

Frequency Windows of Absolute Negative Conductance in Josephson Junctions

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Abstract. We report on anomalous conductance in a resistively and capacitively shunted Josephson junction which is simultaneously driven by ac and dc currents. The dependence of the voltage across the junction on the frequency of the ac current shows windows of absolute negative conductance regimes, i.e. for a positive (negative) dc current, the voltage is negative (positive).

Keywords: anomalous transport, Josephson junctions, absolute negative conductance

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INTRODUCTION

A system in thermodynamic equilibrium responds to small external stimuli in the way predicted by linear response theory. For instance the current in an ohmic resistor increases with increasing voltage. This intuitive situation where the effect follows the cause may change when the system is far from equilibrium. In the case of an electric circuit, it can happen that the increase of the voltage diminishes the current or even induces a current in the opposite direction so that the conductance becomes negative. Such phenomenon is usually referred to as an 'anomalous' response, in contrast to the 'normal', ohmic-like response. There exists a broad variety of physical systems which can exhibit such 'anomalous' behavior. One of them is the so-called ratchet effect [1]. Next is the negative differential mobility [2, 3] of a massive particle moving in spatially periodic structures, driven by a symmetric unbiased time-periodic force and thermal equilibrium fluctuations [4]. Further examples are absolute negative conductance (ANC) or mobility (ANM) which were experimentally confirmed in p-modulation-doped multiple quantum-well structures [5] and semiconductor superlattices [6]. ANC (ANM) was also studied theoretically for ac-dc-driven tunnelling transport [7], in the dynamics of cooperative Brownian motors [8], for Brownian transport in systems of a complex topology [9] and in some stylized, multi-state models with state-dependent noise [10], to name but a few. Recently, ANC was discovered in relatively simple symmetric periodic systems [11] like a Josephson junction. In this paper, we continue to study the same system as in [11], and demonstrate multiple manifestations of ANC in the range of ac-current frequency.

STEWART-MCCUMBER MODEL

The Stewart-McCumber model of a Josephson junction is well known in literature [12]. In this model, a current through the junction consists of a Josephson supercurrent characterized by the critical current I_0 , a normal current characterized by the resistance R and a displacement current accompanied with the capacitance C . Thermal equilibrium noise is the Johnson noise associated with the resistance R . The dynamics of the phase difference $\phi = \phi(t)$ across the junction is described by the following equation [12]

$$\left(\frac{\hbar}{2e}\right)^2 C \ddot{\phi} + \left(\frac{\hbar}{2e}\right)^2 \frac{1}{R} \dot{\phi} = -\frac{\hbar}{2e} I_0 \sin(\phi) + \frac{\hbar}{2e} I_d + \frac{\hbar}{2e} I_a \cos(2\pi\nu t) + \frac{\hbar}{2e} \sqrt{\frac{2k_B T}{R}} \xi(t), \quad (1)$$

where the dot denotes differentiation with respect to time t , I_d and I_a are the amplitudes of the applied dc and ac currents, respectively, $2\pi\nu$ is the angular frequency of the ac driving. The parameter k_B denotes the Boltzmann constant and T stands for temperature of the system. Thermal equilibrium fluctuations are modeled by δ -correlated Gaussian white noise $\xi(t)$ of zero mean and unit intensity.

The dimensionless form of the above equation reads

$$\ddot{x} + \gamma \dot{x} = -\sin(2\pi x) + F_0 + a \cos(\omega s) + \sqrt{2\gamma D_0} \Gamma(s), \quad (2)$$

where $x = \phi/2\pi$, the dot denotes differentiation with respect to the dimensionless time $s = t/\tau_0$ and $\tau_0 = 2\pi\sqrt{(\hbar/2e)C/I_0}$ [13]. The remaining re-scaled parameters become: the friction coefficient is $\gamma = \tau_0/R$, the amplitude and the angular frequency of the alternating current are denoted by $a = 2\pi I_a/I_0$ and $\omega = 2\pi\nu\tau_0$, respectively. The rescaled bias load reads $F_0 = 2\pi I_d/I_0$, the rescaled zero-mean Gaussian white noise $\Gamma(s)$ has the auto-correlation function $\langle \Gamma(s)\Gamma(u) \rangle = \delta(s-u)$, and the noise intensity $D_0 = (2e/\hbar)k_B T/I_0$. The rescaled stationary voltage

$$V = \langle \dot{x} \rangle, \quad (3)$$

where the brackets $\langle \dots \rangle$ denote an average over the initial conditions, over all realizations of the thermal noise and over one cycle of the external ac driving. The dimensional voltage reads $\mathcal{V} = (\hbar/2e)\omega_0 V$, where $\omega_0 = (2/\hbar)\sqrt{E_J E_C}$ is the Josephson plasma frequency expressed by the Josephson coupling energy $E_J = (\hbar/2e)I_0$ and the charging energy $E_C = e^2/C$.

Eq. (2), in the classical mechanics terms, describes a Brownian particle moving in the spatially periodic potential $U(x) = U(x+L) = -\cos(2\pi x)$ of unit period $L = 1$, driven by the time-periodic force and the constant force F_0 .

ABSOLUTE NEGATIVE CONDUCTANCE

The noiseless deterministic dynamics determined by (2) shows rich behavior in dependence of the system's parameters and often exhibits chaotic dynamics [12]. By adding noise, the dynamics becomes diffusive i.e. stochastic escape events among possibly co-existing attractors are possible. It follows from the symmetry properties that the voltage

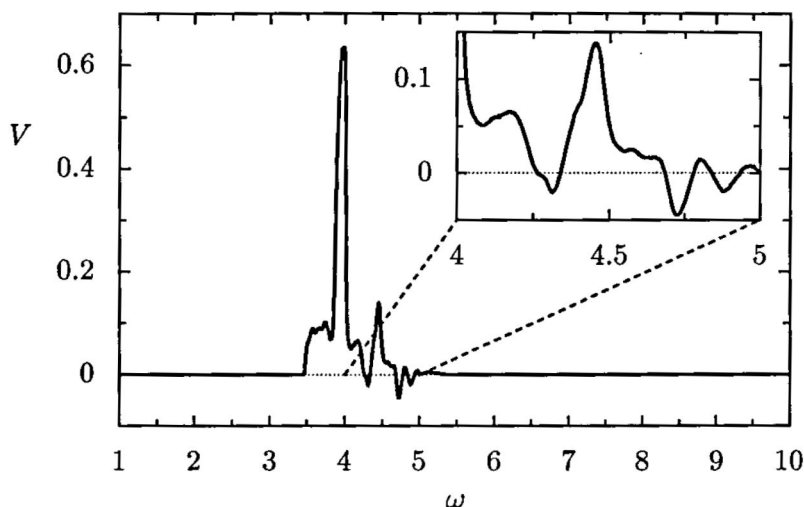


FIGURE 1. The rescaled voltage V versus rescaled angular frequency ω of the applied ac current. Other parameters are set as following: $a = 4.2$, $\gamma = 0.9$, $D_0 = 0.001$ and the positive dc current $F_0 = 0.1$. Remarkable are three regimes of frequencies where ANC occurs (see inset).

is zero if the dc current is zero ($F_0 = 0$). If the dc current is applied ($F_0 \neq 0$), the reflection symmetry $x \rightarrow -x$ is broken and the voltage can be non-zero. Most often, the signs of current and voltage coincide. Its sign is usually the same as the sign of F_0 . Sometimes, however, the dynamics turns out to be very counterintuitive and regimes of ANC occur.

The occurrence of ANC may be governed by different mechanisms. In some regimes ANC is induced by thermal equilibrium fluctuations, i.e. the effect is absent in absence of thermal fluctuations. In other regimes, ANC can occur in the noiseless, deterministic system and the effect disappears with increasing temperature. [11]. Both situations though have its origins in the deterministic structure of the dynamics governed by stable and unstable orbits.

Here we report on a search for an optimal robust set of parameters where slight changes of the system's parameters do not destroy the effect of ANC. Based on the original set of parameters ($a = 4.2$, $\gamma = 0.9$, $F_0 = 0.1$, see [11] for details) we scrutinize the interesting range of frequencies of the ac current. We have carried out extensive numerical simulations in order to find as large as possible regimes of pronounced ANC. Though simulations also are time-consuming, they still require much less effort than performing a real experiment involving Josephson junctions.

In Figure 1 we present the average dimensionless voltage V versus frequency of the external, time-periodic current. Within the displayed range of frequencies one can distinguish three interesting regions where ANC occurs. Up to frequency around $\omega = 3.3$ the average current is zero valued (numerically the current is never zero, but it is smaller than a line thickness). Starting from the mentioned value the transport sets in and later the voltage reaches a maximum of $V \approx 0.6337$ at $\omega \approx 3.985$. For higher frequencies, the voltage stays positive up to the value of $\omega \approx 4.260$ and then changes sign for the first

time, resulting in an anomalous response - the voltage becomes *negative* although the external dc current is in fact *positive*. This situation holds up to $\omega \approx 4.34$ and then the voltage turns to positive values again. The same scenario is repeated twice - between the values of $\omega \approx 4.69 \div 4.79$ and $4.84 \div 4.94$. It means that the phenomenon of ANC is not very rare and therefore gives more than one possibility of adjusting feasible frequencies of the applied ac current in real experiments involving a single Josephson junction. We also checked the system response for the signal of other frequencies from the interval $\omega \in [10^{-4}, 10^2]$. However, we have not detected the ANC appearance.

In conclusion, we have shown that there are frequency windows of ANC in a resistively and capacitively shunted Josephson junction driven by ac current and constant dc current. This phenomenon is quite robust against variation of the system parameters which is desirable from the experimental point of view.

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