitz and Wigger (2000) and Zhang and Zhang (2003) with a focus on the effect of different benefit formula, Sanchez-Losada (2000) and Lambrecht, Michel, and Vidal (2005) studying the role of altruism, Boldrin and Montes (2005) with credit-constrained households, or Echevarria and Iza (2006) with a focus on ageing. However, these authors do not compare different reform proposals of the pension scheme.

İmrohoroğlu, and Joines (1995) and de Nardi, İmrohoroğlu, and Sargent (1999) who focus on reform scenarios within the pay-as-you-go system and assess their welfare implications. With respect to their results, our findings suggests that i) the welfare effects are much larger if economic growth is endogenous, and ii) the welfare ranking of reform proposals may depend on whether economic growth is exogenous as in their papers or endogenous as in our study.

This paper is organized as follows. Section 2 presents the model and establishes the stationary equilibrium. Section 3 develops important channels that drive our calibration results in a simplified setting with two-period lived individuals. Section 4 provides the detailed description of our calibration strategy. Our main findings concerning the interplay between the demographic transition and economic growth appear in Section 5. In particular, we establish that the feedback from population growth onto innovation incentives generates faster labor productivity growth. In Section 6, we turn to the comparative institutional analysis of pay-as-you-go pensions schemes with different characteristics. Section 6.1 considers three pension reform proposals and studies their implications for economic growth. Section 6.2 deals with the welfare implications of these reform proposals. Section 7 concludes. Proofs are relegated to the Appendix. The latter also includes some additional material that complements topics dealt with in the main text.

2 The Model

This section describes our large-scale OLG model with heterogeneous agents in the spirit of Auerbach and Kotlikoff (1987) and a production sector extending the setup studied in Irmen (2005).[5]

There is a final-good sector and an intermediate-good sector. Intermediate-good firms undertake innovation investments that increase the productivity of their workers. These productivity gains drive the evolution of the economy's level of technological knowledge and its TFP. Intermediates and capital produce final output that can be consumed or invested. If invested, the current final good may either serve as future capital or as an input in innovation investments. In each period, prices are expressed in units of the current final good.

[5]The production side of Irmen (2005) extends the one of Hellwig and Irmen (2001), which in turn builds on Bester and Petrakis (2003).

The government collects taxes, accidental bequests, and social security contributions. In all periods, the budget of the government and the one of the pension scheme are balanced.

2.1 Demographics and Timing

A period, t, corresponds to one year. At each t, a new generation of households is born. Newborns have a real life age of 20 denoted by $s = 1$. All generations retire at age 65 ($s = R = 46$) and live up to a maximum age of 94 ($s = J = 75$). At t, all agents of age s survive until age $s + 1$ with probability $\phi_{t,s}$ where $\phi_{t,0} = 1$ and $\phi_{t,J} = 0$.

Let $N_t(s)$ denote the number of agents of age s at t. The population $N_t(s)$ of the periods $t = 0, 1, ..., 450$ is calibrated as the actual and predicted US population of the years 1950 to 2400 using the data of Krüger and Ludwig (2007).

2.2 Households

Each household comprises one worker. Households maximize intertemporal utility at the beginning of age 1 in period t

$$\max \sum_{s=1}^{J} \beta^{s-1} \left(\Pi_{j=1}^{s} \phi_{t+j-1,j-1} \right) \; u(c_{t+s-1}(s), l_{t+s-1}(s)), \tag{1}$$

where $\beta > 0$ denotes the discount factor, and per-period utility $u(c, l)$ is a function of consumption c and labor supply l

$$u(c, l) = \frac{\left(c^{\gamma}(1-l)^{1-\gamma}\right)^{1-\theta} - 1}{1-\theta}, \quad \theta > 0, \quad \gamma \in (0, 1). \tag{2}$$

Households are heterogeneous with regard to their age, s, their individual labor efficiency, $e(s, j)$, and their wealth, ω. We stipulate that an agent's efficiency $e(s, j) = \bar{y}_s \epsilon_j$ depends on its age, $s \in \mathcal{S} \equiv \{1, 2, ..., 75\}$, and its efficiency type, $\epsilon_j \in \mathcal{E} \equiv \{\varepsilon_1, \varepsilon_2\}$. We choose the age-efficiency profile, $\{\bar{y}_s\}$, in accordance with the US wage profile. The permanent efficiency types ϵ_1 and ϵ_2 are meant to capture differences in education and ability. We use Γ to denote the unique invariant distribution of $\epsilon_j \in \mathcal{E}$.

The net wage income in period t of an s-year old household with efficiency type j is given by $(1 - \tau_w - \tau_{b,t}) w_t e(s, j) l_t(s)$, where w_t denotes the wage rate per efficiency unit in period t. The wage income is taxed at the constant rate τ_w. Furthermore, the worker has to pay contributions to the pension system at rate $\tau_{b,t}$ which may vary over time depending on the pension scheme. A retired worker receives pensions $b(s,j)$ that depend on his efficiency type j. Clearly, $b(s,j) = 0$ for $s < R$.

Households are born without assets at the beginning of age $s = 1$, hence $\omega_t(1) = 0$. Parents do not leave bequests to their children, and all accidental bequests are confiscated by the government. The household earns interest r_t on his wealth $\omega_t \in \mathbb{R}$. Capital income is taxed at the constant rate τ_r. In addition, households receive lump-sum transfers tr_t from the government. As a result, the budget constraint at t of an s-year old household with productivity type j and wealth ω_t is:

$$\begin{aligned} b_t(s,j) + (1 - \tau_w - \tau_{b,t}) w_t e(s,j) l_t(s) + [1 + (1 - \tau_r) r_t]\, \omega_t(s) + tr_t \\ = c_t(s) + \omega_{t+1}(s+1). \end{aligned} \tag{3}$$

2.3 Firms

Firms belong either to the final-good or to the intermediate-good sector. Both sectors are competitive.

2.3.1 Final Goods

The measure of all final-good firms is equal to one. At each t, firms produce output, Y_t, according to the following Cobb-Douglas production function

$$Y_t = K_t^{\alpha} X_t^{1-\alpha}, \quad 0 < \alpha < 1, \tag{4}$$

where K_t is the capital input at t and X_t denotes the amount of the intermediate good used in period-t production. Let p_t denote the price of the intermediate good at t, and $\delta \in [0,1]$ the rate at which capital depreciates within periods. Firms rent capital from households and return $1 + r_t$ units of their output per unit of rented capital. Moreover, they pay $p_t X_t$ to the intermediate-goods producers. Hence, per-period profits are

$$Y_t - r_t K_t - p_t X_t - \delta K_t. \tag{5}$$

Firms take the sequence of prices $\{r_t, p_t\}$ as given and maximize the sum of the present discounted values of profits in all periods. This is equivalent to a series of one-period maximization problems and gives rise to the first-order conditions

$$\frac{\partial Y_t}{\partial K_t} = r_t + \delta = \alpha \tilde{K}_t^{\alpha-1}, \tag{6}$$

$$\frac{\partial Y_t}{\partial X_t} = p_t = (1-\alpha)\tilde{K}_t^{\alpha}, \tag{7}$$

where $\tilde{K}_t \equiv K_t / X_t$ denotes the capital intensity in the final-good sector in period t.

2.3.2 Intermediate Goods

The set of all intermediate-good firms is represented by the set $\mathbb{R}_+$ of nonnegative real numbers with Lebesgue measure.

Technology All firms have access to the same technology and face a capacity limit of 1.[6] The output of the intermediate good, x_t, is given by

$$x_t = \min\left\{1, a_t\, l_t^d\right\}, \tag{8}$$

where a_t and l_t^d denote the firm's labor productivity and its labor input, respectively. Firms hire effective labor, l_t^d, defined as the product of the efficiency factor $e(s, j)$ and the 'number' of working hours $l_t(s)$. Labor is assumed to be divisible between firms.

The individual firm's labor productivity at t depends on both the indicator of the economy-wide level of labor productivity A_{t-1} of period $t-1$ and its individual productivity growth rate q_t according to

$$a_t = A_{t-1}(1 + q_t). \tag{9}$$

In order to achieve productivity growth at rate q_t, the firm must invest $i(q_t)$ units of the final good in period $t-1$. The input requirement function is given by

$$i(q) = v_0 q, \quad \text{with } v_0 > 0. \tag{10}$$

[6]The capacity limit of 1 is a shortcut that has no bearing on the evolution of any magnitude of interest in the present model. It may be endogenized along the lines set out in Hellwig and Irmen (2001).

Following Hellwig and Irmen (2001), the innovation reflected in a_t is proprietary knowledge of the firm only for production in period t; afterwards it becomes common knowledge. A consequence of this assumption is that an intermediate-good firm can produce at t with a technology involving $a_t = A_{t-1}$ if it decides not to undertake an innovation investment in $t-1$. The evolution of A_t over time is described below.

Profit Maximization and Zero-Profits A firm's innovation investment fully depreciates after one year. Therefore, the cost of an innovation investment undertaken at $t-1$ in units of the final good at t is $(1+r_t)i(q_t)$. With $w_t l_t^d$ denoting a firm's wage bill, a production plan $\{x_t, l_t^d, q_t\}$ for period t yields the profit

$$\pi_t = p_t\, x_t - w_t\, l_t^d - (1+r_t)\, i(q_t). \tag{11}$$

Firms take the sequence of real prices $\{p_t, w_t, r_t\}$ as well as the sequence of aggregate productivity indicators $\{A_{t-1}\}$ as given and maximize the sum of the present discounted values of profits in all periods. Since production choices for different periods are independent of each other, for each t they choose the plan $\{x_t, l_t^d, q_t\}$ to maximize the profits π_t of (11).

If a firm innovates then it incurs an investment cost $(1+r_t)\, i(q_t)$ that is independent of the level of output. Hence, an innovation investment can only be profit-maximizing if the margin per unit of output is strictly positive, i. e., $p_t - w_t/a_t > 0$. Since this margin is independent of the amount of produced output, an innovating firm produces the capacity output $x_t = 1$. Accordingly, (8) delivers its demand for effective labor as

$$l_t^d = \frac{1}{A_{t-1}(1+q_t)}. \tag{12}$$

Upon substitution of the latter in (11), the profit-maximizing productivity growth rate is

$$\hat{q}_t = \arg\max_{q \geq 0} \left[p_t - \frac{w_t}{A_{t-1}(1+q)} - (1+r_t)\, i(q) \right]. \tag{13}$$

The corresponding first-order condition balances the advantage of a lower wage bill and the disadvantage of a higher investment cost. It is sufficient for a maximum and given by

$$\frac{w_t}{A_{t-1}(1+\hat{q}_t)^2} \leq (1+r_t)\, v_0, \tag{14}$$

with strict inequality only if $\hat{q}_t = 0$.

This condition shows that, indeed, the incentive to engage in labor saving innovation investments depends on (relative) factor prices. If firms face a higher wage, then, ceteris paribus, an increase in q implies a larger reduction in the wage bill. Similarly, a higher real interest rate reduces innovation incentives since, ceteris paribus, it raises marginal investment costs. This is the sense in which profit-maximizing firms adjust their innovation investments to changing factor prices as conjectured by Hicks: if $w_t/(1+r_t)$, i. e., the relative price of labor, increases they invest more. However, since the input requirement for the first marginal unit of q_t is not infinitesimally small, firms may find it optimal not to innovate if the relative price of labor is too small.

To complete the analysis of the profit maximum, we must consider the case where a firm's margin is not strictly positive, i. e., $p_t \leq w_t/a_t$. Then, the firm will not invest. In case of a zero-margin, any plan $(x_t, l_t, 0)$ with $x_t \in [0,1]$ and $l_t = x_t/A_{t-1}$ maximizes π_t. Without loss of generality and to simplify the notation, we assume for this case that intermediate-good firms still produce the capacity output. If a firm faces a strictly negative margin, it won't produce and $(0,0,0)$ is the optimal plan.

Denote n_t the measure of the set of all active firms in the intermediate-good sector at t. Since all firms have access to the same technology and face identical prices, all active firms choose the same production plan $\{1, 1/\left(A_{t-1}\left(1+\hat{q}_t\right)\right), \hat{q}_t\}$.

In equilibrium, active intermediate-good firms must earn zero profits, i. e.,[7]

$$\pi_t\left(\hat{q}_t; p_t, w_t, r_t, A_{t-1}\right) = 0. \tag{15}$$

2.3.3 Consolidating the Production Sector

The following lemma shows that the conditions for profit-maximization and zero-profits in the final-good and the intermediate-good sector relate the growth rate of labor productivity q_t to the capital intensity in the final-good sector, $\tilde{K}_t$.

[7]This follows since the labor supply in each period is bounded. Therefore, the set of producing intermediate-good firms that employ more than some $\epsilon > 0$ units of effective labor must have bounded measure, i. e., it must be smaller than the set of all intermediate-good firms. Since inactive intermediate-good firms must be maximizing profits just like active ones, we need that maximum profits of intermediate-good firms at equilibrium prices are equal to zero. See Hellwig and Irmen (2001) for more details.

Lemma 1 *If all firms in the economy maximize profits and earn zero-profits, i. e., equations (6), (7), (14) and (15) hold, then there is a continuous map $g : \mathbb{R}_+ \to \mathbb{R}_+$ and a critical value $\tilde{K}_c > 0$ such that $q_t = g\left(\tilde{K}_t\right)$ with $g\left(\tilde{K}_t\right) = 0$ for $\tilde{K}_t \leq \tilde{K}_c$ and $g\left(\tilde{K}_t\right) > 0$ for $\tilde{K}_t > \tilde{K}_c$ with $g'\left(\tilde{K}_t\right) > 0$.*

Lemma 1 provides the link between the equilibrium incentive to innovate and the capital intensity. Intuitively, profit-maximization in the final-good sector as stated in (6) and (7) means that a higher $\tilde{K}_t$ lowers the real interest rate and raises the price of the intermediate good. Both price movements raise the wage that is consistent with zero-profits in the intermediate-good sector. The function $g\left(\tilde{K}_t\right)$ reflects the effect of these price movements onto the profit-maximizing level of innovation activity implied by (14). We shall later see that in equilibrium the capital intensity, $\tilde{K}_t$, is a measure of the relative abundance of capital with respect to effective labor. Thus, Lemma 1 links the relative abundance of factors to innovation incentives as envisaged by Hicks: if capital is abundant relative to effective labor then the incentive to engage in labor-saving innovation investments is high. As mentioned above, the input requirement for the first marginal unit of q_t is not infinitesimally small. Therefore, profit-maximization and zero-profits of intermediate-good firms are only compatible with $q_t > 0$ if $p_t/(1+r_t)$ is sufficiently high, i. e., if $\tilde{K}_t > \tilde{K}_t^c$.

Turning to the evolution of the economy-wide indicator of technological knowledge, we assume that $A_t = \max\{a_t(n) = A_{t-1}\,(1+q_t(n))\,|\,n \in [0, n_t]\}$. Since in equilibrium the productivity of all producing intermediate-good firms is $a_t = A_{t-1}(1+q_t)$, we have $A_t = a_t$ and

$$A_t = A_{t-1}\,(1+q_t). \tag{16}$$

Moreover, in equilibrium the aggregate demand for innovation investment at $t-1$ is $n_t i(q_t)$, the aggregate demand for effective labor at t is $n_t l_t^d = n_t/\left(A_{t-1}\,(1+q_t)\right)$, and the aggregate supply of the intermediate good at t is n_t.

2.4 Government

The government collects income taxes T_t in order to finance its expenditure on government consumption G_t and transfers Tr_t. In addition, it confiscates all accidental bequests Beq_t. The government budget is balanced in every period t, i. e.,

$$G_t + Tr_t = T_t + Beq_t. \tag{17}$$

In view of the tax rates τ_w and τ_r, the government's tax revenue is

$$T_t = \tau_w w_t L_t + \tau_r r_t \Omega_t, \tag{18}$$

where Ω_t is aggregate wealth at t.

Government spending is a constant fraction of final-good output

$$G_t = \bar{g} Y_t. \tag{19}$$

2.5 Social Security

The social security system is a pay-as-you-go system. The social security authority collects contributions from the workers to finance its pension payments to the retired agents. Pensions are a constant fraction of the net labor income of the productivity type j

$$b_t(s,j) = \begin{cases} 0 & s < R \\ \zeta(1-\tau_w-\tau_{b,t}) w_t \epsilon_j \bar{l}_t & s \geq R, \end{cases} \tag{20}$$

where $\bar{l}_t$ denotes the average hourly labor supply of all workers in period t.

In equilibrium, the social security budget is balanced and will be defined below. As to the institutional characteristics of the pension scheme we will distinguish three different policy scenarios that will become effective in period t_0:[8]

1. constant replacement rate: $\zeta_t \equiv \frac{b_t}{(1-\tau_w-\tau_b) w_t \bar{l}_t} = \zeta$,
2. constant contribution rate starting in period t_0: $\tau_{b,t} = \tau_{b,t_0}$ for $t \geq t_0$,
3. constant replacement rate and later retirement at age 70, i. e. $R = 51$.

Under the first policy, the pension benefits of the retired worker with productivity type j remain constant relative to the average wage income of all type-j workers. Moreover, the contribution

[8] In our computations t_0 corresponds to the year 2010. Prior to this period, the contribution rate $\tau_{b,t}$ adjusts in every period t to balance the social security budget. For computational reasons, we will assume below that the economy is in steady state during all periods $t < t_{-1}$. All generations born between period t_{-1} and t_0 adjust their behavior in line with the anticipated policy changes. In our computation, t_{-1} corresponds to the year 1950.

rate $\tau_{b,t}$ adjust in every period t to balance the social security budget. The second policy has a constant contribution rate $\tau_{b,t}$ after period t_0. Accordingly, pension benefits adjust to keep the social security budget balanced. Under the third pension policy, the pension benefits of the retired workers remain constant relative to the wage income of all type-j workers, and workers have to be active for an additional 5 years. As under the first policy, the pension contribution rate $\tau_{b,t}$ adjusts to balance the social security budget. However, the contribution rate can be much lower here, since, on average, the contribution period is longer and the benefit period is shorter.

2.6 Stationary Equilibrium

In the stationary equilibrium, individual behavior is consistent with the aggregate behavior of the economy, firms maximize profits, households maximize intertemporal utility, and factor and goods' markets clear. To express the equilibrium in terms of stationary variables only, we have to divide aggregate quantities by X_t and individual variables and prices by A_t. Therefore, we define the following stationary aggregate variables

$$\tilde{Beq}_t \equiv \frac{Beq_t}{X_t}, \quad \tilde{T}_t = \frac{T_t}{X_t}, \ \tilde{G}_t = \frac{G_t}{X_t}, \ \tilde{\Omega}_t = \frac{\Omega_t}{X_t}, \ \tilde{C}_t = \frac{C_t}{X_t}, \ \tilde{Y}_t = \frac{Y_t}{X_t},$$

and stationary individual variables

$$\tilde{c}_t \equiv \frac{c_t}{A_t}, \ \tilde{w}_t \equiv \frac{w_t}{A_t}, \ \tilde{b}_t \equiv \frac{b_t}{A_t}, \ \tilde{\omega}_t \equiv \frac{\omega_t}{A_t}, \ \tilde{tr}_t \equiv \frac{tr_t}{A_t}.$$

Let f_t denote the cross-section measure of households in period t.

A *stationary equilibrium* for a government policy $\{\tau_r, \tau_w, \tau_{b,t}, \bar{g}, \zeta_t, tr_t\}$, an initial value $A_0 > 0$, and initial measures f_0 in period 0 corresponds to a price system, an allocation, and a sequence of aggregate productivity indicators $\{A_t\}$ that satisfy the following conditions:

1. Population grows at the rate $\lambda_t = \frac{N_{t+1}}{N_t} - 1$.
2. The 'number' of intermediate-good firms, n_t, is equal to the total output of intermediates as each firms produces one unit of output, $x_t = 1$

$$n_t = X_t. \tag{21}$$

3. The aggregate productivity indicator, A_t evolves according to (16).

4. Labor market equilibrium: Aggregate effective labor supply, L_t, is equal to the aggregate labor demand of all intermediate-good firms

$$L_t = n_t l_t^d. \tag{22}$$

5. Capital market equilibrium: aggregate wealth is equal to aggregate capital plus aggregate innovation investment

$$\Omega_t = K_t + n_t i_t(q_t).$$

6. Households maximize the intertemporal utility (1) subject to the budget constraint (3) and $l \in [0, 1]$. This gives rise to standard first-order conditions for $\tilde{c}_t(s)$ and $l_t(s)$. Moreover, there is a transversality condition requiring $\omega_t\,(J+1) = 0$.

   As a result, for each period t individual labor supply $l_t(\tilde{\omega}, s, j)$, consumption $\tilde{c}_t(\tilde{\omega}, s, j)$ and optimal next period assets $\tilde{\omega}'_t(\tilde{\omega}, s, j)$ are functions of the individual state variables $\tilde{\omega}$, j, and s. Moreover, they also depend on t.

7. Firms maximize profits satisfying (6), (7) and (14). In equilibrium, firm profits are zero. Using (16), the zero-profit condition (15) can be rewritten as follows

$$\tilde{w}_t = (1-\alpha)\tilde{K}_t^{\alpha} - (1+r_t)i(q_t). \tag{23}$$

8. Aggregate variables are equal to the sum of the individual variables

$$\begin{aligned}
L_t &= \int e(s,j) l_t(\tilde{\omega}, s, j)\; f_t(d\tilde{\omega} \times ds \times dj), \\
\tilde{\Omega}_t &= \tilde{K}_t + i(q_t) = \frac{1}{L_t} \int \tilde{\omega} f_t(d\tilde{\omega} \times ds \times dj), \\
\tilde{B}eq_{t+1} &= \frac{A_t}{X_{t+1}} \int (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}'_t(\tilde{\omega}, s, j)\; f_t(d\tilde{\omega} \times ds \times dj) \\
&= \frac{1}{L_{t+1}} \frac{1}{1+q_{t+1}} \int (1-\phi_{t+1,s+1})(1+r_{t+1}(1-\tau_r))\tilde{\omega}'_t(\tilde{\omega}, s, j)\; f_t(d\tilde{\omega} \times ds \times dj), \\
\tilde{C}_t &= \frac{1}{L_t} \int \tilde{c}_t(\tilde{\omega}, s, j)\; f_t(d\tilde{\omega} \times ds \times dj), \\
\tilde{T}_t &= \tau_w \tilde{w}_t + \tau_r r_t(\tilde{K}_t + i(q_t)).
\end{aligned}$$

9. The government budget is balanced

$$\bar{g}\tilde{K}_t^{\alpha} + \tilde{t}r_t \frac{N_t}{L_t} = \tilde{T}_t + \tilde{B}eq_t. \tag{24}$$

10. The budget of the social security system is balanced

$$\frac{1}{L_t}\int \tilde{b}_t(s,j)\, f_t(d\tilde{\omega}\times ds\times dj) = \tau_{b,t}\tilde{w}_t, \tag{25}$$

11. The market for the final good clears.

12. The cross-sectional measure f_t evolves as

$$f_{t+1}(\tilde{\mathcal{W}}\times\mathcal{S}\times\mathcal{E}) = \int\, P_t\left((\tilde{\omega},s,j),\tilde{\mathcal{W}}\times\mathcal{S}\times\mathcal{E}\right)\, f_t(d\tilde{\omega}\times ds\times dj)$$

for all sets $\tilde{\mathcal{W}}$, $\mathcal{S}$, $\mathcal{E}$, where the Markov transition function P_t is given by

$$P_t\left((\tilde{\omega},s,j),\tilde{\mathcal{W}}\times\mathcal{S}\times\mathcal{E}\right) = \begin{cases} \phi_t(s) & \text{if } \tilde{\omega}_t'(\tilde{\omega},s,j)\in\tilde{\mathcal{W}}, \\ & j\in\mathcal{E}, s+1\in\mathcal{S} \\ & \\ 0 & \textit{else}, \end{cases}$$

and for the newborns

$$f_{t+1}(\tilde{\mathcal{W}}\times 1\times\mathcal{E}) = N_{t+1}(1)\cdot\begin{cases} \Gamma(\mathcal{E}) & \text{if } 0\in\tilde{\mathcal{W}} \\ & \\ 0 & \textit{else}. \end{cases}$$

3 Population, Pensions, and Endogenous Economic Growth in a Simplified Setting

In this section, we show that the relationship between population growth and endogenous economic growth depends on the characteristics of the pay-as-you-go pension scheme. We make this point in a simplified setting that combines the production sector of Section 2.3 with a household sector à la Diamond (1965). We establish the existence of a steady state and study the steady-state response of the economy's growth rate to a decline in its population growth rate. We find that the rise of the steady-state growth rate caused by a decline in population growth is larger under a pension scheme that keeps the contribution rate constant than under a scheme with a constant replacement rate. We shall see in Section 6 that the calibration exercises bring out these qualitative findings, too.

Consider two-period lived households. Generations comprise homogeneous individuals. The labor supply when young is exogenous and normalized to one, the one when old is zero. The population growth rate, λ, is constant over time and coincides with the growth rate of aggregate labor supply, i. e., $\lambda = L_{t+1}/L_t - 1$.

3.1 Households and the Pay-As-You-Go Pension Scheme

Lifetime utility of a member of cohort t is

$$\ln c_t^y + \beta \ln c_{t+1}^o, \quad \beta \in (0,1), \tag{26}$$

where c_t^y and c_{t+1}^o denote per-capita consumption when young and old, respectively. The maximization of (26) is subject to the per-period budget constraints

$$c_t^y + s_t = (1-\tau_b)w_t, \quad \text{and} \quad c_{t+1}^o = s_t(1+r_{t+1}) + b_{t+1}, \tag{27}$$

where b_{t+1} denotes the expected benefit when old. Optimal savings of a young at t results as

$$s_t = \frac{\beta}{1+\beta}(1-\tau_b)w_t - \frac{1}{1+\beta}\left(\frac{b_{t+1}}{1+r_{t+1}}\right). \tag{28}$$

First, we consider a pay-as-you-go system with a constant contribution rate $\tau_b \in (0,1)$. Leaving intragenerational heterogeneity and taxes on wage income aside, the benefits at $t+1$ implied by the balanced budget satisfy $b_{t+1}\, L_t = L_{t+1}\tau_b\, w_{t+1}$ such that

$$b_{t+1} = (1+\lambda)\, \tau_b\, w_{t+1}. \tag{29}$$

3.2 The Dynamical System

We use $\tilde{K}_t$ as the state variable of the dynamical system. To express the savings of the young as a function of this state variable we employ Lemma 1 to find the equilibrium real wage as

$$w_t \;=\; (1-\alpha)\tilde{K}_t^{\alpha}\, A_{t-1}\left(1+g\left(\tilde{K}_t\right)\right)\left[1-\left(\frac{\alpha}{1-\alpha}\right)\frac{i\left(g\left(\tilde{K}_t\right)\right)}{\tilde{K}_t}\right]. \tag{30}$$

Accordingly, a rise in the capital intensity $\tilde{K}_t$ affects the wage in three ways. First, a higher $\tilde{K}_t$ raises the marginal product of the intermediate good (see equation 7). Second, a rise in $\tilde{K}_t$ renders labor scarcer such that the productivity growth rate $g(\tilde{K}_t)$ rises. Third, faster productivity growth increases investment outlays. The first two channels increase the wage, the last can be shown to decrease it. Observe that (30) gives the same real wage as for a neoclassical production sector with costless exogenous productivity growth if we set $g\left(\tilde{K}_t\right) = g > 0$ and $i(.) = 0$.

Expressed in efficiency units at t we have

$$\frac{w_t}{A_{t-1}\left(1+g\left(\tilde{K}_t\right)\right)} \;=\; \tilde{K}_t^{\alpha} - \alpha\,\tilde{K}_t^{\alpha-1}\left(\tilde{K}_t + i\left(g\left(\tilde{K}_t\right)\right)\right) \equiv \tilde{w}\left(\tilde{K}_t\right). \tag{31}$$

In these units, the real wage at t is equal to the difference between final-good output and overall debt service payments per unit of effective labor, respectively.

In equilibrium, the savings of the young must be equal to aggregate investment, i. e.,

$$s_t\,L_t = K_{t+1} + n_{t+1}\,i\left(q_{t+1}\right). \tag{32}$$

Using Lemma 1, optimal savings (28), the budget of the pension scheme (29), equilibrium wages (30) for periods t and $t+1$, and the full employment condition (22) in (32) gives the equation of motion for $\tilde{K}_t$ as

$$\xi_1\,\tilde{K}_t^{\alpha}\left(1 - \frac{\alpha}{1-\alpha}\frac{i\left(g\left(\tilde{K}_t\right)\right)}{\tilde{K}_t}\right) = \left(1+g\left(\tilde{K}_{t+1}\right)\right)\left[\xi_2\,\tilde{K}_{t+1} + \xi_3\,i\left(g\left(\tilde{K}_{t+1}\right)\right)\right], \tag{33}$$

where

$$\xi_1 \;\equiv\; \frac{\beta(1-\alpha)(1-\tau_b)}{(1+\beta)(1+\lambda)} \in (0,1), \tag{34}$$

$$\xi_2 \;\equiv\; 1 + \frac{\tau_b}{1+\beta}\frac{1-\alpha}{\alpha} > 1, \tag{35}$$

$$\xi_3 \;\equiv\; 1 - \frac{\tau_b}{1+\beta} \in (0,1). \tag{36}$$

The dynamical system of the economy is given by (33) in conjunction with an initial value $\tilde{K}_0 > 0$. The three statements of the following theorem establish the key results that we need for the analysis of the link between population, pensions, and endogenous economic growth. For ease of exposition, we state and prove the theorem assuming particular parameter values which are sufficient but by no means necessary for the results.

Theorem 1 *Let $\delta = 1$ and $\alpha = 1/2$.*

1. *Equation (33) gives rise to a function $\tilde{K}_{t+1} = \phi\left(\tilde{K}_t\right)$ with $\phi'\left(\tilde{K}_t\right) > 0$ for all $\tilde{K}_t > \tilde{K}_c$.*
2. *There is at least one locally stable steady state $\tilde{K}^* = \phi\left(\tilde{K}^*\right) > \tilde{K}_c$ if $v_0 < (\xi_1/\xi_2)^2$.*
3. *There is a function $\tilde{K}^* = \tilde{K}^*(\lambda, \tau_b)$ with $\partial\tilde{K}^*/\partial\lambda < 0$ and $\partial\tilde{K}/\partial\tau_b < 0$.*

Statement 1 assures a unique equilibrium value $\tilde{K}_{t+1}$ given $\tilde{K}_t$ in the regime with strictly positive innovation investments. Statement 2 assures the existence of at least one locally stable steady state. Hence, without a closed form solution at hand, the focus of our analysis on steady-state effects is admissible.

The comparative statics of Statement 3 reveal that faster population growth and a higher contribution rate reduce the steady-state capital intensity. Intuitively, in a steady state savings per unit of effective labor are such that all generations are equipped with the same amount of capital and innovation investments per unit of effective labor. Moreover, since the steady state is locally stable, an increase in $\tilde{K}^*$ augments savings per unit of effective labor by less than the investment needs per unit of effective labor. Then, a higher population growth rate has two reinforcing effects on $\tilde{K}^*$. First, it increases the amount of aggregate investments necessary to maintain a given steady-state capital intensity. Given savings per unit of effective labor this reduces $\tilde{K}^*$ (*Solow effect*). Second, savings per unit of effective labor falls since, ceteris paribus, faster population growth means a higher pension benefit in accordance with (29). The second finding of Statement 3 follows from the observation that the contribution rate affects the individual incentive to save through the well-known income effects. An increase in τ_b means a lower net wage when young and a higher benefit when old. Both effects reduce savings per unit of effective labor. If such a hike occurs in a locally stable steady state the value of $\tilde{K}^*$ must fall.

Finally, observe that the steady-state growth rate of all per-capita magnitudes in this economy is equal to $q^* = g\left(\tilde{K}^*\right)$. This follows from Lemma 1, the updating condition (16), and the equilibrium conditions for the intermediate good market and for the labor market, (21) and (22), respectively. Equilibrium output at t becomes $Y_t = K_t^{\alpha}\,(A_t L_t)^{1-\alpha}$ such that output per worker at t may be written as $Y_t/L_t = \tilde{K}_t^{\alpha}\,A_t$. Since $\tilde{K}_t$ is constant in the steady state, Y_t/L_t grows at rate $g\left(\tilde{K}^*\right)$.

3.3 Population, Pensions, and Endogenous Economic Growth

What is the effect of population growth on the steady-state growth rate of the economy? It turns out that the answer depends on particular properties of the pension scheme. The following proposition has the result for the pension scheme with a constant contribution rate discussed above.

Proposition 1 *Suppose the economy is in a locally stable steady state $\tilde{K}^* > \tilde{K}_c$. If the pension system has a constant contribution rate, then $q^* = g\left(\tilde{K}^*\left(\lambda, \tau_b\right)\right)$ and it holds that*

$$\left.\frac{dq^*}{d\lambda}\right|_{\tau_b = const.} < 0. \tag{37}$$

Hence, a decline in the population growth rate increases the steady-state growth rate of the economy, q^*. The intuition is straightforward. According to Statement 3 of Theorem 1 a decline in the population growth rate implies a higher steady-state capital intensity (*Solow effect*). Moreover, in accordance with Lemma 1, the increased relative scarcity of labor means faster productivity growth (*Hicks effect*).

Next, we consider the same economy except that the pay-as-you-go pension system stipulates a constant replacement rate and not a constant contribution rate. How does the steady-state growth rate of such an economy respond to a demographic change?

Consider, in accordance with (25), a constant replacement rate $\zeta \in (0, 1)$ that ties the benefit when old to the net wage of the current young, i. e.,

$$b_{t+1} = \zeta\, w_{t+1}(1 - \hat{\tau}_{b,t+1}), \tag{38}$$

where $\hat{\tau}_{b,t+1}$ adjusts such that the budget of the pension scheme is balanced. The $\hat{}$ -notation distinguishes the contribution rate under a constant replacement rate from τ_b of the constant-contribution-rate case. At all t the contribution rate consistent with a balanced budget of (29) and (38) satisfies $(1 + \lambda)\hat{\tau}_{b,t} = \zeta\,(1 - \hat{\tau}_{b,t})$, or, suppressing the time argument, we have

$$\hat{\tau}_b = \frac{\zeta}{\zeta + 1 + \lambda} \equiv \hat{\tau}_b\left(\lambda, \zeta\right), \quad \text{with} \quad \frac{\partial \hat{\tau}_b}{\partial \lambda} < 0 \quad \text{and} \quad \frac{\partial \hat{\tau}_b}{\partial \zeta} > 0. \tag{39}$$

Since the population growth rate is constant, so is the contribution rate. Faster population growth reduces the dependency ratio such that a given replacement rate can be supported with a lower contribution rate. A higher replacement rate requires a higher contribution rate. On the one hand, such an increase reduces benefits when old, on the other hand, it increases the contributions paid by the young.

Another important implication of (39) is that Statement 1 and 2 of Theorem 1 also apply here. Indeed, given $\tilde{K}_0 > 0$ the evolution described by the dynamical system of (33) is the same under a constant contribution rate and under a constant replacement rate if the right-hand sides of (29) and (38) are equal, i. e., if

$$(1+\lambda)\tau_b = \zeta(1-\hat{\tau}_b). \tag{40}$$

Statement 3 of Theorem 1 needs to be modified. In view of (39) the relationship between the steady-state capital intensity and population growth is now described by $\tilde{K}^* = \tilde{K}^*\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right)\right)$. It follows that

$$\frac{d\tilde{K}^*}{d\lambda} = \frac{\partial \tilde{K}^*\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right)\right)}{\partial \lambda} + \frac{\partial \tilde{K}^*\left(\lambda, \hat{\tau}\left(\lambda, \zeta\right)\right)}{\partial \hat{\tau}_b}\frac{\partial \hat{\tau}_b\left(\lambda, \zeta\right)}{\partial \lambda}. \tag{41}$$

According to Theorem 1 and (39), all three partial derivatives are negative. Hence, the overall effect of a decline in population growth on the steady-state capital intensity is ambiguous. The reason is to be found in the relationship between changes in the population growth rate and the incentive to save. Under a fixed contribution rate, a decline in population growth reduces the expected pension benefit (see, (29)). Accordingly, savings increase. Under a constant replacement rate this effect is weakened since a lower population growth rate also raises the contribution rate. Indeed, from (29) and (39), we have $b_{t+1} = (1+\lambda)\,\hat{\tau}_b\left(\lambda, \zeta\right)\, w_{t+1} \equiv b_{t+1}\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right), w_{t+1}\right)$. Therefore, $db_{t+1}/d\lambda = \partial b_{t+1}/\partial \lambda + \left(\partial b_{t+1}/\partial \hat{\tau}_b\right)\left(\partial \hat{\tau}_b\left(\lambda, \zeta\right)/\partial \lambda\right)$, where the first term is positive and the second is negative. Of course, (38) implies that the overall effect remains positive such that the incentive to save is weaker. Moreover, since a higher contribution reduces the net wage of the current young, they also save less.

As to the effect of population growth on the steady-state growth rate of the economy under a constant replacement rate we obtain the following result.

Proposition 2 *Consider an economy in a locally stable steady state $\tilde{K}^* > \tilde{K}_c$. If the pension system has a constant replacement rate, then $q^* = g\left(\tilde{K}^*\left(\lambda, \hat{\tau}_b\left(\lambda, \zeta\right)\right)\right)$ and it holds that*

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} < 0. \tag{42}$$

Proposition 2 holds since the direct effect of λ on $\tilde{K}$ in (41) dominates the indirect effect through $\hat{\tau}_b$ such that the overall effect is negative. Hence, under a constant replacement rate we expect the economy's steady-state growth rate to increase following a decline in the population growth rate just as under a constant contribution rate. However, according to the following proposition, the strength of the steady-state response of the economy's growth rate to a demographic change depends on the institutional design of its pension scheme.

Proposition 3 *Consider an economy in a locally stable steady state $\tilde{K}^* > \tilde{K}_c$. Suppose (40) is satisfied such that $q^* = g\left(\tilde{K}^*(\lambda, \tau_b)\right) = g\left(\tilde{K}^*(\lambda, \hat{\tau}_b(\lambda, \zeta))\right)$. Then, the impact of a permanent decline in the population growth rate on the steady-state growth rate is greater under a constant contribution rate than under a constant replacement rate, i. e.,*

$$\left| \frac{dq^*}{d\lambda}\bigg|_{\tau_b = const.} \right| > \left| \frac{dq^*}{d\lambda}\bigg|_{\zeta = const.} \right|. \tag{43}$$

Hence, the prediction is that a decline in the population growth rate increases the steady-state growth rate of the economy by more if the pension scheme keeps the contribution rate constant. The necessary increase of the contribution rate under a constant replacement rate discourages savings. Hence, the induced change in $\tilde{K}^*$ will be less pronounced. We shall confirm this result in our evaluation of the quantitative model in Section 6 (see, in particular Figure 5).

4 Calibration Strategy

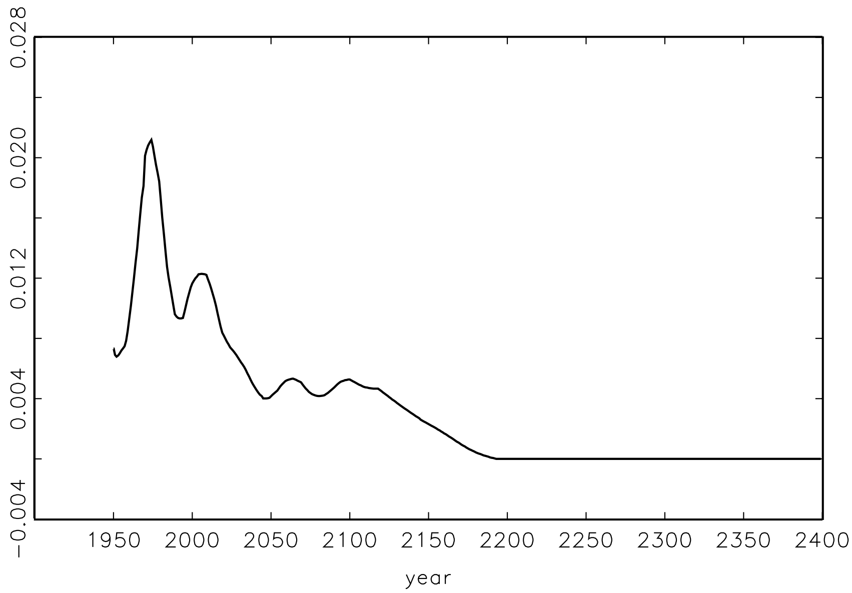
In this section, we describe our calibration of the model parameters.

4.1 Demographics

Our projection of the US population demographics for 1950-2050 is taken from United Nations (2002). The forecast for the US population development until 2400 is taken from Krüger and Ludwig (2007). These projections are based on the assumptions that life time expectancy increases at constant rates until the year 2100 and that the number of newborns is constant after 2200. Figure 1 illustrates the time profile of the population growth rate 1950-2400.[9] We take this evolution as exogenous and study the consequences for economic growth, pensions, and welfare.

[9]The depicted growth rates are computed for the population aged 20-94 in compliance with our model of Section 2.

Figure 1: Actual and Predicted US Population Growth Rates



4.2 Endowments and Preferences

We choose the discount factor $\beta = 1.011$ in accordance with the empirical estimates of Hurd (1989) who explicitly accounts for mortality risk.[10] For our choice of β, the real interest rate before taxes amounts to 6.96% (6.59%) during 1980-2005 (at the end of the demographic transition). We set the coefficient of risk aversion $\theta = 2.0$. The parameter $\gamma = 0.32$ of the utility function is calibrated so that the average labor supply of the workers is approximately 0.30.

The s-year old household of type j has the productivity $e(s, j) = \bar{y}_s \epsilon_j$. The age-efficiency profile $\{\bar{y}_s\}_{s=1}^{45}$ is taken from Hansen (1993), interpolated to in-between years and normalized to one. For those aged 65 - 69, we linearly extrapolated the data of Hansen (1993). The set of the equally distributed productivity types $\{\epsilon_1, \epsilon_2\} = \{0.57, 1.43\}$ is taken from Storesletten, Telmer, and Yaron (2004). With this calibration, we are able to replicate the empirical distribution of US wages. In our model, the Gini coefficient of the wage distribution is equal to 0.301 which compares favorably with empirical values reported by, e. g., Díaz-Giménez, Quadrini, and Ríos-Rull (1997).

[10]Related research that uses such a value for β includes İmrohoroğlu, İmrohoroğlu, and Joines (1995) and Huggett (1996). In the literature, the discount factor β has often been set smaller than one. Therefore, we provide a detailed sensitivity analysis for $\beta = 0.99$ in Heer and Irmen (2009). Besides an unrealistically high interest rate both our qualitative and our quantitative results remain virtually unaffected.

4.3 Production

The labor income share and the depreciation rate are set in accordance with Prescott (1986) implying the values $\alpha = 0.35$ and $\delta = 0.08$, respectively. The labor income (capital income) share in our model amounts to 64.0% (35.0%), while research expenditure relative to GDP are equal to 1.0%.[11] The parameter v_0 is set equal to 0.93 implying an average annual total factor productivity growth rate of 1.05% over the years 1980-2005 which corresponds to the US experience.

4.4 Government

The government share $\bar{g} = G/Y$ is set equal to the average ratio of government consumption in GDP, $\bar{g} = 0.195$ in the US during 1959-93 according to the Economic Report of the President (1994). The tax rates $\tau_w = 24.8\%$ and $\tau_r = 42.9\%$ are computed as the average values of the effective US tax rates over the time period 1965-88 that are reported by Mendoza, Razin, and Tesar (1994). The pension net replacement rate $\zeta = 0.50$ is taken from İmrohoroğlu, İmrohoroğlu, and Joines (1995). Government transfers, tr, and the social security contribution rate, τ_b, are computed using the equilibrium condition of the government budget (17) and the social security budget (25).

4.5 Computation of the Transition Dynamics

To compute the transition dynamics presented in the next section, we assume that the US economy attains a new steady state with zero population growth in the year 2400.[12] In addition, we assume an arbitrary steady state with 1.1% population growth in the year 1950. Given that we simulate the economy starting in 1950, the initialization is found to have a small effect on the transition after 2000.

In a first step, we make an initial guess of the time path of $\{\tilde{K}_t, \bar{l}_t, \tau_{b,t}, \tilde{t}r_t, q_t\}_{t=1950}^{t=2400}$. We iterate backwards in time and compute the household decisions of the newborn generation in each

[11] One reason why the fraction of research expenditure in GDP is relatively small is that the model does not explicitly account for above average salaries of researchers.

[12] In fact, the deviation of the steady-state capital intensity in 2400 from the computed capital intensity at the end of the transition in period 2399 is less than 0.001%. Details on the transition until 2400 are given in the Appendix.

period $t = 2400, 2399, \ldots, 1950$.[13] In each period, we aggregate savings of all generations and compute new values for $\{\underline{\tilde{K}}^i_t, \underline{\bar{l}}^i_t, \underline{\tau}^i_{b,t}, \underline{\tilde{tr}}^i_t, \underline{q}^i_t\}$. We update the time path using the Gauss-Seidel-Quasi-Newton algorithm presented by Ludwig (2007). In particular, we use the derivatives of the final steady-state equilibrium conditions with respect to $\tilde{K}$, $\bar{l}$, τ_b, $\tilde{tr}$, and q to initialize the Jacobian matrix in the Broyden algorithm. Moreover, we solve the model blockwise by first solving the transition for $\{\tilde{K}^1_t\}_{t=1950}^{t=2400}$ given the initial guess $\{\bar{l}^0_t, \tau^0_{b,t}, \tilde{tr}^0_t, q^0_t\}$. In the next step, we solve the transition for $\{\bar{l}^1_t\}$ given the remaining variable values $\{\tilde{K}^1_t, \tau^0_{b,t}, \tilde{tr}^0_t, q^0_t\}$ and so forth. When we tried to solve the model for all endogenous aggregate variables simultaneously, the program did not converge. Therefore, computational time is considerable and amounts to several hours on a Pentium III, 860 MHz, for an accuracy chosen to be equal to 10^{-5} for both the individual decision rules and the time path of the endogenous aggregate variables.

5 Demographic Transition and Economic Growth

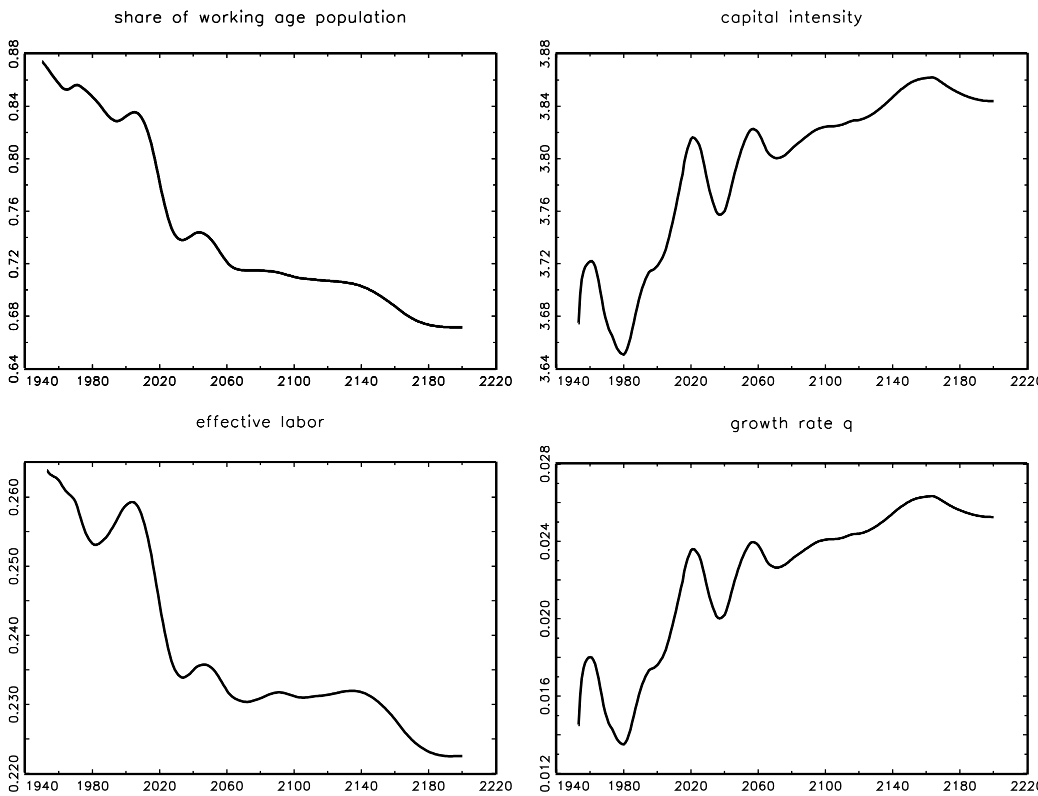
In this section we study the evolution of the US economy implied by the demographic transition depicted in Figure 1. As to the pension scheme we assume a constant pension replacement rate equal to $\zeta = 50\%$ such that the social security contribution rate $\tau_{b,t}$ adjusts to balance the budget of the social security system. We refer to this setup as the benchmark economy. In the following analysis, we establish and interpret the evolution of the labor force, the capital intensity, the factor prices, and the growth rate of technological knowledge, q, for the period 1950-2200.

The transition from 1950 until 2200 is graphed in Figures 2 and 3 for the benchmark economy. As the population ages, the share of the working age population falls from 88% in 1950 to 67% in 2200. Therefore, the contribution rate to the social security system $\tau_{b,t}$ increases from 5.1% to 14.8%.

The dynamics of the capital intensity, $\tilde{K} = K/AL$, seems to reflect the evolution of the labor force share; as the aggregate effective labor supply, L, declines, $\tilde{K}$ increases. However, the capital intensity is also influenced by general equilibrium effects that impinge on, e. g., the ability and willingness of a declining labor force to save and the evolution of the level of technological knowl-

[13] To compute the individual household problem, we have to solve a non-linear equation system in two variables which are the capital stocks of the 21-year old households with productivity ϵ_1 and ϵ_2, respectively. The methods are described in more detail in Heer and Maussner (2009).

Figure 2: Transition in the Benchmark Economy, Results I



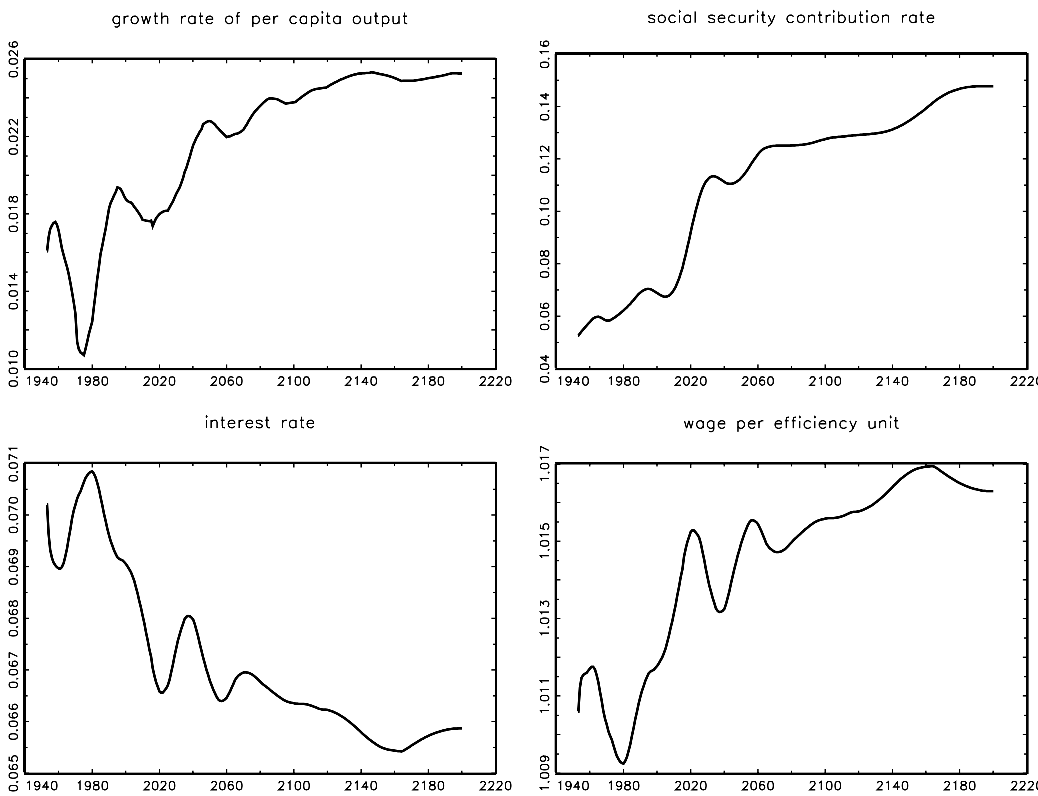
edge, A. Overall, the capital intensity is predicted to increase over time, though the evolution is not monotonic. The dynamics of factor prices reflects the one of the capital intensity; while the real wage rate w increases in $\tilde{K}$, the real interest rate r falls.

Since the growth rate of technological knowledge is tied to the capital intensity by the monotonic function $q_t = g\left(\tilde{K}_t\right)$, the evolution of q mimics the one of $\tilde{K}$. The lower right diagram in Figure 2 shows that q_t increases from an average rate of 1.63% during 1980-2005 to 2.41% in 2100. As we approach the steady state after 2200, q_t rises even further.[14] In the steady state $q^* = 2.52\%$ is constant and coincides with the growth rate of labor productivity, TFP, and all per-capita variables.

Hence, as our first main result, we conclude that the demographic transition is associated with

[14] All endogenous variables of the model like the capital stock per capita or the aggregate effective labor supply per capita stabilize around the year 2200. Indeed, in 2200 the transition is almost complete. The entire transition of $\tilde{K}_t$ and q_t until the year 2400 is shown in Figure 7 in Appendix 8.2.

Figure 3: Transition in the Benchmark Economy, Results II



an economically significant increase of the long-run growth rate of labor productivity from an average annual growth rate over 1990-2000 of 1.74% by 0.78 percentage points.

We may use these observations to shed some light on the sources of labor productivity growth. From Lemma 1, the updating condition (16), and the equilibrium conditions for the intermediate good market and for the labor market, (21) and (22), respectively, we find the equilibrium output at t to equal $Y_t = K_t^{\alpha} \, (A_t L_t)^{1-\alpha}$. Then, output per effective hour worked at t may be written as $Y_t/L_t = \tilde{K}_t^{\alpha} \, A_t$. Suppressing time subscripts and denoting the growth rate of some variable x by g_x, the growth rate of Y/L at all t is

$$g_{Y/L} = \alpha \, g_{\tilde{K}} + g\left(\tilde{K}\right). \tag{44}$$

Hence, growth of labor productivity has two sources. First, there is the direct contribution of growth in $\tilde{K}$, second there is the indirect contribution of $\tilde{K}$ on the growth of technological knowledge. As population and effective labor decline, both channels contribute to the growth of labor productivity. However, the impact of a rise in $g(\tilde{K})$, i. e., the Hicks effect, is quantitatively much larger. For instance, the model predicts average annual growth rates of labor productivity and technological knowledge between 2000 and 2100 of 2.23% and 2.22%, respectively. Accordingly, growth in Y/L is almost entirely due to the Hicks effect.

6 Population, Pensions, and Growth

In the previous section, we found that a decline in the population growth rate increases the growth rate of technological knowledge. In this section, we ask whether and how the design of the pension system affects this result. In Section 6.1 the focus is on possible growth effects. Since faster growth is no end in itself, Section 6.2 turns to the comparison of individual welfare under different designs of the pension scheme.

6.1 Comparative Analysis of Pension Reform Proposals

We consider the three policy scenarios of the pay-as-you-go pension scheme already introduced in Section 2:

1. constant replacement rate, i. e., $\zeta_t = \frac{b_t}{(1-\tau_w-\tau_b)w_t\bar{l}_t} = \zeta = 50\%$. This is the benchmark economy (solid line in the following figures),
2. constant contribution rate after 2010 (broken line). More precisely, the contribution rate adjusts until 2010 before it remains constant at the level of $\tau_b = 7.0\%$. We assume that this policy change is fully anticipated by all economic agents.[15]
3. constant replacement rate and later retirement at age 70, i. e., $R = 51$, for those agents born in 2010 and afterwards (dotted line).

Figure 4 illustrates the dynamics of the US economy for the three pension reform scenarios under endogenous growth. In the scenarios 1 and 3 (2), the contribution rate τ_b (the replacement rate of the pensions ζ) adjusts in order to balance the social security budget. Consequently, pensions fall in scenario 2 as ζ is reduced from 50.0% to 23.5%. In scenario 3, agents retire at age 71 if they

[15] Appendix 8.4.1 has the case where this reform is not anticipated.

are born after 2010. Therefore, the burden on the pension system declines significantly during 2055-59 when the cohorts of the 66 through 70-year old workers remain in the labor market. During this period, the effective labor supply increases (see upper right picture of Figure 4) and the dependency ratio falls. The social security contribution rates in the new steady states amount to 14.8%, 7.0%, and 11.2% in the scenarios 1, 2, and 3, respectively.

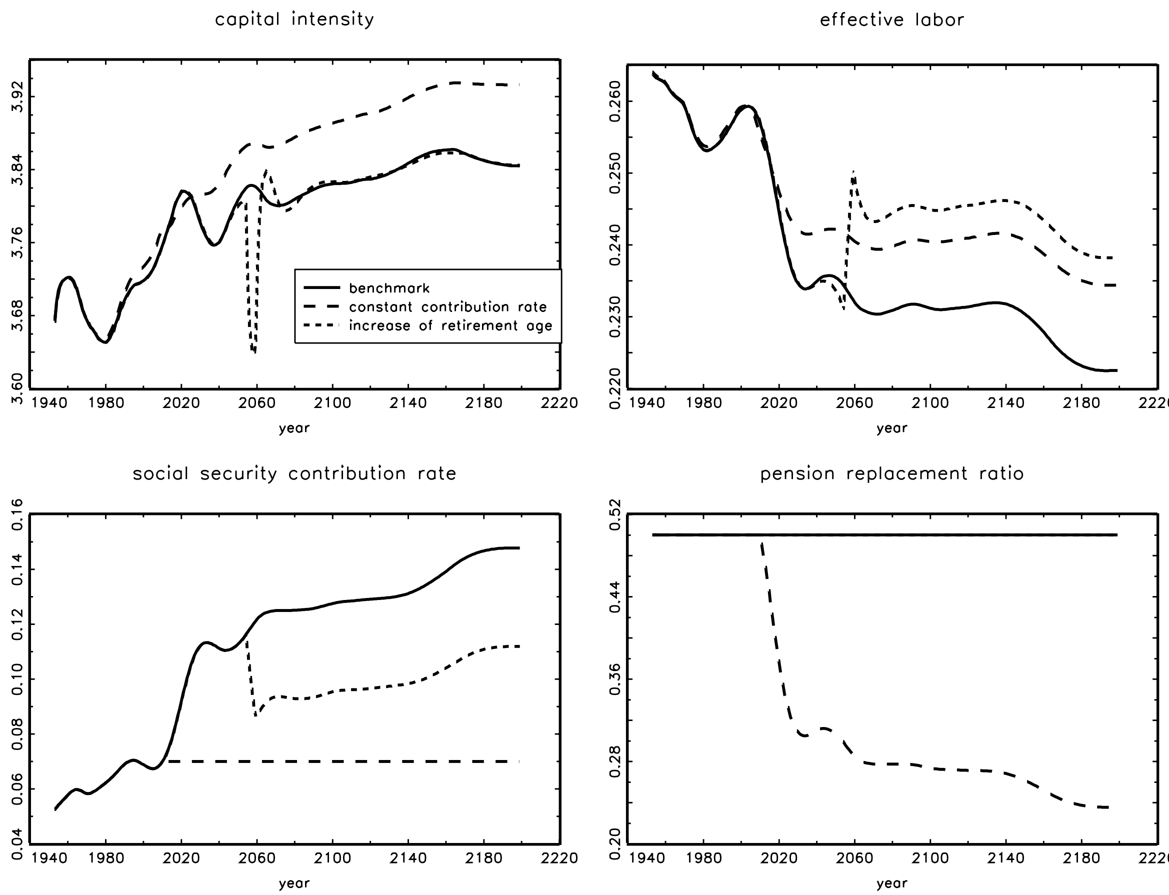
In the scenarios 2 and 3, the lower social security contribution rates also increase the incentives to supply labor as the net wage rate increases and the substitution effect dominates the income effect. Therefore, the aggregate supply of effective labor increases. In scenario 2, households also accumulate higher savings for old age, while in scenario 3 households accumulate savings over a longer working life. As a consequence, savings are also much higher in the scenarios 2 and 3. Since both savings and the labor force increase, the net effect on the capital intensity is ambiguous. As can be seen from the upper left picture in Figure 4, the capital intensity is higher in scenario 2 than in the benchmark case, while it is approximately equal in the benchmark case and in the scenario 3 with the exception of the period 2055-70.

In all three scenarios, the evolution of the growth rate of technological knowledge q mimics the evolution of $\tilde{K}$. Therefore, the long-run change in q is largest in scenario 2 that keeps the contribution rate τ_b constant after 2010. As can be seen from Figure 5, q_t rises from 1.71% during 1990-2000 to 3.06% in 2200.[16] In both scenarios 1 and 3, q_t is approximately at 2.53% in 2200. Since in 2200 the transition is almost complete we may interpret these numbers as the steady-state growth rates of the respective scenarios. Then, the calibration exercise confirms the qualitative predictions of Proposition 3: the effect of the demographic transition on the steady-state growth rate of the economy is larger under a constant contribution rate (scenario 2) than under a constant replacement rate (scenario 1).

Summing up, we state as our second main result that the design of the pension system may have large quantitative effects on the economy's growth rate.

[16] However, even though the growth rate increases significantly in scenario 2, it takes another 169 years until the absolute level of the pensions overtakes the one in the benchmark case. The effect of the lower replacement rate dominates and lasts for many decades. For example in 2100, pensions are still 20.9% lower in scenario 2 than compared to the benchmark.

Figure 4: Transition Dynamics for Different Public Pension Reform Scenarios under Endogenous Growth

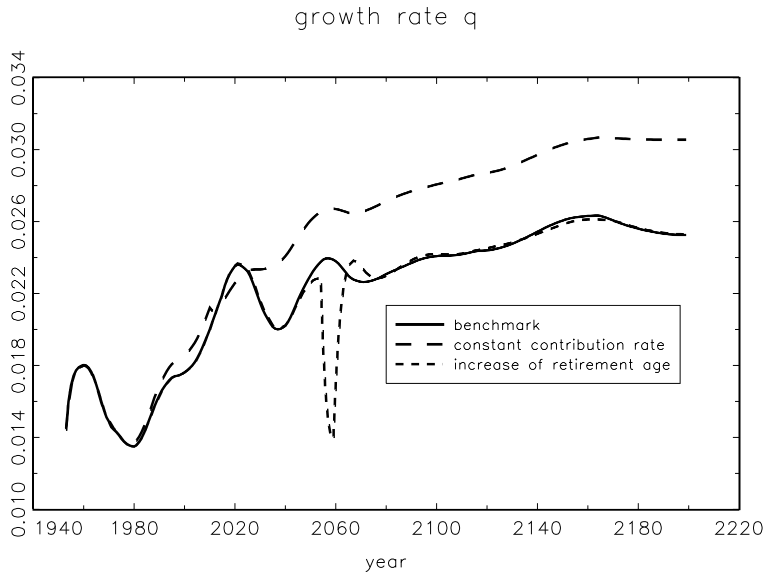


6.2 Welfare under Endogenous and Exogenous Growth

This section addresses two questions. First, we ask how the three pension reform designs compare in terms of welfare. Second, we want to know how the ranking of these proposals depends on whether the growth rate is endogenous or exogenous. Our results suggest that such a ranking can be established and that the negligence of growth rate effects of the social security system matters for the rank order of the proposals.[17]

[17]In Appendix 8.3, we study the welfare role of social security in the steady state with exogenous growth and find that social security is welfare-improving if the replacement ratio ζ is not too high. In particular, the optimal replacement ratio in the final steady state is found to be equal to 23.0%.

Figure 5: Growth Rate Effects of Different Public Pension Reform Scenarios



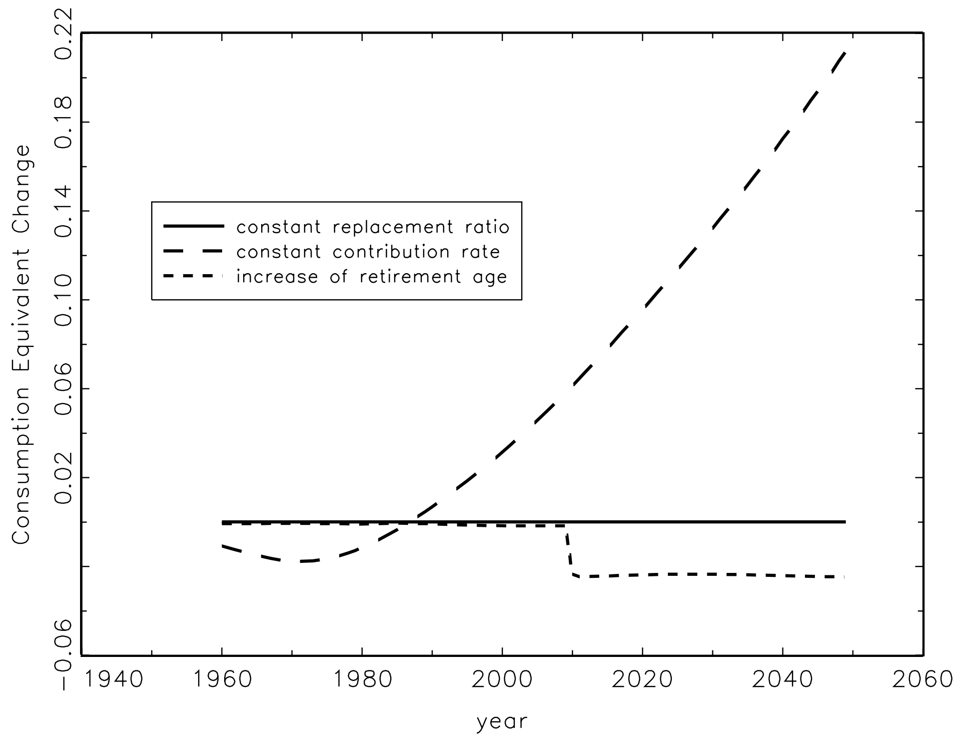
To address these questions, we hypothesize that the average newborn of each generation knows her lifetime utility associated with a life under each scenario. Equipped with this knowledge, she is able to rank the scenarios according to the lifetime utility they deliver. Figure 6 displays this welfare ranking for each generation born between 1960 and 2050. To provide an interpretable measure of welfare, the vertical axis states the consumption equivalent change relative to the benchmark economy with endogenous growth and a constant replacement rate. In other words, for all 3 scenarios it states the percentage change of the life-time consumption profile for the average newborn that is necessary to make her indifferent to a life in the benchmark economy.[18]

Obviously, the welfare effects are large and public pension reform matters. For example, consider the generation born in the year 2050. The compensation in consumption necessary to make the average newborn indifferent between the benchmark economy and the economy with a constant contribution rate amounts to 21.2% of total lifetime consumption, the difference between the upper broken line and the solid horizontal line at 2050. Of course, these huge welfare effects represent differences in the growth effects of the individual pension reforms.

Our results with regard to the welfare effects of the public pension reform can be summarized as follows:

[18]Similarly, de Nardi, İmrohoroğlu, and Sargent (1999) use wealth equivalent changes.

Figure 6: Welfare of Newborn Generations – Consumption Equivalent Change



1. Under a constant social security contribution rate from 2011 onwards, all agents born after 1986 benefit. The welfare effects are quantitatively substantial. While the losses for the generations born between 1960-1986 are also significant, they do not exceed 1.7% of total consumption; the gains accruing to the generations born after 1986 are large and increase over time.

2. If the retirement age increases from 65 to 70, the welfare of all workers affected by this change in legislation declines by approximately 2.4% of total consumption. This finding suggest that an increase in the age of retirement is not a worthwhile policy.

In Appendix 8.4.2 we compare the effect of pension reform proposals under endogenous and exogenous growth. This comparison shows that the quantitative welfare effects of public pension reform proposals are much larger when growth is endogenous. Moreover, we find that quantitative welfare results may depend on the assumption of endogenous or exogenous growth.

7 Concluding Remarks

In an economy where the production technology is endogenous, the investment decisions of firms are guided by profit incentives. The evolution of the labor force, taken as exogenous by firms, is then an important determinant of aggregate investment and growth. Moreover, a declining labor force also affects an economy's ability to save and invest. The purpose of the present paper is to study the interdependency of these aspects in a dynamic general equilibrium model and to assess quantitatively the resulting effects on economic growth, pensions, and welfare for the US economy.

We find that, as a consequence of the demographic transition, the US labor productivity growth rate increases from 1.74% (1.82%) during 1990-2000 to 2.41% (2.81%) in 2100 if the pension replacement rate (contribution rate) is kept at its current level. From a policy perspective, we find that the current pension system dominates a reform that increases the age of retirement. Switching to a constant contribution rate after 2010 generates winners and losers: welfare decreases for all generations born until 1986 and increases for all generations born after 1986. Overall, these results reflect the insight that the institutional characteristics of the pension system affect the relative scarcity of labor with respect to capital and, thereby, the incentives to engage in labor-saving technical change. At the conceptual level, our results suggest that allowing for TFP growth to be endogenous affects both the positive and the normative implications of large-scale OLG models employed to study policy reform proposals of the social security system.

Our results are subject to several caveats. They include the following. First, our analysis assumes a constant labor force participation rate. However, over the last three decades, this rate has increased in the US from 58.9% in 1966 to 67.1% in 1998. Since 1998, it has declined to 66.0% in 2005 basically due to cyclical effects (see, McEwen, Orrenius, and Wynne (2005)). Intuitively, the labor force participation rate is likely to depend on the demographic transition and the design of the public pension system. While a rising participation rate reduces the relative scarcity of labor it is likely to increase aggregate savings and capital accumulation through a rise in the economy's wage incomes. The net effect on investment incentives is hitherto undetermined.

Second, we study pension reforms in a closed economy. However, globalization and market liberalization in many emerging countries, notably China and India, account for a rising global labor force. As a matter of fact, population aging is already a predominant phenomenon in many of these countries including China, Japan, and Europe. In an open economy, these developments are likely to affect wages and interest rates in the US. While immigration to the US renders labor more abundant, capital inflows make it scarcer.[19] Krüger and Ludwig (2007) find that the decline in the labor force in Europe and Japan implies a small increase of the US relative price of labor. Allowing for the Hicks effect in their framework such a small increase is likely to raise the growth rate of the US economy. The role of this channel for the assessment of pension reforms is yet to be determined.

[19]Storesletten (2000) emphasizes the role of immigration to the US for fiscal policy. Recent contributions which account for open economy aspects in the context of pension reform include Huang, İmrohoroğlu, and Sargent (1997), Attanasio, Kitao, and Violante (2007), and Heijdra and Romp (2009b).

8 Appendix

8.1 Proofs

8.1.1 Proof of Lemma 1

Consider equations (6), (7), (14) for a maximizer $\hat{q}_t > 0$, and (15). Upon combining the latter two, we obtain

$$p_t = \left[(1+q_t)\, i'(q_t) + i(q_t)\right] (1+r_t) \tag{45}$$

Using the input requirement function (10) and acknowledging the dependency of p_t and r_t on $\tilde{K}_t$ through (6) and (7) in (45) gives

$$\frac{(1-\alpha)\tilde{K}_t^{\alpha}}{1-\delta+\alpha\tilde{K}_t^{\alpha-1}} = v_0(1+2q_t). \tag{46}$$

Taking into account that the latter is only valid as long as $q_t > 0$, we obtain the function $g\left(\tilde{K}_t\right)$ in closed-form as

$$q_t = g\left(\tilde{K}_t\right) = \max\left\{0, \frac{1}{2}\left(\frac{(1-\alpha)\tilde{K}_t^{\alpha}}{v_0\left(1-\delta+\alpha\tilde{K}_t^{\alpha-1}\right)} - 1\right)\right\}. \tag{47}$$

The critical capital intensity $\tilde{K}_c$ solves

$$\frac{(1-\alpha)\tilde{K}_c^{\alpha}}{v_0\left(1-\delta+\alpha\tilde{K}_c^{\alpha-1}\right)} = 1. \tag{48}$$

Obviously, there is a unique value $\tilde{K}_c > 0$ at which $g\left(\tilde{K}_t\right)$ is continuous. The remaining properties of the function $g\left(\tilde{K}_t\right)$ stated in the Lemma follow immediately from (47). ■

8.1.2 Proof of Theorem 1

We prove each statement of the theorem separately starting with Statement 1. Recall that $i(q) = v_0 q$, $\delta = 1$, and $\alpha = 1/2$ such that $\tilde{K}_c = v_0$.

1. Implicit differentiation of (33) gives

$$\left[\xi_1\,\alpha\tilde{K}_t^{\alpha-1}\left(1+\frac{i(g(\tilde{K}_t))}{\tilde{K}_t}-\frac{i'\left(g(\tilde{K}_t)\right)g_{\tilde{K}}(\tilde{K}_t)}{1-\alpha}\right)\right]d\tilde{K}_t = Z(\tilde{K}_{t+1})d\tilde{K}_{t+1}. \tag{49}$$

Here,

$$Z(\tilde{K}_{t+1}) \equiv g_{\tilde{K}}\,\left[\xi_2\,\tilde{K}_{t+1}+\xi_3\,i\right]+(1+g)\left[\xi_2+\xi_3\,i'\,g_{\tilde{K}}\right], \tag{50}$$

and the arguments of i is g and the one of g is $\tilde{K}_{t+1}$. For our parameter constellation, (46) delivers

$$\frac{d\hat{q}_t}{d\tilde{K}_t} \equiv g_{\tilde{K}}(\tilde{K}_t) = \frac{1}{2v_0} > 0, \quad \text{for all } \tilde{K}_t > \tilde{K}_c. \tag{51}$$

This leads to the conclusion that $Z(\tilde{K}_{t+1}) > 0$. Hence,

$$\frac{d\tilde{K}_{t+1}}{d\tilde{K}_t} = \frac{\xi_1\,\tilde{K}_t^{-\frac{3}{2}}\left(\tilde{K}_t-\tilde{K}_c\right)}{4Z(\tilde{K}_{t+1})} \equiv \phi'(\tilde{K}_t) > 0, \quad \text{for all } \tilde{K}_t > \tilde{K}_c. \tag{52}$$

Accordingly, for $\tilde{K}_t > \tilde{K}_c$ the function $\phi(\tilde{K}_t)$ exists and is strictly increasing.

■

2. The steady state is determined by (33) for $\tilde{K}^* = \tilde{K}_t = \tilde{K}_{t+1}$. First, we show that the right-hand side, $RHS(\tilde{K})$, and the left-hand side, $LHS(\tilde{K})$, of (33) intersect on $\tilde{K} > \tilde{K}_c$ at least once if $v_0 < (\xi_1/\xi_2)^2$. Then, we tackle local stability.

First, consider

$$RHS(\tilde{K}) \equiv \left(1+g(\tilde{K})\right)\left[\xi_2\,\tilde{K}+\xi_3\,i(g(\tilde{K}))\right]. \tag{53}$$

This function satisfies $RHS(\tilde{K}_c) = \xi_2\,\tilde{K}$. Moreover, it is bounded from below, i. e., $RHS(\tilde{K}) > \xi_2\,\tilde{K}$ for all $\tilde{K} > \tilde{K}_c$, and strictly increasing for all $\tilde{K} > \tilde{K}_c$ since $RHS'(\tilde{K}) = Z(\tilde{K}) > 0$.

Next, consider

$$LHS(\tilde{K}) \equiv \xi_1\,\tilde{K}^{\alpha}\left(1-\frac{\alpha}{1-\alpha}\frac{i(g(\tilde{K}))}{\tilde{K}}\right) = \frac{\xi_1}{2}\tilde{K}^{\frac{1}{2}}\left(1+\frac{\tilde{K}_c}{\tilde{K}}\right). \tag{54}$$

This function satisfies $LHS(\tilde{K}_c) = \xi_1\tilde{K}_c^{\frac{1}{2}}$. It is bounded from above, i. e., $\xi_1\,\tilde{K}^{\alpha} > LHS(\tilde{K})$ for all $\tilde{K} > \tilde{K}_c$. Moreover, form (52), it is strictly increasing for $\tilde{K} > \tilde{K}_c$.

Next observe that the condition $v_0 < (\xi_1/\xi_2)^2$ assures that $LHS(\tilde{K}_c) > RHS(\tilde{K}_c)$. Since $\xi_1 \tilde{K}^\alpha > LHS(\tilde{K})$ and $RHS(\tilde{K}) > \xi_2 \tilde{K}$ for $\tilde{K} > \tilde{K}_c$, the two functions $LHS(\tilde{K})$ and $RHS(\tilde{K})$ must intersect at least once. Since the two bounds intersect at $(\xi_1/\xi_2)^2$, there is at least one steady state $\tilde{K}^* \in \left(\tilde{K}_c, (\xi_1/\xi_2)^2\right)$ that satisfies $LHS'(\tilde{K}^*) < RHS'(\tilde{K}^*)$. At this $\tilde{K}^*$, we have $\phi'(\tilde{K}^*) = LHS'(\tilde{K}^*)/RHS(\tilde{K}^*) < 1$ such that this steady state is locally stable. ■

3. Consider equation (33) at $\tilde{K}_t = \tilde{K}_{t+1} = \tilde{K}^* > \tilde{K}_c$ and define the left-hand side of this equation as $LHS\left(\tilde{K}^*, \xi_1\right)$ and its right-hand side as $RHS\left(\tilde{K}^*, \xi_2, \xi_3\right)$. Taking the differential with respect to $d\tilde{K}^*$ and $d\xi_1$ gives

$$\frac{d\tilde{K}^*}{d\xi_1} = \frac{-LHS_{\xi_1}\left(\tilde{K}^*, \xi_1\right)}{LHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_1\right) - RHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_2, \xi_3\right)} > 0. \tag{55}$$

The sign follows since $LHS_{\xi_1} > 0$ and local stability implies $LHS_{\tilde{K}^*} < RHS_{\tilde{K}^*}$.

Then, the negative relationship between $\tilde{K}^*$ and λ obtains from the fact that $\partial\xi_1/\partial\lambda < 0$ such that

$$\frac{d\tilde{K}^*}{d\lambda} = \frac{d\tilde{K}^*}{d\xi_1}\frac{\partial\xi_1}{\partial\lambda} < 0. \tag{56}$$

Turning to the relationship between $\tilde{K}^*$ and τ_b we need to take into account that ξ_1, ξ_2, and ξ_3 depend on τ_b. Total differentiation of (33) with respect to $\tilde{K}^*$ and τ_b delivers

$$\frac{d\tilde{K}^*}{d\tau_b} = \frac{-LHS_{\xi_1}\left(\tilde{K}^*, \xi_1\right)\frac{\partial\xi_1}{\partial\tau_b} + RHS_{\xi_2}\left(\tilde{K}^*, \xi_2, \xi_3\right)\frac{\partial\xi_2}{\partial\tau_b} + RHS_{\xi_3}\left(\tilde{K}^*, \xi_2, \xi_3\right)\frac{\partial\xi_3}{\partial\tau_b}}{LHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_1\right) - RHS_{\tilde{K}^*}\left(\tilde{K}^*, \xi_2, \xi_3\right)}. \tag{57}$$

Again, due to local stability the denominator is negative. The first term of the numerator is positive since $\partial\xi_1/\partial\tau_b < 0$. The sum of the second and third term is also positive. To see this, observe that

$$\begin{aligned} RHS_{\xi_2}\frac{\partial\xi_2}{\partial\tau_b} + RHS_{\xi_3}\frac{\partial\xi_3}{\partial\tau_b} &= \frac{1+g(\tilde{K}^*)}{1+\beta}\left[\frac{1-\alpha}{\alpha}\tilde{K}^* - i\left(g(\tilde{K}^*)\right)\right] \\ &= \frac{1+g(\tilde{K}^*)}{1+\beta}\frac{1-\alpha}{\alpha}\tilde{K}^*\left[1 - \frac{\alpha}{1-\alpha}\frac{i\left(g(\tilde{K}^*)\right)}{\tilde{K}^*}\right] \end{aligned} \tag{58}$$

From (54) the term in brackets is positive. Hence, it follows that $d\tilde{K}^*/d\tau_b < 0$. ■

8.1.3 Proof of Proposition 1

From Lemma 1 and Theorem 1, we have

$$\left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.} = g_{\tilde{K}}(\tilde{K}^*(\lambda,\tau_b))\,\frac{\partial K^*(\tilde{\lambda},\tau_b)}{\partial\lambda} < 0. \tag{59}$$

■

8.1.4 Proof of Proposition 2

Under a constant replacement rate we have $q^* = g(\tilde{K}^*(\lambda,\hat{\tau}_b(\lambda,\zeta)))$. Hence,

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} = g_{\tilde{K}}\left(\tilde{K}^*\right)\left[\frac{\partial\tilde{K}^*}{\partial\lambda} + \frac{\partial\tilde{K}^*}{\partial\hat{\tau}_b}\frac{\partial\hat{\tau}_b(\lambda,\zeta)}{\partial\lambda}\right], \tag{60}$$

where the argument of $\tilde{K}^*$ is $(\lambda,\hat{\tau}_b(\lambda,\zeta))$. To proof the proposition, we show that the term in brackets of (60) is negative for all admissible parameter values.

Define $\hat{\xi}_i \equiv \xi_i|_{\tau_b=\hat{\tau}_b(\lambda,\zeta)}$, $i=1,2,3$. Using (56) and (57) we find that $d\tilde{K}^*/d\lambda < 0$ holds if and only if

$$-LHS_{\hat{\xi}_1}\frac{\partial\hat{\xi}_1}{\partial\lambda} + \left(-LHS_{\hat{\xi}_1}\frac{\partial\hat{\xi}_1}{\partial\hat{\tau}_b} + RHS_{\hat{\xi}_2}\frac{\partial\hat{\xi}_2}{\partial\hat{\tau}_b} + RHS_{\hat{\xi}_3}\frac{\partial\hat{\xi}_3}{\partial\hat{\tau}_b}\right)\frac{\partial\hat{\tau}_b}{\partial\lambda} > 0, \tag{61}$$

where the argument of LHS is $(\tilde{K}^*,\hat{\xi}_1)$, the one of RHS is $(\tilde{K}^*,\hat{\xi}_2,\hat{\xi}_3)$, and the one of $\tilde{K}^*$ is $(\lambda,\hat{\tau}_b(\lambda,\zeta))$. From the definitions of $\hat{\xi}_i$, $i=1,2,3$ and the fact that the derivatives are evaluated at the steady state, we derive that (61) can be written as

$$\frac{\hat{\xi}_2\tilde{K}^* + \hat{\xi}_3 i}{1+\lambda} + \left(\frac{\hat{\xi}_2\tilde{K}^* + \hat{\xi}_3 i}{1-\hat{\tau}_b} + \tilde{K}^*\frac{1-\alpha}{\alpha(1+\beta)} - \frac{i}{1+\beta}\right)\frac{\partial\hat{\tau}_b}{\partial\lambda} > 0. \tag{62}$$

Using (39) to substitute for $\hat{\tau}_b$, condition (62) becomes

$$\frac{\hat{\xi}_2\tilde{K}^* + \hat{\xi}_3 i}{1+\lambda}\left(1 - \frac{\zeta}{\zeta+1+\lambda}\right) + \left(\tilde{K}^*\frac{1-\alpha}{\alpha(1+\beta)} - \frac{i}{1+\beta}\right)\frac{-\zeta}{(\zeta+1+\lambda)^2} > 0. \tag{63}$$

With $\hat{\xi}_i$, $i=1,2,3$, one readily verifies that inequality (63) holds for all admissible parameter values. Hence, the term in brackets of (60) is negative. With Lemma 1 the proposition follows. ■

8.1.5 Proof of Proposition 3

Under a constant replacement rate where $q^* = g(\tilde{K}^*) = g(\tilde{K}^*(\lambda, \hat{\tau}_b(\lambda, \zeta)))$, we have

$$\left.\frac{dq^*}{d\lambda}\right|_{\zeta=const.} = g_{\tilde{K}}(\tilde{K}^*)\left[\frac{\partial \tilde{K}^*(\lambda, \hat{\tau}_b(\lambda,\zeta))}{\partial \lambda} + \frac{\partial \tilde{K}^*(\lambda, \hat{\tau}_b(\lambda,\zeta))}{\partial \hat{\tau}_b}\frac{\partial \hat{\tau}_b(\lambda,\zeta)}{\partial \lambda}\right] \tag{64}$$

$$= g_{\tilde{K}}(\tilde{K}^*)\frac{\partial \tilde{K}^*(\lambda, \hat{\tau}_b(\lambda,\zeta))}{\partial \lambda} + g_{\tilde{K}}(\tilde{K}^*)\frac{\partial \tilde{K}^*(\lambda, \hat{\tau}_b(\lambda,\zeta))}{\partial \hat{\tau}_b}\frac{\partial \hat{\tau}_b(\lambda,\zeta)}{\partial \lambda} \tag{65}$$

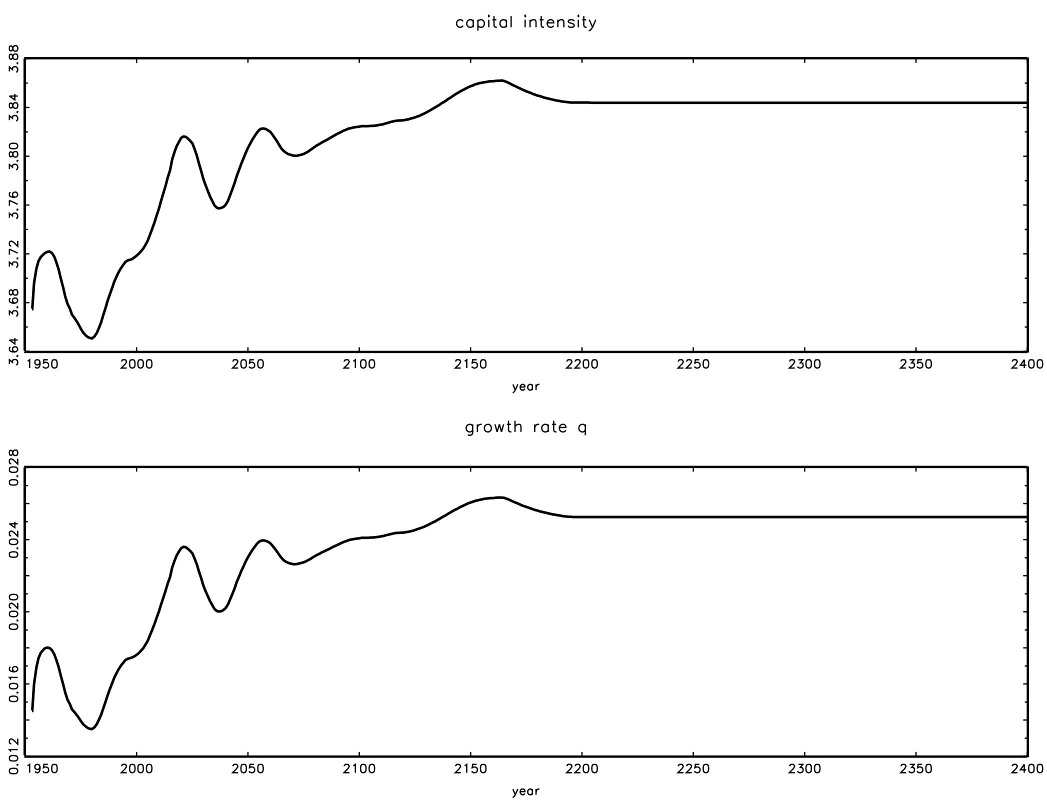
$$= \left.\frac{dq^*}{d\lambda}\right|_{\tau_b=const.} + g_{\tilde{K}}(\tilde{K}^*)\frac{\partial \tilde{K}^*(\lambda, \hat{\tau}_b(\lambda,\zeta))}{\partial \hat{\tau}_b}\frac{\partial \hat{\tau}_b(\lambda,\zeta)}{\partial \lambda}, \tag{66}$$

where the last step makes use of Proposition 1 and the fact that $\tau_b = \hat{\tau}_b(\lambda, \zeta)$. Since the last term in (66) is positive the proposition is proved. ■

8.2 Computation Details

As mentioned in the main text, we assume that the transition is complete in the year 2400. Figure 7 presents the complete transition from 1950 to 2400 for the capital intensity, $\tilde{K}_t$, and the endogenous growth rate, q_t. These time paths suggest that the economy is in its new steady state from 2200 onwards.

Figure 7: Complete Transition in the Benchmark Economy



8.3 The Welfare Role of the Pension Policy

This section addresses the welfare role of the pension policy.[20] The purpose is to figure out whether the presence of a pension scheme is welfare-improving in the benchmark calibration of the US economy, i. e., for the model in Section 2. Moreover, we want to get an idea of the approximate optimal pension policy in this framework.[21]

To approach this question, we focus on the setting with exogenous growth and confine attention to the steady state of the year 2400. For this state, we compute the expected life-time utility of the newborn who ignores his efficiency type. Using the calibration with $\zeta = 50\%$ (which implies $\tau_b = 14.8\%$) as our benchmark, we compute the welfare gains in terms of consumption equivalent changes that arise for replacement ratios $\zeta < 50\%$.

Figure 8: Optimal Pension Policy

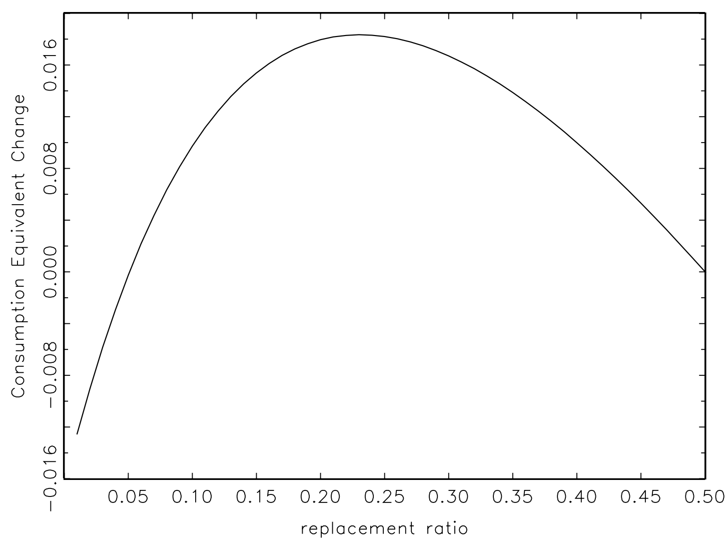


Figure 8 presents our results. The optimal policy is found to be given by $\{\zeta, \tau_b\} = \{23.0\%, 7.60\%\}$.

[20] We would like to thank an anonymous referee for suggesting the analysis of this section.

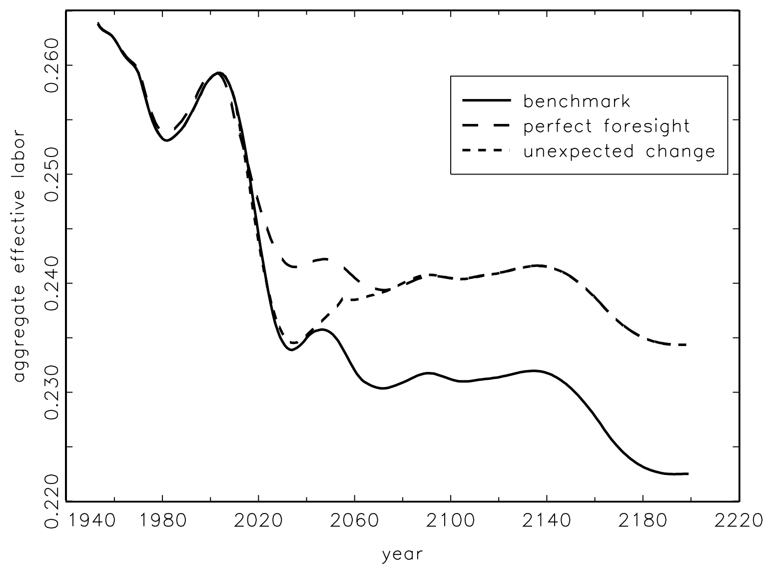
[21] In a model related to ours, İmrohoroğlu, İmrohoroğlu, and Joines (1995) show that the welfare gains from social security are primarily driven by dynamic inefficiency. Therefore, social security is a more efficient way to finance retirement consumption than private saving. These authors also point to the role of bequest income. With endogenous bequest income, welfare gains from a social security system are lower. See, Caliendo, Guo, and Hosseini (2013) for a theoretical refinement of the underlying arguments. See, Conesa and Garriga (2008) for the derivation of the optimal social security system in a Ramsey setup. In contrast to our analysis, their analysis has an optimization over the entire policy space including tax rates and debt policy.

The consumption equivalent gain under the optimal pension policy compared to the scenario without a pension scheme, i. e., where $\{\zeta, \tau_b\} = \{0, 0\}$, is equal to 3.47%. Hence, there is a welfare-improving role for social security in our setup.

8.4 Sensitivity Analysis

8.4.1 Perfect Foresight versus Unexpected Policy Changes

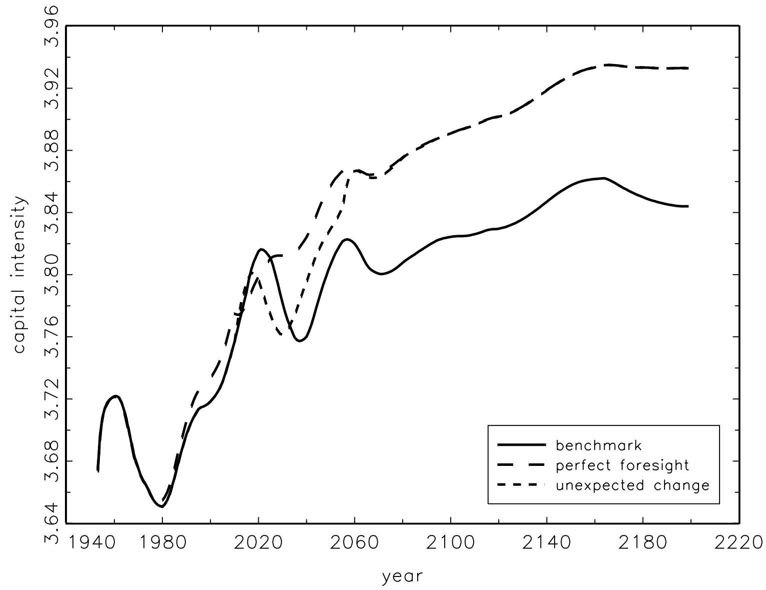
Figure 9: Effects of Policy Announcement on Aggregate Effective Labor



In Section 6, we analyze the pension reform that switches from a constant replacement rate of pensions relative to net average wages ($\zeta_t = 0.5$ for $t \leq 2010$), to a constant contribution rate ($\tau_{b,t} = \tau_{b,2010}$ for $t > 2010$) in the year 2010. In our computations, we assume that the change in the pension policy is fully anticipated. In this Appendix, we study the alternative scenario where the households do not learn about this policy change until the end of the year 2010.

In Figures 9 and 10, the transition dynamics of the aggregate supply of effective labor, L, and the capital intensity, $\tilde{K}$, are illustrated for the benchmark case (with constant replacement rate $\zeta_t = 0.5$ for all t), and for the two cases of an unexpected and an expected policy change involving a constant contribution rate for $t > 2010$. If the policy change is unexpected, the time paths of the two variables coincide with the one of the benchmark case until the year 2010. Afterwards, L

Figure 10: Effects of Policy Announcement on the Capital Intensity



decreases temporarily (the dotted line falls below the solid line until 2020), even though only to a very small extent. Households increase their labor supply in old age at the expense of their labor supply in young age because, in comparison to the benchmark case, the contribution rates $\tau_{b,t}$ in old age are much lower so that the net wages are higher. As the substitution effect is stronger than the income effect, labor supply in the later years of their working life increases.

In addition, households learn in 2010 that their pensions will be lower than expected. As a consequence, they increase savings, especially in old age. Compared to the case with perfect foresight (broken line), the capital intensity $\tilde{K}$ is always lower. As the growth rate, q_t, is a monotone function of the capital intensity, growth is also higher in the case of an expected policy change. In 2031, the two transition paths display the maximum difference between the two growth rates amounting to 0.27 percentage points (not illustrated). Clearly, the unexpected change in policy results in a short and medium-run decline of the compounded growth rate.

8.4.2 Welfare Effects of Pension Reform Proposals under Exogenous Growth

In this section, we study the sensitivity of the welfare effects for the three pension scenarios if productivity growth is exogenous and chosen such that the average annual total factor productivity growth rate during 1980-2005 is 1.05% (compare Section 4.3). In addition, we set investment cost equal to zero, i. e., $i \equiv 0$. As a consequence, the demographic transition has no effect on total factor productivity.

Figure 11: Pension Reform Proposals and Exogenous Growth – Welfare of Newborn Generations

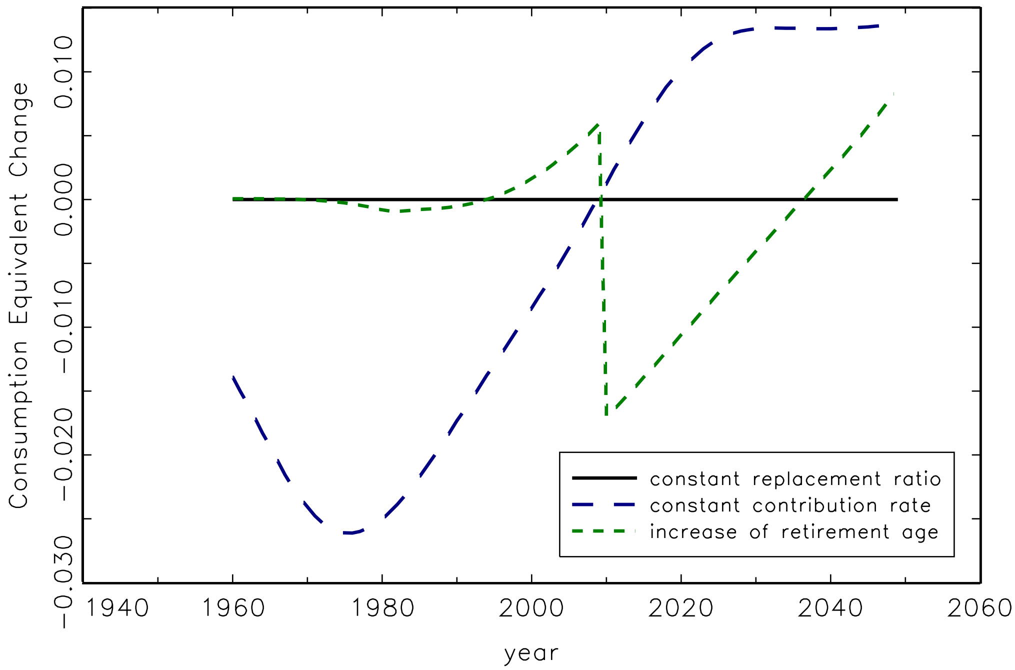


Figure 11 presents our results. Under exogenous growth, the increase in the retirement age (broken line) does not unanimously decrease the welfare of all generations. Compared to the benchmark case with a constant replacement rate (solid line), a higher retirement age introduced for all those born in the year 2010 and later increases the welfare of the generations born during 1995-2009. For example, the life-time utility of the generation born in 2009 increases by an amount equivalent to 0.6% of annual consumption. Welfare declines only for those generations born during 1973-1994 and 2010-2037. In the long run, both pension reform proposals, i. e., a constant contribution rate and a higher retirement age increase the welfare of the newborn generations. A constant contribution rate after 2010 only harms generations born prior to 2010.

Compared to the welfare results under endogenous growth presented in Figure 6, the quantitative welfare effects are much smaller here. More importantly, the assessment of each generation concerning the preferred pension reform hinges on whether growth is endogenous or exogenous.

References

Attanasio, O., S. Kitao, and G. L. Violante (2007): "Global Demographic Trends and Social Security Reform," *Journal of Monetary Economics*, 54(1), 144–198.

Auerbach, A. J., and L. J. Kotlikoff (1987): *Dynamic Fiscal Policy*. Cambridge University Press, Cambridge, MA.

Barro, R. J., and X. Sala-í-Martin (2004): *Economic Growth*. MIT Press, Cambridge, MA, 2nd edn.

Bester, H., and E. Petrakis (2003): "Wage and Productivity Growth in a Competitive Industry," *Journal of Economic Theory*, 109, 52–69.

Boldrin, M., and A. Montes (2005): "The Intergenerational State Education and Pensions," *Review of Economic Studies*, 72, 651–664.

Bös, D., and R. K. von Weizsäcker (1989): "Economic Consequences of an Aging Population," *European Economic Review*, 33, 345–354.

Caliendo, F. N., L. Guo, and R. Hosseini (2013): "Social Security is NOT a Substitute for Annuities," *mimeo, Utah State University*.

Conesa, J. C., and C. Garriga (2008): "Optimal Fiscal Policy in the Design of Social Security Reforms," *International Economic Review*, 49, 291–318.

Conesa, J. C., and D. Krüger (1999): "Social Security Reform with Heterogeneous Agents," *Review of Economic Dynamics*, 2, 757–795.

Cutler, D. M., J. M. Poterba, L. M. Sheiner, and L. H. Summers (1990): "An Aging Society: Opportunity or Challenge?," *Brookings Papers on Economic Activity*, 21(1), 1–73.

de Nardi, M., S. İmrohoroğlu, and T. J. Sargent (1999): "Projected U.S. Demographics and Social Security," *Review of Economic Dynamics*, 2, 575–615.

Diamond, P. A. (1965): "National Debt in a Neoclassical Growth Model," *American Economic Review*, 55(5), 1126–1150.

Díaz-Giménez, J., V. Quadrini, and J. Ríos-Rull (1997): "Dimensions of Inequality: Facts on the U.S. Distributions of Earnings, Income, and Wealth," *Federal Reserve Bank of Minneapolis Quarterly Review*, 21, 3–21.

Echevarria, C. A., and A. Iza (2006): "Life Expectancy, Human Capital, Social Security and Growth," *Journal of Public Economics*, 90(12), 2323–2349.

Fuster, L., A. İmrohoroğlu, and S. İmrohoroğlu (2007): "Elimination of Social Security in a Dynastic Framework," *Review of Economic Studies*, 74, 113–145.

Habakkuk, J. (1962): *American and British Technology in the Nineteenth Century: Search for Labor-Saving Inventions*. Cambridge University Press, Cambridge, England.

Hansen, G. (1993): "The Cyclical and Secular Behavior of the Labor Input: Comparing Efficiency Units and Hours Worked," *Journal of Applied Econometrics*, 8, 71–80.

Heer, B., and A. Irmen (2009): "Population, Pensions, and Endogenous Economic Growth," *Center for Economic Policy Research (CEPR), London, Discussion Paper, No. 7172*.

Heer, B., and A. Maussner (2009): *Dynamic General Equilibrium Modeling*. Springer Verlag, Heidelberg, 2nd edn.

Heijdra, B. J., and W. E. Romp (2009a): "Human Capital Formation and Macroeconomic Performance in an Aging Small Open Economy," *Journal of Economic Dynamics and Control*, 33, 725–744.

——— (2009b): "Retirement, Pensions, and Ageing," *Journal of Public Economics*, 93(3-4), 586–604.

Hellwig, M., and A. Irmen (2001): "Endogenous Technical Change in a Competitive Economy," *Journal of Economic Theory*, 101, 1–39.

Hicks, J. R. (1932): *The Theory of Wages*. Macmillan, London, 1st edn.

Huang, H., S. İmrohoroğlu, and T. J. Sargent (1997): "Two Computations to Fund Social Security," *Macroeonomic Dynamics*, 1, 7–44.

Huggett, M. (1996): "Wealth Distribution and Life-Cycle Economies," *Journal of Monetary Economics*, 38, 469–494.

Hurd, M. D. (1989): "Mortality Risk and Bequests," *Econometrica*, 57(4), 779–813.

İmrohoroğlu, A., S. İmrohoroğlu, and D. H. Joines (1995): "A Life Cycle Analysis of Social Security," *Economic Theory*, 6, 83–114.

Irmen, A. (2005): "Extensive and Intensive Growth in a Neoclassical Framework," *Journal of Economic Dynamics and Control*, 29(8), 1427–1448.

Kelley, A. C., and R. M. Schmidt (1995): "Aggregate Population and Economic Growth Correlations: The Role of the Components of Demographic Change," *Demography*, (32), 543–555.

Kemnitz, A., and B. U. Wigger (2000): "Growth and Social Security: The Role of Human Capital," *European Journal of Political Economy*, 16, 673–683.

Kormendi, R. C., and P. G. Meguire (1985): "Macroeconomic Determinants of Growth: Cross-Country Evidence," *Journal of Monetary Economics*, 16(2), 141–163.

Krüger, D., and A. Ludwig (2007): "On the Consequences of Demographic Change for Rates of Returns to Capital, and the Distribution of Wealth and Welfare," *Journal of Monetary Economics*, 54, 49–87.

Lambrecht, S., P. Michel, and J.-P. Vidal (2005): "Public Pensions and Growth," *European Economic Review*, 49(5), 1261–1281.

Lucas, R. E. (1988): "On the Mecanics of Economic Development," *Journal of Monetary Economics*, 22(5), 3–42.

Ludwig, A. (2007): "The Gauss-Seidel-Quasi-Newton Method: A Hybrid Algorithm for Solving Dynamic Economic Models," *Journal of Economic Dynamics and Control*, 31, 1610–1632.

Ludwig, A., T. Schelkle, and E. Vogel (2012): "Demographic Change, Human Capital and Welfare," *Review of Economic Dynamics*, 15(1), 94–107.

McEwen, H., P. Orrenius, and M. Wynne (2005): "Opting out of Work: What is Behind the Decline in Labor Force Participation," *Federal Reserve Bank of Dallas, The Southwest Economy*, (November/December), 1–8.

Mendoza, E. G., A. Razin, and L. L. Tesar (1994): "Effective Tax Rates in Macroeconomics: Cross-Country Estimates of Tax Rates on Factor Incomes and Consumption," *Journal of Monetary Economics*, 34, 297–323.

Prescott, E. C. (1986): "Theory Ahead of Business Cycle Measurement," *Federal Reserve Bank of Minneapolis Quarterly Review*, 10, 9–22.

Romer, P. M. (1987): "Crazy Explanations for the Productivity Slowdown," in *NBER Macroeconomics Annual 1987*, ed. by S. Fischer, pp. 163–202. MIT Press, Cambridge, MA.

Sanchez-Losada, F. (2000): "Growth Effects of an Unfunded Social Security System when there is Altruism and Human Capital," *Economics Letters*, 69(1), 95–99.

Solow, R. M. (1956): "A Contribution to the Theory of Economic Growth," *Quarterly Journal of Economics*, 70(1), 65–94.

Storesletten, K. (2000): "Sustaining Fiscal Policy through Immigration," *Journal of Political Economy*, 108(2), 300–323.

Storesletten, K., C. I. Telmer, and A. Yaron (2004): "Consumption and Risk Sharing over the Business Cycle," *Journal of Monetary Economics*, 51, 609–633.

United Nations (2002): "World Population Prospects: The 2002 Revision," *United Nations, New York*.

——— (2013): "World Population Prospects: The 2012 Revision, CD-ROM Edition," *United Nations, Department of Economic and Social Affairs, Population Division, New York, http://esa.un.org/unpd/wpp/Excel-Data/population.htm (accessed on August 14, 2013)*.

Zhang, J. (1995): "Social Security and Endogenous Growth," *Journal of Public Economics*, 58(2), 185–213.

Zhang, J., and J. Zhang (2003): "Long-Run Effects of Unfunded Social Security with Earnings-Dependent Benefits," *Journal of Economic Dynamics and Control*, 28(3), 617–641.