

**Campisi *et al.* Reply:** The logarithmic oscillator possesses a spectacular property: its heat capacity is infinite, hence it can lead a second system to Gibbs equilibrium by means of weak interactions [1]. The criticism of Meléndez *et al.* is that this can neither be implemented in simulations nor in experiments due to the length and time scales involved [2]. That our method can be employed in computer simulations is an incontrovertible fact that both we (see Figs. 2 and 3 in Ref. [1]) and, as well, the authors of the Comment (see Figs. 1 and 3 of Ref. [3]) have convincingly demonstrated with the number of particles ranging from  $N = 1$  to  $N = 18$ . In Table I below we show that this is also experimentally feasible.

The table reports data referring to a one-dimensional (1D) implementation in which  $N$  Rb atoms ( $m = 85.4678$  amu) move in a 1D box, and collide with themselves and with a particle subject to the potential  $\varphi_b(x) = T \ln \sqrt{1 + x^2/b^2}$ . This setup can be implemented with current cold-atom physics technology [4]. In the first column we have the number  $f$  of degrees of freedom of the system. In the second column we give the accuracy with which the actual distribution  $p(v)$  of the absolute value  $v$  of any of the  $f$  velocities approximates the target Maxwell distribution  $p_\beta(v) = (\pi/2\beta m)^{-1/2} e^{-\beta m v^2/2}$ . In Fig. 3 of our Letter [1], the red solid line is  $p(v)$  and the black dashed line is  $p_\beta(v)$ . The accuracy is here calculated as the Kolmogorov-Smirnov distance  $H_{KS}[p|p_\beta] = \max_u |\int_0^u dv [p(v) - p_\beta(v)]|$  [5]. In the third and fourth columns we have, respectively, the corresponding ratio  $E_{\text{tot}}/T$ , and trap lengths calculated as  $L = 2b\sqrt{e^{2E_{\text{tot}}/T} - 1} \simeq 2be^{E_{\text{tot}}/T}$ , with a cutoff length of  $b = 10^{-8}$  m [6]. In the fifth column we report the corresponding collision times  $\tau = L/N\bar{v}$  where  $\bar{v} = \sqrt{k_B T/m}$  is the average velocity. Following Meléndez *et al.* we use  $T = 1$  K. The upper part of the table is for fixed  $f$  and varying accuracy  $H_{KS}$ . It shows how the length and time scales vary accordingly. The lower part is for fixed accuracy  $H_{KS}$  and varying  $f$ . In agreement with our estimate [1], the ratio  $E_{\text{tot}}/T$  scales approximately as  $E_{\text{tot}}/T \sim f/2$ . Note that, accordingly, the box size scales exponentially with  $f$ , i.e.,  $L \sim 2be^{f/2}$ , which, as stressed in our Letter, limits the applicability to small systems [1]. As the table clearly shows, for  $f$  sufficiently small, good accuracies can be achieved with experimentally accessible length and time scales [7].

We also respond to the second criticism raised in the Comment. Since in our method the temperature appears as a parameter in the Hamiltonian, one can use it to study the response to a temporally varying temperature, using the theory of fluctuations in time-dependent Hamiltonians [1,8]. The authors of the comment instead studied the issue

TABLE I. Length ( $L$ ) and time ( $\tau$ ) scales of possible implementations of logarithmic oscillators as thermostats for cold Rb atoms confined into a 1D trap, depending on the number  $f = N$  of atoms and the required accuracy  $H_{KS}$ .

$f$	$H_{KS}$	$E_{\text{tot}}/T$	$L$ [m]	$\tau$ [sec]
20	0.005	16.45	$2.78724 \times 10^{-1}$	$1.41295 \times 10^{-3}$
20	0.01	14.8	$5.35289 \times 10^{-2}$	$2.71358 \times 10^{-4}$
20	0.02	13.1	$9.77885 \times 10^{-3}$	$4.95726 \times 10^{-5}$
20	0.03	11.9	$2.94533 \times 10^{-3}$	$1.4931 \times 10^{-5}$
20	0.04	11.05	$1.25888 \times 10^{-3}$	$6.38172 \times 10^{-6}$
10	0.02	7.75	$4.64314 \times 10^{-5}$	$4.70756 \times 10^{-7}$
20	0.02	13.1	$9.77885 \times 10^{-3}$	$4.95726 \times 10^{-5}$
30	0.02	18.1	$1.45131 \times 10^0$	$4.90482 \times 10^{-3}$
40	0.02	23.1	$2.15393 \times 10^2$	$5.45955 \times 10^{-1}$
50	0.02	28	$2.89251 \times 10^4$	$5.86529 \times 10^1$

of a spatially varying temperature. Not only is this second criticism not pertinent to our Letter, but it is also neither sufficiently documented nor conclusive [9].

Michele Campisi,<sup>1</sup> Fei Zhan,<sup>2</sup> Peter Talkner,<sup>1</sup> and Peter Hänggi<sup>1</sup>

<sup>1</sup>Institute of Physics, University of Augsburg  
Universitätsstrasse 1, D-86135 Augsburg, Germany  
<sup>2</sup>Centre for Engineered Quantum Systems  
School of Mathematics and Physics  
University of Queensland, Brisbane 4072, Australia

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- [6] The choice  $b = \sigma = 10^{-10}$  m of Meléndez *et al.* is presently too small to be experimentally achievable.
- [7] In clear contrast, the choice of 26 particles in three dimensions, i.e.,  $f = 3 \times 26 = 78$ , used in the Comment, yields, due to the exponential growth, astronomical length and time scales. Such a choice evidently violates our criterion of “smallness” [1].
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