Hedge Funds

Alternative Investment Strategies and Portfolio Models

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Datum der mündlichen Prüfung: 22. Juli 2005
Dissertation zur Erlangung des Grades eines Doktors der
Wirtschaftswissenschaften (Dr. rer. pol.) durch die Wirtschaftswissenschaftliche
Fakultät der Universität Augsburg
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<td>SfP</td>
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<td>The Mortgage Corporation</td>
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<td>United Kingdom</td>
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<td>US</td>
<td>United States</td>
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<tr>
<td>VaR</td>
<td>Value-at-Risk</td>
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<tr>
<td>VC</td>
<td>Venture Capital</td>
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<tr>
<td>vw</td>
<td>value weighting</td>
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<td>$AD(.)$</td>
<td>Anderson-Darling statistic</td>
</tr>
<tr>
<td>$\hat{c}(.)$</td>
<td>density of the empirical copula function</td>
</tr>
<tr>
<td>$C(.)$</td>
<td>copula function</td>
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<tr>
<td>$\hat{C}(.)$</td>
<td>empirical copula function</td>
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<tr>
<td>$C_C(.)$</td>
<td>Clayton copula function</td>
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<td>$C_F(.)$</td>
<td>Frank copula function</td>
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<tr>
<td>$C_G(.)$</td>
<td>Gumbel copula function</td>
</tr>
<tr>
<td>$c_N(.)$</td>
<td>density of the Gaussian or Normal copula</td>
</tr>
<tr>
<td>$C_N(.)$</td>
<td>Gaussian or Normal copula</td>
</tr>
<tr>
<td>$Cov$</td>
<td>covariance</td>
</tr>
<tr>
<td>$CoV$</td>
<td>coefficient of variation</td>
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<tr>
<td>$C_P(.)$</td>
<td>product copula</td>
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<tr>
<td>$c_t(.)$</td>
<td>density of the t-copula</td>
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<tr>
<td>$C_t(.)$</td>
<td>t-Copula</td>
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<td>CVaR(.)</td>
<td>Conditional Value-at-Risk function</td>
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<tr>
<td>$f_i(.)$</td>
<td>probability density function for returns of asset $i$</td>
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<tr>
<td>$\hat{f}_i(.)$</td>
<td>estimated probability density function for returns of asset $i$</td>
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<tr>
<td>$F_i(.)$</td>
<td>cumulative probability density function for returns of asset $i$</td>
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<tr>
<td>$\hat{F}_i(.)$</td>
<td>estimated cumulative probability density function for returns of asset $i$</td>
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<tr>
<td>$F^{-1(.)}$</td>
<td>quantile function or inverse cumulative density function</td>
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<td>Symbol</td>
<td>Definition</td>
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<tr>
<td>$h$</td>
<td>bandwidth parameter of the kernel estimator</td>
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<tr>
<td>$i, j$</td>
<td>indices</td>
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<tr>
<td>$IAD(.)$</td>
<td>Integrated Anderson-Darling statistic</td>
</tr>
<tr>
<td>$K(.)$</td>
<td>kernel function</td>
</tr>
<tr>
<td>$KU$</td>
<td>sample kurtosis</td>
</tr>
<tr>
<td>$\hat{KU}$</td>
<td>estimated kurtosis</td>
</tr>
<tr>
<td>$KU_i$</td>
<td>kurtosis for returns of asset $i$</td>
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<tr>
<td>$\mathbb{L}$</td>
<td>lattice for the empirical copula</td>
</tr>
<tr>
<td>$L(.)$</td>
<td>log-likelihood function</td>
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<tr>
<td>$M(.)$</td>
<td>function for the upper probability bound</td>
</tr>
<tr>
<td>$MAD(.)$</td>
<td>mean-absolute deviation function</td>
</tr>
<tr>
<td>$P_t$</td>
<td>asset price at time in point $t$</td>
</tr>
<tr>
<td>$r_t$</td>
<td>continuous return for period $t$</td>
</tr>
<tr>
<td>$r_{P,t}$</td>
<td>continuous portfolio return for period $t$</td>
</tr>
<tr>
<td>$r_{i,t}$</td>
<td>continuous return of asset $i$ for period $t$</td>
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<tr>
<td>$R_t$</td>
<td>discrete return for period $t$</td>
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<tr>
<td>$R_{P,t}$</td>
<td>discrete portfolio return for period $t$</td>
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<tr>
<td>$R_{i,t}$</td>
<td>discrete return of asset $i$ for period $t$</td>
</tr>
<tr>
<td>$\hat{R}_T$</td>
<td>geometric average return over period $T$</td>
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<tr>
<td>$R^*_t$</td>
<td>unsmoothed discrete return for period $t$</td>
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<tr>
<td>$R_{Target}$</td>
<td>target return for the shortfall probability</td>
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<tr>
<td>$R_P$</td>
<td>discrete portfolio return</td>
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<tr>
<td>$R_{P_{\text{min}}}(.)$</td>
<td>minimum portfolio return function</td>
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<tr>
<td>$\mathbf{R}$</td>
<td>vector of asset returns</td>
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<tr>
<td>$\mathbf{R}_P$</td>
<td>vector of discrete portfolio returns</td>
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<tr>
<td>$\tilde{\mathbf{R}}$</td>
<td>vector of random discrete returns</td>
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<tr>
<td>$\bar{\mathbf{R}}$</td>
<td>vector of expected discrete returns</td>
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<tr>
<td>$\mathbf{R}^s$</td>
<td>return vector for scenario $s$</td>
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<td>Symbol</td>
<td>Definition</td>
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<tr>
<td>$S$</td>
<td>number of scenarios</td>
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<tr>
<td>$\text{SD}(.)$</td>
<td>standard deviation function</td>
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<tr>
<td>$SE$</td>
<td>standard error</td>
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<tr>
<td>$\text{SfP}(.)$</td>
<td>shortfall probability function</td>
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<tr>
<td>$SK$</td>
<td>sample skewness</td>
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<tr>
<td>$\overline{SK}$</td>
<td>estimated skewness</td>
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<tr>
<td>$SK_i$</td>
<td>skewness for returns of asset $i$</td>
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<tr>
<td>$t$</td>
<td>index</td>
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<tr>
<td>$T$</td>
<td>number of periods or number of realized historical returns</td>
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<tr>
<td>$T_i(.)$</td>
<td>transformation function for random variable $i$</td>
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<tr>
<td>$T_{\nu}^{-1}(.)$</td>
<td>inverse of the distribution function of the univariate t-distribution</td>
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<tr>
<td>$T_{\rho,\nu}^{n}(.)$</td>
<td>cumulative density function of the $n$-variate t-distribution function with correlation matrix $\rho$ and $\nu$ degrees of freedom</td>
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<tr>
<td>$u_i$</td>
<td>standard-uniformly distributed random variable</td>
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<tr>
<td>$U(.)$</td>
<td>utility function</td>
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<tr>
<td>$V$</td>
<td>end of period wealth</td>
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<tr>
<td>$V_0$</td>
<td>initial wealth</td>
</tr>
<tr>
<td>$Var$</td>
<td>variance</td>
</tr>
<tr>
<td>$\text{VaR}(.)$</td>
<td>Value-at-Risk function</td>
</tr>
<tr>
<td>$\text{VaR}_0(.)$</td>
<td>Value-at-Risk function</td>
</tr>
<tr>
<td>$w$</td>
<td>vector of portfolio weights</td>
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<tr>
<td>$w_i$</td>
<td>portfolio weight of asset $i$</td>
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<tr>
<td>$W(.)$</td>
<td>function for the lower probability bound</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
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<td>--------</td>
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</tr>
<tr>
<td>$\alpha$</td>
<td>quantile corresponding to a confidence level of $1-\alpha$</td>
</tr>
<tr>
<td>$\beta_C$</td>
<td>parameter of the generator for a Clayton copula</td>
</tr>
<tr>
<td>$\hat{\beta}_C$</td>
<td>parameter estimate of the generator for a Clayton copula</td>
</tr>
<tr>
<td>$\beta_F$</td>
<td>parameter of the generator for a Frank copula</td>
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<tr>
<td>$\hat{\beta}_F$</td>
<td>parameter estimate of the generator for a Frank copula</td>
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<tr>
<td>$\beta_G$</td>
<td>parameter of the generator for a Gumbel copula</td>
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<tr>
<td>$\hat{\beta}_G$</td>
<td>parameter estimate of the generator for a Gumbel copula</td>
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<tr>
<td>$\gamma$</td>
<td>vector of parameters for a marginal distribution</td>
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<tr>
<td>$\hat{\gamma}$</td>
<td>vector of parameter estimates for a marginal distribution</td>
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<tr>
<td>$\gamma(.)$</td>
<td>autocovariance function</td>
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<tr>
<td>$\Gamma(.)$</td>
<td>gamma function</td>
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<tr>
<td>$\lambda$</td>
<td>smoothing coefficient</td>
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<td>$\mu$</td>
<td>expected portfolio return</td>
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<tr>
<td>$\hat{\mu}$</td>
<td>sample mean</td>
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<tr>
<td>$\mu_i$</td>
<td>expected return of asset $i$</td>
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<tr>
<td>$\nu$</td>
<td>degrees of freedom of a t-distribution</td>
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<tr>
<td>$\xi$</td>
<td>vector of normal inverses or inverses from a t-distribution</td>
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<td>$\phi$</td>
<td>vector of copula parameters</td>
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<tr>
<td>$\hat{\phi}$</td>
<td>vector of copula parameter estimates</td>
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<td>$\varphi(.)$</td>
<td>generator function for Archimedean copulas</td>
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<td>$\varphi^{-1}(.)$</td>
<td>pseudo-inverse of a generator function for Archimedean copulas</td>
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<tr>
<td>$\Phi_\rho(.)$</td>
<td>standardized multivariate normal distribution function with correlation matrix $\rho$</td>
</tr>
<tr>
<td>$\Phi^{-1}(.)$</td>
<td>inverse of the standardized univariate normal distribution function</td>
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<tr>
<td>$\rho$</td>
<td>linear correlation coefficient or correlation matrix</td>
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<td>$\rho(.)$</td>
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<td>$\sigma$</td>
<td>sample standard deviation</td>
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<td>$\sigma_i$</td>
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<tr>
<td>$\chi^2$</td>
<td>Chi-Square test</td>
</tr>
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</table>
1 Introduction

1.1 Motivation

The hedge fund industry is moving out of the alternative corner and into the mainstream. Investments in hedge funds have experienced considerable and steady growth in recent years, both in terms of the number of funds and asset volume. In the years from 2002 to 2005 net asset flows to hedge funds of 70 to 100 billion dollars were reported on an annual basis. According to leading hedge fund data providers, the estimated assets under management reached more than one trillion dollars in 2005.1 This amount is invested in approximately 6100 single hedge funds and 1800 funds of hedge funds.2

Investments from institutional investors in particular have increased considerably. Some well-known and sophisticated investors already hold significant proportions of hedge funds.3 While high net worth individuals are currently believed to own two thirds of the global hedge fund capital, research institutes predict substantial growth in the institutional area. It is expected that up to 50% of hedge fund assets will be owned by institutional investors by 2007.4

1 Two thirds of this amount are invested in hedge funds with offshore domiciles, like the Caymans, the Channel Islands, or the Bahamas.
3 For example, many universities report substantial investments in hedge funds, see Reserve Bank of Australia (1999), p. 3.
4 See Mercer Oliver Wyman (2005), p. 15.
While institutional investors in some countries already allocate substantial parts of their portfolio to hedge funds, institutional investors in Germany are rather reserved about this new asset class. The German hedge fund market was stimulated by the so-called “Investmentmodernisierungsgesetz” of 2004. Prior to this, hedge fund managers were prohibited from operating in Germany and investors were subject to punitive capital gains tax. In 2004 investment regulations were relaxed and single-strategy funds and funds of hedge funds were permitted to be domiciled in Germany. Furthermore, domestic investors were allowed to allocate money to domestic and offshore funds. The new investment legislation gives equal tax treatment to domestic and foreign funds, but the requirements to be met by hedge funds in order to obtain this taxation status (so-called “tax transparency”) are relatively strict.

More than 20 domestic hedge funds are currently authorized by the German “Bundesanstalt für Finanzdienstleistungen” (BaFin) but by 2005 only about 1.5 billion dollars’ worth of funds had been sold to investors in German single hedge funds and funds of hedge funds. Institutional investors, especially insurance companies, one important group of investors, are reluctant to invest in hedge funds as the BaFin has placed restrictions on the amounts which may be invested in hedge funds and has also introduced strict risk management requirements. These regulatory restrictions apart, however, the majority of institutional investors are suspected of not being adequately informed about how hedge funds and the strategies they employ really work.

7 Retail products in the form of structured certificates were already on offer before the regulatory reform. These certificates enable investors to participate in the performance of hedge fund investments but without the tax transparency requirements. See Mader/Echter (2004) for an overview of the German market for hedge fund certificates.
1 Introduction

1.2 Objectives and Structure

There are several reasons for the growing interest in hedge funds. A major motivation for many investors is the historically attractive return profile of such funds. These returns have triggered massive capital inflows and, as a consequence, a need for new funds and managers. Whether there is adequate supply to meet the growing demand remains an open question. Market observers estimate that only approximately 5 - 10% of hedge fund managers are able to add significant value on an after-fee basis.\(^9\) Identifying those managers who are able to provide such additional value is probably the main task investors have to face.

In addition to this central problem of selecting skilled hedge fund managers, investors face a variety of other challenges: legal and accounting issues, regulatory requirements, compensation structures, trading strategies, portfolio considerations, performance measurement, etc. These issues are addressed by several research institutes,\(^10\) some academic journals and also by a number of publications for institutional investors that concentrate on hedge funds in particular and alternative investments in general.

The challenges faced by investors form the starting point for this work, and its main objectives are to analyze different sources of risk in hedge fund investing and to improve the quantitative modeling of hedge fund investments in the portfolio context. Figure 1.1 on the following page gives an overview on the different topics covered.

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\(^{10}\) Examples of research institutes that focus on hedge funds or alternative investments in general are the “Centre for Hedge Fund Research and Education” at London Business School, the “Center for International Securities and Derivatives Markets” (CISDM) at the University of Massachusetts, the “Edhec Risk and Asset Management Research Centre” at the Edhec Business School of Nice, and the “Alternative Investment Research Centre” at the Cass Business School of London.
Figure 1.1: Structure of this work

The work covers a variety of aspects, with the four main chapters combining qualitative and quantitative information on the hedge fund industry. Generally speaking, Chapters 2, 3, and the first part of Chapter 4 provide qualitative hedge fund research while quantitative aspects are discussed in the second part of Chapter 4 and in Chapter 5.

The basic characteristics of hedge funds and alternative investments in general are outlined in Chapter 2. In this chapter, hedge fund industry standards and the characteristics typically exhibited by hedge funds are described in detail and compared to those in the traditional investment universe. The chapter also concentrates on the definition of hedge funds and the alternative investment universe.

After discussing the special features of the hedge fund industry, we analyze which sources of risk may justify the historically very attractive returns achieved by the different hedge fund strategies. Although the relevant literature contains few systematic overviews of individual hedge fund strategies and the corresponding trades,
such an analysis of individual strategies and funds of hedge funds is essential when investment in hedge funds is considered. In Chapter 3 systematic and unsystematic risk factors in hedge fund investing and the risk premia captured by hedge fund strategies are introduced. Furthermore, the relevant trades and the corresponding risk factors for several classical hedge fund strategies are described. Since new investors in hedge funds typically choose the fund of hedge funds structure instead of direct investment in single hedge funds\textsuperscript{11}, Chapter 3 also analyzes the fund of hedge funds structure. This way of structuring hedge fund investments offers investors some advantages but also has some drawbacks.

Chapter 4 addresses the need for reliable hedge fund return data. When investors consider allocating money to hedge funds they form an expectation of the development of future returns. Since historical data is usually their starting point in this, Chapter 4 examines in detail the most important hedge fund index data providers and the relevant indices and takes a closer look at the different hedge fund databases and the different index methodologies. After discussing several data biases and their implications, the statistical properties of fund of hedge funds indices are analyzed. Such an analysis of historical behavior is a vital issue for an investor interested in the quantitative modeling of the risk return characteristic and the impact of hedge funds. This in-depth analysis also forms the starting point for the analysis of a portfolio including hedge funds.

The implications of the statistical properties of hedge fund index returns for portfolio selection are discussed in the first sections of Chapter 5. We introduce an alternative one-period modeling approach for portfolio returns that is flexible enough to capture these return characteristics. While Markowitz’s\textsuperscript{12} classical portfolio selection framework involves rather strict assumptions about the structure of asset returns

\textsuperscript{11} See State Street (2005), p. 5. 35\% of global hedge fund assets are currently held by funds of hedge funds, see Mercer Oliver Wyman (2005), p. 16.

\textsuperscript{12} See the original work of Markowitz (1952).
or utility functions, the approach presented in this work is very flexible. In a first step we model the marginal return distributions of the individual assets in order to reproduce the univariate return characteristics of the assets under consideration. In a second step, we focus on the dependence structure of the asset returns in the portfolio. This modeling of dependencies with multivariate copula functions delivers more realistic results than the classical portfolio selection framework. Combining the marginal return distributions and the dependence structures delivers a multivariate model for asset returns. This model is applied in a case study which analyzes a traditional portfolio in which hedge funds are included. This is a major issue for hedge fund investors and, with the model for portfolio returns at hand, we are able to derive optimal allocations to asset classes coupled with an optimization with respect to different risk measures.

Chapter 6 concludes with a brief summary of the most important results. In addition, we make suggestions for future research.
2 Hedge Funds

The history of hedge fund investment dates back to the 1940s. But over the last ten years the interest in and the depth of this market segment have increased significantly. This has lead to a high degree of heterogeneity within the hedge fund industry of today. Therefore, thorough analysis of this industry is very important. This chapter provides an introduction to hedge funds and outlines some of the characteristics of this investment opportunity. First, Section 2.1 introduces the term “hedge fund”. In Section 2.2 a categorization of hedge funds is provided and Section 2.3 specifies some important hedge fund characteristics that are fundamental for the understanding of this investment opportunity.

2.1 Hedge Funds Defined

Various definitions of the term “hedge funds” circulate in the relevant literature. The term itself could be misleading in two ways. On the one hand “hedging” risk is not necessarily the main focus of hedge fund managers. The widespread misunderstanding that hedge funds in general pursue high risk strategies is just

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3 See UBS (2004), pp. 112-117, for an interesting and sometimes amusing survey of hedge fund definitions.
as false as the perception that a hedge fund manager hedges all risk factors in his
investment strategy. As (ex-ante) returns are theoretically a function of risk, hedging
all the relevant risk factors would imply an expected return on the level of riskfree
assets.\footnote{See for example UBS (2004), p. 111.} On the other hand the declaration as “fund” could be misleading as the
legal structure is not identical to common (mutual) funds.\footnote{See Bekier (1998), p. 75.}

As there is no exact legal definition of hedge funds there are many partly contradic-
tory definitions circulating. A very simple and sensible definition of “hedge funds”
is given in Bookstaber (2003).\footnote{See Bookstaber (2003), p. 19.} When the universe of all tradable securities and
possible trading strategies is regarded, then hedge funds are described as the whole
investment universe minus a small slice that is referred to as traditional investments.\footnote{See Bookstaber (2003), p. 19.}

These traditional investment funds and/or investment strategies are characterized
by three restrictions. At first the leverage in the fund structure is limited (smaller
than 1), furthermore these funds are long only, meaning they do not employ short
selling on a big scale. Finally these traditional funds are restricted to traditional
assets like stocks and bonds that are traded in developed markets\footnote{See Section 3.2.9 on page 77 for a detailed discussion of emerging vs. developed markets.}.\footnote{See Bookstaber (2003), p. 19.}

As a consequence hedge funds (or alternative investments in general) can be “non-
traditional” in two ways. Opposed to investments that consist of unlevered long
positions in stocks, bonds, money market products, or currencies there are differ-
ences regarding the asset classes invested in and/or the strategies pursued.\footnote{See for example Barra RogersCasey (2001), p. 3-4, or Jaffer (2000), p. 225.}

On the one hand, the hedge fund manager can allocate money in alternative asset
classes such as private equity, real estate or commodities. These alternative assets
are described in the following Section 2.2. On the other hand, the classification as
alternative can stem from the non-traditional investment strategy employed by a fund manager. Traditional investment strategies consist of long positions in assets and use derivatives for hedging purposes only. Reasons for this limited flexibility in traditional investment strategies can be found in the regulatory environment for most common funds. Alternative investment strategies offer by far more flexibility as they combine long and short positions and make use of leverage. Within these alternative strategies, derivatives are widely employed for hedging, for speculation purposes and/or for providing leverage. Figure 2.1 gives a consolidated interpretation of the hedge fund definition.

![Figure 2.1: Traditional investments in the investment universe](image)

It becomes obvious that hedge funds or alternative investments in general are the investment universe minus a small slice, which is called traditional investments. The intersection of the restrictions in strategies and asset classes represents the traditional investment world, while the remaining investment universe is the workplace for hedge fund managers. So what is called hedge fund is really the full spectrum

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12 See the definition in Bookstaber (2003), p. 19.
of investment strategies.\textsuperscript{13} This definition also points out that hedge funds are not a homogeneous group. We will adopt this extensive definition of the term “hedge fund” which sets up the basis of the analysis in the following chapters.

\section*{2.2 Hedge Funds in the Alternative Investment Universe}

The alternative investment universe offers a very broad range of different assets and products to invest in. The position of hedge funds in this part of the investment universe shall be outlined in this section. In Figure 2.2 on the next page a systematization of alternative investments is given. The universe of alternative investments can be divided in alternative asset classes\textsuperscript{14} and alternative investment strategies. Hedge funds are members of both alternative categories.\textsuperscript{15}

Single hedge funds are rather investment strategies than an asset class, as they are very heterogeneous.\textsuperscript{16} The classification of single hedge funds and managed futures funds\textsuperscript{17} as “alternative” usually stems from the strategies employed by a fund manager.\textsuperscript{18} These alternative investment strategies are also commonly referred to as “skill-based strategies”.\textsuperscript{19} The assets a fund invests in are very often exchange-
traded bonds or stocks. But some managers also implement long-only strategies with other alternative asset classes or less liquid privately traded products.\textsuperscript{20}

![Alternative Investments](image)

**Figure 2.2: Systematization of alternative investments\textsuperscript{21}**

Fund of hedge funds that invest in these alternative investment strategies in order to diversify and avoid any directional exposure to risk factors (so-called non-directional fund of hedge funds) can be categorized as alternative asset class.\textsuperscript{22} These funds are much more homogeneous and fulfill some important asset class features. Especially the properties of an estimable risk profile, a low correlation with other asset classes and an adequate liquidity are met by non-directional fund of hedge funds.\textsuperscript{23}

\textsuperscript{20} Strategies with traditional long-only investments in such alternative asset classes are for example Regulation D (see Section 3.2.13 on page 90) and Distressed Securities (see Section 3.2.6 on page 64).

\textsuperscript{21} See for example Jaeger (2002), p. 19, for a related systematization.

\textsuperscript{22} See Section 3.3 on page 91 for more details on fund of hedge funds.

\textsuperscript{23} See CRA Rogers Casey (2003), p. 18.
2.3 Key Characteristics of Hedge Funds

This section covers some important hedge fund characteristics that are common to most or even all hedge funds. The partially overlapping categories that are looked at in greater detail are the financial instruments that are employed by hedge funds, the investment approaches, the absolute return targets of most managers, the incentive fees that are very common and the management investment in hedge funds. The last topic that is covered in this section is the liquidity of hedge fund investments. See Figure 2.3 for a summary of these key characteristics.

<table>
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<th>Key Characteristics of Hedge Funds</th>
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<tr>
<td>Broad Range of Financial Assets</td>
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<td>Co-Investment by Fund Managers</td>
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Figure 2.3: Key characteristics of hedge funds

2.3.1 Financial Instruments

The investment approach of hedge funds is very flexible.\textsuperscript{24} In general the full range of securities and derivatives is available to the hedge fund manager. Positions can be set up with equity, fixed income, currency and commodity instruments. Other

\textsuperscript{24} See for example Bekier (1998), pp. 89-91.
rather new fields for hedge funds are trading or investing in credit derivatives and structured products, like for example collateralized debt obligations (CDOs). The financial instruments a hedge fund holds can either be traded at an exchange or in the over-the-counter (OTC) market. Often the liquidity of hedge fund positions is rather low in order to capture some kind of illiquidity premium\textsuperscript{25,26}. The most common strategies and the corresponding securities, trades and risk factors are outlined in Chapter 3.

As mentioned above, most hedge funds make use of short selling and/or leverage. While traditional money managers are usually only allowed to set up long positions, a hedge fund is much more flexible in its investment policy. A hedge fund manager can easily set up a negative exposure to the market, an index and/or the price of a certain security. This is usually done by selling borrowed securities\textsuperscript{27}. The usage of leverage differs among strategy groups and individual hedge funds. Very often hedge funds have overall leverage limits\textsuperscript{28}. Table 2.1 gives an idea of the amount of leverage for different hedge fund strategies and strategy groups\textsuperscript{29}. It is obvious that these strategies employ derivatives and debt financing differently\textsuperscript{30}. Within a hedge fund structure there are different ways to build up such a leverage. The most obvious leverage facility is the use of external (debt-) financing to enlarge the fund’s capital. Another way to increase a fund’s capital base are repurchase agreements, options, swaps or futures, as these derivative contracts offer implied leverage\textsuperscript{31}.

\textsuperscript{25} See Section 3.1.2 for details on liquidity risk.
\textsuperscript{26} See Bekier (1998), pp. 106-107.
\textsuperscript{27} Different possibilities to establish short positions or in general negative exposures are outlined in Section 3.2.8 on page 72.
\textsuperscript{28} See Cottier (1998), p. 49.
\textsuperscript{29} Osterberg/Thomson (1999) report slightly lower numbers. For 85% of the hedge funds a leverage of two or less is estimated.
\textsuperscript{30} See Chapter 3 for details on these strategies and specific trades.
\textsuperscript{31} See for example Kaiser (2004), pp. 30-33.
\textsuperscript{32} Data: VanHedge, December 2003.
Table 2.1: Usage of leverage with different hedge fund strategies

2.3.2 Sources of Return

With the flexibility in instruments, leverage and short selling (outlined in Section 2.3.1) at hand, hedge fund managers focus on capturing risk premia and/or on detecting and exploiting market inefficiencies. Here the terminology of capital market models is used to differentiate between the two sources of return that are usually labeled with “beta” for the risk premia and “alpha” for returns in consequence of inefficiencies.

A risk premium is offered in the market in exchange for taking on systematic risks (like market risk, credit risk or liquidity risk, see Section 3.1.2 on page 31). These risk premia are permanent price effects, only the amount of the reward per unit of risk fluctuates over time. These exposures to systematic risk factors, which are measured with the corresponding beta factors, are important sources of hedge fund returns. According to Jaeger (2003) such systematic risk premia generate about

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See Jaeger (2003).

See for example Wallmeier (1997), pp. 56-86, or Steiner/Bruns (2002), pp. 22-37, for details on the Capital Asset Pricing Model (CAPM) and multifactor models like the Arbitrage Pricing Model (APT).
80 percent of the hedge fund industry’s return.

Excess returns above such a compensation for bearing risk can be attributed to manager skill (or luck). Here the manager was able to detect opportunities where it is possible to earn more than the relevant risk premia, for example with mispriced securities or superior price relevant information.\textsuperscript{35} Following Jaeger (2003) the exploitation of these inefficiencies makes up for about 20 percent of the hedge funds’ returns.

### 2.3.3 Absolute Return Targets

The absolute return approach of (most) hedge fund managers is quite different to the traditional relative return approach in asset management. A relative return manager looks for opportunities to outperform passive market benchmarks or indices in increasing and decreasing market environments. Therefore, even with negative absolute returns a relative return strategy could be successful. This relative return approach also fits with traditional performance evaluation techniques. Here risk is measured as tracking or active risk. If the fund return departs from the benchmark return this is perceived as source of risk. As Ineichen (2003) puts it the benchmarking approach can be viewed as a method of limiting the potential for surprises, either positive or negative.\textsuperscript{36} The main problem with this relative return approach is that there is no incentive for the management to preserve the wealth of investors in down markets. The relative return fund manager tolerates negative returns brought about by the market volatility and leaves risk management (besides the tracking risk) to the investor.\textsuperscript{37} The main differences between classical relative return and absolute return funds are outlined in Figure 2.4 on the following page.

\textsuperscript{35} See Jaeger (2002), pp. 24-25.
\textsuperscript{38} See for example Ineichen (2004), p. 23.
The absolute return approach tackles some of the problematic issues with the relative benchmark perspective. The objective of absolute return managers is to earn a consistent rate of return regardless of the bond or stock market environment. Typically hedge funds have such absolute return targets. Here the interest of investors for capital preservation and steady profits without relying on the market environment is aligned with management interest by incentives for the hedge fund managers that are outlined in sections 2.3.4 and 2.3.5. Therefore, very common return benchmarks for hedge funds are the returns (or multiples) of risk free assets like the three-month T-bill sometimes with a risk premium on top. These returns are often reflected in a fund’s hurdle rate. This focus on absolute, positive returns

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[43] For more details on hurdle rates, see Section 2.3.4, p. 19.
entails a different understanding of risk. Contrary to the relative return world the relevant risk for an absolute return manager is total risk, which describes the problem of facing uncertain absolute results.\footnote{See Ineichen (2004), p. 23.}

\section*{2.3.4 Performance Fees}

A characteristic feature of hedge funds is the incentive system for the management. In order to attract the industry’s best managers there are very interesting compensation schemes in place for hedge fund managers.\footnote{See Tarrant (1996), p. 146.} While the compensation of traditional fund managers is solely based on a management fee as percentage of assets under management, hedge funds usually charge additional performance or incentive fees.\footnote{Under various jurisdictions incentive fees are not allowed for traditional investment funds, see Cottier (1998), p. 36. But they are prevalent in the real estate sector and the venture capital sector.} As the management fees for hedge funds are essentially intended to meet operating costs, the incentive fees are meant to encourage managers to achieve maximum returns or at least to perform well.\footnote{See Lhabitant (2002), p. 17.} The size constraint\footnote{See Section 2.3.6.} in some hedge fund strategies gives another reasoning for performance fees. As the management should not increase asset size by any means in order to collect management fees there must be an incentive to produce positive returns and resist net growth.\footnote{See for example Goetzmann et al. (2003), p. 1716.} According to Grinold/Rudd (1987) these performance or incentive fees in general are suitable for unskillful investors and good or at least average managers. But bad managers and sophisticated investors should rather avoid such performance fees.\footnote{See Grinold/Rudd (1987), p. 37.}

The option-like payoff structure within this compensation scheme might be problematic, as the manager participates in profits but not in losses. If the fund performs
well, the manager is able to collect the additional performance fee. Conversely, if the fund performs poorly, the manager collects the management fee. This sets an incentive for the manager to increase the volatility of the fund’s net asset value in order to maximize fees. Especially when the option-like incentive contract is out of the money, managers have a strong incentive to increase the fund’s volatility. Here the co-investment outlined in Section 2.3.5, a high watermark and hurdle rates are able to reduce these negative side effects to some extent.

Usually a hedge fund with a performance fee has also a so-called high watermark. With this clause the manager commits to charging performance fees only if past losses have been recovered. Therefore, the performance fee is only charged on value added to the investor. These high watermark provisions in hedge fund contracts limit the value of the optionality of performance fees for the management, as the exercise price is determined by the watermark. As this combination of performance fees and high watermarks has the effect of penalizing losses this has a moderating effect on risk-taking. But it usually can not discourage excessive risk taking when the fund is already substantially below the historic watermark. In this case investors usually withdraw capital from the fund. This is consistent with the common (voluntary) termination of funds when there is no reasonable possibility of meeting the high watermark provision in the incentive contract. Based on

51 See Bailey (1990), p. 36.
53 For a detailed analysis of these moral hazard problems and related issues, see Signer (2003), pp. 119-194.
54 See Cottier (1998), p. 37, for an example on the calculation of performance fees with new subscriptions and redemptions influencing a fund’s net asset value.
56 In Goetzmann et al. (2003) the authors provide a closed-form solution for the high-watermark contract under certain model conditions. The results show that managers have a smaller incentive to take risks than without a high watermark.
research by Liang (1999), such a high watermark is a determinant of fund performance as hedge funds with high watermarks show significantly higher returns than funds without.\textsuperscript{59}

The hurdle rate of a hedge fund is another mechanism to align the hedge fund manager’s interest with those of the investors.\textsuperscript{60} Such a hurdle rate is a rate of return that has to be achieved before the performance fee is paid. Only above this hurdle rate a hedge fund manager begins taking incentive fees. Here a soft or hard hurdle can be installed in the incentive contract. A soft hurdle rate allows incentive fees on the gross fund performance, while with hard hurdle rates the incentive fee is calculated on the fund performance above the hurdle. As an adequate minimum return often a reference rate, like the three-month T-bill, or a stock market index return, like the S&P 500, is employed. But also fixed rates are common as hurdle rates.\textsuperscript{61} This hurdle rate should ensure some kind of minimum return for the investor. However historically most hedge funds did not have such a hurdle rate.\textsuperscript{62} The study of Liang (1999) did not find significant influence of hurdle rates on fund performance.\textsuperscript{63}

\section*{2.3.5 Co-Investment}

On the one hand the hedge fund management should act in the interest of the investors, but on the other hand the managers try to maximize their own wealth. With the performance fee introduced in Section 2.3.4 the investor has to account for the problem of moral hazard.\textsuperscript{64} As the management has an incentive to maximize the fee revenue, it is tempted by an option-like compensation with limited downside

\begin{itemize}
  \item \textsuperscript{59} See Liang (1999), p. 75.
  \item \textsuperscript{60} See Ineichen (2000), p. 87.
  \item \textsuperscript{61} See Cottier (1998), p. 36.
  \item \textsuperscript{63} See Liang (1999), p. 75.
  \item \textsuperscript{64} See Signer (2003), pp. 128-132.
\end{itemize}
risk like the performance fee. A reasonable strategy for this compensation scheme could be either alternating positive and negative returns in order to capture the performance fee for up-movements or excessive risk-taking as outlined in Section 2.3.4. And after a period of good performance the manager might be tempted to avoid any risk-taking until the incentive fee is paid.\textsuperscript{65}

In order to align the interest of management and investors and to avoid the outlined problem of moral hazard, managers usually make significant investments in the fund they operate. According to data from VanHedge for the year 2003 about 82\% of the managers have a minimum of $500,000 invested in their fund.\textsuperscript{66} With an investment in their own fund, managers are interested in protecting the fund from (large) downside movements as their own capital is at stake as well.\textsuperscript{67} Especially in cases where the track record of the fund is very short this is a confidence-building measure that is able to align economic interests between the manager and external investors.\textsuperscript{68}

\section*{2.3.6 Liquidity of Hedge Fund Investments}

The life-cycle of hedge fund investing is another characteristic feature of this industry. From the initial investment to the final disinvestment there are some characteristics of hedge funds that have to be considered by investors as these features affect liquidity. In this section we will introduce minimum investments, subscription and redemption frequencies, and capacity constraints.

A \textbf{minimum investment limit} is common with hedge funds.\textsuperscript{69} A (historical) rea-


\textsuperscript{66} This number is stable over time. In 1995 78\% of the managers had a minimum investment of $500,000, see Van (1996), p. 223.

\textsuperscript{67} See Bekier (1998), p. 94.


\textsuperscript{69} See for example Financial Services Authority (2002a), p. 8.
son for this fact can be found in the US regulation of hedge funds. Depending on
the structure of the hedge fund, the number of US investors is limited. Therefore,
only a minimum investment is able to ensure a sizeable pool of money even with
a smaller amount of customers.\textsuperscript{70} Under some jurisdictions, especially in Europe,
explicit minimum investment limits for hedge funds have been determined by regu-
laratory authorities.\textsuperscript{71} According to data from the index provider HFR 35 percent of
all hedge funds require minimum investments above $500,000.

\textbf{Subscriptions} in hedge funds are restricted to certain dates. In the case of closed-
end funds investors can deposit money only during the initial issuing period or if
the fund is reopened later on for additional investments. The subscription possi-
bleties with open-end funds are much more frequent. Open-end funds usually allow
investors to deposit money at fixed dates, mostly at the end of each month. These
subscriptions are usually based on the latest net asset value.\textsuperscript{72}

Similar to the subscription the \textbf{redemption} of hedge fund investments is also limited
to certain dates.\textsuperscript{73} Most funds allow a monthly redemption or at least multiple
redemption opportunities each year.\textsuperscript{74} After depositing money, investors of most
funds have to wait a certain period of time, the so-called \textbf{lock-up period}, before
the investment can be withdrawn again. On average this lock-up period is around
80-90 days.\textsuperscript{75} But some hedge funds do allow the withdrawal of money within the
lock-up period against a lock-up provision.\textsuperscript{76} When the lock-up period is over, many

\textsuperscript{71} Such minimum investments have for example been installed in the investment laws of France,
Ireland or Italy. For a survey of minimum investment amounts see PriceWaterhouseCoopers
\textsuperscript{73} Some hedge funds have a so-called \textbf{key-man clause}, giving the right to terminate the agreement
immediately if an important manager is leaving the fund, see Kaiser (2004), p. 99.
\textsuperscript{74} See for example Ackermann et al. (1999), p. 840.
\textsuperscript{76} See CRA Rogers Casey (2003), p. 6.
funds require a written advance notice to the administrator prior to a redemption. The corresponding **notice period** extends the whole redemption period in order to reduce managing cost and ensure the necessary liquidity in the fund.\(^{77}\) The length of the required notice period is on average around 30 days.\(^{78}\) Finally the **redemption period** until the actual redemption of the invested money completes the so-called **restriction period**\(^{79}\) that also includes the notice period. This restriction period can be interpreted as a measure for the liquidity of the hedge fund investment.\(^{80}\) Figure 2.5 on the next page illustrates the different dates and periods from the initial hedge fund investment until the final disinvestment.

As mentioned before, the restrictions for redeeming hedge fund investments shall reduce managing cost and ensure a long-term focus of investors. Scarce possibilities for redemption result in lower cash reserves and should therefore increase the performance of the fund.\(^{81}\) Empirical evidence on this hypothesis can be found for example in Cottier (1998) and Das (2003).\(^{82}\) As hedge funds very often operate in illiquid market environments, the nature of hedge fund investments implies some degree of illiquidity, too.\(^{83}\)

The recent studies by Agarwal et al. (2004) and Getmansky (2004) point out that the performance of hedge funds is dependent on asset size and that an optimal asset size of hedge funds does exist, especially when they hold illiquid assets, have limited market opportunities and a high market impact of trades.\(^{84}\) Examples are especially found in the strategy-groups convertible arbitrage or emerging markets.\(^{85}\)

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\(^{77}\) See Amin/Kat (2002b), p. 9.


\(^{79}\) See Agarwal et al. (2004), p. 3.


\(^{81}\) See Lhabitant (2002), pp. 16-17.


\(^{85}\) See Getmansky (2004), p. 29.
These findings are consistent with the very common capacity constraints in the hedge fund industry. As the opportunities for hedge fund managers in terms of mispricings or very high risk premia are limited, most hedge funds that pursue special trading strategies close their fund to new investors when they reach a certain size. These limits to the investment strategies ensure an adequate return for the investors.\textsuperscript{86} In this respect hedge fund managers behave differently from traditional money managers. In the case of common mutual funds positive performance is usually followed by dramatic capital inflows. With hedge funds this pattern can not be revealed.\textsuperscript{87}

\textsuperscript{86} For a related discussion of marginal returns in the hedge fund industry see Deutsche Bundesbank (1999), pp. 34-37.

\textsuperscript{87} See for example Goetzmann et al. (2003), pp. 1711-1714.
2.4 Summary

The key characteristics of the hedge fund industry or alternative investments in general introduced in this chapter are only the first step into an interesting part of the investment universe. These distinctive features are common to most hedge funds, but the following Chapter 3 will point out that the hedge fund universe is not that homogeneous. There exists a broad variety of different trading approaches within this hedge fund universe, and the resulting risk (and return) characteristics are manifold. Therefore, thorough analysis of hedge funds has to focus on the different traded assets and the corresponding trading strategies in order to give a complete picture on this part of the investment industry. Such a detailed analysis of the most important hedge fund strategies is part of the following Chapter 3.
3 Hedge Fund Strategies

Often investors consider hedge funds as a homogeneous investment class. But as already mentioned in Chapter 2, not all hedge funds are of similar type. The hedge fund universe consists of funds with completely different trading approaches. These manifold active investment strategies of hedge funds show very different risk profiles. In this chapter the different risk factors in hedge fund investing will be introduced and the implications for various strategies are documented. At first we will describe some central risk factors which originate from the hedge fund industry characteristics itself. Then common risk factors in hedge fund trading strategies will be specified as these factors determine the profile of alternative investment strategies. In Section 3.2 some important hedge fund strategies are introduced and the sources of risk and return are addressed. The last section of this chapter deals with fund of hedge funds. These funds invest in target hedge funds. Section 3.3 discusses some important characteristics and outlines the investment process of a fund of hedge funds.

3.1 Risks in Hedge Fund Investing

There are two major sources of financial risk in hedge fund investing. On the one hand risk arises from the hedge fund sector and its special characteristics.\(^1\) In the following these sources of uncertainty are called \textit{industry-inherent risk factors}.

\(^1\) See Chapter 2 for some important characteristics.
On the other hand some risk factors in hedge fund investing are associated with hedge fund trading strategies and the resulting exposures. These sources of risk are aggregated as **strategy-specific risk factors**. The most important risk factors that are introduced in the following sections are systematized in Figure 3.1.

![Risk Factors in Hedge Fund Investing](image)

**Figure 3.1: Systematization of risk factors in hedge fund investing**

### 3.1.1 Industry-Inherent Risk Factors

Before we introduce special investment strategies, the risk factors of the alternative investment industry in general have to be considered. This industry-inherent risk is to a large extent due to the information asymmetry between investors and managers. Therefore, the limited transparency in the hedge fund industry is one of the central sources of risk.²

The industry-inherent risk factors in hedge fund investing are usually independent of the strategies applied by the fund management. In a sufficiently broad hedge

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fund portfolio (like a fund of hedge funds\(^3\)) these risk factors can be diversified to some extent. But in a smaller portfolio with capital allocation in a few hedge funds only, these sources of risk have to receive a great deal of the investor’s attention and must be controlled and managed actively.\(^4\) In the following we will introduce some of the most important risk factors in the alternative investment industry, but this list is not exhaustive.\(^5\)

As mentioned before, **lack of transparency** is perhaps the central source of risk in hedge fund investing. It is the source for many of the risk factors in hedge fund investing. While only a permanent and full disclosure of hedge fund positions would prevent investors from risk factors like strategy or fraud risk, information especially on current positions is held back by the hedge fund managers.\(^6\) A possible solution to this transparency problem is that the hedge fund discloses the current exposure to different risk factors or at least that the fund manager reveals his (historical) positions with an appropriate time lag.\(^7\) Managed accounts, where the investor has turned over direction of his account to a manager, can also reduce this intransparency. This set-up provides the investor with full disclosure of the trades and therefore it addresses the basic problem of transparency and the resulting problems of strategy and fraud risk.\(^8\) But too much transparency might also pose a threat to a hedge fund. When confidential information about the fund’s significant positions reaches other market participants this entails market risk for the fund.\(^9\) Therefore, transparency that reveals the hedge fund’s risk profile regarding some predefined risk factors but no information about individual positions or trades is a suitable

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\(^3\) See Section 3.3 on page 91 for details on fund of hedge funds structures.

\(^4\) See Patel et al. (2002), p. 89.

\(^5\) Further potential risk factors that can be subsumed in the following categories, are mentioned for example in Jaeger (2002).


\(^8\) See Jaeger (2002), pp. 222-223.

\(^9\) See Reynolds Parker/Warsager (2000), p. XXVI.
instrument for risk reduction.

When constructing a portfolio with allocations to hedge funds, the investor expects a certain risk and correlation or dependence profile concerning other investments. A change in the hedge fund’s strategy focus will expose the investor to a new set of risk factors and is therefore a general source of risk, the so-called strategy risk.\textsuperscript{10} A typical example for such a style drift is the equity market neutral fund gaining directional exposure or moving to new market sectors.\textsuperscript{11} A further source of strategy risk is the leverage which the manager employs to pursue his investment strategy.\textsuperscript{12} The probability for changes in the strategy or leverage is especially high in times of bad performance or changing markets and after the drop out of key personell.\textsuperscript{13}

The lack of transparency in the hedge fund industry also leads to so-called fraud risk. The unregulated industry facilitates fraud in many ways like for example illegal pyramid schemes or false performance reports.\textsuperscript{14} According to SEC numbers, there was a maximum of 5 cases of fraud per year from 1998 through 2001 and 12 documented cases in 2002, so fraud is a rather rare event.\textsuperscript{15} The detection of fraud by the hedge fund management is a very challenging task faced by hedge fund investors. Third party verification and auditing have to ensure the accuracy of reported numbers and facts.\textsuperscript{16}

Management risk or human risk is especially important for hedge funds as they are very often lead by small teams or even individuals.\textsuperscript{17} So the return and risk profile can be seen as a function of the managers’ skill.\textsuperscript{18} There are three main

\begin{itemize}
\item \textsuperscript{10} See Moix (2004), pp. 10-11.
\item \textsuperscript{11} See Patterson (2003), p. 46.
\item \textsuperscript{12} See Moix (2004), p. 11.
\item \textsuperscript{13} See Cottier (1998), p. 47.
\item \textsuperscript{14} See Jaeger (2002), p. 140.
\item \textsuperscript{15} See Rettberg (2003), p. 37.
\item \textsuperscript{16} See Patterson (2003), p. 46.
\item \textsuperscript{17} See Patel et al. (2002), p. 93.
\item \textsuperscript{18} See Jaeger (2002), p. 138.
\end{itemize}
aspects of human risk. The first and most important source of management risk is the ability of the management to pursue a really successful trading strategy.\textsuperscript{19} The length of a management’s (successful) track record might serve as proxy for measuring this ability. Secondly an adequate compliance system and an attractive incentive structure should be in place in order to motivate the management and to avoid irrationalities. Key-men (or even key-man) dependency is a third aspect of management risk. One or more head traders leaving the hedge fund could be unfavorable to the fund’s perspective when the fund’s expertise in pursuing a special strategy shrinks to zero. This problem could be encountered by long term incentive fees or the implementation of a proprietary trading software.\textsuperscript{20}

Fund age is also influences the risk profile of a hedge fund investment. Survivial risk addresses the problem that young funds (2-3 years of age) have a significantly higher attrition rate than established funds.\textsuperscript{21} New entrants to the hedge fund industry usually face a strong incentive to implement very risky strategies with their performance-based funds. An option-like payoff to the manager rewards successful high-risk bets while the manager’s downside only consists of closing the fund and starting a new venture.\textsuperscript{22} As setting up and funding an investment platform needs special organizational skills and the applied trading strategy (or strategies) has to be successfully implemented in a new environment, there is also a high level of risk for young funds not to survive the first years of their independence due to structural or organizational problems.\textsuperscript{23}

The larger the equity base of a hedge fund is getting, the more difficult it usually becomes to earn an acceptable or promised return on the employed capital.\textsuperscript{24}

\textsuperscript{19} See for example Rutkis (2002), p. 46.
\textsuperscript{21} See for example the study of Howell (2001).
\textsuperscript{22} See Section 2.3.4 on page 17 for more details on performance fees.
\textsuperscript{23} See Patel et al. (2002), p. 94.
\textsuperscript{24} See Tarrant (1996), p. 147.
This problem, so-called size risk, arises especially when arbitrage, convergence or relative-value strategies are implemented. If the manager invests in new positions in order to accommodate the capital inflow, the fund’s return may decline as suboptimal investment decisions are made. When adding to existing positions, the capital becomes less liquid as negative size effects are associated with the market impact of large transactions. In order to downsize and to remain flexible, some managers avoid such problems by returning capital to their investors. As managers will tend to close their program when certain strategy immanent capacity limits are reached, some hedge fund investors might also face the problem of not being able to participate in high quality hedge funds.

Operational risk comprises a diverse set of risks and can be broadly defined as risk of losses resulting from inadequate or failed internal processes, people and systems, or from external events. One aspect of the human factor in operational risk was already mentioned above where management risk was discussed. Further influences on operational risk are back-office problems, money transfer risk, clearance risk and systems risk. As hedge funds very often do have a higher transaction turnover and fewer back-up staff, operational problems, like for example limit exceedance or erroneous executions, are more likely than with traditional investment funds. Because of the sophistication of the strategies and their timely nature, technical systems and computing infrastructure are also extremely important for hedge funds. Especially in the offshore domiciles of hedge funds basic infrastructure requirements are not

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25 For directional strategies the opposite may hold due to the market impact of large traders.
3.1.2 Strategy-Specific Risk Factors

When looking at the resulting net positions from manifold hedge fund trading strategies several well known risk factors can be identified. This broad range of risk factors is the source for risk premia which a hedge fund manager looks for. According to Jaeger (2003) these risk premia account for 80 percent of hedge fund returns. Taking these systematic risks is rewarded with higher (expected) returns but the ex post realization of the risk premia is uncertain.

Besides traditional risk factors, like market or credit risk, there are various sources of risk that are typical for or very pronounced in hedge fund investment strategies, for example event or model risk. On the following pages we will introduce the central risk factors in most hedge fund strategies: market risk, liquidity risk, credit risk, event risk and model risk.

Arising from unexpected (adverse) changes in the market prices or rates of financial instruments, market risk is the most common risk factor in the investment industry. Potential movements in the prices of equity, interest rates, credit spreads, currency and commodities markets are captured by this risk category. Regarding market risk, we can further differentiate between specific and systematic sources of risk. While specific risk focuses on price changes of individual securities, the systematic component refers to general changes in market prices. As there is the possibility

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34 The remaining 20 percent earned by fund managers are contributed by the detection and exploitation of market inefficiencies, see Jaeger (2003).
36 As it is not possible to define all the risk categories without intersection there will be partially overlapping risk factors in this listing.
of net short positions in financial instruments, a negative exposure to various market
prices may be the case with hedge funds (as opposed to traditional long-only mutual
funds). Changes in dependency structures, for example correlations, or volatility of
returns or market prices are also included in this category.\footnote{39} Regarding hedge funds
in comparison to traditional investment funds the impact of market risk is much
more severe as hedge fund strategies very often make intense use of leverage.\footnote{40}

Liquidity risk is one (or the) central risk factor with hedge fund investing.\footnote{41,42} Liquidity risk could be categorized as a sub-component of general market risk.\footnote{43} But as it is a central risk factor to most hedge fund strategies, we will treat it as
a separate risk category. Liquidity risk can be split into funding or cash flow risk
and market liquidity risk.\footnote{44} Funding liquidity risk refers to the possibility that a
(financial) firm will be unable to meet its obligations as they come due because
of an inability to liquidate assets or obtain adequate funding.\footnote{45} The hedge fund
investors are a source of risk as they are also influencing the fund’s liquidity. Short
term investors with a significant stake in the fund could cause harm to long term
strategies when their capital is recalled. Therefore, a broad diversification of the
client base and/or sufficient lock-up periods are very important for hedge funds.\footnote{46}
Market liquidity risk arises when financial managers become unable to adjust a
position in a timely manner at a reasonable price (for example as bid-ask spreads
widen).\footnote{47} As less liquid positions offer premiums, they are frequently a (major) part

\footnote{39} See Duffie/Singleton (2003), p. 4.
\footnote{40} See Cottier (1998), p. 44, and Section 2.3.1 on page 12.
\footnote{42} See for example Blanco et al. (2005) on the important issue of liquidity risk modeling.
\footnote{44} See for example Jorion (1997), pp. 15-16.
\footnote{45} Risks from margin calls or from the redemption policy of hedge funds belong to this risk-class
as well. See for example Cottier (1998), p. 46.
\footnote{47} See Jameson (2001), p. 2. This part of liquidity risk could also be categorized in the market
of hedge fund portfolios and the resulting risk profile.

**Credit risk** originates from the possibility that a borrower (issuer risk) or counterparty will fail to perform on an obligation. It can be defined as risk of default or reduction in value associated with unexpected changes in credit quality of issuers or counterparties.\(^{48}\) Counterparties include debt and equity holders. The risk from investors as counterparties withdrawing funds is categorized among liquidity funding risk. As funding support and collateral management are vital issues for hedge funds, changes in policies of prime brokers could have devastating consequences to a fund.\(^{49}\) Here the fund manager has to ensure that the counterparty to a transaction will be able to fulfil its end of the obligation.\(^{50}\) The hedge fund manager should be aware of, mitigate and diversify this risk component.\(^{51}\) Sovereign risk as well as the risk from potential changes in credit quality of clearing brokers and the probability of default of counterparties in OTC trades are also included in this category.\(^{52}\) Deduced risk factors like the widening of credit spreads are subsumed in this risk class.

As most of the hedge fund strategies are set up in very special market segments, they are sensitive to events in their market environment. Examples for the so-called **event risk** are prepayments with mortgage-backed securities arbitrage (see Section 3.2.4 on page 55), shareholder denials or management decisions in the case of merger arbitrage (see Section 3.2.5 on page 60), but also extraordinary political or economic events.\(^{53}\) Legal risk is also summarized under event risk. Legal risk considers different influences like court orders, changes in tax laws, interventions from regulatory authorities and prospectus risk. Such decisions and conditions are at

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\(^{48}\) See Duffie/Singleton (2003), pp. 3-4.

\(^{49}\) See the example in Jaeger (2002), p. 139.

\(^{50}\) See Press/Stamatelos (1996), p. 207.


the bottom of many hedge fund strategies\textsuperscript{54} and these events are therefore primarily responsible for the success of such transactions. The extent of this event risk depends on the traded securities and implemented strategies. In most cases it is simply impossible to quantify this risk factor with mathematical models since it is based on idiosyncratic events.\textsuperscript{55}

Many arbitrage trades are identified and/or hedged with quantitative models. The crucial point is that these models must accurately predict pricing relationships. The model risk originates from imperfect or unrealistic assumptions, when a quantitative model does not build on realistic estimates or when it is not properly reassessed and adjusted to market conditions. As a definition it is the risk that theoretical models used in pricing, hedging or estimating risk will produce misleading results.\textsuperscript{56} According to Allen (2003) the reasons for this risk factor can be found in the incorrect implementation of models, in ignoring key sources of risk during the modeling processes or in using uncertain input parameters to calculate the model prices.\textsuperscript{57} As many hedge fund trades are identified using models, this is an important factor in the fund’s risk profile.

### 3.2 Hedge Fund Strategies

This section introduces classical hedge fund strategies. The strategies that are examined in the following paragraphs cannot give a complete summary of all possible hedge fund trades. Rather there will be given a review for the most important hedge fund strategies and the resulting positions.\textsuperscript{58} After a brief strategy description we

\textsuperscript{54} See for example distressed securities (Section 3.2.6 on page 64) or regulation D arbitrage (Section 3.2.13 on page 90).

\textsuperscript{55} See Bookstaber (2003), p. 21.

\textsuperscript{56} See Allen (2003), p. 97.

\textsuperscript{57} See Allen (2003), pp. 102-110.
will outline major trades and describe the relevant risk factors/risk premia for the respective hedge fund category.

3.2.1 Equity Hedge

3.2.1.1 Equity Hedge Subgroups

Equity hedge is a very broad hedge fund category. According to numbers from the index provided HFR for the first quarter of 2005 approximately one third of the assets under management in hedge fund strategies is invested in equity hedge strategies. These strategies in general combine long holdings of equities with short positions in stocks.\(^{59}\) The equity hedge universe can be divided in a number of subcategories. Based on the net position of the resulting equity hedge portfolio, the different hedge fund subcategories are equity market neutral, long/short equity and long or short biased strategies, see Figure 3.2 on the following page. The analysis of the equity non-hedge strategy short selling can be found in Section 3.2.8 on page 72. Levered long-only funds are very similar to traditional long-only mutual funds. In addition to long positions in stocks these funds employ leverage to increase their capital base.

In equity hedge approaches managers purchase stocks that are considered relatively cheap, while simultaneously selling short stocks that are considered expensive. The main difference between these strategy subgroups is the degree to which the resulting portfolios are hedged concerning overall equity market risk.\(^{60}\)

**Equity market neutral** strategies are designed to profit from equity market in-

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\(^{58}\) Niche strategies like trading in energy-, weather- and credit-derivatives, options arbitrage, voting versus non-voting shares, or stub trading will not be covered in this section.

\(^{59}\) Often stock (index) derivatives are also used to set up the necessary exposures, see Ineichen (2001), p. 9, and Mogford (2005), pp. 31-32.

\(^{60}\) See Parnell (2001b), p. 62.
efficiencies with zero or negligible exposure to the market. The market neutrality (beta-neutrality, currency-neutrality or both) is achieved by balanced long and short equity positions in the hedge fund’s portfolio. In contrast to other equity hedge strategies, an equity market neutral investment approach requires frequent rebalancing in order to sustain the desired neutrality.

**Long/short equity** hedge fund managers maintain net market exposures. They have a more flexible investment policy as they adjust their net exposure (long or short bias) to the equity market risk depending on the manager’s preference and the prevailing or expected market conditions. Depending on the market conditions their portfolios may be anywhere from net long to net short. A net long position in rising markets and a net short portfolio in bear markets would be the optimal use of this additional flexibility. So-called sector specialists are a special type of

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62 See Lhabitant (2002), p. 82.
long/short equity funds. Here the manager analyzes and exploiting opportunities in some industry sectors only.\textsuperscript{65}

Investment strategies with fixed \textbf{long or short bias} have a dedicated exposure to one side of the equity market and are therefore less flexible than general long/short equity approaches.

### 3.2.1.2 Investment Approaches

There are three basic approaches to portfolio composition in equity hedge investing: fundamental analysis, quantitative multifactor analysis and statistical arbitrage. \textbf{Fundamental analysis} is the tool for investors with a rather long-term investment horizon. With fundamental analysis the stock selection is discretionary and aims at the identification of fundamental mis-valuations.\textsuperscript{66} Quantitative but also qualitative factors, like for example accounting practices and management ability or credibility have important influence on the investment decision.

A second approach to equity hedge investing is the use of \textbf{factor models} to identify or trade risk premia. Quantitative multifactor analysis looks at risk or return factors that affect or explain stock prices. The detection of under- and overvalued securities with quantitative research is often based on value or growth investing strategies.\textsuperscript{67} Other investment approaches try to exploit well known anomalies like the size effect\textsuperscript{68}, the book-to-market effect\textsuperscript{69} or human irrationalities known from behavioral finance\textsuperscript{70}. These investment styles or return factors can be found in most implementations of stock picking algorithms for detecting over- and/or undervalued


\textsuperscript{67} See for example Reiss (1996) or Nicholas (2000), pp. 223-225, for further details on these investment approaches in equity hedge funds.

\textsuperscript{68} For the firm size effect see the work of Banz (1981) and Reinganum (1981).

\textsuperscript{69} See for example Rosenberg et al. (1985).

\textsuperscript{70} See Blatter (2003), p. 12.
Tradeable market factors are utilized in quantitative multifactor models in order to determine optimal long/short portfolios that generate attractive returns, usually within a medium-term holding period.

As a third approach to portfolio composition statistical arbitrage strategies with equities are based on the expected persistence of historical relationships (technical analysis) or in general on the predictability of movements between equities or groups of equities. While hedge fund managers with fundamental approaches are basically stock pickers, the statistical arbitrage is model based. These strategies make use of statistical techniques to identify and exploit temporary pricing imbalances that are very often supply- or demand-driven. Common tools in this discipline are classical time series techniques, including for example cointegration models, bayesian models with a priori information, autoregressive models, vector error correction models, pattern recognition or statistical decision theory. The trading frequency with these strategies is very high and the investment horizon is usually short term.

Often hedge funds traders combine the approaches outlined above. A manager could for example use a top-down approach to identify a market and sector focus. Then the fund might employ fundamental bottom-up techniques to pick out the most promising stocks within the chosen market or sector. Finally statistical or technical analysis could be applied to time entry and exit points.

### 3.2.1.3 Equity Hedge Trades

Classical statistical arbitrage trades are pair trades with long and short positions in two related shares of a particular industry or sector. The objective in a pair trade

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72 See Parnell (2001b), p. 63. Sometimes statistical arbitrage with equities is considered as an individual hedge fund strategy, see for example the HFR index definitions.


75 See Blatter (2003), pp. 10-11.
is to profit from one stock outperforming another. These stocks are often identified with mean-reversion models where their current market price is compared to the historical development or trend. Therefore, the hedge fund attempts to profit from the likelihood that prices will trend toward a historical norm. So the hedge fund manager has a positive expectation for one of these stocks which is held long in the portfolio and a negative or less positive forecast for the other company resulting in a corresponding short position. Realizing the resulting long/short spread is the objective of the hedge fund’s bet. To avoid market and even industry risk, the two companies involved in this trade are often competitors within the same industry.

In statistical index arbitrage an index, exchange traded fund or index future is traded against constituent stocks. The replication of broad market indices (usually plus a spread) with few stocks is a very suitable hedge fund strategy for statistical cointegration models as the cointegration analysis aims at minimizing the tracking error and maximizing the stationarity of the resulting (net) time series. This stationary time series results from the difference of two integrated time series: index (plus spread) and replicating portfolio. The profit is generated from superior returns with less volatility and a smaller turnover than with other replication strategies.

Intersectoral arbitrage is another field for statistical analysis. With econometric approaches the influence of key economic indicators on cross-sector spreads is analyzed and forecasts are developed. Depending on the modeling results the spread is judged

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76 For different assessments or categorizations of the terms “pair trading” and “statistical arbitrage” see Campell et al. (1999), p. 16, and Ineichen (2001), pp. 6-7.

77 The identification of the components of a pair trade may also be done by fundamental methods, see for example Kao (2002), p. 33, or Ineichen (2001), p. 6.


79 Cointegration in financial markets refers to co-movements in asset prices instead of co-movements in returns like in other models. The econometric toolbox offers cointegration analysis to detect common trends in various time series of stock prices.

80 For the application of cointegration analysis in equity hedge strategies see for example Alexander et al. (2001), Ionescu (2002), or Rehkugler/Jandura (2002).
as overvalued or undervalued. The resulting investment in the spread is built up with long and short equity positions from these sectors.

<table>
<thead>
<tr>
<th>trade</th>
<th>resulting positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>long/short portfolio</td>
<td>long/short over- and undervalued securities</td>
</tr>
<tr>
<td>pair trades</td>
<td>long/short positions in two related stocks</td>
</tr>
<tr>
<td>index arbitrage</td>
<td>long/short indices and constituent stocks</td>
</tr>
<tr>
<td>intersectoral arbitrage</td>
<td>cross sectoral long/short positions</td>
</tr>
</tbody>
</table>

Table 3.1: Typical equity hedge trades

The desired positions for the different equity hedge trades in Table 3.1 are usually established with cash instruments (stocks), with derivatives like futures and options or with over-the-counter contracts.

3.2.1.4 Equity Hedge Risk Factors

Various risk factors are associated with equity hedge investing (see Table 3.2 on the following page). An important source of hedge fund returns with equity strategies is market risk. Due to positions in different stocks the portfolio is exposed to the specific risk of these companies but the manager can also take risk on a sector or market level. This stock picking risk includes for example changing profit margins or market shares, regulatory issues, research and development and management ability.

Equity market risk is usually hedged on a revolving basis in equity market neutral

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strategies but when the long and short position do not match perfectly as with long/short strategies the portfolio is also exposed to systematic market movements. For the short leg of long/short trades the risk factors in Section 3.2.8 on short selling have to be considered. Liquidity risk may arise when the hedge fund is engaged in stocks with a small capitalization. When long (or short) positions in small capitalization stocks shall be sold (or bought back) in the market, there might be liquidity problems. These capacity constraints arise especially if the size of the position is large, compared to the company’s overall market capitalization. A breakdown of long term relationships or patterns can cause serious problems with the statistical methods. As statistical arbitrage techniques usually try to indicate future convergence or divergence based on the analysis of historic movements this is a source of model risk. Extraordinary factors or a less liquid market environment can have a severe impact on the relationship of the financial instruments under consideration. Another source of model risk is the fundamental method employed in the valuation of individual companies. Table 3.2 sums up the central risk factors in equity hedge strategies.

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - equities</td>
<td>company specific price risk systematic net long/short bias in the portfolio</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>market squeezes</td>
</tr>
<tr>
<td>model risk</td>
<td>model based detection of opportunities</td>
</tr>
</tbody>
</table>

Table 3.2: Important risk factors in equity hedge strategies

When compared to traditional long-only funds, the equity hedge portfolio is slightly

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86 Primarily these capacity problems are encountered on the short side, see Tisdale (2000), p. 172.
87 See Nicholas (2000), pp. 228-229.
more concentrated. The focus of single trades or parts of the portfolio can be on regions, sectors, market capitalization and/or other exposures but the (equity) hedge fund portfolio is usually diversified across a number of these market segments. Leverage is often deployed to some amount but is used altogether scantily. The main performance variable in equity hedge investing is the stock picking ability of the hedge fund manager on the long and on the short side. Size is another key variable for the success of equity hedge funds or strategies. When the amount of assets under management is too large, the performance of market neutral equity hedge strategies might suffer.\textsuperscript{89}

3.2.2 Convertible Arbitrage

The strategy group convertible arbitrage manages about five percent of the capital invested in hedge funds. Convertible arbitrage managers try to realize profits from positions in mispriced convertible securities. Convertible securities are fixed income instruments\textsuperscript{90} and they are usually issued as convertible bonds or as convertible preferred stock\textsuperscript{91} which are exchangeable into common stock of the issuer or another company.\textsuperscript{92} Another category of convertible securities are mandatory convertibles which must be converted at the issuer’s option.\textsuperscript{93} For the issuer, convertibles are a funding vehicle with a minor impact on cash flows and also a strategic tool in mergers and acquisitions.\textsuperscript{94}

As the holder of a convertible security has the right to exchange a fixed income se-


\textsuperscript{90} Therefore, convertible arbitrage is sometimes categorized among the fixed income arbitrage strategies, see Section 3.2.3.

\textsuperscript{91} Convertible preferred stock represents equity rather than debt and is therefore subordinated to debt of the issuing company.

\textsuperscript{92} See for example Tremont (2000a), pp. 10-11.

\textsuperscript{93} The following analysis is primarily focused on convertible bonds. In the US convertible bonds made up about 50 percent of the convertibles market, see Tremont (2000a), p. 8.

\textsuperscript{94} See Ahn/Wilmott (2003), p. 55.
curity for a fixed number of shares, the value of a convertible bond can be notionally divided into a fixed income component and a so-called equity kicker containing the optionality’s value. The fair value of such hybrid instruments has to be determined with pricing models, so the modeling assumptions and the required estimates are potential sources of subjective views or errors. These models have to consider the price process of the underlying security and the development of interest rates. Consequently, hedge fund managers try to isolate individual risk premia and exploit pricing inefficiencies between convertible bonds and related shares, bonds and/or derivative securities.

Convertible bonds are often undervalued compared to their theoretical value. Especially two reasons may account for this gap in primary offerings as well as in the secondary market: the analysts’ coverage of convertible bonds and the issuers rating. As many convertible bond issues are small in size only few analysts follow them and therefore the transparency in this market segment is limited. The second reason can be found in the non-investment grade rating of many convertible bond issuers. This makes them suitable for a limited set of (institutional) investors causing a lower liquidity in such convertibles.

The most intuitive strategy with these undervalued securities is to buy and hold

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95 Besides this (usually American type) call option convertible securities very often contain a put option as they are callable, allowing the issuer to force the conversion into equity, see for example Lhabitant (2002), pp. 84-85.


97 See Ammann et al. (2003) for a detailed analysis of the French market for convertibles. In this study the theoretical values for the analyzed convertibles are on average 3 percent higher than their observed market prices. The longer the time to maturity the more convertible bonds tend to be underpriced and the market also significantly underprices convertibles that are at-the-money and out-of-the-money, see Ammann et al. (2003), pp. 648-651.

98 This theoretical value is usually based on market and credit risk, but does not take liquidity risk into account.


them until their conversion, carrying various market and credit risks. Hedge funds usually realize more sophisticated strategies (see Table 3.3 on page 47 for various hedge fund strategies). A first convertible arbitrage hedge fund strategy is to hold convertible bonds while hedging equity market risk with short positions in the underlying stocks.\(^{101}\) When such a position is delta neutral\(^{102}\) the portfolio profits from movements in the underlying stock’s market price. Thus the portfolio is long equity volatility.\(^{103}\) Returns with these so-called cash-flow trades\(^{104}\) are generated from the convertible bond’s coupon, the rebate on the short position in the underlying stock and the delta neutral trading strategy.

The term convergence trading with convertibles aims at the detection of under- or overpriced convertible securities. The long or short position in the convertible is hedged synthetically with equity and interest rate instruments. Over time the values of the hedge portfolio and the convertible security should converge and the manager might be able to lock in a profit from the pricing discrepancies.\(^{105}\) This profit is simply the difference between the original convertible value and the hedging cost.\(^{106}\)

But often even the delta hedge is left incomplete, for example when fewer stocks are sold short in the beginning or the adjustments after a change in market prices are missed out. The result is a net long position in the underlying stock which is held in the portfolio exposing the strategy to additional equity risk.\(^{107}\) This so-called bullish hedge profits from increasing prices in the underlying stock. A bearish

\(^{102}\) Delta refers to the sensitivity of the convertible bond’s price to changes in the underlying stock.
\(^{103}\) With the delta hedging of convertibles a straddle-like portfolio (long gamma) is created, see for example Parnell (2001a), p. 55, or Calamos (2003).
\(^{104}\) See for example Yee (2004), pp. 22-23.
\(^{105}\) See for example Yee (2004), p. 25.
\(^{106}\) These trades conform with the strategy label “convertible arbitrage” as a real arbitrage is focused.
hedge leaves a negative delta in the portfolio and is set up by selling short more underlying stocks than for a delta neutral position. Therefore, a decreasing stock price is a source of profit for this alteration of the classical delta (neutral) hedge. In these at least partially delta hedged portfolio constructions interest rate and credit risk is still carried by the hedge fund.\(^{108}\)

Distressed trading with convertible securities focuses on convertibles issued by companies that are threatened by financial distress. This is a more event-oriented trading strategy as the manager bets on the restructuring process of the target company.\(^{109}\) Here the convertible bond’s price does not behave like a common convertible (which is highly dependent on the underlying equity) because the inherent call option is usually deep out-of-the-money.\(^ {110}\) When setting up the trade, the distressed convertible is bought and the equity market risk is hedged with a short position in the underlying stock.\(^ {111}\) Here the manager sometimes takes a larger short position betting on a further depreciation of the stock or the bankruptcy of the company (in analogy to a bearish hedge).\(^ {112}\)

In a Convertible Bond Asset Swap (CBAS), different arbitrage trades are combined.\(^ {113}\) The basic elements of such a hedging transaction can be summarized as follows.\(^ {114}\) The hedge fund manager acquires a convertible bond in the market and sells this security to a so-called credit investor, usually an investment bank. In exchange he receives the value of the convertible’s bond floor and an OTC American

\[\text{\textsuperscript{108}}\text{To hedge these risk factors, credit and interest rate derivatives can be used. Hedging the interest rate risk can be easily done with interest rate derivatives like futures, forwards or swaps while the credit risk component which arises when convertible bonds are unsecured or subordinated, can be hedged with derivatives like credit default swaps, see Yee (2004), p. 25.}\]

\[\text{\textsuperscript{109}}\text{See Section 3.2.6 on page 64 for the related event-driven strategy “distressed securities”.}\]

\[\text{\textsuperscript{110}}\text{See Tremont (2000a), p. 13.}\]

\[\text{\textsuperscript{111}}\text{See Ineichen (2000), p. 25.}\]

\[\text{\textsuperscript{112}}\text{See Yee (2004), p. 25.}\]

\[\text{\textsuperscript{113}}\text{Such asset swaps have become a industry standard in hedging risk, see for example Blatter (2002a), p. 12.}\]

\[\text{\textsuperscript{114}}\text{As the agreements can be very complex, the basic structure is open to numerous variations.}\]
type call option on the convertible bond. The strike price of this option is the bond’s present value calculated with a (recall) credit spread that is slightly smaller than the spread for the evaluation of the bond floor. The resulting position is still exposed to equity market risk that can be delta hedged by the manager. Credit and interest rate risk is transferred to the credit investor.\footnote{With the credit swap transaction the trade is exposed to counterparty risk which could be severe in a credit crisis like the Russian debt default in 1998, see Tremont (2000a), p. 16.} The hedge fund’s net position after a delta (neutral) hedge is a long position in the option on the convertible bond (which is identical to the (cheap) option component in the convertible bond) while the downside potential is limited to the option premium. This strategy will profit from tightening credit spreads as the hedge fund will exercise his call option on the convertible bond and establish a new asset swap at a higher bond floor.\footnote{See Lhabitant (2002), pp. 88-90.} Table 3.3 on the following page sums up some of the more important convertible arbitrage trades.

The risk factors in different strategies of convertible arbitrage investing are listed in Table 3.4 on page 48. They are straightforward when the positions resulting from the trading strategies mentioned above are examined in detail. For some trading strategies equity market risk is associated with the net position in the portfolio. The very common long bias in the equity market creates a positive exposure while a net short delta position has a negative correlation with this risk factor. Further an at least partly delta neutral strategy is short (realized) equity volatility as the resulting profit depends on the actual hedging strategy and transaction costs on the underlying.\footnote{If the delta hedging is done with matching standard options, this problem might be less severe.} As the value of the conversion option in the portfolio is associated with the anticipated implied volatility (reflected in the options market price and also incorporating other market frictions), the portfolio NAV increases with the value of the option component.\footnote{See Ineichen (2000), p. 24.}
### Table 3.3: Typical convertible arbitrage trades and sources of profit

<table>
<thead>
<tr>
<th>trade</th>
<th>positions</th>
<th>sources of profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>delta hedging</td>
<td>long convertible bond,</td>
<td>coupon,</td>
</tr>
<tr>
<td></td>
<td>short underlying stock,</td>
<td>short rebate,</td>
</tr>
<tr>
<td></td>
<td>long implied volatility,</td>
<td>stock price movements,</td>
</tr>
<tr>
<td></td>
<td>short equity volatility</td>
<td>conversion premium</td>
</tr>
<tr>
<td>bullish/bearish hedging</td>
<td>additionally long/short equity market</td>
<td>increasing or decreasing stock price</td>
</tr>
<tr>
<td>convergence trading</td>
<td>fully hedged convertible</td>
<td>pricing discrepancies</td>
</tr>
<tr>
<td>CBAS</td>
<td>long convertible volatility,</td>
<td>tightening credit spreads,</td>
</tr>
<tr>
<td></td>
<td>short underlying stock</td>
<td>stock price movements,</td>
</tr>
<tr>
<td></td>
<td></td>
<td>conversion premium</td>
</tr>
</tbody>
</table>

If the bond component is held in the hedge fund’s portfolio, interest rate, market risk and credit risk influence the success of the convertible arbitrage strategy. This is not the case for CBASs where credit and interest rate exposure is avoided or exchanged into counterparty risk.

Convertible arbitrage is especially dependent on valuation models. The detection of opportunities and the hedging of the very often complex conversion features is done employing option pricing models and is therefore subject to model risk.

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market risk - equities  long (or short) underlying company, 
               long implied volatilities, 
               short realized volatilities

market risk - interest rates long convertible bond

credit risk  
               long (or short) underlying company

model risk  
               model based detection 
               of opportunities and hedging

liquidity risk  
               large bid/ask in convertibles, 
               short squeeze in underlying

event risk  
               regulatory and prospectus risk

Table 3.4: Important risk factors in convertible arbitrage hedge funds

Often, the pricing of the convertible portfolio components also relies on these models which gives rise to another form of model risk, the mark-to-model risk. The hedging process in most convertible arbitrage strategies typically requires short sales in order to adjust the portfolio’s delta. A common risk factor from short selling is a short squeeze which forces the manager to buy back shares at inflated prices. The illiquidity of the convertibles market is a chance for arbitrageurs and funds willing to capture this risk premium but on the other hand it is also a source of risk.

121 See for example Ayache et al. (2002) or Wilmott (2000b), pp. 597-610, for convertible bond pricing models.
124 In cases where a liquid option market on the relevant underlying exists this point is usually less severe. The prices or implied volatilities of standardized option contracts can serve as proxy for the conversion option. But as the features of plain vanilla options differ significantly from the exotic construction of most convertibles, these modeling inputs have to be treated very carefully.
Legal risk may arise due to intransparent or complicated details in the convertible’s prospectus (for example concerning the treatment of dividends) or as a consequence of regulatory intervention.\textsuperscript{126}

Diversification in a convertible arbitrage hedge fund’s portfolio is usually achieved by careful consideration of exposures to factors like industry, sector, credit quality, implied volatility and event risk.\textsuperscript{127} The use of leverage in most cases ranges from 2 to 10 times equity.\textsuperscript{128} A key factor for convertible arbitrage hedge fund’s profits is the supply with cheap convertible bonds although historically bullish hedges (with a net long position in the equity market) were a source of substantial gains, too.\textsuperscript{129}

### 3.2.3 Fixed Income Arbitrage

In this section we focus on hedge fund arbitrage strategies with interest paying instruments (or their derivatives). There is a variety of investment strategies that can be summarized under this broad category. According to HFR numbers a total of five percent of assets under management is invested in this strategy group. The core of these trades can be described as buying cheap fixed income instruments and selling short (relatively) expensive securities. In the following we focus on the bond/old-bond spread, basis trading, yield curve arbitrage, bond options trading and spread arbitrage.\textsuperscript{130}

A traditional hedge fund trade is the on-the-run and off-the-run bond arbitrage. This issuance-driven trade holds a long position in an “old” bond from a recent

\textsuperscript{130} The strategies convertible arbitrage and mortgage backed securities arbitrage (see Section 3.2.2 on page 42 and Section 3.2.4 on page 55) are sometimes classified as fixed income arbitrage strategies as they involve interest bearing instruments.
issuance and a short position in the corresponding “new” bond from the current auction or issuance. The reason for this set-up is the striking characteristic that very often a significant premium is attached to the new bond. As time passes the spread between the old and the new bond will converge toward zero. Therefore, shorting the expensive new bond, earning the repo rate on the sale proceeds and purchasing the old bond in order to lock in the position on the spread has the potential to generate trading profits. Usually this spread position is held until the next auction date and then the position is unwind at a smaller spread.

Basis trades are classical arbitrage transactions. Usually futures contracts on (government) bonds are sold and a bond that fits the delivery clause is bought simultaneously. As it is not sure which bond will be the cheapest at the delivery date and if the supply in this cheapest-to-deliver (CtD) bond will meet the amount demanded for the expiring futures contracts, there are various opportunities for arbitrage profits. A net profit can be locked in when the bought bond’s price (including financing costs and accrued interest) is smaller than the sold futures contract. This trade sounds simple but requires very sophisticated tools when all the delivery-related options in the futures contract are considered or valued.

Yield-curve arbitrage focuses on mispriced yield differences for various maturities. This group consists of trades like arbitrage in different maturities or the anticipation

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132 The spread between on-the-run and off-the-run bonds is used as explanatory variable in numerous multifactor models for hedge fund returns, see for example Edwards/Caglayan (2001b), pp. 1009-1010.
133 For a recent analysis of the bond/old-bond spread with the 30-year Treasury bond see Krishnamurthy (2002), pp. 464-465.
134 In the case of the 30-year Treasury these bonds are usually highly liquid, see for example Beder et al. (1998), p. 296.
135 See Krishnamurthy (2002), pp. 472-478, for an in-depth analysis of profits from this strategy with 30-year Treasury bonds over the period 1995 to 1999.
137 See for example Wong (1993), pp. 196-202, for the different option features.
of yield curve movements. When single bonds or maturities show mispricings relative
to the yield curve (graphically the yield curve might show kinks) the arbitrage trade
is usually realized with very close maturities. An assumption with most trades is
that the shape of the yield-curve remains unchanged for the duration of the trade.\textsuperscript{138}
An example for yield-curve arbitrage is a butterfly trade where a bond is identified
as expensive relative to other bonds. Shorting the too expensive bond would create
a high amount of interest rate market risk. A butterfly trade minimizes the risk of
changes in the yield curve as there are two “wings” - one trade with a longer and one
trade with a shorter maturity against the mispriced security. Therefore, one long
position will profit from a steeper yield curve while the other long position bets on a
flattening yield curve.\textsuperscript{139} Trading directly on yield curve movements (with so-called
yield curve shape trades) is much more risky than this. When a hedge fund manager
bets on a decreasing or increasing slope of the yield curve, there is usually no effective
hedge in place. One bond is held long and another bond with a significantly different
maturity is sold short. Profits are made when the forecast, which is often conducted
with the assistance of statistical analysis (to isolate seasonalities or irregularities),
holds true.\textsuperscript{140}

Bond options (or options on bond futures) are a further playing field for hedge fund
managers. Arbitrage strategies include the exploitation of bond option mispricings
via put-call-parity, the purchase (sale) of cheap (expensive) options, combined with
delta hedging or the trading of implied volatilities of options.\textsuperscript{141} Embedded option-
like features in bonds are another field that is open to arbitrage trading. If these
implicit bond options (for example call features) are mispriced, they can be isolated
and picked up by the arbitrageur.

\textsuperscript{139} See for example Tuckman (2002), p. 77-82.
\textsuperscript{140} See for example Wong (1993), p. 206.
most common spread is observable between corporate and government bonds. As this bond spread can be partially attributed to different influences like the corporate issuer’s credit risk, taxes on corporate bonds and systematic risks\(^\text{142}\) (further influences might be a specific bond’s liquidity or call features), the trader’s subjective views about the development of these factors decide about the positions he should enter. Expecting a widening spread, the hedge fund will enter a long position in government bonds or bond-futures and sell short the respective corporate bond. The spread arbitrage can also take the form of a relative-value trade when the trade is established with two different corporate bonds.\(^\text{143}\) The term TED-spread (Treasury/Eurodollar-spread) originally referred to the difference between yields on US Treasury Bills and yields on Eurodollars (certificates of deposit in US dollars in a non-US bank).\(^\text{144}\) Nowadays the TED-spread often stands for general positions in (global) government bonds and par swaps.\(^\text{145}\) The TED-trades are usually constructed by trading Treasury bill (or note) futures against Eurodollar futures.\(^\text{146}\) When the spread is expected to widen, Treasury bill futures are bought and the equivalent amount of Eurodollar futures is sold. Asset swap spread trades with corporate bonds are very popular nowadays. In such an asset swap transaction a bond is purchased and the bond’s fixed-rate cash flows are immediately exchanged for floating-rate cash flows with an interest rate swap. If the floating rate is above the bond’s financing costs, there is a potential profit in this trade. The floating-rate will bear some spread over a (risk-free) reference rate, for example LIBOR.\(^\text{147}\) This spread is the reward for (or a function of) credit risk inherent in the bond position.

\(^{142}\) See the study of Elton et al. (2001) for details on these influences.

\(^{143}\) See for example Lhabitant (2002), p. 93.

\(^{144}\) So this spread reflects views of the relative credit quality of the US Treasury and high quality international financial institutions, see Lhabitant (2002), p. 94.

\(^{145}\) See Nicholas (2000), p. 211.

\(^{146}\) See CBOT (2002) for an implementation with interest rate swap futures. Kawaller (1997) provides further information on the implementation of corresponding trades.

\(^{147}\) See for example Hull (2003), pp. 618-619.
The introduced fixed income trades are listed in Table 3.5.

Interest rate risk is the central market risk for this basket of fixed income trades. Usually it is hedged in fixed income arbitrage strategies, but when yield curve arbitrage trades are considered there is no extensive hedge in place. The main components of this risk factor are parallel shifts in the yield curve, steepening or flattening of the yield curve, increased or decreased curvature of the yield curve and the volatility of interest rates.\(^{149}\) As second market risk residual currency exposure has to be examined when securities denominated in different currencies are arbitraged. They are usually hedged with currency futures contracts, but in practice these hedges are not always perfect.\(^{150}\) Another risk factor is credit risk. Especially in spread arbitrage trades this is the central risk factor. Differences in default risk cause the relevant spreads, and changes in the estimated ability to repay interest and principal are responsible for increasing or decreasing spreads.\(^{151}\) Liquidity risk arises in two


\(^{149}\) See for example Tremont (1999), p. 5.


<table>
<thead>
<tr>
<th>trade</th>
<th>resulting positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>on- and off-the-run trade</td>
<td>long/short two close maturities</td>
</tr>
<tr>
<td>basis trade</td>
<td>long CTD bond, short future</td>
</tr>
<tr>
<td>butterfly trade</td>
<td>long/short three close maturities</td>
</tr>
<tr>
<td>yield curve movements</td>
<td>long/short different maturities</td>
</tr>
<tr>
<td>bond option trading</td>
<td>long/short (implicit) options and underlying</td>
</tr>
<tr>
<td>spread arbitrage</td>
<td>long/short different credit qualities</td>
</tr>
<tr>
<td>bond asset swap</td>
<td>long bond, interest rate swap</td>
</tr>
</tbody>
</table>

Table 3.5: Typical fixed income arbitrage trades\(^{148}\)
forms. On the one hand the success of most strategies or trades is contingent on low net financing costs, especially when short positions are set-up. On the other hand fixed income arbitrageurs usually hold less liquid bonds as they trade at a discount and can therefore be arbitrated against liquid securities trading at a premium.\footnote{See Ineichen (2000), p. 28.}

The usage of quantitative modeling is very pronounced in fixed income arbitrage hedge funds. Mispricings are detected with sophisticated models and apart from true arbitrage trades, the success of trades relies on realistic modeling assumptions and accurate predictions.\footnote{See Nicholas (2000), p. 214.} Event risk with fixed income instruments includes government policy moves like the issuance of new bonds or changes in tax laws. But both risk factors can also open up new opportunities for hedge funds. The different sources of the risk premia for fixed income arbitrage hedge funds are summarized in Table 3.6 on the next page.

The margins in fixed income arbitrage are usually very small in relative numbers,\footnote{See Ineichen (2000), pp. 24-25, Cottier (1998), p. 129, Smith (2001), p. 3, and Nicholas (2000), pp. 209-218.} so the positions have to be levered in order to offer attractive returns.\footnote{Ineichen (2000) talks about 3 to 20 basis points, see Ineichen (2000), p. 29.} The leverage is achieved through borrowing, repurchase transactions or derivatives. This extensive leverage ranges from 20 to 30 times equity\footnote{See Wong (1993), pp. 239-240.} (and the fund volume can also reach values of 150 times equity\footnote{See Ineichen (2000), p. 29.}) confirming the importance of attractive financing conditions in fixed income investing.

\footnote{See Cottier (1998), p. 129.}
3 Hedge Fund Strategies

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - interest rates</td>
<td>usually hedged, existent in yield curve arbitrage</td>
</tr>
<tr>
<td>market risk - currencies</td>
<td>residual exposure to different currencies</td>
</tr>
<tr>
<td>credit risk</td>
<td>long/short default risk, short positions usually in better ratings</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>low financing costs, long positions in illiquid instruments</td>
</tr>
<tr>
<td>event risk</td>
<td>issuance of new government bonds, changes in tax laws, etc.</td>
</tr>
<tr>
<td>model risk</td>
<td>model based detection of opportunities and hedging</td>
</tr>
</tbody>
</table>

Table 3.6: Important risk factors in fixed income arbitrage hedge funds\(^{154}\)

3.2.4 Mortgage-Backed Securities Arbitrage

Mortgage-backed securities (MBS) arbitrage hedge funds invest in mortgage-backed securities and corresponding derivatives.\(^{159}\) MBS differ from regular bonds with respect to their cash flow structure, the amortization of principal and prepayment possibilities.\(^{160}\) Because of such special features, this is an interesting area for arbitrageurs.\(^{161}\) The US MBS market with an outstanding amount of more than 4 trillion dollars\(^{162}\) is especially attractive due to various factors. In the first quarter of 2005 MBS hedge funds reflect nearly three percent (HFR estimate) of the global hedge fund capital. Positive influences are for example the governmental (or quasi-

\(^{161}\) The markets for MBS are fully developed in the US and in the UK, see Cottier (1998), p. 131.
governmental) standardization of the issues and the therefore high credit quality, the tax relief for mortgage interest and the availability of consumer credit information.\textsuperscript{163} Like convertible arbitrage (see Section 3.2.2 on page 42), this strategy is sometimes classified as a subcategory of fixed income investing (see Section 3.2.3 on page 49) although their cash flows depend on a broad range of influences.\textsuperscript{164}

There exists a variety of mortgage-backed securities. The starting point of all these structures are loans on residential and/or commercial mortgages. Loans with similar characteristics are pooled into asset-backed securities (ABS) with pass-through structures. The pooling and the issue of residential MBS is usually done by government-sponsored organizations like the Federal National Mortgage Association (FNMA) and the Federal Home Loan Mortgage Corporation (FHLMC), or by government agencies like the Government National Mortgage Association (GNMA) in the US and The Mortgage Corporation (TMC) in the UK.\textsuperscript{165} Numerous private institutions such as investment banks or home builders also issue residential and commercial mortgage-backed securities, which are referred to as non-agency or private label MBS.\textsuperscript{166} Most MBS are of the highest rating class and sometimes in addition guarantee capital.\textsuperscript{167} Pooling these MBS and/or original loans in so-called collateralized mortgage obligations (CMO) creates a derivative structure that is sold to investors in different tranches or classes with different cash flow characteristics.\textsuperscript{168} These tranches carry individual risk profiles and receive different portions of the interest and principal (so-called sequential-pay structure). There are numerous variations of these CMO structures.

\textsuperscript{163} See Tremont (1999), p. 3.
\textsuperscript{164} See for example Fung/Hsieh (2002b) for this categorization.
\textsuperscript{165} See Wong (1993), pp. 38-39 or Nicholas (2002), pp. 131-132 for a brief description of these organizations.
\textsuperscript{166} See for example Friedland (2001).
The coupon rates of CMO floaters are periodically adjusted to some underlying index (usually Libor). Super floaters adjust to a multiple of an index while the coupon of inverse floaters reacts converse to the reference index. As the prepayment risk is high with the underlying loans, investors are attracted by planned amortization class (PAC) bonds offering significant prepayment protection. This protection is achieved when other tranches or classes in the ABS structure absorb all the prepayment cash flows and thus all the prepayment risk. Other CMO variations are the very common interest-only (IO) and principal-only (PO) bonds. Here the payments in the ABS collateral are stripped into interest and principal cash flows. Therefore, they can serve as natural hedging instruments for mortgage portfolios. A Z-Bond is a tranche of the CMO that receives no interest or principal payments until the preceding classes are fully paid. So this type of CMO does not offer regular coupon and it is sold at a discount (similar to zero coupon bonds). Inverse IO securities combine the features of floating securities with the IO characteristic. Therefore, the inverse IO coupon rises with decreasing interest rates and the average life and cash flow falls with increasing prepayments.

There are various possibilities for trades with MBS. Directional investment in MBS tries to exploit views on the development of interest rates and/or the amount of prepayments with long positions in different MBS instruments (for example with inverse floaters). Market neutral or hedged strategies with MBS are very common among hedge funds. They capture the spread between different MBS or CMO structures or the spread between MBS and a reference like for example Libor or the yield on government bonds. For some of these trades and the resulting positions see

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169 See page 58 for more details on prepayment risk.
<table>
<thead>
<tr>
<th>trade</th>
<th>resulting positions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Directional MBS</td>
<td>positions in interest rates and/or prepayment risk</td>
</tr>
<tr>
<td>MBS hedging</td>
<td>long/short different pass-through MBS</td>
</tr>
<tr>
<td>MBS arbitrage</td>
<td>long MBS and short government bonds or derivatives</td>
</tr>
<tr>
<td>CMO arbitrage</td>
<td>long/short CMO and pass-through MBS</td>
</tr>
<tr>
<td></td>
<td>or long PAC and short other MBS or CMOs</td>
</tr>
<tr>
<td>Inverse IO trading</td>
<td>positions in inverse IO</td>
</tr>
<tr>
<td>Distressed MBS</td>
<td>long/short distressed MBS</td>
</tr>
</tbody>
</table>

Table 3.7: Typical mortgage-backed securities arbitrage trades\textsuperscript{175}

Table 3.7.

Risk factors in MBS arbitrage trades are listed in Table 3.8 on the next page. As MBSs in general offer fixed income features, they are subject to interest rate risk. The risk of prepayment for MBS can be further divided into general turnover risk, mortgager’s cost changes and changes in the mortgagers’ alertness.\textsuperscript{176} The prepayment feature of the underlying loans creates an inverse exposure to interest rates. When interest rates decline, a substantial part of the homeowners or corporations will start to refinance their mortgages causing prepayment cash flows in the collateral pool of mortgage-related ABS. Credit risk in MBS depends on the respective issuer. As most mortgage securities are arranged by government institutions and offer guarantees of principal, they attain a credit quality similar to treasury securities.\textsuperscript{177} Other “private” placements may be more exposed to this risk factor.\textsuperscript{178} Liquidity risk arises with some securities in the MBS market. There are very liquid


\textsuperscript{176}See Tremont (1999), p. 5.

\textsuperscript{177}See Lhabitant (2002), p. 95.

\textsuperscript{178}See Tremont (1999), p. 6.
segments that are comparable to the corporate bond market, but there are also some CMO structures (like for example inverse IOs) that lack liquidity and offer opportunities to arbitrageurs.

A further crucial point to MBS investing is the tax legislation. If the deductability of mortgage interest would be limited or abolished this would have a severe impact on the MBS-market. Another major risk factor can be found in the modeling of MBS. In the course of detecting opportunities, MBS hedge funds have to apply sophisticated pricing models for example in order to embed prepayment features or interest rate scenarios, which exposes them to significant model risk.

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - interest rates</td>
<td>long/short fixed income securities</td>
</tr>
<tr>
<td>credit risk</td>
<td>depending on issuer and guarantees</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>more complicated CMOs lack liquidity</td>
</tr>
<tr>
<td>event risk</td>
<td>prepayment risk and tax legislation</td>
</tr>
<tr>
<td>model risk</td>
<td>complex pricing models are used</td>
</tr>
</tbody>
</table>

Table 3.8: Important risk factors in mortgage-backed securities arbitrage

Because of these risk factors, MBS trade at a yield premium of up to one or two hundred basis points above comparable government bonds. A central source of return in MBS arbitrage is the complexity of the mortgage related securities. Another main influence is the ability to predict the mortgager’s behavior, as irrational human

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179 See Friedland (2001).
decisions due to non-financial reasons can be a major performance factor.\textsuperscript{186} Further contributions to the performance are the behavior of spreads or credit mispricings. Leverage with MBS arbitrage is usually less than three times equity. The diversification in MBS hedge funds is achieved with investments in numerous positions across a variety of mortgage sectors and different MBS structures.\textsuperscript{187}

3.2.5 Merger Arbitrage

Merger arbitrage hedge funds pursue classical convergence trades. They focus on the stocks of companies which are or will be involved in an acquisition or merger process.\textsuperscript{188} As there is some risk or probability that such a transaction will not be completed, the target company’s securities trade at a discount, expressing the market’s view about success or failure.\textsuperscript{189} This is the spread or risk premium merger arbitrage hedge fund managers are interested in.\textsuperscript{190} The time horizon of merger arbitrageurs is short term. Although the spreads in merger arbitrage are typically narrow, the returns are significant on an annualized basis as the arbitrage transactions usually can be realized in a short period of time.\textsuperscript{191} In 2005 the amount invested in merger arbitrage hedge funds is about 1.5 percent of the net asset value in the hedge fund universe.

In general there are two different types of mergers, stock mergers and cash mergers, depending on what a bidder offers in exchange for the target companies equity.\textsuperscript{192}

\textsuperscript{188}Some definitions differentiate between merger and risk arbitrage. In Ineichen (2001) risk arbitrage includes merger arbitrage as well as special (corporate) situations, see Ineichen (2001), p. 24.
\textsuperscript{189}See Cornelli/Li (2001) for the role of merger arbitrageurs in takeovers.
\textsuperscript{190}See Yang/Branch (2001) for a detailed review of academic literature on merger arbitrage.
\textsuperscript{191}See Lhabitant (2002), p. 112.
\textsuperscript{192}More complicated situations may arise when warrants, preferred stocks, debentures or multiple bids are involved, see for example Lhabitant (2002), p. 111.
In a cash merger\textsuperscript{193} (also called “cash tender offer”) a hedge fund is simply long the stock of the company being acquired. This aims at realizing the spread between the market price at which the stock currently trades and the tender price realized when the deal is completed (the premium offered by the acquiring company). The dividend paid during the holding period on the target company’s stock is also realized by the arbitrageur.\textsuperscript{194}

In a stock merger (also called “stock for stock merger” or “stock swap merger”\textsuperscript{195}) an arbitrageur is usually short the stock of the acquiring company and long the stock of the acquisition target. This is more complicated than in a cash merger as the offer in such a transaction is not fixed but depends on the bidding company’s stock price.\textsuperscript{196} When the spread between the stocks involved in this transaction disappears, a profit from the long and short positions can be realized.\textsuperscript{197} Another source of profit can be the dividends of the stocks held long, but this should be offset by the dividends for the short position in the acquiring company.\textsuperscript{198} Finally interest on the short sale proceeds is a source of profits in stock merger arbitrage.\textsuperscript{199} Table 3.9 on the following page shows the positions in typical merger arbitrage trades.

As the success of a merger or acquisition depends on the approval by the shareholders of the acquisition target, the regulation authorities (Departement of Justice, Federal Trade Commission, European Competition Bureau) and management actions,
problems during the negotiations have to be taken into account. This transaction-based risk is a source of returns for hedge fund managers as this uncertainty causes the required spreads which will often fluctuate as the market changes its opinion on the probability of consummation.\textsuperscript{201} Herein lies the fund managers’s opportunity. A successful merger or acquisition process will force these spreads to disappear. But in case a deal is called off, the spreads will quickly widen again. In such a situation the long position will loose value (the premium will disappear) and the short position has to be covered at higher prices as several arbitrageurs will try to cover their short sales at the time the merger proposal is terminated.\textsuperscript{202} These losses will usually be much greater than the potential gains if the deal succeeds.\textsuperscript{203} In the case of stock mergers this event or breakup risk can be influenced by market risk. When equity markets are falling, the target company’s shareholders may refuse their consent as they would be compensated in shares.\textsuperscript{204} Therefore, the strategy is exposed to a long position in equity volatility.\textsuperscript{205} In case of an unsuccessful takeover due to a sharp decline in the price of the bidder’s stock the gain in the short position would compensate for the losses in the target company’s equity. A hedge fund manager must also pay attention to the target company’s specific risk as a decline in value

\begin{table}[h]
\centering
\begin{tabular}{lcc}
\hline
trade & positions & sources of profit \\
\hline
cash merger & long target & spread, dividends, (opportunity cost) \\
stock merger & long target, short bidder & spread, net dividends, short rebate \\
\hline
\end{tabular}
\caption{Typical merger arbitrage trades and sources of profit\textsuperscript{200}}
\end{table}

\textsuperscript{200} See Parnell (2000), p. 3.
\textsuperscript{203} See Lhabitant (2002), p. 112.
\textsuperscript{204} See Ineichen (2000), p. 34.
might also cause a breakup of negotiations.

Besides this legal or breakup risk another source of uncertainty is a delay of the transaction, which causes the annualized return (resulting from the fixed spread) to drop significantly. Regarding the net position in the arbitrageurs portfolio, further risks can be identified. In a cash tender offer, the hedge fund is usually net long the acquisition’s target and therefore in case of an unsuccessful takeover also exposed to market risk when the premium vanishes. In a successful takeover the market price should equal the tender price thus eliminating market risk. A stock merger normally results in an arbitrage portfolio with a small short bias. The long and short positions are usually not cash-balanced and thus not completely market neutral. The invested volume in the long position is a bit smaller than the short position as the values (when the exchange-ratio is considered) are expected to converge. Therefore, the stock merger arbitrage portfolio is usually short delta to a small extent.\textsuperscript{206} In order to reduce transaction costs, some managers use options instead of buying stocks which makes them sensitive to changes in (implied) volatility. The central risk factors in merger arbitrage hedge funds are summarized in Table 3.10.\textsuperscript{207}

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - equities</td>
<td>positions in target (and bidder) company,</td>
</tr>
<tr>
<td></td>
<td>net short position (stock merger),</td>
</tr>
<tr>
<td></td>
<td>position in implied and/or realized volatility</td>
</tr>
<tr>
<td>event risk</td>
<td>approval of regulators and/or shareholders</td>
</tr>
</tbody>
</table>

Table 3.10: Important risk factors in merger arbitrage hedge funds\textsuperscript{208}

Merger arbitrage hedge funds diversify their portfolio by investing in several deals at


\textsuperscript{207} See for example Paulson (2000) for more details on risk in Merger Arbitrage.

The same time.\footnote{See for example Ineichen (2000), p. 34, or Drippe/Eyrick (2001), p. 70.} They also limit the size of capital allocation in individual deals and often invest in transactions which involve different risk factors in order to have a low correlation of success or failure in the portfolio of merger bets.\footnote{See Nye et al. (1996), p. 77.} The use of leverage in merger arbitrage hedge funds is not standardized. Many managers employ at least some amount of leverage depending on the arbitrage opportunities.\footnote{See for example Nicholas (2000), p. 205, or Drippe/Eyrick (2001), p. 69.} An adequate amount of mergers and acquisitions is naturally the core of the strategies in order to build a diversified merger arbitrage portfolio. This quantity is a main performance variable.\footnote{See Lhabitant (2002), p. 112, and Ineichen (2000), p. 34.} In times of an economic upturn or a rising equity market, merger and acquisition situations tend to occur very frequently.\footnote{See Drippe/Eyrick (2001), pp. 67 and 70.} Another key factor in merger arbitrage is a large or sufficient premium on successful transactions in order to compensate for losses in unsuccessful takeovers. Further a declining interest rate environment is a catalyst for this strategy.\footnote{See Drippe/Eyrick (2001), p. 70.}

### 3.2.6 Distressed Securities

When following a distressed securities strategy, hedge funds invest in debt or equity instruments of companies which face difficulties. Both, companies with financial and/or operational problems, as well as enterprises which are already unable to fulfill their liabilities or are in bankruptcy are counted among the distressed category.\footnote{See Ineichen (2000), p. 37.} Therefore, the portfolio constituents of distressed securities investors predominantly show ratings in the categories C (imminent default) and D (actual default).\footnote{See Parnell/Matos (2001), p. 31.}

The spreads that attract hedge funds may result from absolute or relative pricing

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inefficiencies.\textsuperscript{217} Absolute inefficiencies occur between a security’s fundamental or intrinsic value and its market price while relative mispricings can come up between the prices of two securities issued by the same company which is currently in distress.\textsuperscript{218}

The market for securities of distressed companies is mainly a buyer’s market.\textsuperscript{219} Most individual investors are unwilling to bear the risks associated with securities from companies in distress,\textsuperscript{220} and many institutional investors face legal restrictions, which force them to sell bonds rated below investment grade.\textsuperscript{221} Finally the required know-how for distressed securities investing, the time consuming monitoring process of the distressed assets and the scarce coverage by analysts even convince holders of distressed securities to sell below their fundamental value.\textsuperscript{222} In such a market hedge funds that invest in distressed securities act as liquidity providers.

The strategies employed by distressed securities hedge funds are numerous, but there are two main categories of distressed securities investing. Active hedge funds try to gain significant influence as creditors in the restructuring and refinancing process, while passive distressed securities managers buy “cheap” equity and/or debt of distressed companies and simply hold until their investment pays off.\textsuperscript{223}

When exploiting relative mispricings of distressed securities, i.e. inefficiencies between the prices of two securities issued by the same company, the trades are often classified among capital structure arbitrage. During a restructuring process the cap-

\begin{thebibliography}{99}
\bibitem{218} This leads to so-called capital structure arbitrage, see for example Parnell/Matos (2001), p. 33. Some convertible arbitrage trades (see Section 3.2.2 on page 42) could also be classified among capital structure arbitrage as different instruments from one company are traded against each other.
\bibitem{221} See Lhabitant (2002), p. 100.
\bibitem{222} See Lhabitant (2002), p. 100.
\end{thebibliography}
Hedge Fund Strategies

3

ital structure arbitrage specialists purchase the undervalued security and take short trading positions in the overpriced asset to generate an arbitrage profit. The classical trade is to identify companies where the market value of equity is at a too high level versus debt and to implement a trade consisting of a long debt and a short equity position.

Distressed investing includes various instruments from secured debt to common stock, for example debtor-in-possession loans, trade claims, secured bank debt, real-estate loans, mortgages, senior or subordinated debt, letters of credit, (busted) convertible bonds, preferred or common stock. In these securities many hedge funds very often only take long positions as it is difficult, risky or even impossible to short distressed or even defaulted securities. Some typical trades with distressed securities are summarized in Table 3.11 on the next page.

In the past, most investment opportunities for distressed securities hedge funds were offered by US corporations. For the US Chapter 11 of the US Bankruptcy Code provides a market oriented bankruptcy process which provides relief from creditor claims for companies in financial distress. Depending on the filing, distressed securities can be categorized in one of four stages of their life cycle. The pre-bankruptcy period prior to the bankruptcy filing, the early-stage bankruptcy starts immediately after the filing for bankruptcy protection and finally the middle- and late-stages with due diligences and the distribution of new security baskets.

Due to two major drawbacks, the European market has not been very attractive

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225 Busted means that the conversion feature is virtually worthless, see for example Parnell/Matos (2001), p. 33.
229 If companies file for Chapter 7 of the US Bankruptcy Code they prepare for liquidation and look for supervision in the liquidation process.
Table 3.11: Typical distressed securities trades

<table>
<thead>
<tr>
<th>Trade</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>active debt/equity core positions</td>
<td>influence on the restructuring process</td>
</tr>
<tr>
<td>long term passive debt/equity positions</td>
<td>buy and hold</td>
</tr>
<tr>
<td>investing in so-called orphan equities after reorganization</td>
<td>buy and hold</td>
</tr>
<tr>
<td>passive short term positions</td>
<td>anticipation of a specific event</td>
</tr>
<tr>
<td>capital structure arbitrage</td>
<td>different securities of one company</td>
</tr>
<tr>
<td>long term positions under partial hedging</td>
<td>risk until reorganisation</td>
</tr>
</tbody>
</table>

for distressed securities specialists. In Europe, under various jurisdictions, it is harder to force a default as very often distressed securities are issued by holding companies, whereas in the US the issuers are operating companies. A second reason for the less liquid distressed securities market is the lack of transparency in most of the national European bankruptcy laws.

There is a variety of risk factors that affect distressed securities investing. Specific risk of the distressed company (or companies) is the most obvious and essential risk. Model risk is also a very important part of the overall risk profile. The search for under- or misvalued securities the detection is usually model-based. As marking-to-market is often impossible, net asset values are usually model-based. This may result in lower (historical) risk figures due to performance smoothing. Equity market risk of stock positions held long is to some extent hedged with short positions

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231 See for example Paetzmann (2003) for problems with distressed investing in Germany.
in the distressed company’s sector or futures contracts, but cannot be fully avoided. The strategy is usually long debt instruments with a low rating classification which implicates credit risk.\textsuperscript{235} Liquidity risk accounts for another significant portion of overall risk, as the hedge fund’s positions are generally illiquid. On the one hand, after a restructuring, a controlling position cannot be sold immediately and on the other hand, the road to the restructuring in itself may already take it’s time. This leads to a strict redemption policy of distressed securities hedge funds with lock-up periods of up to one year.\textsuperscript{236} The legal risk with distressed securities investing comes from the often needed court decisions before the final restructuring. Positions in distressed debt are usually long term and thus subject to interest rate risk, as rising interest rates will decrease the strategy’s returns.\textsuperscript{237} A short review of the risks associated with distressed securities investing is given in Table 3.12.

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk</td>
<td>long distressed company, long duration position</td>
</tr>
<tr>
<td>model risk</td>
<td>model based detection of opportunities, model based NAV calculation</td>
</tr>
<tr>
<td>credit risk</td>
<td>long default risk</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>illiquid positions, long lock-up periods</td>
</tr>
<tr>
<td>event risk</td>
<td>court decisions</td>
</tr>
</tbody>
</table>

Table 3.12: Important risk factors in distressed securities hedge funds\textsuperscript{238}

Diversification in order to reduce portfolio risk is achieved by investing in various

\textsuperscript{236}See Lhabitant (2002), p. 104.
securities, companies and sectors. The leverage used with distressed securities investing is usually low.\textsuperscript{239} Key factors for success in distressed securities investing are a superior ability to value a distressed company’s assets, negotiating and bargaining skills and a thorough understanding of the relevant investment risks.\textsuperscript{240} As economic downturns with deficient corporate returns often entail financial distress, for many companies the economic environment is another key influencing factor on distressed hedge funds.\textsuperscript{241} The market share of distressed securities hedge funds in 2005 is approximately five percent.

\subsection*{3.2.7 Global Macro}

Macro or global macro hedge funds are very flexible in their investment approach and try to profit wherever they identify opportunities. The investment process of macro funds is the least restricted in the hedge fund industry.\textsuperscript{242} These funds usually try to capture shifts in international monetary, political and economic policymaking, which impacts interest rates and currency, stock and bond markets.\textsuperscript{243} Depending on the macroeconomic environment managers follow completely different strategies.\textsuperscript{244} The strategies have a macroeconomic view in common and are applied in developed and emerging markets.\textsuperscript{245} Very often macro hedge funds pursue a base strategy (for example long/short equity) and take on different opportunities or bets depending on the manager’s views.\textsuperscript{246} Due to spectacular trades, they were the hedge fund industry’s biggest players and account(ed) for most media attention.\textsuperscript{247}

\footnotesize
\begin{itemize}
\item \textsuperscript{240}See Tremont (2000b), p. 9.
\item \textsuperscript{241}See for example Tremont (2000b), p. 24.
\item \textsuperscript{242}See Ineichen (2003), p. 319.
\item \textsuperscript{244}See for example Strome (1996), pp. 27-28.
\item \textsuperscript{245}Immature markets like those for electricity or weather are new fields for macro hedge fund managers, see Blackfish (2001) or Ahn et al. (2002).
\item \textsuperscript{246}See Ineichen (2000), pp. 42-43.
\end{itemize}
3 Hedge Fund Strategies

The resulting strategies can be assigned to the three different classes directional macro, long/short macro and macroeconomic arbitrage which are summarized in Table 3.13 on page 72. Expectations on economic variables that influence the prices of different assets are in the heart of directional macro trading. Positions (long or short) in various assets are set up in order to profit from anticipated (global) trends or movements in currencies, interest rates or emerging markets. These bets on future divergences are the central strategy for global macro hedge funds. Here are many intersections with managed futures strategies (see Section 3.2.10) as directional macro managers and managed futures traders often employ trend-following models that are based on macroeconomic data.

Long/short macro trading stands for simultaneous long and short positions in different assets. Based on macroeconomic views, the direction or development of such spreads is extrapolated. This strategy can be successfully implemented in trending and non-trending markets, a feature that directional macro trading cannot offer. So the long/short macro strategy is less correlated with the reference markets than pure directional bets.

In directional macro as well as in long/short macro trading, models are very important tools to identify opportunities in different markets. Simple statistical mean-reversion plays a central role in such macro models. Recent trading models apply more sophisticated macroeconomic modeling approaches in order to detect disequilibria in financial markets. The decisions on these trades are usually discretionary.

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247 According to HFR data for the first quarter of 2005, 11 percent of the hedge fund capital is invested in macro strategies. In the year 1990 HFR even reported a market share of 71 percent.
250 Often literature on hedge funds mentions this strategy as the only macroeconomic approach, see for example Lhabitant (2002), pp. 115-116.
but the timing of entry and exit dates is very often supported by technical analysis. Examples for specific long/short macro trades can be found in the area of currency options trading or in currency convergence trades.

The term macroeconomic arbitrage was introduced by Burstein (1999). Subjective macroeconomic views are no longer the fundament for this group of trades. Macroeconomic arbitrage is quantitative in nature and involves fundamental and technical analysis. Macroeconomic arbitrage is defined as identification and exploitation of macroeconomic mispricings. Such a mispricing can be the divergence of the market price ratio between different assets or groups of assets and a corresponding relation of influential macroeconomic factors. For example, the ratio of two sector indices of stocks and the ratio of two driving macroeconomic variables (like the gross domestic products (GDP) of two countries) can track each other for a long time. When a significant divergence in these ratios gets obvious, a macroeconomic arbitrage opportunity is detected. A correction or convergence in these relations to the long term equilibrium can realize a handsome profit. Besides long/short positions in related instruments the exploitation of inefficiencies can also lead to directional (long or short positions) bets. An example for such a trade is the development of an asset or stock index relative to a macroeconomic variable. When suddenly a divergence in the macroeconomic variable and the underlying price or index value occurs, there is also an opportunity for a long/short arbitrage trade.

As the mandates of macro hedge fund managers are very flexible we will not extract separate risk factors for this group of strategies. The central success factor is the

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254 See for example Ahl (2001) for a short introduction.
256 See Burstein (1999), pp. 74-75.
257 See Burstein (1999), p. 44.
258 See Burstein (1999), p. 50.
259 Not necessary on the same level but usually with a fixed difference.
260 See further examples in Burstein (1999).
strategy | description
---|---
directional macro | directional bets in different markets
long/short macro | bets on the spread of different instruments
macroeconomic arbitrage | long and/or short positions based on macroeconomic variable(s)

Table 3.13: Macroeconomic arbitrage strategies

ability of the manager to identify and make adequate use of trends or divergences.\textsuperscript{262} Besides the use of the right instruments, putting such strategies into action also requires strong timing abilities of the manager.\textsuperscript{263} The use of leverage (directly or through derivatives) is very common and usually extensive with macro strategies.

### 3.2.8 Short Selling

Short selling is an integral element of many hedge fund strategies.\textsuperscript{264} Most market neutral strategies or at least partially hedged strategies make use of this opportunity.\textsuperscript{265} But short selling is also constituting a self-contained investment discipline in equities or whole equity indices when dedicated-short hedge funds focus to profit from declining stock prices.

With short selling strategies a negative exposure to the market, an index and/or the price of a single stock is set up. There are various possibilities to attain such

\textsuperscript{263} See Strome (1996), p. 32.
\textsuperscript{264} For examples on other users and uses of short selling see Financial Services Authority (2002b), pp. 12-13.
\textsuperscript{265} See for example the equity hedge strategies in Section 3.2.1 on page 35.
a negative exposure like selling borrowed securities, selling (index or single stock) futures, buying put options or selling call options (see the trades in Table 3.14 on the following page.).\textsuperscript{266} However the main instrument for hedge funds is the physical short sale,\textsuperscript{267} consisting of the borrowing and sale of stock with the intention to buy the shares back at some future date at a lower price.\textsuperscript{268} Short selling faces various restrictions depending on the market place. In many emerging markets short sales are completely prohibited\textsuperscript{269} and major US exchanges apply the uptick rule (Rule 10a-1 of the Securities Exchange Act) which prevents short sales unless the stock’s last trade was at the same or higher price than the previous trade. Another point is the tax discrimination of capital gains from short selling as these profits are fully taxed.\textsuperscript{270}

In an equity short sale transaction the resulting position has an equity and a fixed income component and is therefore not the exact opposite of a long position in the respective stock.\textsuperscript{271} This is a result of the technical short selling process. As opposed to buying stocks, a short sale requires no investment or borrowed money but has to be collateralized, usually with liquid government bonds like US treasury securities. The revenue from the sold stocks is held in a restricted cash account earning an interest rebate.\textsuperscript{272} The total profit or loss of such a short sale transaction is influenced by four factors:\textsuperscript{273} the isolated result of the short stock position, the

\textsuperscript{266} See D’Avolio (2002) for an excellent description and analysis of the US securities borrowing (and lending) market.

\textsuperscript{267} See Angel et al. (2003) for an interesting analysis of short selling on the NASDAQ. The authors detected a higher frequency of short selling activity among stocks with high returns and among actively traded stocks.

\textsuperscript{268} The major tools for identifying short sale candidates are described in Section 3.2.1 on equity hedge strategies. See for example Apfel et al. (2001) for a brief illustration of short sales.

\textsuperscript{269} See Charoenrook/Daouk (2003) on short sale constraints and put options trading in different developed and emerging equity markets.


\textsuperscript{272} This is the so-called short rebate.
3 Hedge Fund Strategies

additional margin interest on this profit or loss,\textsuperscript{274} the dividend payments to the security’s lender\textsuperscript{275} and the short rebate earnings.\textsuperscript{276}

\begin{table}[h]
\begin{tabular}{|l|l|}
\hline
trade & description \\
\hline
short selling & physical short sales of overvalued stocks \\
future selling & selling stocks to a future date, selling index or single stock futures \\
equity option trading & long put options or put warrants, short call options or call warrants \\
total return swaps & synthetic short exposure to an index or stock \\
contracts for difference & short position in CFD (synthetic swaps) \\
\hline
\end{tabular}
\end{table}

Table 3.14: Trades involving explicit or implicit short sales\textsuperscript{277}

A negative exposure due to implicit or explicit short sales is set up with the trades listed in Table 3.14. Total return swaps pay total return on some reference asset with a life longer than the swap (for example a stock market index or single stocks). This includes interest, dividends or fees and capital gains/losses in exchange for floating interest payments. When a total return index (with reinvested dividends) is considered, the contract can also be categorized as equity swap.\textsuperscript{278} Contracts

\textsuperscript{273} The interest income on the collateral is not considered, as this profit can not be attributed to the short selling strategy. For an opposite view see Ringoen (1996).

\textsuperscript{274} Depending on the development of the underlying stock, cash is released from the restricted account, then earning a higher rate of return (declining stock price), or the restricted credit has to be increased through a margin loan at a higher interest rate (increasing stock price), see Ringoen (1996), p. 92.

\textsuperscript{275} See Schmittmann/Schäffler (2002) for tax implications concerning dividend payments under the German tax legislation.

\textsuperscript{276} See Ringoen (1996), pp. 90-94.


\textsuperscript{278} See for example Hull (2003), pp. 601-602 and 644-645.
for difference (CFD), also known as synthetic swaps, are equity derivatives allowing investors to participate in stock price, stock index or exchange traded funds (ETF) movements without buying and selling the shares themselves. The value of the contract is defined as a number of shares multiplied by the share price. Therefore, the profit or loss in CFD is the difference between opening and closing values for the contract. The central benefits of CFD are the exemption from the UK stamp duty that is levied on standard equity trades (this is the origin of the financial contract) and the relative ease of selling short.\footnote{See for example Mattinson (2002) and Connolly (2000).}

Share prices can fall to a value of zero but can (theoretically) reach an infinite value. This unlimited downside potential in combination with the passive portfolio strategy of backing losers and diminishing winners\footnote{In a long only portfolio the passive strategy would overweight winning and underweight loosing stocks.} requires active management of the short selling portfolio.\footnote{See Ringoen (1996), pp. 95-96.} Risk factors in short selling strategies with equity instruments are summarized in Table 3.15 on the following page. The main impact on the risk profile of short selling hedge funds has equity market risk. The exposure to the equity market development is negative and therefore momentum is a large risk factor to short sellers as overvalued stocks can continue to outperform.\footnote{See Ringoen (1996), p. 94, and Ineichen (2000), p. 45. In the bull markets of the 1990’s many short selling managers migrated to long/short or short biased equity hedge strategies (Section 3.2.1 on page 35), see Lhabitant (2002), p. 120.} Credit risk for the collateral is usually very low but the credit quality of the shorted company influences the outcome of a short sale. The hedge fund is short default risk as the trade will profit from the short position’s default.\footnote{See Ineichen (2000), p. 45.} Therefore, an upgrade in the company’s default risk will affect the short position in a negative way.

Liquidity risk plays an important role with short selling activities. When hedge funds focus on large capitalization stocks, liquidity risk is less severe than with small and

\footnote{See Ringoen (1996), pp. 95-96.}
mid caps as these stocks can be borrowed very efficiently.\footnote{See Ineichen (2000), p. 45.} But also with these intensely traded securities a short squeeze can occur for example in periods of rising prices when short sellers even boost demand in order to cover their positions.\footnote{See for example Financial Services Authority (2002b), p. 10.} The resulting high prices force short selling hedge funds to close their positions. In general, liquidity increases when prices go up, so short sellers usually have to cover their positions at higher prices in order to avoid (further) losses on their positions.\footnote{See Ringoen (1996), p. 97.} Another source of liquidity risk are active managers calling in the shares they were lending to the short sellers. This is sometimes called an “engineered” short squeeze.\footnote{This enables active portfolio managers to fight back against short sellers that band together against constituents of his portfolio, see for example Arulpragasam/Chanos (2000), p. 242.} Therefore, re-calling the borrowed stock in a physical short selling transaction is a particularly important risk factor for short selling funds as this might force the manager to close positions.\footnote{See Financial Services Authority (2002b), p. 10, and Ineichen (2000), p. 45.} Interest rate risk is a minor influence. A fluctuation of interest rates has some impact on the short selling as it could reduce the short rebate on the restricted cash account.\footnote{See Ineichen (2000), p. 45.}

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - equities</td>
<td>unlimited downside potential, momentum</td>
</tr>
<tr>
<td>market risk - interest rates</td>
<td>short rebate</td>
</tr>
<tr>
<td>credit risk</td>
<td>short position</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>risk of a short squeeze</td>
</tr>
<tr>
<td>event risk</td>
<td>recall of the borrowed stocks</td>
</tr>
</tbody>
</table>

Security selection and the general market environment are the key factors influencing returns of this strategy.\textsuperscript{291} In addition short selling requires very efficient stock borrowing facilities and timing skills.\textsuperscript{292} Diversification in short selling hedge funds is achieved by positions in a significant number of stocks.\textsuperscript{293} Specific influences to be diversified are for example exposures to certain industries, short-interest stocks, companies near bankruptcy or stock split candidates.\textsuperscript{294} Leverage could be created within the technical short selling process. As the securities in the collateral usually account for at least the value of the stocks borrowed (due to the legal structure of the borrowing), there is no real leverage involved in this transaction.\textsuperscript{295} When derivatives are used to set up, the short positions leverage might be generated depending on the margining regulations. In a portfolio context, short selling hedge funds are very often used to balance long-biased managers in fund-of-funds structures.\textsuperscript{296}

### 3.2.9 Emerging Markets

Emerging market hedge funds are grouped by a geographic relationship as they focus on some specific geographic areas but not on a narrow band of trades. The term “emerging” refers to economies as a whole, and in the hedge fund context it is based on criteria like a medium gross national income (GNI) per capita\textsuperscript{297}, future growth

\textsuperscript{293} Ringoen (1996) speaks of 100 to 200 stocks in order to provide effective diversification, see Ringoen (1996), p. 96.
\textsuperscript{295} See Financial Services Authority (2002b), p. 10.
\textsuperscript{296} See Saerfvenblad (2002).
\textsuperscript{297} The former measure for country classification used by the World Bank was gross domestic product (GDP) per capita. Nowadays the world bank ranks economies in four GNI per capita groups. The groups according to 2002 GNI per capita are: low income ($ 735 or less), lower middle income ($ 736 - $ 2,935), upper middle income ($ 2,936 - $ 9,075) and high income, (above $ 9,076), see World Bank (2004). The category upper middle income was previously known as the emerging markets class.
potential and the existence of a public securities market with reliable data. Most economies with such growth markets are located in Eastern Europe, Asia, Latin America or the Middle East.\textsuperscript{298} The focus of hedge fund managers is on exploiting inefficiencies in all types of securities. Reasons for these opportunities in emerging markets are political and economic by nature.\textsuperscript{299} The low interest from international investors is caused by political uncertainties and the low liquidity of many emerging market securities.\textsuperscript{300}

Hedge fund strategies in emerging markets have to compete with an increasing number of traditional long-only fund managers looking for high returns and/or diversification. The Fund Rating Agency Lipper currently monitors over 200 emerging market mutual funds, compared with about 80 emerging market mutual funds in 1995.\textsuperscript{301} So these markets are no longer an investment novelty, but the early 1990’s expectations of growing liquidity, falling volatility and low correlations with developed markets were not completely fulfilled.\textsuperscript{302} In the first quarter of 2005 HFR reportes a market share of emerging markets hedge funds of 3.5 percent.

Emerging market hedge funds take positions in emerging markets equity, corporate or government debt and/or in corresponding derivative securities.\textsuperscript{303} Opposed to macro hedge funds (see Section 3.2.7 on page 69), emerging markets managers do not take on large macroeconomic bets. Emerging markets hedge funds buy and sell financial instruments and try to hedge residual risk such as currency exposure or market risk. The portfolios of most emerging markets hedge funds predominantly consist of long positions. At least they exhibit a substantial long bias\textsuperscript{304} as most

\textsuperscript{298} See Lhabitant (2002), pp. 116-117.
\textsuperscript{300} See Speidell (2002), p. 5.
\textsuperscript{303} See for example Cottier (1998), p. 137.
\textsuperscript{304} See Ineichen (2003), p. 349.
of the emerging markets allow for only limited short selling and do not offer liquid futures contracts or other derivative instruments.\textsuperscript{305} So, hedge funds often have to move to OTC trades (like for example equity swaps) in order to hedge risk factors or set up negative exposures.\textsuperscript{306}

Emerging market funds employ a mixture of fixed income trades, convertible arbitrage and equity hedge trades with a regional focus. Therefore, the trades in this category very often overlap with the strategies mentioned before in sections on equity hedge (Section 3.2.1), convertible arbitrage (Section 3.2.2) and fixed income arbitrage strategies (Section 3.2.3). Some characteristic trades of emerging market hedge funds are summarized in Table 3.16.

<table>
<thead>
<tr>
<th>trade/strategy</th>
<th>description</th>
</tr>
</thead>
<tbody>
<tr>
<td>long/short countries</td>
<td>long and/or short positions in different countries</td>
</tr>
<tr>
<td>equity trades</td>
<td>equity hedge strategies or levered long positions</td>
</tr>
<tr>
<td>convertible trading</td>
<td>long undervalued convertible bonds</td>
</tr>
<tr>
<td>debt trades</td>
<td>broad spectrum of fixed income trades, high yield trading, credit arbitrage</td>
</tr>
</tbody>
</table>

Table 3.16: Emerging markets trades\textsuperscript{307}

It is obvious that emerging market hedge funds can be exposed to the full range of risk factors as the managers are not restricted to a certain asset class. Numerous risk factors in emerging market trading relate to the market environment. Emerging markets depend on capital inflows in order to finance their economic growth. If investors get more risk averse and withdraw their capital from emerging markets or stop depositing funds, this might lead to severe problems in these economies and


\textsuperscript{306}See Lhabitant (2002), p. 117.

the corresponding capital markets.\textsuperscript{308} Due to the common long positions in equities, hedge funds are often exposed to market risk. As the dependence structure in emerging markets is usually less stable than in developed economies, changes in correlation are another important (market) risk factor. Furthermore, the correlations among different stocks are significantly higher than in developed markets. This leads to smaller diversification benefits in emerging market equity portfolios.\textsuperscript{309} When the hedge fund trades debt instruments, the results are also sensitive to changes in interest rates. But the main risk factor in emerging markets fixed income trades is credit risk. Investors are usually long the issuer’s and the emerging economy’s default risk.\textsuperscript{310} As these emerging markets are usually less liquid than their counterparts in developed economies, liquidity risk plays a major role with emerging market hedge funds, too. Emerging market funds have long (biased) positions in inefficient emerging markets through the traded (less liquid or even illiquid) securities.\textsuperscript{311} They provide and enhance liquidity and are therefore subject to liquidity risk.\textsuperscript{312} Furthermore, political and legal uncertainties in emerging economies are sources of risk as subcategories of event risk. This event risk is high in many emerging markets hedge funds.\textsuperscript{313} The most important risk factors in emerging markets hedge funds are listed in Table 3.17 on the next page.

\textsuperscript{308} See Signer (2003), p. 31.
\textsuperscript{309} See Ineichen (2003), p. 349.
\textsuperscript{311} A further threat to the liquidity of emerging markets is the global trend to float weighted benchmarks as this will have a significant effect on capital allocation due to the small free float in emerging markets, see Speidell (2002), p. 5. For the market capitalization of (Asian) emerging markets in the MSCI World free float index see for example Gruenig (2002), p. 10.
\textsuperscript{312} See Ineichen (2003), p. 349.
### Risk Factors in Emerging Markets Hedge Funds

<table>
<thead>
<tr>
<th>Risk Factor</th>
<th>Description/Position</th>
</tr>
</thead>
<tbody>
<tr>
<td>Market Risk - Equities</td>
<td>Long or at least long biased positions, increasing volatility, correlation jumps</td>
</tr>
<tr>
<td>Market Risk - Interest Rates</td>
<td>Trading emerging markets debt</td>
</tr>
<tr>
<td>Credit Risk</td>
<td>Long the issuer’s and the country’s default risk</td>
</tr>
<tr>
<td>Liquidity Risk</td>
<td>Long market and mispriced securities</td>
</tr>
<tr>
<td>Event Risk</td>
<td>Political and legal risk</td>
</tr>
</tbody>
</table>

Table 3.17: Important risk factors in emerging markets hedge funds

### 3.2.10 Managed Futures

Managed futures funds take long and short positions in (liquid) futures and options contracts on financial instruments (currencies, interest rates, stock market indices) and various physical commodities. Some managed futures funds additionally trade OTC derivatives. Managed futures investments are sometimes referred to as derivatives funds, futures funds, managed derivatives or leveraged funds, and they are seen as a distinct asset class by some authors. The money managers are in most cases commodity trading advisors (CTA) or commodity pool operators (CPO) that are registered with the US Commodity Futures Trading Commission (CFTC) through membership in the National Futures Association or registered with the Securities and Futures Authority in the UK. Further managed futures vehicles are, for example, the so-called public commodity funds. CTAs manage single managed futures funds while CPOs coordinate commodity pools that are organized as private partnerships by selecting professional traders and/or CTAs. The registration of

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managed futures is a major difference to hedge funds as these are largely exempt from government regulations.

There is a natural link between several hedge fund strategies and managed futures trading. The link to emerging market hedge funds (see Section 3.2.9 on page 77) is established as currencies of the emerging markets segment are frequently traded in managed futures funds. Furthermore there are intersections with global macro strategies (see Section 3.2.7 on page 69) because managed futures trades also analyze macroeconomic information in order to anticipate the moves of certain markets or securities. Currency trading funds are also very similar to managed futures or global macro strategies. Both types of funds trade in foreign exchange markets employing currency spot and derivatives products.

The universe of managed futures strategies can be divided in three broad categories (see Table 3.18 on the following page). **Systematic trend-following managers** try to identify trends in prices using quantitative models. According to Lungarella (2002) and Lhabitant (2002) about 60 to 70 percent of managed futures funds follow such a systematic trend-based approach. They are usually active in very liquid markets with low transaction costs. The individual trades depend on the systematic application of models and the resulting signals to buy or sell. These decisions are often based on trendfollowing models employing momentum indicators, but counter-trend and spread models are also widely employed. The computerized trading systems often work with protective stops that are usually adjusted on a daily basis in order to close losing positions quickly. Although most trades from such an

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324 For an in-depth quantitative analysis of trend-following strategies see Fung/Hsieh (2001).
325 See Lungarella (2002), p. 11, and Lhabitant (2002), p. 120.
automated system are unprofitable, this stop loss strategy is able to produce attractive returns as the managers is only keeping at profitable trades.\textsuperscript{328} Discretionary managers or funds make use of fundamental analysis and/or superior information. Very often these funds focus on a market or sector where the manager has extensive experience. A discretionary trader has to be familiar with the factors that can affect prices in the particular market. In addition to knowledge about such fundamentals for this market and the changing relationship of demand and supply, managed futures traders often make use of technical trend-models to determine the optimal timing for entry or exit.\textsuperscript{329} A third passive futures strategy is introduced in Jaeger et al. (2000). The strategy tries to capture the premium commercial hedgers are willing to pay for insurance on different markets. When the manager identifies trends, he offers supply in hedging instruments matching the demand of potential investors. Therefore, these passive strategies usually depend on long-term trendfollowing models.\textsuperscript{330}

<table>
<thead>
<tr>
<th>strategy</th>
<th>description</th>
</tr>
</thead>
</table>
| systematic trend-following      | technical trend-, countertrend-
|                                 | or spread-based trading                           |
| discretionary strategies        | fundamental (and technical) approaches in special segments |
| passive futures strategies      | capturing hedging premia                          |

Table 3.18: Managed futures strategies\textsuperscript{331}

As the trades of systematic trend-following and discretionary traders are ranging from short term trend- or countertrend-following over directional volatility trading

\textsuperscript{328} See Jaeger (2002), p. 97.
\textsuperscript{330} See Jaeger et al. (2000) and Jaeger (2002), p. 102-106.
to currency arbitrage, these trades are subject to a very diverse set of risk factors.\(^{332}\)

Market risk moves are the dominant risk factor with most managed futures trades since they usually have a significant exposure to different markets. But especially in non-trending market environments with frequent price reversals such trend-following strategies are problematic because the implemented models need persistent price trends to be profitable.\(^{333}\) In general, managed futures tend to perform well under extreme (trending) market conditions.\(^{334}\) Liquidity risk is also an important issue with managed futures as the funds provide liquidity to different derivatives markets. This risk premium is an important part of the fund’s return for long-term futures strategies.\(^{335}\) For systematic trend-following managers, model risk is an important component of the funds risk profile. As the structure and dependencies of financial markets continuously change the quantitative models that have been fitted (and probably over-fitted) to a historic data set and market environment must be adjusted on a regular basis.\(^{336}\) Finally, event risk has to be considered with managed futures. Especially with discretionary trading strategies, the impact of external economic or political decisions on the fund’s returns can be devastating. These risk factors are aggregated in Table 3.19 on the next page.

### 3.2.11 Closed-end Fund Arbitrage

Closed-end fund arbitrageurs try to capitalize on the difference between the net asset value and the trading price of closed-end fund shares.\(^{338}\) A closed-end fund

\(^{332}\) See Jaeger (2002), pp. 97 and 100, for examples of trend-following and discretionary trades.


\(^{334}\) See Harding/Nakou (2004), p. 64.


\(^{337}\) See Jaeger (2002), pp. 100-106.
Table 3.19: Important risk factors in managed futures

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk</td>
<td>long or short positions in commodities or financial instruments</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>managed futures are liquidity providers</td>
</tr>
<tr>
<td>model risk</td>
<td>flawed or unsuited trading models</td>
</tr>
<tr>
<td>event risk</td>
<td>economical and political events</td>
</tr>
</tbody>
</table>

is one of the basic types of investment companies and is registered under the US Investment Company Act of 1940. In Section 5 of the Investment Company Act, a closed-end company is defined as management company other than an open-end company. The fund’s capital is mainly raised through an initial public offering with a fixed number of shares. The limited number of outstanding shares is the main difference to open-end funds where the quantity of shares constantly changes as they can be issued or redeemed at the fund’s particular net asset value. The number of closed-end fund shares can in fact change for some specific reasons, for example if the fund offers rights to purchase additional shares, if the fund buys-back and retires shares or if the fund issues shares in a dividend reinvestment program. If the fund trades at a premium, there is also the possibility of secondary stock offerings. Therefore, closed-end fund shares have to be traded on the secondary market, where no instantaneous arbitrage mechanism is in place in order to adjust

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338 See Lhabitant (2002), p. 95. Net asset value arbitrage in open-end funds is often connected with illegal or at least unwanted trading practices like late trading (prohibited by the Martin Act and SEC regulations (rule 22c-1)) or timing strategies and is therefore not subject to this section. For an insightful investigation concerning the Canary hedge fund see Supreme Court of The State of New York (2003) and for the consequences of timing activities in mutual funds see for example Zitzewitz (2003). Zitzewitz (2003) estimates a dilution to long-term mutual fund shareholders of over $4 billion per year caused by timing arbitrage.


the fund’s price to its intrinsic value. So, market forces determinate the closed-end share price, which may be greater or less than the shares’ net asset value, depending on the relation between demand and supply.

Closed-end funds invest in equity, convertible or fixed income securities with the whole spectrum of different investment philosophies. In 2002, the closed-end fund industry included over 500 funds with approximately $150 billion in assets. Despite of some advantages of closed-end over open-end funds, a large part of these closed-end funds trade at a discount to their net asset value.

The core of an arbitrage strategy with closed-end funds is about identifying closed-end funds trading at substantial discounts to their net asset value, whose underlying portfolio can be easily hedged in order to attain market neutrality. Abnormal discounts (and premiums) represent market inefficiencies. The identification of real mispricings is based on a thorough analysis of the fund, examining the fund’s holdings, dividend yield, performance and management, expenses, number of shareholders and its historical discount (or premium). Trading strategies with closed-end funds usually result in long positions of closed-end funds under, at least partial, hedging of the fund’s assets. To a very large extent, such strategies focus on timing as buying closed-end funds is especially promising in some situations. Deep dis-

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341 The types of closed-end funds enclose municipal bond funds (54 percent), taxable bond funds (17 percent), international equity funds (12 percent), international bonds funds (8 percent) and US equity funds (9 percent), see Closed-End Fund Association (2002), p. 12 (data: invested capital according to Wiesenberger, a Thompson Financial Company, July 2002). See Fredman/Scott (1991) for a review of different fund types.

342 See Gabelli (2002).

343 Advantages of closed-end funds are for example lower marketing and distribution expenses, the possibility of using leverage and a stable pool of capital with lower cash flow risk allowing for long term investments and hight total return. Disadvantages can be seen in the premium at the shares public offering, the lack of liquidity in some funds’ secondary market or in the increased risk due to leverage, see Fredman/Scott (1991), Bush (2003), Maier/Brown (2000) and Closed-End Fund Association (2002).

counts are offered in generally oversold markets or at the end of the year when other investors realize their potential tax losses in funds. Special situations in a fund’s life could eliminate the common discount in trading prices. Such developments are for example a share repurchase, the replacement of managers, the (forced) conversion to an open-end fund or the liquidation of the whole fund.\footnote{See Fredman/Scott (1991), pp. 381-387.} Even if no immediate or complete correction in the closed-end fund’s price occurs, the income dividends or capital gains distribution with these undervalued closed-end funds is at least higher in relative terms (as they are paid on the assets as a whole).\footnote{See Fredman (2000) and Closed-End Fund Association (2002).}

With the historical observation that most funds do not change between discount and premium, other long/short strategies with closed-end funds have been developed. Long positions in high discount funds while shorting high premium funds is betting on (slight) reversals in the premium or discount of closed-end funds. Under the assumption that the market offers a premium for good and a discount for bad performing funds, buying closed-end funds with a low premium and shorting funds with a low discount is an appropriate strategy.\footnote{See Dauenhauer et al. (2000) for an empirical study on trading strategies.}

<table>
<thead>
<tr>
<th>risk factor</th>
<th>description/position</th>
</tr>
</thead>
<tbody>
<tr>
<td>market risk - equities</td>
<td>usually hedged</td>
</tr>
<tr>
<td>market risk - interest rates</td>
<td>usually hedged</td>
</tr>
<tr>
<td>liquidity risk</td>
<td>small issues show low liquidity</td>
</tr>
<tr>
<td>event risk</td>
<td>management opposition, anti-takeover provisions</td>
</tr>
</tbody>
</table>

Risk factors in closed-end fund arbitrage are summarized in Table 3.20 on the preceding page.\textsuperscript{349} Market risk emerges with equity and fixed income closed-end funds but hedge fund managers attempt to hedge both risk components. The volatility of closed-end prices is usually higher than the volatility of comparable mutual (open-end) funds as they will almost never trade at their net asset value. In addition, the leverage amplifies the volatility. As closed-end funds are exchange-traded, liquidity is not a major risk factor for most large funds, but some small sized issues do not trade significantly imposing a greater extent of liquidity risk.\textsuperscript{350} Legal risk with closed-end trades references to the opposition of the closed-end fund managers in general and the specific anti-takeover provisions for most funds. Funds that are relatively easy to convert to open-end funds usually trade at narrower discounts than their harder to open pendants with a larger amount of this legal risk.\textsuperscript{351}

### 3.2.12 Depository Receipts Arbitrage

Arbitrage with depository receipts (DRs) comes close to the real meaning of (riskless) arbitrage, the simultaneous buying and selling of a security at two different prices in two different markets. As depository receipts arbitrage involves positions in equity instruments, it could also be classified among equity hedge strategies (see Section 3.2.1 on page 35).

A DR is a physical certificate evidencing ownership in a specified number of depository shares from a company outside the market in which the DR is traded. Depository shares represent several underlying shares that are deposited with a custodian bank in the issuer’s home market. The main advantages of depository receipts are that there is no currency conversion in trading and in receiving dividends and

\textsuperscript{349} See Closed-End Fund Association (2002) or Fredman/Scott (1991) for further details on investment risk with closed-end funds.


\textsuperscript{351} See Fredman/Scott (1991), pp. 390-391.
they help in minimizing higher overseas transaction costs and custodial fees. They are frequently identified by the markets in which they are available. DR that are primarily available to US investors are called American depository receipts (ADRs), privately placed DR are named Rule 144A ADRs and global depositary receipts (GDRs) are offered in two or more markets outside the issuer’s home country. GDRs are usually listed on the London or the Luxembourg stock exchange, while ADRs are listed on US exchanges. For investors, DRs provide an alternative to investing in foreign equities directly without the inconveniences such as currency conversion and foreign settlement procedures.

Taking into account the current exchange rate, these depository receipts’ trading price sometimes differs from the value of the underlying shares in the company’s home or some other market. Explanations for gaps in the trading prices are transaction cost and non-synchronous trading times between the different markets. In these situations a hedge fund’s manager will bet on the convergence of the prices with long and short positions in these securities. The two-way fungibility (DRs can be converted into underlying shares and foreign shares can be re-converted into DRs) offers another direct arbitrage possibility. An investor, who compares the DR price with the price of the underlying share, can lock in a profit if the price differential (in both directions) is sufficiently large to cover transactions costs.

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352 Rule 144A of the Securities Act of 1933 regulates private resales of securities.
354 See for example Alaganar/Bhar (2001), p. 98.
355 Even though the depository receipts are dollar denominated they do not eliminate currency risk.
358 See for example Citibank (1999), pp. 4-7.
359 For a detailed analysis of tax effects and tax-induced trading activity with ADRs see Callaghan/Barry (2003).
Hedge fund managers following Regulation D or so-called PIPE strategies make private placements in public companies in need of financing. These private equity investments that are placed with small capitalization companies, are very often directly negotiated and usually take the form of equity issues, debenture issues, convertible issues or the issue of warrants in return for the capital allocation. In a direct equity issue, the hedge fund will purchase the company’s stock at a discount with respect to the market price or to an anticipated market price. Convertibles can be issued with a floating strike price or subject to a look-back provision, making the private placement market neutral. The convertible securities or debentures with the right to be exchanged into common stock are usually issued at a discount or bear high coupons. Hedge fund trades involving Regulation D securities consist of long positions in the private placements. In the case of fixed exercise prices for convertibles, the stock’s market risk is often hedged with short positions in the underlying shares.

Regulation D trades are profitable because of the discount in the issued securities. This discount is offered in exchange for the risk factors an investor has to bear. The key risk factor with Regulation D debenture trades is default risk, if the issuer is unable to meet it’s principal or coupon obligations. During a limited period of time after the securities’ issuance they are only tradeable among accredited investors until the registration statements are filed with the SEC. The liquidity risk

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360 The Regulation D is part of the Securities Act of 1933 since 1982 and offers an exemption from registration requirements for private placements.
361 PIPE stands for Private Investments in Public Equities.
362 Most US hedge funds make use of Regulation D themselves when placing their securities directly to individual investors, see Lhabitant (2002), p. 119.
363 See for example Jaeger (2002), p. 73.
366 This period ranges from 75 to 180 days, see Jaeger (2002), p. 74, and Lhabitant (2002), p. 120.
with these securities is especially high in this pre-registration period when there is no market to liquidate the position.\textsuperscript{367} As the issue’s capitalization very often represents a significant part of the firm’s overall market capitalization, the securities are also subject to post-registration liquidity risk as a consequence of the market impact.\textsuperscript{368} Equity market risk is a relevant risk factor for convertibles with a fixed exercise price, but if the underlying stocks are available, this market risk is usually hedged.

3.3 Fund of Hedge Funds

To screen the hedge fund universe, to select and monitor managers is a difficult and quite time consuming task for investors. The evaluation process is very complex and usually a lot of experience is necessary to analyze the organizational structure of hedge funds and the implemented trading strategies. At this point funds of hedge funds can be an interesting investment opportunity as they provide an alternative access to hedge funds.\textsuperscript{369} A fund of hedge funds manager blends funds from the same strategy group or funds that pursue different strategies in order to diversify over risk factors and unsystematic risk. Fund of hedge funds therefore allow investors to obtain an instant exposure to a diversified hedge fund portfolio.

According to hedge fund databases, around 25 percent of the reporting funds were classified as fund of hedge funds.\textsuperscript{370,371} The question is whether fund of hedge funds managers are really able to add value for the investor. Finding the right or best fund

\textsuperscript{367} See Jaeger (2002), p. 77.
\textsuperscript{368} See Jaeger (2002), p. 77.
\textsuperscript{369} See UBS (2004), pp. 17-18, for recent figures on the fund of hedge funds industry.
\textsuperscript{370} As of the first quarter of 2005 according to HFR.
\textsuperscript{371} It is interesting to note that the fund of hedge fund universe is very concentrated. The 20 largest funds of hedge funds have a market share of about 80\%, see Mercer Oliver Wyman (2005), p. 17.
of funds manager is still a challenging task, but the fund of hedge funds universe is much smaller than the universe of hedge funds. At first we will examine some advantages and disadvantages of fund of hedge funds. The major benefits and drawbacks of fund of hedge funds are summarized in Figure 3.3. Then we will take a closer look at the investment process for hedge funds that is important for the understanding of the fund of hedge funds management and the selection of hedge funds in general.

Figure 3.3: Advantages and disadvantages of funds of hedge funds

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Disadvantages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficient Allocation and Diversification</td>
<td>Double Fees Structure</td>
</tr>
<tr>
<td>Risk Control and Return Predictability</td>
<td>Lack of Control</td>
</tr>
<tr>
<td>Access to Closed Funds</td>
<td>Dependence on Other Investors</td>
</tr>
<tr>
<td>Smaller Minimum Investment</td>
<td></td>
</tr>
<tr>
<td>Accounting and Reporting</td>
<td></td>
</tr>
</tbody>
</table>

3.3.1 Advantages of Funds of Hedge Funds

Several advantages of fund of hedge funds can be identified. Some of these advantages are close to the arguments for rather investing in mutual funds than in single stocks. But we will also introduce some benefits of fund of hedge funds that deal with the characteristics of the hedge fund industry.

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See for example Stemme/Slattery (2002), p. 66-68, for the investment process with funds of hedge funds.
Investors try to diversify their hedge fund exposure and allocate money to several funds and different hedge fund strategies. An argument for funds of hedge funds is that they are able to provide **efficient allocation and diversification** in a large number of hedge funds. The selection of hedge funds by fund of funds managers is usually much more efficient as fund of hedge funds constantly screen the large hedge fund universe and provide standardized due diligence procedures. The same holds true for the ongoing monitoring process with target hedge funds. Additionally, fund of hedge funds managers provide investors with their expertise in optimizing hedge fund allocations. Due to their experience, most fund of hedge funds managers should be able to estimate return distributions and return dependencies more accurately than the individual investor. Besides the selection, optimization and monitoring of hedge fund allocation, fund of hedge funds managers also simplify the process of allocating money to target funds. In the process of changing the exposure to a diversified portfolio of hedge funds, a fund of hedge funds investor does not have to deal with the different investment or subscription policies of every single target fund. The same holds true for the disinvestment process. In contrast to an investment in the same number of target funds, a fund of hedge funds investor faces only one redemption clause.

**Risk control and return predictability** are important reasons for fund of hedge funds investments. Active management of the fund of funds should provide risk con-
and ensure capital preservation to conform with the absolute return approach of hedge fund investing (see Section 2.3.3). In addition, the results of fund of hedge funds are usually more stable over time and therefore better to predict than the returns of single funds.\footnote{See for example Jaeger (2000) for a general discussion of risk measurement in fund of hedge funds.}

Capacity constraints are common in the hedge fund industry (see Section 2.3.6 on page 23). Especially successful hedge fund managers close their funds to new investors when they reach a certain size.\footnote{See Ineichen (2000), p. 94.} Here fund of hedge funds could again offer benefits to investors as they can provide \textbf{access to closed funds}. If the fund of hedge funds is already invested in a closed fund, an allocation in the fund of hedge funds implies an exposure to the desired hedge fund.\footnote{See Section 3.1.1 on page 29 for more details on size risk.} In the case of a so-called “soft close”, single hedge funds are officially closed to new investors, but still accept capital from high-quality investors with a long-term focus and a sizeable commitment.\footnote{An important consequence is a dilution effect for existing investors, as the fund of hedge funds manager is unable to allocate the capital inflow in the desired hedge fund in order to keep its weighting constant. Therefore, fund of hedge funds are often closed when their core underlying funds are closed, see for example Lhabitant (2002), p. 200.} Here a fund of hedge funds manager might be able to allocate capital while the requested amount is too large for an individual investor.\footnote{See Ineichen (2003), p. 409, and Brown et al. (2004), p. 1.}

In addition, funds of hedge funds provide diversification at a \textbf{smaller minimum investment}.\footnote{See Lhabitant (2002), p. 200.} The minimum investment for fund of funds is usually only a fraction of the minimum investments for single hedge funds.\footnote{Investors can not take this advantage of funds of hedge funds in so-called “private fund of hedge funds” products that are customized for the needs of the individual investor, see Hennessee/Gradante (2002).} For an investor who composes a portfolio with sufficient diversification, the minimum investment is multiplied and

\footnote{See Fothergill/Coke (2001), p. 12.}
therefore even less attractive.\textsuperscript{389}

**Accounting and standardized reporting** are further operational aspects that advantage fund of hedge funds.\textsuperscript{390} The fund of hedge funds investor does not have to collect and aggregate data from the target funds which is quite cumbersome and time consuming. Usually there are systems in place to ensure a standardized reporting from target funds to the fund of hedge funds manager. This allows funds of hedge funds to report on different levels of aggregation (for example by position, strategy or risk factor allocation).\textsuperscript{391} The information that is needed by investors can therefore be delivered in time. Standardized (monthly) reports usually include figures and ratios on the performance development for the fund of hedge funds and the target fund level (net asset values and risk/return figures), a description of the current situation and an outlook for the funds and the market environment.\textsuperscript{392} Some sophisticated funds of hedge funds managers even offer web-based access to allow their investors to monitor the portfolio construction.\textsuperscript{393}

### 3.3.2 Disadvantages of Funds of Hedge Funds

Besides the advantages of funds of hedge funds that have been outlined in Section 3.3.1, there are also a few drawbacks of a fund of hedge funds structure that investors have to consider.

The **double fees structure** is often seen as a major disadvantage of fund of funds structures.\textsuperscript{394} The service and the advantages of a fund of hedge funds management

\textsuperscript{389} An investor that is diversifying across 20 single hedge funds with a minimum investment of $500,000 each, would need at least an amount of $10,000,000 for a diversified hedge fund allocation.
\textsuperscript{390} See Mahadevan/Schwartz (2002), p. 49.
\textsuperscript{391} See GlobeOp (2003), p. 70.
\textsuperscript{393} See Wilmot-Smith (2003), p. 66.
comes at a certain price. A fund of hedge funds imposes additional management fees and usually also performance fees. These fees are therefore charged twice, on the target fund and on the fund of funds level.\textsuperscript{395,396} The degree of technical knowledge that is required to properly understand and evaluate individual hedge funds is sizeable.\textsuperscript{397} Therefore, this second level of fees can be compared to cost savings in order to determine if a value is added by fund of hedge funds managers.\textsuperscript{398} Empirical research in this field determines a significant underperformance (absolute and risk-adjusted) of funds of hedge funds compared to portfolios of single hedge funds that can not be explained with research costs or biases.\textsuperscript{399}

**Lack of control** over the actual allocation is another problem with fund of hedge funds structures. The exposures to target funds are solely determined by the management of the fund of hedge funds. An individual investor is (usually) unable to influence the strategy or target fund allocations. Therefore, the fund of hedge funds manager could for example miss an interesting fund in the search process, screen too restrictively or overlook qualitative risk factors in target funds.\textsuperscript{400} Selling or redeeming the stake in the fund of hedge funds would be the only way to change the investors risk profile. Moreover the fund of hedge funds management itself has only limited control over the trades of target funds. Therefore, problems like the cancellation of trades in the fund of hedge funds portfolio (because of long and short positions in different target funds) or the duplication of trades in target funds might

\textsuperscript{395} See for example Barra Rogers Casey (2001), p. 12.
\textsuperscript{396} There are also some other streams of income for a fund of hedge funds: If the target fund shares management or performance fees with the fund of funds (retrocession agreement), if the fund of funds forces the target fund to use its clearing broker usually a commission is paid (kickback), if a target fund offers a trailing fee when a fund of funds remains invested after the lock-up period. See Lhabitant (2002), p. 202, and Fothergill/Coke (2001), p. 13.
\textsuperscript{399} See for example Brown et al. (2004), pp. 8-13, and Liang (2003).
\textsuperscript{400} See Mahadevan/Schwartz (2002), p. 49.
come up.\textsuperscript{401}

The \textbf{dependence on other investors} is a disadvantage especially for allocations in smaller funds of hedge funds. Because of the advantageous redemption policies (in comparison to the target funds, see Section 3.3.1), the fund of hedge funds management has to maintain a higher level of liquidity.\textsuperscript{402} This allocation (money market account) does not offer the same return to the investor as is expected from the hedge fund portfolio. Even worse consequences for investors might arise if the fund is forced to liquidate target fund positions to meet redemptions as this destroys performance.\textsuperscript{403} The problem of this dependence on other investors could be avoided if the fund of hedge funds gets listed on a secondary market.\textsuperscript{404,405}

\section*{3.3.3 Hedge Fund Selection}

As already mentioned, selecting hedge funds is a challenging task. For fund of hedge funds as well as for direct hedge fund investors, a structured investment approach is essential. This section describes the investment process for hedge funds that is usually implemented in fund of hedge funds structure to avoid at least some of the industry specific risks outlined in Section 3.1.1.\textsuperscript{406} Figure 3.4 on the following page introduces an idealistic investment process which includes the most important components concerning the investment process and the selection criteria for hedge funds.

The \textbf{hedge fund universe} sets the frame for searching hedge fund managers. The universe has grown substantially over the last years and is estimated to contain

\begin{itemize}
\item \textsuperscript{401} See Lhabitant (2002), p. 205.
\item \textsuperscript{402} See Lhabitant (2002), pp. 203-204.
\item \textsuperscript{403} See Spitz (2001), p. 61.
\item \textsuperscript{404} See Fothergill/Coke (2001), p. 12.
\item \textsuperscript{405} See for example ABN AMRO (2004), for an overview on exchange-listed funds of hedge funds.
\item \textsuperscript{406} See Seides (2002) for an interesting discussion of risk transparency in hedge funds.
\end{itemize}
about 6100 single hedge funds. The problem is to identify possible target fund managers as they are not allowed to promote their funds in public. Nevertheless there are many sources of information on hedge funds.

![Hedge Fund Selection Process Diagram](image)

**Figure 3.4: Typical hedge fund selection process**

Publicly available hedge fund databases (see Section 4.1 on hedge fund databases) are a starting point for the hedge fund search. But none of these databases can offer the complete universe of hedge funds as the registration and reporting happens on a voluntary basis. Additionally, many top hedge funds do not report to any of the commercial data providers. Therefore, investors, fund of hedge funds managers

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408 See for example Lhabitant (2002), p. 186, for a related figure.
and consultants usually extend databases from several information providers by actively searching for new hedge funds. Manager and fund names are gleaned from different sources like for example industry contacts, industry journals, recommendations, conferences and seminars or even cold calls from hedge fund managers.\textsuperscript{410,411} Such an ongoing \textbf{screening process} is usually followed by a pre-selection of funds according to some quantitative and qualitative \textbf{minimum requirements}.\textsuperscript{412} Filtering according to criteria like fund strategy (especially when a fund of hedge funds is limited to certain strategies), minimum assets under management, fund domicile and management experience leads to the \textbf{pool of hedge funds} that a potential investor examines in greater detail.\textsuperscript{413}

Before individual funds are selected from such a reduced database, the investor has to determine his \textbf{views} on the economic development in general, on the market environment or even on individual risk premia. This is a very important part of the hedge fund selection process as it predetermines the strategy allocation. The second step of such a top-down approach is therefore to analyze the attractiveness of certain strategies or strategy groups in the anticipated market environment or in stress scenarios.\textsuperscript{414} Here favorable and unfavorable market conditions and their consequences on returns and the dependence structure of different strategies are examined. Like the screening of the hedge fund universe, this is an ongoing process, and major changes in the market environment should directly influence the portfolio allocation.\textsuperscript{415}

Based on the investor's assessment of different markets, further quantitative and

\textsuperscript{411} Some skilled investment managers are even identified and tracked before they actually launch a hedge fund, see Ineichen (2003), p. 440.
\textsuperscript{412} See for example Périsse (2004), p. 17.
\textsuperscript{413} See Lhabitant (2002), p. 185.
\textsuperscript{414} See Jaeger (2002), p. 236.
qualitative analysis is carried out in order to condense the manager pool and arrive at the so-called hedge funds short list.

**Quantitative** hedge fund analysis relies on historical time series of net asset values. It is therefore usually standardized and not too costly. In order to reveal significant information behind the time series, the statistical evaluation of net asset values needs a certain length of a fund managers track record. For very short histories of net asset values or returns this quantitative part of the analysis should not be overly weighted. The time series of returns are usually analyzed on a stand alone basis in order to obtain descriptive statistics and risk measures like volatility or maximum drawdown. But more important is the analysis of hedge fund returns in the context of market indices, risk factors and the returns of strategy benchmarks. The fund’s returns in different market situations might reveal interesting results especially when the desired diversification in a broader portfolio context with bonds and stocks tends to break down in extreme market situations. The analysis of risk factors and their explanatory power for fund returns is another important point. In a first step the size and the stability of the fund’s exposures to risk factors can be analyzed over time and used for a peer group comparison. The second step of such a risk factor analysis is to detect the manager’s ability to produce excess returns after the adjustment for risk premia. This analysis of risk premia and the corresponding “alpha” is very important to identify really skilled managers.

As quantitative analysis is a backward-looking process based on historical performance, investment decisions should also consider **qualitative aspects**. These qualitative aspects are a pivotal part of the due diligence process. Sources of information:

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416 See Lhabitant (2002), p. 188.
418 See for example Ang/Chen (2002), Campell et al. (2002), Körberg/Lindberg (2001), and Longin/Solnik (2001), on conditional correlations. The problems with correlations are also addressed in Section 5.2.3 on page 184.
419 See Lhabitant (2002), p. 188.
tion are primarily the offering memorandum of a fund (stating information about the fund’s key people, fees, redemption periods, restrictions on leverage, disclosure policies), annual reports, marketing material, hedge fund databases and simple questionnaires that are sent to the fund management.420 The focus of qualitative analysis at this stage is to obtain a deeper understanding of a hedge fund’s returns and the fund’s investment approach in general. In order to determine the comparative advantage of a hedge fund manager, potential investors usually search for one of the following reasons: better information, better analysis of information, better portfolio implementation, and/or (particularly in private markets) access to proprietary investments.421 Furthermore the quantitative side is supplemented by analyzing the implementation of trades and by ensuring the validity of the track record.422

The result of the combination of investor’s views and the qualitative and quantitative analysis is the so-called short list. The remaining managers on this short list are contacted and monitored on a regular basis but are at first subject to the heart of the hedge fund selection process: the due diligence. The due diligence process can be described as a more comprehensive and more thorough form of a qualitative analysis. The sources of information in such a detailed analysis are due diligence questionnaires, visits to the hedge fund’s offices and interviews with the management team.423 At this point the fund of funds manager tries to rule out most of the industry-specific risk factors outlined in Section 3.1.1.

The qualitative due diligence process can be structured in many different ways. Figure 3.5 distinguishes between four main fields for the due diligence.

As a starting point the manager due diligence takes a look at the investment process and the strategy of the fund. The fund itself has to be considered as well and the

422 See Lhabitant (2002), p. 188.
analysis of the fund’s legal structure is another important point. Here the terms of the hedge fund are in the focus of the due diligence. A sound risk management system might play a vital role for the hedge fund investment, therefore the portfolio constraints and the risk monitoring are also subject to thorough investigation in the process of a due diligence. Finally the infrastructure of the fund, a more technical matter, is also examined. The investor should assure that the implemented infrastructure does not impose any constraints on the fund’s development.\footnote{See for example Jaeger (2002), pp. 237-246, for a very detailed description of the due diligence process.}

The results of the due diligence are usually aggregated in a scoring system which allows the ranking of funds on the short list. A formal report should document the resulting decision to invest or not to invest or certain recommendations.\footnote{See Lhabitant (2002), p. 192.} The due diligence information is kept up to date with monthly performance reviews and

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Figure 3.5: Important topics in a hedge fund due diligence\textsuperscript{424}
management contacts. Usually the broad-based due diligence process is updated every year. If a fund has been selected and added to the portfolio, an ongoing monitoring process should be started. This permanent due diligence should support the decision making on the question whether the investment should be left unchanged, more capital should be committed or assets in whole or part should be redeemed.

3.4 Summary

In this chapter a broad range of different hedge fund strategies was introduced. For a useful analysis of hedge fund strategies an investor has to distinguish industry-inherent and strategy-specific risk factors. While almost every hedge fund bears at least some of the industry-inherent risk factors, the risk profile of hedge funds considering strategy-specific risks is very different. Therefore, we outlined the most important trading strategies and the corresponding range of risk factors for a broad variety of strategies.

Furthermore, funds of hedge funds were introduced. These funds allow investors to profit from an already diversified portfolio of several single hedge funds. We described the most important advantages and disadvantages of these investment vehicles and illustrated a typical fund selection process.

Altogether, this chapter was focused on qualitative aspects of hedge fund investments. In order to make some statements on hedge funds that are based on actual data the next chapter will take a look at the world of hedge fund indices.

4 Description and Analysis of Hedge Fund Data

This chapter gives a general review on hedge fund data. At first most of the different database providers and the corresponding hedge fund indices are introduced. Most of the hedge fund index universe is described in detail in this section. Biases in the resulting time series are the object of Section 4.2. As hedge fund databases will never be able to mirror the whole hedge fund population, we have to consider systematic biases in hedge fund data. As these biases will affect the indices that are calculated on database samples.

After the description of data bases and the corresponding problems with biases we will focus on the analysis of some of the resulting index series. Because of different problems and biases fund of hedge fund data will be used to proxy the performance of hedge fund investments. In Section 4.4 the statistical properties of fund of hedge fund index data including unconditional return distributions and time series behaviour will be examined in greater detail.

4.1 Hedge Fund Databases and Index Providers

In the following the main providers of hedge fund databases and indices are described in more detail. The next section gives a brief overview on desirable properties of
hedge fund indices while Sections 4.1.2 and 4.1.3 contribute information on the index providers and the index methodology, respectively.¹

### 4.1.1 Desirable Properties of Hedge Fund Indices

In this section we will briefly describe the usage of hedge fund indices and desirable properties for these indices. Hedge fund indices serve as benchmarks for hedge fund investments.² Only if such a benchmark for relative comparisons is available, the risk adjusted alpha of a hedge fund investment can be determined.³ Therefore, hedge fund indices provide a service in measuring performance. In order to allow “passive” investments in this part of the alternative investment universe, indexing is also necessary. With diversified index products investors could achieve a broad-based, efficient and (relatively) low cost hedge fund exposure.⁴ Furthermore indices clarify hedge fund strategies and the corresponding risk and return characteristics. In general indices are able to enhance the hedge fund industry’s transparency.⁵

There is a variety of desirable properties for hedge fund indices.⁶ The construction of hedge fund indices should consider most of these competing objectives in order to obtain a useful benchmark. A summary over these benchmark objectives and the corresponding problems within the hedge fund industry is found in Figure 4.1 on the following page.

A first desirable attribute for a hedge fund benchmark is the completeness of such

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¹ The information on hedge fund database providers and the calculated indices used in the following sections was collected from the internet, official documents and company representatives.

² Other market indices (like for example stock market indices) are not able to capture the nature of hedge fund investing.

³ See for example Zask et al. (2004).

⁴ See Kohler (2003), p. 5.

⁵ See Zask et al. (2004).

⁶ See for example Ruckstuhl et al. (2004) for an overview on construction requirements.

Figure 4.1: Objectives and problems of hedge fund benchmark construction

an index. The index should represent the hedge fund universe accurately. In the index all of the available instruments within the hedge fund industry should be represented directly or the index should at least approximate them. This full coverage is essential for hedge fund indices. There are various problems with this demand for representativeness. At first there is a self-reporting bias in hedge fund data, as some managers will not submit data to index providers. Multi-strategy funds are also a problem, as they are difficult to classify in the set of available hedge fund strategy groups. So they are often not included in the index-calculations. Finally a representative index should comprise hedge funds that are open to new investors and closed hedge funds that do not accept further capital. These closed funds are still active hedge funds but they are not investable and therefore they violate the investability objective of hedge fund benchmark construction.

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8 In Amenc/Martellini (2002) the authors estimate that only half of the existing hedge funds report their performance data to at least one major index provider, see Amenc/Martellini (2002), p. 5.
As a second attribute of a good index, the accuracy of the calculated index is very important. The price discovery process of the index should be reliable, consistent and verifiable by an independent third party. For the underlying hedge fund data the same holds true. This performance information should be readily available and reliable. This is a problematic point in the hedge fund industry where most managers report unaudited data to hedge fund database providers.\(^9\)

The calculated indices should be transparent in order to be easily replicable. Therefore, the index should be well and unambiguously defined, and the index construction should follow clear and objective guidelines that are known in advance. When hedge fund indices are calculated, the index construction methodology is not always well-defined and articulated. Often index committees are involved in the selection of index funds and the underlying strategies are not always unambiguously classified.\(^10\)

A useful benchmark should be investable. Investors should be able to achieve a benchmark exposure easily. Therefore, the indices should be designed to facilitate product development.\(^11\) The indices should represent a well-defined (systematic) risk premium that is available through passive investing. As mentioned above a main problem with this investability objective is the resulting exclusion of closed funds from the index calculation.\(^12\) A problem with open funds might be their limited capacity to accept new investors. So the benchmark (or some constituents) might become un-investable when funds reach their capacity limits. Besides these capacity constraints the limited liquidity of hedge fund investments in general (concerning minimum investment limits, lock-up and redemption periods) is problematic.\(^13\)

\(^12\) See Patel (2003).
\(^13\) For the trade-off between representation and investability see for example Patel et al. (2003), p. 62.
\(^14\) See Kohler (2003), p. 8, Jaeger (2004), and Section 2.3.6 on page 20.
As we see, these objectives are partially competing and in the hedge fund industry not all of them can be met at the same time. Therefore, the calculated indices always represent a trade-off between the various objectives.\footnote{See for example Patel (2003).} \footnote{For a list of alternative strategy specific hedge fund benchmarks, see for example Jaeger (2004).}

4.1.2 Hedge Fund Index Providers

In this section 21 hedge fund index providers are described in detail. We will take a closer look at the corresponding hedge fund databases and the particular index methodologies.

**Alternative Asset Center**

The Alternative Asset Center (AAC) provides fund of hedge fund data.\footnote{See \url{www.aa-center.net}.} The index offered by AAC is calculated since 2001 and offers a history beginning in 1996. The equal weighted index is published monthly with a time lag of 5 to 8 weeks. The indices are not investable that means the funds in the database might be closed to new investors. The company claims to operate the largest fund of hedge funds database in the world. This database currently features information on 1700 on- and offshore funds of hedge funds. All of these funds are included in the published index.

**ABN AMRO Eurekahedge Index**

ABN AMRO and Eurekahedge (ABN/EH) calculate an index series since 2002 with a history that dates back to 1999.\footnote{See \url{www.eurekahedge.com}.} The index series is focused on Asian hedge funds and offers a composite and two additional regional indices. The Asian hedge fund

\footnote{See for example Patel (2003).} \footnote{For a list of alternative strategy specific hedge fund benchmarks, see for example Jaeger (2004).} \footnote{See \url{www.aa-center.net}.} \footnote{See \url{www.eurekahedge.com}.}
4 Description and Analysis of Hedge Fund Data

database from Eurekahedge currently comprises 468 funds and the non-investable indices represent 147 hedge funds. The final equal weighted index values are calculated with a time lag of two months.

Altvest/InvestorForce

The database provider Altvest started in 1996 to publish its hedge fund indices. In 1999 it was acquired by InvestorForce. The original Altvest indices offered data going back to 1993 but the Altvest sub-indices prior to the year 2000 had to be recalculated due to biases. The monthly values are published from the beginning of the following month on and are permanently updated for the data of funds reporting their recent figures. On the last day of the following month the final index values are calculated. The indices are not investable. The database comprises about 2600 hedge funds and the composite index combines 2245 of these funds. Besides this composite index, 13 strategy indices are calculated. In all of the indices the hedge funds are equally weighted.

Barclay Trading Group/Global HedgeSource

Global HedgeSource was founded in 2002 by Barclay Trading Group. The first indices were calculated in 2003 and the history was filled until 1997. A total of 18 hedge fund indices and additionally a top ten fund ranking for all the strategies is published every month. Index estimates are released the next or the next but one month and the final index values are released three months after. The database consists of about 3000 funds and all of these funds are included in the index calculations for one composite, one fund of funds and 16 strategy indices. All of these indices are equally weighted.

19 See www.investorforce.com.
Bernheim Index

Since 1995 Dome Capital Management publishes the so-called Bernheim Index.\textsuperscript{21} From 1995 to 1999 the index was calculated on a quarterly basis and since 1999 it is updated monthly. The index values are available with a time lag of 2 to 5 weeks. The published index is based on the well-known US Offshore-Funds Directory (OFD) which includes about 1000 offshore hedge funds but only 18 representative (\textquotedblleft leading\textquotedblright) fund managers are included in the equally weighted Bernheim Index. Besides this composite index there are no strategy indices available.

BlueX

The investable BlueX (Blue Chip Hedge Fund Index) is the result of a cooperation between the Austrian Stock Exchange and Benchmark Capital Management.\textsuperscript{22} The index started in July 2002 and is calculated back to December 2001. The Benchmark database comprises 2500 hedge funds but only 30 to 40 (blue chip) hedge funds that are managed and controlled by large investment banks or leading financial organizations are included in the BlueX index. The estimated index is calculated on a weekly basis and the final index is determined on a monthly basis with a maximum delay of 25 days.\textsuperscript{23} The calculation scheme of the index is a mix of value and equal weighting of the individual funds.

CISDM/MAR

The Managed Accounts Reports (MAR) index series was established in 1994 and the calculated index values started in 1990.\textsuperscript{24} The hedge fund index series was then sold

\textsuperscript{21} See \url{www.hedgefundnews.com}.

\textsuperscript{22} See \url{www.bluex.org} and Benchmark Advisory (2004).

\textsuperscript{23} When the final monthly index value is calculated a maximum of 15\% estimated fund values is used to determine the BlueX.
to Zurich Capital Markets and since August 2002 the hedge fund indices are calculated by the Center for International Securities and Derivatives Markets (CISDM) at the University of Massachusetts.\textsuperscript{25} The database which is still maintained by MAR contains about 1200 single hedge funds and about 500 fund of hedge funds. Additionally the database includes 1500 inactive funds. A total of 20 indices is calculated from all active hedge funds. There are 9 strategy indices and 11 corresponding sub-indices that represent the median of the funds in the strategy groups. The monthly indices are calculated with a time lag of two months.

CSFB/Tremont

The joint venture CSFB/Tremont was initiated by Credit Suisse First Boston and Tremont Capital Management.\textsuperscript{26} CSFB/Tremont was established in 1999 and the index history dates back to 1994. The monthly indices are value weighted and published on the 15th of the following month. The funds are taken from the Tremont TASS database with data on over 3000 hedge funds managed by more than 1300 managers.\textsuperscript{27} The number of hedge funds from the TASS database that is used for the calculation of the composite index and the 10 sub-indices is about 430.

Since August 2003 CSFB/Tremont also offers a series of investable indices. The CSFB/Tremont Investable Hedge Fund Index (CSFB inv) is calculated back to the year 1999 and it is investable through an index-tracker fund.\textsuperscript{28} The index is based on the funds for the common CSFB/Tremont index series. From these funds (a total of about 430) 60 funds are selected for the investable index. The investable index series comprises 10 sector indices. From every sub-index category the 6 largest funds

\textsuperscript{24} See \url{www.marhedge.com}.
\textsuperscript{25} See \url{http://cisdm.som.umass.edu}.
\textsuperscript{26} See \url{www.hedgeindex.com}, CSFB/Tremont (2002), CSFB/Tremont (2004a) and CSFB/Tremont (2004b).
\textsuperscript{27} See \url{www.tremont.com}.
\textsuperscript{28} See for example Goricki/Söhnholz (2004), p. 34, for a brief discussion of index-tracker funds.
that meet certain conditions are included in the Investable Hedge Fund Index. US domiciled hedge funds are not part of the investable funds universe.

**Dow Jones Hedge Fund Indices**

Dow Jones Hedge Fund Indices is a relatively new name in the hedge fund industry. Since 2004 an investable hedge fund index series is calculated that is based on a managed accounts platform. This series comprises five strategy indices, the so-called Dow Jones Hedge Fund Strategy Benchmarks, and a portfolio index that is allocated in these strategies, the Dow Jones Hedge Fund Balanced Portfolio Index. The monthly values of the equal weighted strategy indices start at the end of 2001 and daily index values are available since the beginning of 2004. For the Balanced Portfolio Index daily and weekly data is provided. The final values of the indices are published with a time lag of two months. In order to select the currently 35 funds for the index series Dow Jones obtains fund information from different information providers.

**Evaluation Associates Capital Markets**

Evaluation Associates Capital Markets (EACM) calculates hedge fund indices since 1996. The EACM100 Index which only comprises US hedge funds was launched in 1996 and was originally calculated back to the year 1990. But due to biases the

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29 See CSFB/Tremont (2004a) for more details on eligible funds for the CSFB/Tremont Investable Hedge Fund Index.
31 In fact there are two indices, the Dow Jones Hedge Fund Balanced Portfolio Index AX and BX, that reflect the development of a hedge fund portfolio. They differ as the Dow Jones Hedge Fund Balanced Portfolio Index BX documents the performance of an investment that is hedged in Euro.
32 Data on the strategy index “Equity Market Neutral” is only available since 2003.
33 See www.eacm.com.
index data before 1996 is no longer published. Since 2004 Evaluation Associates Capital Markets also offers the “EACM100 Index - Offshore Funds” which focuses only on offshore hedge funds. For this index no historical performance information is available before the year 2004. The values of both equal weighted indices are published with one month delay and they are based on 100 hedge funds that have been directly selected. Fund managers are grouped into 5 broad investment strategies, which are further cut into 12 sub-strategies.

EDHEC

Since march 2003 EDHEC, a French business school, calculates a series of hedge fund indices that is different from other index concepts in the industry. As suggested by Amenc/Martellini (2002), EDHEC publishes a series of “indices of indices” because the resulting index series is less biased and more representative for the hedge fund universe. Therefore, the virtual EDHEC database is the sum of the databases of other index providers. Namely the indices of Altvest, Barclay, CISDM, CSFB/Tremont, EACM, Hennessee Group, HEDGEFUND.NET, HFR, Standard&Poors and VAN are currently used in the EDHEC calculations. The weighting of these different indices is determined by principal components analysis (PCA). The published index series with 13 strategy indices starts in 1997 and the final monthly values are available with a delay of one month.

Feri Trust

The Absolute Return Investable Index (ARIX) series that is published by Feri Trust is calculated since 2003 and the index history dates back to 2002. The monthly equal weighted indices are published with a time lag of up to 6 weeks. The Feri

35 See www.feritrust.de.
4 Description and Analysis of Hedge Fund Data

database is a collection of TASS, VanHedge and HF Net data and additional funds. According to Feri there are currently more than 6000 funds and managers included. 41 hedge funds form the ARIX composite index. The ARIX series offers 2 master and 4 strategy indices.

**FTSE Hedge Index**

FTSE publishes a broad range of financial indices and since 2004 the FTSE Hedge Index is calculated with a history beginning in 1997. The FTSE composite index is made up of 3 style indices which can be further divided into 8 strategy indices. The composite index includes a total of 40 hedge funds. The monthly final values of FTSE indices are published with a delay of about six weeks. Additionally FTSE offers the daily indices with a time lag of 3 days. The weighting of the funds in these indices is done according to the fund’s investability measured as remaining fund capacity. The FTSE database is collected from different hedge fund database providers and contains additional FTSE data.

**Hennessee Group**

The Hennessee Group has a long tradition calculates hedge fund indices. Since 1987 Hennessee Group computes hedge fund indices and currently publishes an index history beginning in 1993. A total of 24 indices is calculated, a composite index (Hennessee Hedge Fund Index) and 23 strategy indices. Index values are published with a two week delay but final index values are made public not until the beginning of the following year. The Hennessee database contains 3000 on- and offshore funds and 750 of these funds are included in the index calculations.

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36 The ARIX composite index is also calculated as value weighted index but this index is not published.


38 See [www.hennesseegroup.com](http://www.hennesseegroup.com).
Hedge Fund Intelligence

London-based Hedge Fund Intelligence (HFI) currently determines 3 index series, the EuroHedge-, the AsiaHedge- and the InvestHedge-Index with different numbers of sub-strategies. These indices were first published in 2001, 2002, and 2003 and are all calculated back to 1998. The new index values are distributed the next month. The database for the EuroHedge (EH) series comprises 1000 hedge funds, for the AsiaHedge (AH) series about 400 funds and the InvestHedge (IH) series database includes 1300 (fund of) hedge funds. In the course of the actual index calculation 850, 290 and 900 funds are used when the median values are determined.

HEDGEFUND.NET

Another source for hedge fund indices is the online provider HEDGEFUND.NET (HF-NET) which offers the so-called Tuna index series. Founded in 1997 HEDGEFUND.NET started calculating indices in 1998 and offers a history for some series that dates back to 1976. HEDGEFUND.NET monthly publishes 5 composite indices and a broad range of 33 strategy indices that are released at the beginning of the next month. As the index values are permanently updated, there are no final index values available. The equal weighted Tuna index calculations are based on data from more than 3900 hedge funds that report to HEDGEFUND.NET. All of these funds are included in the published indices.

Hedge Fund Research

Hedge Fund Research (HFR) offers a well-established index series. The monthly HFR indices (HFRI) are calculated since 1994 with an index history beginning in

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40 See www.hedgefund.net.
41 See www.hfr.com.
1990. The individual hedge funds in these indices are equally weighted. In 2003 HFR launched a series of daily indices that are investable. These HFRX indices offer a history beginning in the year 2000 and the index values are published the next day. For the HFRI series the final index values are made public with a time lag of one month. Hedge Fund Research works with a database of about 4000 hedge funds and 1650 of these funds are included in the calculation of the HFRI series. HFRX indices are calculated from the same basic database but no information on the number of funds used in the averaging is given. Hedge Fund Research publishes 36 strategy indices for the HFRI and 8 strategy indices for the HFRX.

**Mondo Hedge Index**

The Italian Mondo Hedge Index was created in 2003 and offers an index history beginning in 2001. The focus of Mondo Hedge is to provide a benchmark for Italian fund of hedge fund managers. Therefore, data from 107 Italian funds of hedge funds is currently included in the database for the Mondo Hedge Index series and the results of 102 funds are used in the process of index calculation. The index series that is published with a time lag of about five weeks offers a composite fund of hedge funds index and two strategy indices with a total of four sub-indices. The whole index series is calculated on the basis of a value-weighting and an equal weighting scheme.

**MSCI Hedge Fund Indices**

In 2002 Morgan Stanley Capital International (MSCI) started to calculate a hedge fund index series that covers many hedge fund strategies with individual sub-indices. A total of 190 hedge fund indices are calculated and published in different strategy indices.

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42 See [www.mondohedge.com](http://www.mondohedge.com).
groups. The composite index is determined on an equal and a value weighted basis and has an index history until 1994. The indices are based on data from about 1800 hedge funds. About 98% of these funds are included in the index calculations. The final index values are made public with a delay of one month.

In 2003 MSCI in cooperation with Lyxor Asset Management released a series of investable hedge fund indices (MSCI inv). A history of index values has been established that dates back to the year 2000. Based on data of currently 97 hedge funds that is supplied by Lyxor Asset Management weekly index values are published.

**Standard & Poors Hedge Fund Indices**

The well-known index provider Standard & Poors (S&P) started to publish hedge fund indices in 2002.\(^{44}\) Index values are available from 1998 on. The Standard & Poors index series is calculated on a daily basis and published with a time lag of two days. The official monthly indices are released after 3-4 weeks. The database for the investable Standard & Poors index series comprises 3500 funds from proprietary Standard & Poors data and other data providers. For the index calculation 40 hedge funds are included.

**VAN Hedge Fund Indices**

VAN Hedge Fund Advisor was founded in 1988 and started in 1994 to calculate the VAN hedge fund index series that was published for the first time in 1995.\(^{45}\) The three monthly index series Global, US-Onshore and Offshore date back to 1988 and the corresponding indices are calculated with an equal weighting scheme. The final index values are published with a delay of up to one month. The VAN database

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\(^{44}\) See [www.standardandpoors.com](http://www.standardandpoors.com), Standard & Poor's (2003b), Standard & Poor's (2003a), and Standard & Poor's (2004).

\(^{45}\) See [www.vanhedge.com](http://www.vanhedge.com).
comprises more than 6000 hedge funds whereof about 1000 funds are no longer active. The final index values consider more than 1000 funds. The index-tracker fund that makes the Offshore-Index investable comprises about 45 hedge funds.

4.1.3 Hedge Fund Databases and Index Methodologies

For the previously mentioned index and database providers some basic information is summarized in Table 4.1 on the following page. The summary includes information on the launch of the index series and the historical index data. Furthermore the weighting scheme of the indices is considered. The different calculation methods employed are equal weighting (ew), value weighting (vw), an implicit weighting scheme based on principal components analysis (PCA), capacity weighting (cw), and the usage of the statistical median. In an equal weighted index the index values are determined by averaging over a number of funds, that means every hedge fund gets the same weight, while a value weighted index reflects the assets under management (AUM) of the funds in the corresponding fund weights.

Whether an index series is investable or not depends on the hedge funds included in the corresponding database. The column “Investable” contains information wether investors are able to place money in the underlying portfolio or index derivatives, according to the index providers. Finally the last column of Table 4.1 on the following page gives a short summary on the time lag when performance data of hedge fund indices is published.

The main strategy indices that are available from the different index providers are

47 See FTSE (2004b), pp. 8 and 10, for a description of the capacity weighting scheme for FTSE indices.
48 Investment strategies that are implied by different weighting schemes are outlined in Section 4.2.2 on page 139.
<table>
<thead>
<tr>
<th>Index Provider</th>
<th>Start Year</th>
<th>History Year</th>
<th>Weighting</th>
<th>Investable</th>
<th>Published Period</th>
</tr>
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<tr>
<td>AAC</td>
<td>2001</td>
<td>1996</td>
<td>ew</td>
<td>no</td>
<td>5-8 weeks</td>
</tr>
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<td>ABN/EH</td>
<td>2002</td>
<td>1999</td>
<td>ew</td>
<td>no</td>
<td>two months</td>
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<td>1996</td>
<td>1993</td>
<td>ew</td>
<td>no</td>
<td>one month</td>
</tr>
<tr>
<td>Barclay</td>
<td>2003</td>
<td>1997</td>
<td>ew</td>
<td>no</td>
<td>three months</td>
</tr>
<tr>
<td>Bernheim</td>
<td>1995</td>
<td>1995</td>
<td>ew</td>
<td>no</td>
<td>2-5 weeks</td>
</tr>
<tr>
<td>BlueX</td>
<td>2002</td>
<td>2001</td>
<td>ew/vw</td>
<td>yes</td>
<td>two months</td>
</tr>
<tr>
<td>CISDM</td>
<td>1994</td>
<td>1990</td>
<td>median</td>
<td>no</td>
<td>two months</td>
</tr>
<tr>
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<td>1999</td>
<td>1994</td>
<td>vw</td>
<td>no</td>
<td>two weeks</td>
</tr>
<tr>
<td>CSFB inv</td>
<td>2003</td>
<td>1999</td>
<td>vw</td>
<td>yes</td>
<td>six weeks</td>
</tr>
<tr>
<td>Dow Jones</td>
<td>2004</td>
<td>2001/2003</td>
<td>ew</td>
<td>yes</td>
<td>two months</td>
</tr>
<tr>
<td>EDHCEC</td>
<td>2003</td>
<td>1997</td>
<td>PCA</td>
<td>no</td>
<td>one month</td>
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<tr>
<td>Feri</td>
<td>2003</td>
<td>2002</td>
<td>ew</td>
<td>yes</td>
<td>six weeks</td>
</tr>
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<td>FTSE</td>
<td>2004</td>
<td>1997</td>
<td>cw</td>
<td>yes</td>
<td>six weeks</td>
</tr>
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<td>Hennessee</td>
<td>1987</td>
<td>1993</td>
<td>ew</td>
<td>no</td>
<td>year-end</td>
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Table 4.1: Summary on hedge fund indices and index providers
presented in Table 4.2 on the next page. Only the most common hedge fund strategies with a minimum of four representatives from different index providers have been listed. The listing starts with the composite indices and the fund of funds indices. The 12 individual hedge fund strategies considered in Table 4.2 are: equity market neutral, long/short equity, convertible arbitrage, fixed income arbitrage, relative value, merger arbitrage, distressed securities, event driven, global macro, short selling, emerging markets, and managed futures.49

After examining the databases and the calculated strategy indices we will take a closer look at the objectives for benchmark construction that have been outlined in Section 4.1.1.

At first the representativeness of the different hedge fund databases and indices is examined. Table 4.4 on page 123 gives a summary on the representativeness of the hedge fund databases used in the course of index calculation. Here the number of funds in the databases, the number of funds in the resulting index series and the restrictions for the acceptance of funds in the database are outlined.50 Table 4.5 on page 124 offers information on the rules for index construction. The selection of funds from the database by the corresponding index providers is examined according to whether a minimum track record or a minimum amount of assets under management is required and if closed funds are included in the index calculation. Furthermore, Table 4.5 outlines if the strategy categorization is done by the hedge fund manager or the index provider.

The accuracy of the calculated indices with the problem of unreliable and/or unaudited fund performance is addressed in the column “Verification of data” in Table 4.6 on page 125. Furthermore the transparency of the index calculation is analyzed in Table 4.6. Here we examine whether the index selection criteria, the index calculation

49 See Chapter 3 for more information on these hedge fund strategies.
50 The number of funds in the different databases that is reported in Table 4.4 on page 123 is from September and October 2004.
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Table 4.2: Summary on hedge fund index strategies I
## Table 4.3: Summary on hedge fund index strategies II

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*Table 4.3: Summary on hedge fund index strategies II*
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Table 4.4: Summary on hedge fund database representativeness
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Table 4.5: Summary on hedge fund index representativeness
4 Description and Analysis of Hedge Fund Data

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<td>—</td>
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<td>—</td>
<td>—</td>
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<td>yes</td>
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<td>unknown</td>
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<td>yes</td>
</tr>
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<td>known</td>
<td>known</td>
<td>yes</td>
<td>yes</td>
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<td>unknown</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Table 4.6: Summary on hedge fund index transparency
methods and/or the index constituents are published. Another problem regarding the transparency of hedge fund indices is the use of an index committee to revise the index composition. For the introduced index providers the summary on these index committees can be found in the last column of Table 4.6 on the preceding page.

4.2 Biases in Hedge Fund Index Data

After addressing the problems with hedge fund index methodologies in Section 4.1 we will now focus on different biases in hedge fund index data. The data included in different databases represents only subsets of the hedge fund universe and the measurement problems that result from this incompleteness of hedge fund data have to be considered when fund returns are analyzed and interpreted.

In the next sections systematic biases in hedge fund data are explained and the consequences of these findings for hedge fund analysis are documented. In Section 4.2.1 we describe the well known biases in hedge fund databases, and subsequently the problems with different weighting schemes for hedge fund index calculation are outlined in Section 4.2.2. As a strategy index group that is able to cope with at least some of these problems we will finally introduce fund of hedge funds in Section 4.2.3.

4.2.1 Biases in Hedge Fund Databases

Unfortunately hedge fund databases are exposed to a number of biases. As already outlined, there is no central data pool for hedge funds where fund managers have to report their figures to.\(^{51}\) Therefore, the whole population of hedge funds and

\(^{51}\) As hedge funds are frequently organized as offshore or private investment funds, they are usually not even enforced to disclose their results to the public, see for example Fung/Hsieh (2002a), p. 23.
corresponding hedge fund return series is unknown. As a consequence the databases
are not fully representative for the hedge fund population as a whole and samples
from the database will give approximations and biased measures for the performance
of a hedge fund investor. We will discuss the most important biases that are located
on the fund level and on the level of the database provider, respectively.

At first we will take a look at the reasons and explanations for the different biases.
Then we will examine the nature of four biases in greater detail. We will describe
survivorship bias, selection bias, instant-history bias and the stale-price bias. Finally
the consequences for the estimation of return characteristics like expected return,
standard deviation and higher moments from hedge fund databases are discussed in
Section 4.2.1.6.

4.2.1.1 Problems with Hedge Fund Data

There are different characteristics of the hedge fund industry that entail at least
some of the biases outlined in Sections 4.2.1.2 to 4.2.1.5. These biases originate
from problems on different levels of the information flow. We will focus on problems
and the resulting data biases that are a direct result of the data collection process.
The considered biases are therefore usually caused by problems on the level of hedge
funds and on the database level. Figure 4.2 on the next page gives an overview on
some problems and the resulting biases. The researcher which might be another
source of biases is excluded in the further analysis.

To start with the origin of hedge fund data we will take a closer look at hedge
funds themselves. Here the reporting of the data is in the focus of our analysis. As
the reporting to database providers is voluntary and the reported net asset values
or return figures are usually subject to at least slight adjustments by hedge fund
managers, the performance of funds in a database might differ from the actual

\[ 52 \text{ See Signer (2002), p. 29, and Signer (2003), p. 69, for related figures.} \]
performance of the hedge fund population.\textsuperscript{53}

On the level of data providers further problems that result in biases are the beginning of data collection in the mid 1990s, the backfilling of historical data, the inclusion of dead funds in the historical database and the different criteria or minimum requirements for including hedge funds into the database.\textsuperscript{54} These reasons for data problems are outlined in the following sections when the corresponding biases are discussed.

\subsection{4.2.1.2 Survivorship Bias}

Survivorship bias\textsuperscript{55} does arise when a sample of hedge funds includes only funds

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4.2}
\caption{Problems with hedge fund data and resulting biases\textsuperscript{52}}
\end{figure}

\textsuperscript{53} See for example Signer (2002), pp. 29-30.
\textsuperscript{54} See for example Signer (2002), p. 29.
\textsuperscript{55}
that are alive and operating at the end of the sample period.\textsuperscript{56} This bias does occur because of problems with hedge fund databases or because the researcher starts with a list of funds that exist at the end of the analyzed sample period.\textsuperscript{57} The part of the survivorship bias that is influenced by the researcher\textsuperscript{58} is excluded in the following discussion as we focus on database problems and biases.

A first problem that leads to survivorship bias is located in the history of hedge fund databases. For funds that ceased to exist before the start of systematic collection of hedge fund data in the 1990s no extensive dataset is available because these funds predated the existence of most hedge fund databases. Therefore, historical hedge fund data over this time frame is usually based on funds that survived until this early period which leads to survivorship bias. The hedge fund data recorded in these years will usually overestimate the returns for the hedge fund population as a whole.\textsuperscript{59}

Another source of survivorship bias is found in the exclusion of funds that have stopped reporting to database providers. These funds from the hedge fund population are no longer part of the observable database subset. Here we can differentiate funds that ceased operations, that voluntarily stopped reporting and funds that were delisted by the database provider. The funds that stopped reporting and left the database are usually called “defunct” funds as opposed to surviving funds that are still alive and reporting to the database providers.\textsuperscript{60} These different ways to exit

\textsuperscript{55} The “survivorship bias” is also called “survivor bias”, see for example Ackermann et al. (1999), p. 864, Schmeeveis et al. (2001), p. 20, and Howell (2001), p. 58.
\textsuperscript{57} Survivorship bias has been well documented in the mutual fund literature, see for example Grinblatt/Titman (1989), Brown et al. (1992), Brown/Goetzmann (1995) and Malkiel (1995).
\textsuperscript{58} This special research-driven bias is called look-ahead bias or multi-period sampling bias which is a sub-category of survivorship bias. This bias addresses the problem of implicitly selecting funds from a data sample, that survive a number of consecutive periods, see for example Ackermann et al. (1999), pp. 869-870, Baquero et al. (2002), pp. 3-4, and Horst et al. (2001).
the database and the resulting consequences have to be analyzed when survivorship bias in general is measured. The resulting biases are usually aggregated under the so-called “termination bias”\textsuperscript{61} and the percentage of hedge funds which disappear from a database in a given period (generally 1 year) is captured in the so-called “attrition rate”.\textsuperscript{62}

When a fund decides to stop reporting to a hedge fund database provider it usually has reached a capacity or investor limit. The database is no longer needed as a marketing instrument. In these exit situations we would expect the database to underestimate the performance of the hedge fund universe as successful funds usually quickly attract new investors and therefore reach capacity or investor limits more rapidly than the average hedge fund.\textsuperscript{63,64} In the case of a delisting we would expect a rather negative future performance for a fund as database providers usually delist funds that are likely to harm their reputation for providing reliable information to their customers.\textsuperscript{65} When funds that cease operations are no longer part of the database,\textsuperscript{66} this would entail no special biases as the fund would leave the hedge fund universe and the database at the same time. But there is another important bias when these disappearing funds do not report the final periods leading up to and including their liquidation. It is conceivable that hedge funds lose substantial value

\textsuperscript{60} For this terminology see for example Fung/Hsieh (2000), p. 294. The subset of funds that ceased operations is often additionally termed as “dead” funds in the literature on survivorship bias, see for example Fung/Hsieh (2002a), p. 24.

\textsuperscript{61} See for example Ackermann et al. (1999), p. 864.

\textsuperscript{62} See for example Bares et al. (2001), p. 7.

\textsuperscript{63} See Signer (2002), p. 32.

\textsuperscript{64} In some publications on database biases this subset of survivorship bias is called “self-selection bias”, see for example Ackermann et al. (1999), p. 864. We will not use the term “self-selection bias” here because of a possible mix-up with selection bias that is outlined in Section 4.2.1.3 on the following page. The self-selection bias analyzed in these publications considers the problem when hedge funds refuse to report to a database provider.


\textsuperscript{66} According to Liang (2000) poor performance is the main reason for a fund’s disappearance from hedge fund databases, see Liang (2000), p. 315.
in the period subsequent to reporting. Investors in hedge funds that cease operations may experience discounts in value due to the liquidation of the underlying fund holdings, and a delay in the redemption of proceeds.\footnote{See Ackermann et al. (1999), p. 867.} Even studies that explicitly address the termination bias by studying the returns of discontinued hedge funds may suffer from this liquidation bias. In order to minimize this effect database providers usually make efforts including multiple follow-up phone calls and faxes for about one year after disappearance.\footnote{See Ackermann et al. (1999), p. 867.}

\subsection*{4.2.1.3 Selection Bias}

The voluntary reporting to hedge fund databases is another source for data biases. As hedge funds are not allowed to advertise publicly, voluntary reporting is a way to distribute information and attract investors.\footnote{See for example Liang (2000), p. 312.} Because of hedge fund managers that refuse to report their results,\footnote{The restrictive inclusion methodology of database providers could be another reason for not including certain hedge funds to a database, see for example Fung/Hsieh (2002a), p. 24. But we will assume that the relevant hedge fund population does meet the respective inclusion criteria.} the databases do depart from the actual hedge fund universe and are not fully representative for the hedge fund population.\footnote{See Fung/Hsieh (2000), p. 299.} The resulting bias is called “selection” bias or “self-selection” bias.\footnote{This problem does not exist with mutual funds, as mutual funds must publicly disclose their results.}

There are two possible reasons for this selection bias. On the one hand hedge fund managers with a superior track record will be more likely to attract new investors by the usage of a hedge fund database as a marketing tool. A manager with a poor historical performance will think twice if the comparison to other hedge funds in such a database is the right sales channel. As a consequence hedge funds in a database would tend to have better performance than the excluded funds.\footnote{See Fung/Hsieh (2000), p. 299.}
illustrative numerical example on this part of selection bias and the consequences can be found in Lo (2001).74

On the other hand if a hedge fund performs consistently well and does not need additional capital, the manager might also refuse to disclose the numbers to a database provider. So rather attractive funds from the hedge fund universe could not be included by data providers. This reason for not reporting to any database would lead to a database performance that is biased downward as good-performing funds from the population would not be taken into account.75

The two reasons for selection bias partially offset each other which limits the magnitude of this bias. Furthermore the net effect of the selection bias is ambiguous as the sign of this bias depends on which group of hedge funds dominates in the population.76

4.2.1.4 Instant-History Bias

The backfilling or instant-history bias is addressing a problem on the database level. When data providers add a new hedge fund to their database, the historical fund performance is often backfilled into the database.77 If a database provider does ignore this problem and backfills the complete available history of a new database member, these funds enter the databases with so-called instant history78 as the database will include the fund’s return history prior to the fund’s entrance date.79

Because of this instant-history bias the database samples usually overestimate the

78 See Park (1995) for this formulation.
4 Description and Analysis of Hedge Fund Data

performance of the hedge fund population. There are three main reasons for this bias. At first only funds that survived the incubation period are able to enter a database and look for new investors. During this incubation period the investor base is usually small, and if the manager is able to earn acceptable returns on this seed capital, the manager will try to market the fund. The hedge funds that close down before looking for a broader investor base by reporting to a database provider will not be included or backfilled in the historical data of hedge fund database providers.80 This part of instant-history bias is related to the survivorship bias in Section 4.2.1.2 on page 128.

As usually only successful funds with a satisfactory track record will elect to backfill returns, this might be another reason why the databases overestimate the hedge fund population return.81 This part of instant-history bias is related to the selection bias in Section 4.2.1.3 on page 131.

A third source of instant-history bias is the adjustment of the reported and backfilled return series by the hedge fund manager. While the actual performance numbers are usually audited, the hedge fund manager still has the freedom to wait for a good period of performance before requesting the database inclusion or to truncate the performance history in order to provide only the most recent and more successful part.82

4.2.1.5 Stale-Price Bias

The stale-price bias that is also called managed-price bias refers to problems with hedge fund return measurement and is located on the hedge fund level. As many hedge funds trade to at least some degree, illiquid securities, the determination of periodic net asset values might become difficult.83 When assets like illiquid exchange-

traded securities or OTC-securities that are difficult to price, are included in a hedge fund portfolio but do not trade (frequently) near the end of the month, hedge fund managers have considerable flexibility in how they mark their positions for month-end reporting.\textsuperscript{84}

Given this flexibility, it is not unreasonable to assume that hedge fund managers have an incentive to smooth the fund performance in order to report a return series with lower risk (by reducing the reported volatility) and a lower correlation or dependence to other asset classes. Therefore, a hedge fund’s net asset value does not necessarily reflect the true market value of the hedge fund portfolio constituents.\textsuperscript{85}

For illiquid securities that are thinly or infrequently traded the hedge fund manager could use a price at which the security last traded (a so-called “stale” price), estimate a price using proprietary pricing models along with broker-dealer input or simply chose a reasonable price.\textsuperscript{86} All of these alternatives still leave considerable flexibility to determine net asset values with the manager. Even if the portfolio constituents are not that illiquid, the manager is able to smooth net asset values by delayed reporting or by requesting dealer’s prices for evaluation purposes only.\textsuperscript{87} The result of such a smoothing of return figures is an autocorrelated return series.\textsuperscript{88} This topic is outlined in Section 4.4.3.

\textbf{4.2.1.6 Consequences of Hedge Fund Database Biases}

In this section the different biases and the resulting quantitative consequences for the measurement of hedge fund performance are brought together. Figure 4.3 gives

\textsuperscript{84} See Asness et al. (2001), p. 10.
\textsuperscript{86} See for example New (2001), p. 3.
\textsuperscript{87} See Asness (2001), p. 19.
\textsuperscript{88} See Getmansky et al. (2003) and Brunner/Hafner (2005), pp. 130-132.
a review on the different reasons for biases and already indicates the impact of these biases. In the following the direction and magnitude of survivorship bias and backfilling bias are discussed. Selection bias and stale-price bias will not be analyzed as they cannot be addressed directly in empirical research.\(^89\)

![Figure 4.3: Different reasons for biases and the resulting consequences](image)

At first survivorship bias is considered. Different studies estimated the extent of this bias. The methodologies to quantify survivorship bias in these studies slightly differ. In general the survivorship bias of a sample with fund data is measured as the difference in the performance of two portfolios: an “observable” portfolio and a “surviving” portfolio. The observable portfolio does invest equal amounts in each fund in the data sample or database. The portfolio is rebalanced in order to maintain equal investments in the individual funds when a new fund is added to the portfolio or when a fund left the data sample. The capital that is returned from these defunct funds is reinvested in the remaining funds.\(^90\)


The surviving portfolio is invested equally in all funds that are in the database at the end of the analyzed period. Therefore, the surviving portfolio represents a return that an investor would earn if all funds that left the database during the sample period would have been avoided. This definition is used in the empirical work of Bares et al. (2001), Brown et al. (1999), Capocci (2001), Edwards/Caglayan (2001b), Fung/Hsieh (2000), Liang (2000) and Liang (2001). In the studies of Liang (2000) and Liang (2001) the surviving portfolio and the observable portfolio is determined every year. The surviving portfolio therefore includes the funds that exist at the end of each year and the observable portfolio is set up with all funds that have been in the data sample during the year.

For the surviving portfolio some researchers chose a more restrictive definition and also excluded new funds that started reporting during the examined period. This is done in Amin/Kat (2002a) and in Brown et al. (1999). Table 4.7 on the next page summarizes the results of these articles and research papers on survivorship bias in hedge fund databases.

While the first, less restrictive definition of surviving funds is in general appropriate to determine the survivorship bias in the data sample, the second definition, which does exclude new funds, is suitable for investors that are interested in historical parameter estimates for which at least a number of years of data should be available.\textsuperscript{91}

According to Table 4.7 the survivorship bias overestimates hedge fund performance by an amount of 0.60% to 3.00% in annual returns when whole hedge fund databases (not just estimates for different strategies) are considered.\textsuperscript{92} In the mutual fund literature this bias is also well documented and estimated with 0.5% to 1.5% per

\textsuperscript{91} See Amin/Kat (2002a), p. 9.
\textsuperscript{92} A very detailed analysis of survivorship bias can be found in Amin/Kat (2002a). The authors determine the survivorship bias in the TASS-database for different subgroups according to size, age, co-investments, leverage and the strategies of the hedge funds in the sample, see Amin/Kat (2002a), p. 20.
In order to assess the magnitude of backfilling or instant-history bias, the methodology is similar to the estimation of survivorship bias. In this case the difference

<table>
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<th>Bias p.a.</th>
<th>Time frame</th>
<th>New funds</th>
<th>Database</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ackermann et al. (1999)</td>
<td>1.10%</td>
<td>1988-1995</td>
<td>yes</td>
</tr>
<tr>
<td>Amin/Kat (2002a)</td>
<td>1.89%</td>
<td>1994-2001</td>
<td>no</td>
</tr>
<tr>
<td>Bares et al. (2001)</td>
<td>1.30%</td>
<td>1996-1999</td>
<td>yes</td>
</tr>
<tr>
<td>Brown et al. (1999)</td>
<td>0.75%</td>
<td>1989-1995</td>
<td>no</td>
</tr>
<tr>
<td>Brown et al. (1999)</td>
<td>3.00%</td>
<td>1989-1995</td>
<td>yes</td>
</tr>
<tr>
<td>Capocci (2001)</td>
<td>0.60%</td>
<td>1984-1993</td>
<td>yes</td>
</tr>
<tr>
<td>Capocci (2001)</td>
<td>1.20%</td>
<td>1994-2000</td>
<td>yes</td>
</tr>
<tr>
<td>Edwards/Caglayan (2001b)</td>
<td>0.36%</td>
<td>1990-1998</td>
<td>yes</td>
</tr>
<tr>
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<td>3.06%</td>
<td>1990-1998</td>
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</tr>
<tr>
<td>Liang (2000)</td>
<td>0.60%</td>
<td>1994-1997</td>
<td>yes</td>
</tr>
<tr>
<td>Liang (2001)</td>
<td>2.40%</td>
<td>1990-1999</td>
<td>yes</td>
</tr>
</tbody>
</table>

Table 4.7: Survivorship bias in hedge fund databases

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in the performance of an observable portfolio (as defined previously) and a second portfolio which should not be affected by backfilling is measured. A common approach to determine the second portfolio is to eliminate the first one or two years of reported data for every fund as these years should contain the most backfilled data.\textsuperscript{95} The months that are dropped for the second portfolio are usually called “incubation period”.\textsuperscript{96}

<table>
<thead>
<tr>
<th></th>
<th>Bias p.a.</th>
<th>Time frame</th>
<th>Incubation</th>
<th>Database</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ackermann et al. (1999)</td>
<td>0.10%</td>
<td>1988-1995</td>
<td>2 years</td>
<td>MAR/HFR</td>
</tr>
<tr>
<td>Capocci (2001)</td>
<td>0.90%</td>
<td>1984-2000</td>
<td>1 year</td>
<td>HFR/TASS</td>
</tr>
<tr>
<td>Capocci (2001)</td>
<td>1.20%</td>
<td>1994-2000</td>
<td>1 year</td>
<td>HFR/TASS</td>
</tr>
<tr>
<td>Edwards/Caglayan (2001b)</td>
<td>1.17%</td>
<td>1990-1998</td>
<td>1 year</td>
<td>MAR</td>
</tr>
</tbody>
</table>

Table 4.8: Instant-history bias in hedge fund databases\textsuperscript{97}

Table 4.8 summarizes the results of some articles and papers on backfilling bias in hedge fund databases. With the exception of Ackermann et al. (1999), empirical research reports a backfilling bias of 0.90% to 1.40% percent per annum.\textsuperscript{98} As expected, the instant-history bias overestimates the performance of the hedge fund population.\textsuperscript{99}

\textsuperscript{95} See Ackermann et al. (1999), p. 868.
\textsuperscript{96} See for example Fung/Hsieh (2002a), p. 25.
\textsuperscript{98} The results in Ackermann et al. (1999) are not supported by other researchers and are attributed to the particular database used in Ackermann et al. (1999), see for example Brown et al. (2001), p. 1874.
\textsuperscript{99} With most index providers backfilling bias is not a problem as the added performance history of new funds does not affect historical index values.
4.2.2 Weighting Schemes for Hedge Fund Indices

Besides the biases that hedge fund indices inherit from the corresponding hedge fund databases, the weighting scheme of hedge fund indices might entail further problems for the interpretation and the usage of hedge fund index data. We will discuss the two most common weighting schemes in asset management, that is equal weighting and value weighting.\(^{100}\)

As Table 4.1 on page 119 indicates, most index providers offer equally weighted hedge fund indices. The monthly or quarterly index series are calculated as arithmetic average over the returns from funds in the respective category. For a virtual investor in these indices this weighting scheme does imply a certain investment strategy that is periodically rebalancing the portfolio by selling “winners” and buying “losers” from the underlying hedge fund sample. Money that was allocated equally in these hedge funds is shifted within the sample from well-performing to underperforming funds. For the investor the corresponding indices therefore represent contrarian asset allocation strategies on the hedge fund sample.\(^{101}\)

Other index providers offer value-weighted index series, where the weights for different funds are directly connected to the net asset value of the hedge fund. For an investor such a value-weighting of portfolio constituents might be a more common and natural portfolio strategy. This weighting-scheme represents a strategy based on momentum. “Winners” that performed well over the last period will be able to increase their weight in the portfolio or index, while “losers” that underperformed will accordingly reduce their weight.\(^{102}\)

\(^{100}\) Two weighting schemes that are additionally mentioned in Table 4.1 on page 119 are a weighting based on principal components analysis (PCA) and capacity weighting (cw). As both weighting schemes do not represent interpretable portfolio strategies they will not be discussed in the following. In the case of the PCA this implicit weighting is applied on the level of indices with different index methodologies and is therefore not interpretable anyway.


Considering the different weighting schemes therefore reveals another source for biases in hedge fund indices. The differences in equal weighted and value weighted indices are for example analyzed in Fung/Hsieh (2002a) using the composite index from HFR which is equally weighed and the value weighted CSFB/Tremont composite index.\(^{103}\) The differences on an annual basis sum up to an amount of for example \(-9.1\%\) in 1994 and \(+9.5\%\) in 1997, respectively.\(^{104}\)

Using equally weighted indices might be suitable for an analysis concerning the average return of an investment in a particular strategy. Value weighting is focused on a portfolio approach and is therefore more useful in the context of portfolio considerations with the individual funds in the database. Hence, for a broad hedge fund index a value weighting scheme that implies allocations according to the capitalization (or net asset value) might be appropriate.

### 4.2.3 Fund of Hedge Funds Data

For estimating the result from investing in hedge funds it might be useful and natural to look at the results of hedge fund investors that is fund of hedge funds.\(^{105}\) Figure 4.4 on the next page gives an impression on the differences between composite hedge fund indices and fund of hedge fund indices.

Data providers calculate composite and strategy indices from the hedge funds in the database by an explicit weighting scheme, usually equal weighting. This explicit weighting is also applied when it comes to averaging over fund of hedge funds in order to determine an index for this group. The main difference between composite or strategy indices and fund of hedge funds indices is the weighting scheme for the target funds as fund of hedge funds indices implicitly apply value weighting in terms

\(^{103}\) See Section 4.1.2 for a description of these index providers.


of their allocations. So the consequences for a hedge fund investor might be mirrored more appropriately by fund of hedge fund indices than with most composite indices for hedge fund databases.

Compared to general composite indices, fund of fund indices offer numerous advantages when it comes to measuring hedge fund performance. Figure 4.5 on the following page summarizes the most important advantages of fund of hedge fund indices and some of the corresponding drawbacks.

An advantage of fund of hedge fund data over data from individual funds is the accurate and usually audited performance information on a timely basis. This is one of the services the fund of fund management should provide to the investor. A drawback concerning this usually timely reporting can be seen in occasional late reporting due to the aggregation of data from the target funds.\textsuperscript{106}

Another advantage of fund of hedge funds is that the biases and return differ-

Figure 4.5: Advantages and disadvantages of fund of hedge fund indices

Fund of hedge fund data is less vulnerable to biases from data samples. According to Fung/Hsieh (2000) fund of hedge fund data exhibits less than half the survivorship bias and backfilling bias of individual hedge funds. The selection bias is also not a real problem with fund of hedge funds as they invest in a large portfolio of target funds and are therefore less affected by capacity constraints on the fund of hedge funds level.\footnote{See Fung/Hsieh (2000), p. 301.}

Fund of hedge funds also avoid most data problems and biases from individual hedge funds. As fund of hedge funds usually invest in individual funds that cease operations survivorship bias is avoided as dead funds and funds that stopped reporting to database providers remain in the track record of the fund. Selection bias is also not relevant as the performance of individual funds is included in the fund of hedge fund even if the fund manager might choose not to report to a database provider. Instant-history on the level of individual hedge funds is not found in fund of hedge funds from the miscellaneous weighting schemes of various index providers are much smaller when fund of hedge fund indices are considered.\footnote{See Fung/Hsieh (2002a), p. 31, and the data in Fung/Hsieh (2000), p. 302.}
funds data either, as the (audited) performance history is never backfilled when new funds are added to the portfolio. Finally stale or managed price bias should be less severe with fund of hedge funds as the monitoring is usually more sophisticated than with traditional hedge fund investors.\textsuperscript{109}

Besides these favorable characteristics of fund of hedge fund indices there are also some disadvantages of these indices when used as proxies for hedge fund investments. At first only a smaller fraction of the single hedge funds universe is included in fund of hedge funds. The reason is that single hedge funds that are not reporting to fund of hedge fund investors are not part of the measured performance. Furthermore expenses and fees (management fees and performance fees) on the fund of hedge funds level are slightly biasing the reported performance. Reported fund of hedge funds performance is usually measured net of fees. This includes the subtraction of fees on the target fund level and on the level of the fund of fund management. As these fees on the fund of hedge fund level are not typical for a general hedge fund investor the performance is biased slightly downward.\textsuperscript{110}

The cash held by fund of fund managers for redemptions is another source of a minor bias in fund of hedge fund index data.\textsuperscript{111} As an individual hedge fund investor does not need this liquidity the fund of hedge fund performance as a proxy for the performance of a hedge fund investor is biased downwards.\textsuperscript{112}

\textsuperscript{111} See Section 2.3.6 for details on redemption policies.
4 Description and Analysis of Hedge Fund Data

4.3 Data Selection from the Hedge Fund Index

Universe

If we take a look at the variety of hedge fund strategies, the corresponding hedge fund indices and the biases, the question is which index should be used as a benchmark or as a proxy for hedge fund investments in general.

As a result from Section 4.2.2 on weighting schemes we would recommend either equally weighted fund of hedge fund indices or composite indices with value weighting. These value weighted composite indices and fund of hedge fund indices both apply a value weighting on the single hedge fund level and therefore give a useful proxy for the result of actual (diversified) hedge fund allocations.

Because of the problems outlined in Section 2.3.6, a market portfolio of (all observable) hedge funds in a database is not an adequate passive investment alternative because of liquidity issues like minimum investment limits. Furthermore the arguments that where brought forward in Section 4.2.3 suggest the usage of fund of hedge fund indices instead of a general composite index.

Hence we will use fund of hedge fund indices as proxies for the performance of hedge fund investments in the following. Table 4.9 on the next page lists the index providers that where introduced in Section 4.1. This overview gives information if the index provider calculates a fund of hedge funds index and which weighting scheme is employed. Additionally information on the backfilling and the verification of hedge fund data is given.

The column “Backfilling of Data” indicates whether historical performance is backfilled when a fund enters the database. For most indices backfilling is not a problem as newly added hedge fund history does not affect the history of indices. The ver-
<table>
<thead>
<tr>
<th>Fund of Hedge</th>
<th>Funds Index</th>
<th>Weight-</th>
<th>Backfilling</th>
<th>Verification</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>yes</td>
<td>ew</td>
<td>yes</td>
<td>—</td>
</tr>
<tr>
<td>ABN/EH</td>
<td>no</td>
<td>ew</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>yes</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Barclay</td>
<td>yes</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Bernheim</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>BlueX</td>
<td>no</td>
<td>ew/vw</td>
<td>no</td>
<td>—</td>
</tr>
<tr>
<td>CISDM</td>
<td>yes</td>
<td>median</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>CSFB</td>
<td>no</td>
<td>vw</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>CSFB inv</td>
<td>no</td>
<td>vw</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>Dow Jones</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>EACM</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>EDHEC</td>
<td>yes</td>
<td>PCA</td>
<td>—</td>
<td>—</td>
</tr>
<tr>
<td>Feri</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>FTSE</td>
<td>no</td>
<td>cw</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Hennessee</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>HFI</td>
<td>only HFI IH</td>
<td>median</td>
<td>once</td>
<td>yes</td>
</tr>
<tr>
<td>HF-NET</td>
<td>yes</td>
<td>ew</td>
<td>yes</td>
<td>no</td>
</tr>
<tr>
<td>HFRI</td>
<td>yes</td>
<td>ew</td>
<td>no</td>
<td>no</td>
</tr>
<tr>
<td>HFRX</td>
<td>no</td>
<td>ew/vw</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>Mondo</td>
<td>yes</td>
<td>ew&amp;vw</td>
<td>no</td>
<td>—</td>
</tr>
<tr>
<td>MSCI</td>
<td>no</td>
<td>ew/vw</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>MSCI inv</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>S&amp;P</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>yes</td>
</tr>
<tr>
<td>VAN</td>
<td>no</td>
<td>ew</td>
<td>no</td>
<td>no</td>
</tr>
</tbody>
</table>

Table 4.9: Summary on reasons for hedge fund database biases
ification of data by the database provider could reduce stale-price bias on the fund of hedge funds level to some extent. But as this problem is much more severe on the target or individual fund level when composite or strategy indices are calculated, the impact of managed fund of hedge funds prices on the performance measurement should be minimal.

Only eight index providers calculate fund of hedge fund indices, all of which represent equal weighted averages of the underlying performance data. These indices are gathered in Table 4.10 with additional information on the length of the historical time series and whether any restrictions are imposed on the funds for the index.

<table>
<thead>
<tr>
<th>Provider and Index</th>
<th>History</th>
<th>Restrictions or Focus</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC Fund of Hedge Funds Benchmark</td>
<td>Jan 1997 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>Altvest/IF Subindex: FoF</td>
<td>Jan 1993 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>Barclay Fund of Funds</td>
<td>Jan 1998 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>CISDM Fund of Fund Median</td>
<td>Mar 1997 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>EDHEC Funds of Funds</td>
<td>Jan 1997 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>HFI InvestHedge Composite</td>
<td>Jan 2002 - Dec 2004</td>
<td>Asian-Pacific</td>
</tr>
<tr>
<td>HFRI FoF Composite</td>
<td>Jan 1990 - Dec 2004</td>
<td>—</td>
</tr>
<tr>
<td>Mondo Hedge Indice Generale</td>
<td>Jan 2002 - Dec 2004</td>
<td>Italian</td>
</tr>
</tbody>
</table>

Table 4.10: Fund of hedge fund index providers

These indices should give a relatively unbiased estimate for the performance of hedge fund investments, so they are in the focus of Section 4.4 when the statistical properties of hedge fund indices are analyzed. As the fund of hedge fund indices

\[113\] HEDGEFUND.NET is an exception as “historical” index values are recalculated when a fund enters the database with instant history. Therefore, the index series is excluded from further analysis.
from HFI and Mondo offer historical data for a relatively short period of time we will exclude them from the database. Additionally the indices with the shorter return time series are focused on fund of hedge funds from special and therefore not very representative regions like Italy and the Asian-Pacific region. This leaves us with six hedge fund index return series for the data analysis in Section 4.4.
4.4 Statistical Properties of Fund of Hedge Fund Index Data

In this section some important statistical properties of fund of hedge fund index returns are examined. Such a detailed analysis of the return data is essential for the modeling of portfolio returns in Chapter 5. In this section we examine the unconditional distributions and certain time series properties of fund of hedge fund return data. All of these index returns are measured in US Dollar. Figure 4.6 gives an impression on the performance of the six fund of hedge fund indices under consideration since 1997.

![Graphs showing performance of fund of hedge funds indices since 1997](image)

Figure 4.6: Performance of fund of hedge funds indices since 1997\(^{114}\)

From Figure 4.6 we can already see that the development of the historical perfor-

\(^{114}\) All six indices are set to a value of 100 at the beginning of 1997.
mance for five of the six indices is pretty similar over the period beginning in the year 1997. Only the performance of the Altvest/IF fund of hedge fund index differs from other performance histories. The best absolute performance since 1997 is reported for this return series. Since January 1997 a total net return of 180.82% accumulated while most other fund of hedge fund indices offered absolute returns of 80%-100% over this period of eight years. Table 4.11 summarizes the absolute returns of the six hedge fund indices.

<table>
<thead>
<tr>
<th>Provider</th>
<th>Index</th>
<th>Absolute Performance since 1997</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>Fund of Hedge Funds Benchmark</td>
<td>102.76%</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>Subindex : FoF</td>
<td>180.82%</td>
</tr>
<tr>
<td>Barclay</td>
<td>Fund of Funds</td>
<td>82.38%</td>
</tr>
<tr>
<td>CISDM</td>
<td>Fund of Fund Median</td>
<td>78.77%</td>
</tr>
<tr>
<td>EDHEC</td>
<td>Funds of Funds</td>
<td>112.03%</td>
</tr>
<tr>
<td>HFRI</td>
<td>FoF Composite</td>
<td>79.60%</td>
</tr>
</tbody>
</table>

Table 4.11: Fund of hedge fund index performance

In the following we will take a closer look at the monthly returns of these index series. At first in Section 4.4.1 the return definitions that are used in the following sections are introduced. Section 5.2.2 gives more information on the unconditional distribution of fund of hedge fund index returns and Section 4.4.3 describes certain time series properties of these historical index returns.

\footnote{See Section 4.1 for possible reasons for this outstanding performance.}

\footnote{In the case of the Barclay Fund of Funds index we only have seven consecutive years of monthly return data in our sample. For the CISDM Fund of Fund Median the return data is available since March 1997.}
4.4.1 Return Definitions

In financial modeling the most important (random) variable is the “return” of assets or investments in general. This return can be defined in different ways. As the understanding of the underlying return definition is crucial we will introduce the two different return concepts of simple and log returns.

The simple return is applied in many contexts. Denote by $P_t$ the price of an asset or index at date $t$ that is adjusted for any payments, stock splits, etc. The simple (net) return on the asset or index, $R_t$, between dates $t - 1$ and $t$ is defined as

$$ R_t = \frac{P_t}{P_{t-1}} - 1. \quad (4.1) $$

Sometimes the term “discrete” return as opposed to the “continuous” return that is introduced in the following is also employed for this definition. The simple return of a portfolio of assets with weights $w_i$ and returns $R_{i,t}$, $i = 1, ..., N$, is calculated according to

$$ R_{P,t} = \sum_{i=1}^{N} w_i R_{i,t}. \quad (4.2) $$

This linear relationship between simple portfolio returns and the returns of the portfolio components simplifies the calculations and is one of the reasons why this return definition is used in the portfolio model outlined in Chapter 5.

If the compounding happens in continuous time, i.e. interest is paid every instant, a continuously compounded return or so-called log return can be calculated. The log return, $r_t$, for period $t$ is defined as

$$ r_t = \ln \left( \frac{P_t}{P_{t-1}} \right) = \ln (1 + R_t). \quad (4.3) $$


118 $(1 + R_t)$ describes the simple gross return.


121 See for example Copeland et al. (2005), pp. 889-890, for the limiting case of increasing compounding periods.
This return definition is especially suited for continuous time models in option pricing or time series analysis as it is additive over time.\textsuperscript{122} When portfolios of assets are examined the corresponding log return of a portfolio is calculated according to\textsuperscript{123}

\[ r_{P,t} = \ln \left( \sum_{i=1}^{N} w_i e^{r_{i,t}} \right). \]  \hspace{1cm} (4.4)

In many textbooks the approximation

\[ r_{P,t} \approx \sum_{i=1}^{N} w_i r_{i,t} \]  \hspace{1cm} (4.5)

is given.\textsuperscript{124} In empirical research using the approximation from Equation 4.5 instead of the exact result in Equation 4.4 is only a minor problem, as returns that are measured over short periods of time are usually close to zero. Therefore, the numerical difference between the two return definitions is small, as Figure 4.7 illustrates. This absolute return difference is displayed on the right hand plot of Figure 4.7.

![Simple Return vs Log Return](image)

Figure 4.7: Differences between simple and log returns\textsuperscript{125}

As we can see the difference between the two return definitions depends on the absolute value of the returns under consideration. As the simple return can be

\textsuperscript{122} See Dorfleitner (2002), pp. 221-222 and p. 237.

\textsuperscript{123} See for example Bamberg/Dorfleitner (2002), p. 866.

\textsuperscript{124} See for example Campell et al. (1997), p. 12.

\textsuperscript{125} See for example Dorfleitner (2002), p. 219, for a similar figure.
expressed as a function of the log return the difference can be clarified in a Taylor series or more precisely Maclaurin expansion:

\[ R_t = e^{r_t} - 1 = \sum_{i=0}^{\infty} \frac{r_t^i}{i!} - 1 = \sum_{i=1}^{\infty} \frac{r_t^i}{i!} = r_t + \sum_{i=2}^{\infty} \frac{r_t^i}{i!}. \] (4.6)

As Equation 4.6 points out the two return definitions differ in terms of second and higher order. This leads to the common relationship \( R_t \geq r_t \). With returns close to zero the resulting difference is very small.\(^{127}\) The size of these return differences for simple returns is illustrated in Figure 4.7 on the previous page for the interval \([-0.10, 0.10]\).

A major discrepancy between the simple and the log return is their domain. While the support of the log return \( r_t \) is the real line \( \mathbb{R} \), the simple return \( R_t \) can only attain values on \([-1, \infty)\).\(^{128}\) The reason for this domain of simple returns is located in the limited liability assumption for the underlying prices \( P_t \).

In the following the term “returns” will be used for simple returns and “log returns” will describe continuously compounded returns. As Chapter 5 presents a modeling approach which considers only two points in time, we will usually employ (discrete) returns and the corresponding distributions for these returns. The standard textbook assumption of a distribution for portfolio modeling that is defined on \( \mathbb{R} \) is not consistent with the support of the simple return.\(^{129}\) This is an approximation but the errors due to this assumption are negligible when a relatively short period of time which corresponds to minor variation in the returns is considered. Therefore, the probability of simple returns less than \(-1\) becomes very small.\(^{130}\)

\(^{127}\) See Dorfleitner (2002), pp. 219-220.
\(^{130}\) As the error is usually small we will also assume continuous distributions that are defined on the real line \( \mathbb{R} \) for the random variable \( R_t \) in the course of modeling portfolio distributions in Chapter 5.
4.4.2 Unconditional Return Distributions

This section provides a detailed analysis of the unconditional return distributions of fund of hedge fund indices. Here we will not consider the chronology of the different index returns.\textsuperscript{131} At first we give an impression on the empirical return distributions by descriptive statistics. Furthermore distributional parameters for the population of hedge fund index returns are estimated from these historical return realizations. Finally one of the foundations of modern portfolio theory, the assumption of normally distributed returns is checked by statistical test in Section 4.4.2.3.

4.4.2.1 Descriptive Statistics

The purpose of this section is to analyze and compare the descriptive statistics of the indices under consideration. We present aggregate statistics for the realized monthly returns of the index series from the six data providers AAC, Altvest/IF, Barclay, CISDM, EDHEC, and HFRI. At first the descriptive statistics for the empirical distribution of the discrete returns are given in Table 4.12.

<table>
<thead>
<tr>
<th></th>
<th>No.</th>
<th>Mean p.a.</th>
<th>Mean SD</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>96</td>
<td>0.75%</td>
<td>9.36%</td>
<td>1.40%</td>
<td>0.255</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>144</td>
<td>1.29%</td>
<td>16.64%</td>
<td>2.32%</td>
<td>0.903</td>
</tr>
<tr>
<td>Barclay</td>
<td>84</td>
<td>0.73%</td>
<td>9.10%</td>
<td>1.48%</td>
<td>0.413</td>
</tr>
<tr>
<td>CISDM</td>
<td>94</td>
<td>0.63%</td>
<td>7.81%</td>
<td>1.30%</td>
<td>-1.139</td>
</tr>
<tr>
<td>EDHEC</td>
<td>96</td>
<td>0.80%</td>
<td>10.05%</td>
<td>1.75%</td>
<td>0.245</td>
</tr>
<tr>
<td>HFRI</td>
<td>180</td>
<td>0.81%</td>
<td>10.22%</td>
<td>1.62%</td>
<td>-0.255</td>
</tr>
</tbody>
</table>

Table 4.12: Descriptive statistics for fund of hedge fund index returns since inception

\textsuperscript{131}See Section 4.4.3 for a brief analysis of hedge fund data over time.
Table 4.12 reports the descriptive statistics for the indices over the whole available index history. Therefore, the number of monthly index returns in the database differs substantially. While HFR features index data beginning in January 1990 the index series from CISDM offers monthly returns starting in March 1997 and the fund of hedge fund index of Barclay is calculated since January 1998. The calculated descriptive statistics since inception in Table 4.12 show considerable differences in the means of the various return series. Especially the mean of the Altvest/IF index with an index history that begins in January 1993 is obviously above the mean return of other fund of hedge funds indices. The standard deviations (SD) are relatively homogeneous. Again, for the Altvest/IF returns the highest value for the descriptive statistic is derived.

In order to obtain comparable data samples with few special items we will concentrate in the following on the period beginning in January 1997 which leads to sampling distributions with less observations for the HFR and Altvest/IF index. The descriptive statistics for this shorter period are reported in Table 4.13 on the next page.

In Table 4.13 we can see the descriptive statistics for the final sample that is used in the further data analysis. The standard deviations and the skewness coefficients for the two truncated index series published by Altvest/IF and HFRI are slightly higher than before while the descriptive statistic for the kurtosis of the samples does not change dramatically in the case of HFRI. The kurtosis coefficient for Altvest/IF substantially decreases from 3.9 to 3.0 indicating extreme returns over the period before 1997. When we relate the descriptive statistics standard deviation and mean by calculating the coefficients of variation we get a relatively homogeneous picture.

\[ \mu = \frac{1}{T} \sum_{t=1}^{T} R_t, \]
\[ \sigma^2 = \frac{1}{T} \sum_{t=1}^{T} (R_t - \mu)^2, \]
\[ \sigma = \sqrt{\sigma^2}, \]
\[ SK = \frac{1}{T \sigma^3} \sum_{t=1}^{T} (R_t - \mu)^3, \]
\[ KU = \frac{1}{T \sigma^4} \sum_{t=1}^{T} (R_t - \mu)^4 - 3, \]

132 The sample mean \( \mu \) for the return data is given by \( \mu = \frac{1}{T} \sum_{t=1}^{T} R_t \), the sample variance \( \sigma^2 \) by \( \sigma^2 = \frac{1}{T} \sum_{t=1}^{T} (R_t - \mu)^2 \) and the sample standard deviation by \( \sigma \). The sample skewness \( SK \) can be calculated according to \( SK = \frac{1}{T \sigma^3} \sum_{t=1}^{T} (R_t - \mu)^3 \), and the sample (excess) kurtosis \( KU \) by \( KU = \frac{1}{T \sigma^4} \sum_{t=1}^{T} (R_t - \mu)^4 - 3 \), see for example Campell et al. (1997), pp. 16-17.

133 Most indices with a long history suffer biases like those outlined in Section 4.2 especially for their early years.
### Table 4.13: Descriptive statistics for fund of hedge fund index returns since 1997

<table>
<thead>
<tr>
<th></th>
<th>No.</th>
<th>Mean p.a.</th>
<th>Mean SD</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>96</td>
<td>0.75%</td>
<td>9.36%</td>
<td>1.40%</td>
<td>0.255</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>96</td>
<td>1.12%</td>
<td>14.25%</td>
<td>2.68%</td>
<td>1.042</td>
</tr>
<tr>
<td>Barclay</td>
<td>84</td>
<td>0.73%</td>
<td>9.10%</td>
<td>1.48%</td>
<td>0.413</td>
</tr>
<tr>
<td>CISDM</td>
<td>94</td>
<td>0.63%</td>
<td>7.81%</td>
<td>1.30%</td>
<td>-1.139</td>
</tr>
<tr>
<td>EDHEC</td>
<td>96</td>
<td>0.80%</td>
<td>10.05%</td>
<td>1.75%</td>
<td>0.245</td>
</tr>
<tr>
<td>HFRI</td>
<td>96</td>
<td>0.63%</td>
<td>7.81%</td>
<td>1.83%</td>
<td>-0.190</td>
</tr>
</tbody>
</table>

The values for the coefficient of variation (based on the monthly returns) range from 1.9 to 2.9.\(^{134}\)

The range in skewness\(^{135}\) and kurtosis\(^{136}\) values is rather broad over the six fund of hedge fund indices. The descriptive statistics for the skewness parameter give ambiguous information about the shape of fund of hedge funds index returns. While the skewness is negative for the indices from CISDM and HFRI, implying more observations are found in the longer left tail of the empirical distribution, the other index returns series are rather skewed to the positive side. The sample excess kurtosis for all indices indicates that the realized returns might be more peaked and more fat-tailed than a normal distribution would imply.\(^{137}\)

---

\(^{134}\) Here coefficients of variation (CoV) are determined based on \(CoV = \frac{\sigma}{\mu}\).

\(^{135}\) In general the skewness measures the degree of asymmetry of a distribution. This standardized third moment is usually positive when a distribution has a longer positive than negative tail, see for example Thode (2002), pp. 44-45.

\(^{136}\) The kurtosis measures the peakedness of a distribution. It is based on the fourth standardized moment and is usually reported as “excess” kurtosis which is the fourth standardized moment minus 3. This makes the “excess” kurtosis of a normal distribution equal to zero. Higher kurtosis usually implies that more of the variance of a distribution is due to extreme deviations from the mean. These distributions are usually called “fat-tailed”. See for example Thode (2002), pp. 44-45, for different fat-tailed probability densities.
returns are leptokurtic with an excess kurtosis greater than zero.

For the reader who is interested in the distributional characteristics of log returns Table 4.14 gives the descriptive statistics for log returns that have been calculated from the same fund of hedge fund index series.

<table>
<thead>
<tr>
<th></th>
<th>No.</th>
<th>Mean</th>
<th>Mean p.a.</th>
<th>SD</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>96</td>
<td>0.74%</td>
<td>8.84%</td>
<td>1.39%</td>
<td>0.137</td>
<td>3.815</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>96</td>
<td>1.08%</td>
<td>12.91%</td>
<td>2.62%</td>
<td>0.890</td>
<td>2.654</td>
</tr>
<tr>
<td>Barclay</td>
<td>84</td>
<td>0.72%</td>
<td>8.58%</td>
<td>1.47%</td>
<td>0.277</td>
<td>4.340</td>
</tr>
<tr>
<td>CISDM</td>
<td>94</td>
<td>0.62%</td>
<td>7.42%</td>
<td>1.30%</td>
<td>-1.318</td>
<td>9.064</td>
</tr>
<tr>
<td>EDHEC</td>
<td>96</td>
<td>0.78%</td>
<td>9.39%</td>
<td>1.73%</td>
<td>0.108</td>
<td>3.383</td>
</tr>
<tr>
<td>HFRI</td>
<td>96</td>
<td>0.61%</td>
<td>7.32%</td>
<td>1.82%</td>
<td>-0.364</td>
<td>4.639</td>
</tr>
</tbody>
</table>

Table 4.14: Descriptive statistics for index log returns since 1997

Obviously the unconditional distributions of fund of hedge fund index returns are skewed like the unconditional distributions of simple returns and are also fat-tailed.

The histograms for the return series since 1997 in Figure 4.8 on the following page provide further information on the general shape of the empirical distribution of realized returns. The inspection of the plots in Figure 4.8 underscores the results for the descriptive statistics in Table 4.13 on the previous page. The distributions are not symmetrical (especially the unconditional return distributions of the Altvest/IF, EDHEC and HFRI indices) and are more peaked and more fat-tailed than a normal distribution. While especially the empirical distributions of the AAC and the CISDM indices are very peaked, all the return series exhibit more observations in

---

137 There are different definitions of fat-tailedness, see Bamberg/Dorfleitner (2002), pp. 868-869. We are interested in a comparison to the normal distribution therefore we relate kurtosis values to the kurtosis of the normal distribution.
the tails than we would expect for a normally distributed random variable.\textsuperscript{138}

More detailed information on the shape of the sampling distribution is aggregated in Table 4.15 on the next page and Figure 4.9. The quartiles and the 1\%- and 5\%-quantiles for the fund of hedge fund index return distributions are given in Table 4.15. We can infer that the quantiles of the empirical distributions are relatively homogeneous. Only the Altvest/IF hedge fund of funds index shows slightly different extreme quantiles (0.01-quantile or 0.05-quantile).

The corresponding boxplots in Figure 4.9 also take a look at the general shape of the data set and additionally give some information on outliers. These boxplots display the first and third quartiles of the data samples\textsuperscript{139}, the median of the return series\textsuperscript{140}

\textsuperscript{138} See also Figure 4.10 on page 163 for a illustration of the deviations from normality.

\textsuperscript{139} The quartiles are determined by the ends of the boxes.

\textsuperscript{140} The median is the horizontal line in the quartiles box.
and extreme outliers. These outliers are not within 1.5-times the interquartile range (between the 0.25 and 0.75 quantile) from the end of the boxes. For each of the six indices a number of 4 to 6 (positive and negative) outliers is determined. The empirical return distribution for the Altvest/IF, EDHEC and HFRI indices show slightly more extreme outliers than the other three data samples. As Table 4.15 and the boxplots in Figure 4.9 report, the median values for all the fund of hedge fund indices are located in a relatively small range from 0.54% to 0.67%.

Table 4.16 on the following page provides further information on the series of fund of hedge fund index returns. Here the extreme values (minima and maxima) for the time series with the corresponding months are gathered. The best and worst months show certain time patterns for all of the fund of hedge fund indices. The indices had their best months in December 1999 or February 2000 while the worst month was (as expected) August 1998. In these months most of the different available strategy indices and all of the available composite indices with a longer performance history also reported their highest and lowest returns. The minima and maxima in the fund of hedge fund indices are therefore a logical consequence.

---

141 These extreme values are also displayed graphically in the boxplots of Figure 4.9 on the next page as the highest and lowest outliers.

<table>
<thead>
<tr>
<th>Index</th>
<th>0.01-Q</th>
<th>0.05-Q</th>
<th>0.1-Q</th>
<th>0.25-Q</th>
<th>Median</th>
<th>0.75-Q</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>-1.96%</td>
<td>-0.85%</td>
<td>-0.63%</td>
<td>-0.05%</td>
<td>0.54%</td>
<td>1.41%</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>-5.11%</td>
<td>-2.47%</td>
<td>-1.30%</td>
<td>-0.27%</td>
<td>0.67%</td>
<td>1.94%</td>
</tr>
<tr>
<td>Barclay</td>
<td>-2.59%</td>
<td>-0.88%</td>
<td>-0.66%</td>
<td>-0.07%</td>
<td>0.62%</td>
<td>1.41%</td>
</tr>
<tr>
<td>CISDM</td>
<td>-2.40%</td>
<td>-0.92%</td>
<td>-0.49%</td>
<td>-0.03%</td>
<td>0.62%</td>
<td>1.16%</td>
</tr>
<tr>
<td>EDHEC</td>
<td>-2.86%</td>
<td>-1.26%</td>
<td>-0.85%</td>
<td>-0.24%</td>
<td>0.68%</td>
<td>1.47%</td>
</tr>
<tr>
<td>HFRI</td>
<td>-3.58%</td>
<td>-1.58%</td>
<td>-1.08%</td>
<td>-0.35%</td>
<td>0.66%</td>
<td>1.46%</td>
</tr>
</tbody>
</table>

Table 4.15: Quantiles for fund of hedge fund index returns since 1997
Figure 4.9: Boxplots for fund of hedge fund index returns since 1997

<table>
<thead>
<tr>
<th>Fund</th>
<th>Minimum</th>
<th>Month</th>
<th>Maximum</th>
<th>Month</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>-5.05%</td>
<td>August 1998</td>
<td>5.27%</td>
<td>December 1999</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>-5.92%</td>
<td>August 1998</td>
<td>11.54%</td>
<td>February 2000</td>
</tr>
<tr>
<td>Barclay</td>
<td>-5.10%</td>
<td>August 1998</td>
<td>6.05%</td>
<td>December 1999</td>
</tr>
<tr>
<td>CISDM</td>
<td>-6.40%</td>
<td>August 1998</td>
<td>4.24%</td>
<td>February 2000</td>
</tr>
<tr>
<td>EDHEC</td>
<td>-6.16%</td>
<td>August 1998</td>
<td>6.66%</td>
<td>February 2000</td>
</tr>
<tr>
<td>HFRI</td>
<td>-7.47%</td>
<td>August 1998</td>
<td>6.85%</td>
<td>December 1999</td>
</tr>
</tbody>
</table>

Table 4.16: Maxima and Minima for fund of hedge fund index returns since inception
4.4.2.2 Parameter Estimates

To obtain more information on the univariate distributions for the fund of hedge fund index returns the estimates of the moments of the corresponding return populations are calculated and analyzed in this section. Parameter estimates for the different index data samples since 1997 are reported in Table 4.17 on the following page. The estimate for the population mean (or expected value of the distribution) $\hat{\mu}$ is not reported in the table as it equals the sample mean given in Table 4.13 on page 155.\footnote{Statistically this $\hat{\mu}$ is an unbiased estimate while from financial mathematics it is well known that}

An estimate for the standard deviation $\sigma$ of a return population is $\hat{\sigma}$ which is calculated from a sample of returns according to\footnote{See for example Bamberg/Baur (2002), p. 140.}

$$\hat{\sigma} = \sqrt{\frac{1}{T-1} \sum_{t=1}^{T} (R_t - \hat{\mu})^2}. \quad (4.9)$$

The equation for the sample skewness also gives a biased estimate of the population skewness. An estimate of the skewness of returns is

$$\hat{SK} = \frac{T}{(T-1)(T-2)} \sum_{t=1}^{T} \left( \frac{R_t - \hat{\mu}}{\hat{\sigma}} \right)^3. \quad (4.10)$$

Finally an estimate of the population kurtosis is

$$\hat{KU} = \frac{T(T+1)}{(T-1)(T-2)(T-3)} \sum_{t=1}^{T} \left( \frac{R_t - \hat{\mu}}{\hat{\sigma}} \right)^4 - 3. \frac{(T-1)^2}{(T-2)(T-3)}. \quad (4.11)$$

\footnote{Statistically this $\hat{\mu}$ is an unbiased estimate while from financial mathematics it is well known that}

$$\bar{R}_T = \sqrt{\prod_{t=1}^{T} (1 + R_t)} - 1 \quad (4.7)$$

gives the correct (geometric) average return for the time series. But as a result of the Jensen inequality

$$E [\bar{R}_T] < \sqrt{E \left[ \prod_{t=1}^{T} (1 + R_t) \right]} - 1 = E (1 + R_t - 1) = E (R_t) \quad (4.8)$$

$\bar{R}_T$ underestimates the expected value of the return population, see for example Dorfleitner (2002), pp. 223-224.

\footnote{See for example Bamberg/Baur (2002), p. 140.}
The parameter estimates calculated from the index return series are gathered in Table 4.17. In addition the table shows the standard errors (SE)\(^{144}\) and the p-values\(^{145}\) for the estimated skewness and kurtosis parameters.

<table>
<thead>
<tr>
<th></th>
<th>(\hat{\sigma})</th>
<th>(\hat{SK})</th>
<th>(\hat{KU})</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Est.</td>
<td>SE</td>
<td>p-value</td>
</tr>
<tr>
<td>AAC</td>
<td>1.41%</td>
<td>0.259</td>
<td>0.246 29.2%</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>2.70%</td>
<td>1.059</td>
<td>0.246 0.0%</td>
</tr>
<tr>
<td>Barclay</td>
<td>1.49%</td>
<td>0.420</td>
<td>0.263 11.0%</td>
</tr>
<tr>
<td>CISDM</td>
<td>1.31%</td>
<td>-1.158</td>
<td>0.249 0.0%</td>
</tr>
<tr>
<td>EDHEC</td>
<td>1.76%</td>
<td>0.249</td>
<td>0.246 31.2%</td>
</tr>
<tr>
<td>HFRI</td>
<td>1.84%</td>
<td>-0.193</td>
<td>0.246 43.4%</td>
</tr>
</tbody>
</table>

Table 4.17: Parameter estimates for fund of hedge fund index returns since 1997

As the p-values in Table 4.17 indicate, not all of the resulting estimates are different from zero for reasonable levels of significance with a two-sided hypothesis test. When the skewness estimates \(\hat{SK}\) are considered only the fund of hedge fund index returns from Altvest/IF and CISDM deliver skewness estimates that are significantly different from zero for common failure rates of 1% or 5%. In the case of the Altvest/IF fund of hedge fund index we would expect a longer positive tail of the distribution while the results from the CISDM data leads to an asymmetric distribution with a

\(^{144}\) The standard error \(SE_{\hat{SK}}\) for the estimated skewness parameter \(\hat{SK}\) is calculated according to

\[
SE_{\hat{SK}} = \sqrt{\frac{6T(T-1)}{(T-2)(T+1)(T+3)}}
\]

and the standard error \(SE_{\hat{KU}}\) for the estimate of the population kurtosis \(\hat{KU}\) is given by

\[
SE_{\hat{KU}} = \sqrt{\frac{4(T^2-1)}{(T-3)(T+5)}}SE_{\hat{SK}}.
\]

See for example Thode (2002) for sampling moments of the third and fourth moment.

\(^{145}\) The p-values in Table 4.17 are calculated under the assumption of standard normally distributed test statistics.
tail that extends towards more negative index returns. With all other estimates for the skewness parameter in 4.17 the hypotheses of a symmetrical distribution cannot be rejected because of too high standard errors for the corresponding estimates.

When we take a look at the kurtosis estimates the picture is slightly different. All monthly index return samples lead to (excess) kurtosis estimates that deviate significantly from zero. Therefore, return distributions with these estimated parameters are fat-tailed in comparison to a normal distribution. Here we expect more peaked distributions and additionally more extreme values than with common normal distributions.

4.4.2.3 Tests for Normality

The results from Section 4.4.2.2 concern the higher distributional moments that are significantly different from zero lead to an analysis of the index return distributions as a whole. In the following we will take a closer look at the common assumption of normally distributed returns that is one way to justify classical portfolio theory.\textsuperscript{146}

A simple way to compare the sample distribution to the normal distribution is a quantile-quantile-plot. An inspection of the quantile-quantile-plots in Figure 4.10 on the following page already indicates a potential non-normality in the index return series. The spread in the extreme quantiles of the empirical distributions of all the fund of hedge fund indices and the quantiles of a standard normal distribution is indicative of a long-tailed distribution.

After the visual inspection of the plots in Figure 4.10 on the next page we would already doubt the assumption that the analyzed index returns where drawn from (different) normal distributions. In order to obtain statistically significant statements on the distribution of returns we will perform certain statistical tests for normality of the fund of hedge fund index data. Table 4.18 on page 164 reports

\textsuperscript{146}See Chapter 5 for more details concerning this assumption.
p-values on seven different normality tests: the Anderson-Darling (AD) test, the $\chi^2$ test, the Cramer-von Mises (CvM) test, the Jarque-Bera (JB) test, the Kolmogorov-Smirnov test with Lilliefors correction (LKS), the Shapiro-Francia (SF) test and the Shapiro-Wilk (SW) test.\textsuperscript{147}

![Quantile-quantile plots for fund of hedge fund index returns](image)

Figure 4.10: Quantile-quantile plots for fund of hedge fund index returns

The perhaps most common procedure to test whether a univariate sample is drawn from a normal distribution is the Jarque-Bera test.\textsuperscript{148} This test consistently refuses the null hypothesis that the six index return samples are normally distributed for all reasonable failure rates.\textsuperscript{149}

\textsuperscript{147} For a comprehensive overview on univariate tests for normality see Thode (2002). Gnanadesikan (1997) also introduces different methods for assessing univariate normality, see Gnanadesikan (1997), pp. 187-194.

\textsuperscript{148} See for example Amenc/Malaise/Martellini/Stéir (2003), Amin/Kat (2003), Berenyi (2002), Brooks/Kat (2001), and Favre/Galeano (2002) for the usage of the Jarque-Bera test in the context of testing hedge fund returns.

\textsuperscript{149} See Bai/Ng (2005) for a discussion of problems with the Jarque-Bera test when time series are serially correlated.
Other especially useful test for our purposes are the Anderson-Darling test and the Shapiro-Wilk test as they give more weight to extreme deviations.\footnote{See Thode (2002), pp. 143-152 for a discussion of the power of tests for univariate normality.} The Shapiro-Wilk test delivers the same results as the Jarque-Bera test with only slightly higher p-values in the case of the EDHEC and HFRI index return time series and the Anderson-Darling test also refuses all of the six null hypotheses of normality for reasonable levels of significance.

With most other tests the hypothesis of normally distributed fund of hedge fund index returns can also be rejected. At least six of the tests in Table 4.18 always refuse the null hypothesis of a normal distribution when failure rates of less than 1\% are accepted. For the index return series of Altvest/IF, Barclay, CISDM and EDHEC all seven tests (even the Kolmogorov-Smirnov test with Lilliefors correction which gives more weight to “deviations” in the midrange) were able to reject the assumption of normally distributed returns for common levels of significance.\footnote{For the AAC and HFRI index returns the Kolmogorov-Smirnov test with Lilliefors correction is not able to reject the null hypothesis on a failure level of 1\%. This test is especially sensitive to deviations in the midrange and does not put much weight on extreme values. As the smaller deviations are usually not problematic this test should be avoided for evaluation of normality, see Thode (2002), p. 152.}

<table>
<thead>
<tr>
<th></th>
<th>AD</th>
<th>$\chi^2$</th>
<th>CvM</th>
<th>JB</th>
<th>LKS</th>
<th>SF</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>0.01%</td>
<td>0.00%</td>
<td>0.04%</td>
<td>0.00%</td>
<td>1.30%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Barclay</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.02%</td>
<td>0.00%</td>
<td>0.53%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CISDM</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.00%</td>
<td>0.23%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>EDHEC</td>
<td>0.04%</td>
<td>0.00%</td>
<td>0.10%</td>
<td>0.00%</td>
<td>0.71%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>HFRI</td>
<td>0.05%</td>
<td>0.00%</td>
<td>0.17%</td>
<td>0.00%</td>
<td>1.76%</td>
<td>0.00%</td>
<td>0.01%</td>
</tr>
</tbody>
</table>

Table 4.18: P-values of different tests for normally distributed index returns
We additionally take a look at the distributional characteristics of log returns as most publications on hedge funds analyze whether the unconditional empirical distribution of log returns is normally distributed.\textsuperscript{152} Table 4.19 reports the p-values for the six different statistical tests.

<table>
<thead>
<tr>
<th></th>
<th>AD</th>
<th>$\chi^2$</th>
<th>CvM</th>
<th>JB</th>
<th>LKS</th>
<th>SF</th>
<th>SW</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>0.01%</td>
<td>0.00%</td>
<td>0.05%</td>
<td>0.00%</td>
<td>1.75%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.01%</td>
<td>0.00%</td>
</tr>
<tr>
<td>Barclay</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.03%</td>
<td>0.00%</td>
<td>0.73%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>CISDM</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.01%</td>
<td>0.00%</td>
<td>0.16%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>EDHEC</td>
<td>0.05%</td>
<td>0.00%</td>
<td>0.14%</td>
<td>0.00%</td>
<td>1.03%</td>
<td>0.01%</td>
<td>0.01%</td>
</tr>
<tr>
<td>HFRI</td>
<td>0.05%</td>
<td>0.00%</td>
<td>0.18%</td>
<td>0.00%</td>
<td>2.13%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
</tbody>
</table>

Table 4.19: P-values of different tests for normally distributed index log returns

The statistical tests yield slightly higher p-values for all the testing procedures indicating that the empirical distributions of log returns are a bit closer to normality than the unconditional distributions of simple returns. But as the difference between simple and log returns for the monthly data is rather small\textsuperscript{153} the statistical normality tests for log returns also reject all of the normality hypotheses on common levels of significance. Again the highest p-values are reported for the Kolmogorov-Smirnov test with Lilliefors correction, but for a reasonable significance level of 97.5% even this test rejects the hypothesis of normally distributed log returns.

With the results of the statistical tests to the hypothesis of normally distributed returns we lost one possible justification for classical portfolio theory. The second justification involves rather strong assumptions on the preference structure of in-

\textsuperscript{152} However this is not essential in our context as we are interested in a model approach over one period.

\textsuperscript{153} See Section 4.4.1 for the differences between the return definitions.
vestors.\textsuperscript{154} Therefore, these results lead us to the alternative approach to portfolio modeling that is outlined in Chapter 5.

4.4.3 Time Series Analysis

The time series behavior of fund of hedge fund index returns will not be carried out too extensively, as the modeling approach outlined in Chapter 5 is a classical one-period approach. Therefore, we will not go into detail at this point. However we will take a closer look at certain aspects of the index return time series like autocorrelation and a related unsmoothing technique for return series. But at first the stationarity of the return series will be examined as it is a prerequisite for the following analysis.

4.4.3.1 Stationarity

Stationarity of time series is especially important when time series models are considered. As we develop a modeling approach that delivers a one-period result, time series modeling is not in our focus. However the analysis of stationarity is of some relevance as a requirement for the research on autocorrelations in Section 4.4.3.2. Another reason for stationarity analysis on hedge fund data is the use of return series in regression models. In order to determine an appropriate modeling approach, stationarity analysis is essential.

When stationarity of a stochastic process is considered the analyzed time series has to satisfy certain requirements. For the analysis of autocorrelation so-called weak stationarity or covariance stationarity is a necessary assumption.\textsuperscript{155} A stochastic process $R_t$ is called weakly stationary or covariance stationary when it satisfies three

\textsuperscript{154} See Section 5.2.1 for more details on assumptions concerning investor preferences.

\textsuperscript{155} Strong stationarity would require that the joint distribution of all sets of observations is invariant to when the observations are made, see Greene (2000), p. 752.
requirements:\textsuperscript{156} \( \mathbb{E} [R_t] \) is independent of \( t \), \( \text{Var} [R_t] \) is a finite, positive constant, independent of \( t \), and \( \text{Cov} [R_t, R_s] \) is a finite function of \( t - s \), but not of \( t \) or \( s \).\textsuperscript{157} So a stationary time series has time invariant first and second moments.

With test procedures we try to identify trends in the time series under consideration. Deterministic time trends do not make much sense when return series are analyzed and modeled. Therefore, we will concentrate on the identification of possible stochastic trends in the different time series. In order to test stationarity in our data set of fund of hedge fund index returns we first perform an (augmented) Dickey-Fuller test. This test tries to identify unit roots in time series data. Therefore, a Dickey-Fuller test offers the null hypothesis of a unit root in the time series under consideration which means non-stationarity against the alternative hypothesis of stationarity of the time series. Furthermore Table 4.20 also presents the results of the Philips-Perron unit root test. Similar to the Augmented Dickey-Fuller test the Philips-Perron test starts with the null hypothesis that there is a unit root. To another class of stationarity test the Kwiatkowski, Phillips, Schmidt, Shin test (KPSS) belongs to. Here the null hypothesis is that the time series (or the process underlying the time series) is stationary which means that no unit root exists. Table 4.20 summarizes the results for these three statistical testing procedures.

The two unit root tests (Augmented Dickey-Fuller and Philips-Perron) do reject the null hypothesis of unit roots in the different time series at any reasonable significance level.\textsuperscript{158} This holds true for all six indices under consideration. The stationarity test of Kwiatkowski, Phillips, Schmidt, Shin does not reject its null hypothesis of


\textsuperscript{157} The covariance \( \text{Cov} \) of two random variables is \( \text{Cov} [R_t, R_s] = \mathbb{E} [R_t R_s] - \mathbb{E} [R_t] \mathbb{E} [R_s] \) and the variance \( \text{Var} \) (or \( \sigma^2 \)) of a random variable is defined as \( \text{Var} [R_t] = \mathbb{E} [R_t^2] - \mathbb{E} [R_t]^2 \).

\textsuperscript{158} Here the Durbin-Watson test statistic analyzes the autocorrelation in the residuals of the unit root tests. Values close to 2 are reported for all the unit root tests. Therefore, the null hypothesis of no autocorrelation in the regression residuals can not be rejected, see Section 4.4.3.2 for more details. As a consequence the unit root tests are valid.
stationarity for permitted failure rates below 10%. None of the time series of hedge fund index returns comes close to this or any other reasonable level of significance. Therefore, the unit root and stationarity tests we performed would leave us with the result that all the time series of index returns are stationary.

### 4.4.3.2 Autocorrelation

There is a vast amount of literature that detects autocorrelation in the returns of hedge funds or hedge fund indices.\(^{159}\) As already mentioned in Section 4.2.1.5 the reason for such autocorrelated returns can be at least partially found in the problem of so-called stale prices for portfolio assets and the incentives of the hedge fund management to smooth performance numbers.\(^{160}\)

For a covariance stationary time series of returns \(R_t\) the \(\tau\)-th order autocovariance


\(^{160}\) See for example Lo (2001), pp. 28-29, and Section 4.4.3.3 for more details.
function $\gamma(\tau)$ and autocorrelation function $\rho(\tau)$ are defined as:\textsuperscript{161}

$$\gamma(\tau) = \text{Cov}[R_t, R_{t+\tau}]$$  \hspace{1cm} (4.14)

$$\rho(\tau) = \frac{\gamma(\tau)}{\gamma(0)} = \frac{\text{Cov}[R_t, R_{t+\tau}]}{\text{Var}[R_t]}$$  \hspace{1cm} (4.15)

The required stationarity of all the time series under consideration is analyzed and approved in Section 4.4.3.1. We can get a first impression on the time series behavior considering autocorrelation with an autocorrelation plot. With this tool we can visually check the randomness in our return time series. If the returns are completely random we would expect autocorrelations near zero for any time lag $\tau$.\textsuperscript{162} Figure 4.11 displays the autocorrelation plots for the six fund of hedge fund return series with a maximum time lag of 20. From the inspection of the autocorrelation plots in Figure 4.11 we would expect that our time series are not completely random but rather have some degree of autocorrelation between adjacent return observations.

In order to statistically test for autocorrelation in fund of hedge funds index returns we make use of the Durbin-Watson test statistic, the Box-Pierce, and the Ljung-Box test. The null hypothesis for these tests is that the time series exhibit no autocorrelation. The Durbin-Watson test statistic should be located around 2 for a time series with no autocorrelation (for lag 1). It ranges from 0 to 4. Values greater than 2 suggest negative serial correlation, and values less than 2 suggest positive serial correlation.\textsuperscript{163} The Box-Pierce and the Ljung-Box test try to reject the null hypothesis that all the first $\tau$ autocorrelation coefficients are equal to zero. The Ljung-Box test should be better suited for our needs as the test statistic is adjusted for finite samples.\textsuperscript{164} Table 4.21 provides a summary on the autocorrelations in the index return series (for orders 1-3) and on the results of the statistical tests for first order autocorrelation.

\textsuperscript{161} See for example Campell et al. (1997), p. 45.

\textsuperscript{162} Certainly, for a time lag of zero the resulting autocorrelation should always be equal to one.


\textsuperscript{164} See for example Zivot/Wang (2003), p. 62.
The statistical test support the results from the visual inspection of the autocorrelation plots. Considering the Ljung-Box test we would reject the null hypothesis of no first order autocorrelation in the time series for the index return series of AAC, Altvest/IF, Barclay and the HFRI at failure rates below only 5%. For the Barclay and the HFRI index series the results are even significant at significance levels of 98.88% and 99.80%, respectively. Only for the CISDM time series the reported p-value does not allow to reject the null hypothesis at reasonable failure rates.

Therefore, we can conclude that serial correlation is inherent in (most) hedge fund return time series. In order to get an indication of the consequences of performance smoothing\textsuperscript{165} the next section will take a closer look at the characteristics of “un-smoothed” returns series that exhibit no significant autocorrelation.

\textsuperscript{165} Or more generally speaking to get an indication of the consequences of illiquidity of hedge fund positions.
4 Description and Analysis of Hedge Fund Data

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Durbin-Box-Pierce p-value</th>
<th>Box-Pierce p-value</th>
<th>Ljung-Box p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ(1)</td>
<td>ρ(2)</td>
<td>ρ(3)</td>
<td>Watson</td>
</tr>
<tr>
<td>AAC</td>
<td>0.30</td>
<td>0.15</td>
<td>0.03</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>0.25</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td>Barclay</td>
<td>0.37</td>
<td>0.15</td>
<td>0.03</td>
</tr>
<tr>
<td>CISDM</td>
<td>0.23</td>
<td>0.10</td>
<td>-0.04</td>
</tr>
<tr>
<td>EDHEC</td>
<td>0.28</td>
<td>0.10</td>
<td>-0.02</td>
</tr>
<tr>
<td>HFRI</td>
<td>0.34</td>
<td>0.13</td>
<td>-0.02</td>
</tr>
</tbody>
</table>

Table 4.21: Autocorrelation in index return time series

4.4.3.3 Unsmoothing of Hedge Fund Data

As already mentioned in Sections 4.2.1.5 and 4.4.3.2 the smoothing of net asset values is a very common problem with hedge fund data. Especially hedge fund managers that invest in relatively illiquid securities for which there is no recent or observable market price available have a high degree of freedom when it comes to reporting net asset values. To report a return or net asset value figure such funds usually use the last available traded prices or estimates of current market price for the valuation of illiquid instruments. Therefore, a hedge fund manager that trades rather illiquid securities has considerable discretion when marking the portfolio’s value. Regarding the compensation and the fund’s performance statistics a hedge fund manager is likely to smooth returns by marking the portfolio below the actual value in a period of large positive returns in order to create some scope for periods with lower returns.\(^{168}\)


\(^{167}\) Illiquid underlying securities can for example be found in convertible arbitrage (see Section 3.2.2) or mortgage-backed securities (see Section 3.2.4) strategies.

\(^{168}\) See Lo (2001), pp. 28-29.
We assume that it is this smoothing of return figures that results in significant first order autocorrelation in the time series of hedge fund and hedge fund index returns. Therefore, we will make use of an “unsmoothing” technique outlined in Geltner (1991), Geltner (1993), Brooks/Kat (2001) and Davies et al. (2004), in order to obtain an unsmoothed, “real” return series for the six fund of hedge fund indices.

The smoothed observable value of a fund at time $t$ can be expressed as a weighted average of the unsmoothed true value and the smoothed fund value at time $t-1$. If a single exponential smoothing approach is assumed this results in an unsmoothed return series with returns $R_t$ that are based on the smoothed returns $R^*_t$ according to

$$R_t = \frac{R^*_t - \lambda R^*_{t-1}}{1 - \lambda}. \quad (4.16)$$

When the coefficient $\lambda$ is set to the first order autocorrelation coefficient of the smoothed time series the unsmoothed time series will exhibit zero first order autocorrelation. However the mean of the time series will remain the same and as the term “unsmoothing” already implies the newly constructed return series has a higher standard deviation than the smoothed version. Table 4.22 gives a summary on the descriptive statistics of the unsmoothed return series.

As we already anticipated, the standard deviation of the unsmoothed return series is significantly above the standard deviation of the original time series while the

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169 Getmansky et al. (2003) identify the illiquidity exposure and the corresponding smoothing outlined above as most likely explanation of the serial correlation in hedge fund returns. Further influences that are examined in Getmansky et al. (2003) are for example market inefficiencies, time-varying expected returns and time-varying leverage.


171 The observable value of an index is simply the average of many different funds with smoothed observable values.


Table 4.22: Descriptive statistics for unsmoothed hedge fund index returns

<table>
<thead>
<tr>
<th>Fund</th>
<th>Mean p.a.</th>
<th>SD</th>
<th>Skewness</th>
<th>Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>AAC</td>
<td>0.72%</td>
<td>8.98%</td>
<td>0.078</td>
<td>3.260</td>
</tr>
<tr>
<td>Altvest/IF</td>
<td>1.09%</td>
<td>13.93%</td>
<td>0.931</td>
<td>2.309</td>
</tr>
<tr>
<td>Barclay</td>
<td>0.75%</td>
<td>9.38%</td>
<td>0.027</td>
<td>3.510</td>
</tr>
<tr>
<td>CISDM</td>
<td>0.65%</td>
<td>8.11%</td>
<td>-1.303</td>
<td>9.013</td>
</tr>
<tr>
<td>EDHEC</td>
<td>0.77%</td>
<td>9.63%</td>
<td>0.114</td>
<td>2.930</td>
</tr>
<tr>
<td>HFRI</td>
<td>0.59%</td>
<td>7.27%</td>
<td>-0.437</td>
<td>4.122</td>
</tr>
</tbody>
</table>

Table 4.23: Parameter estimates for unsmoothed fund of hedge fund index returns

The estimated standard deviation exceeds the values in Table 4.17 on page 161 by 25 to 40 percent of the smoothed values. The estimates for the higher moments in Table 4.23 do not give such a uniform picture. Some skewness and (excess) kurtosis estimates for the unsmoothed return series are above and some below the smoothed.
values. But in general the significance of the results stays the same as before.

4.4.4 Results

In this section we took a closer look at the statistical properties of representative hedge fund data. Therefore, we analyzed the returns of six fund of hedge fund indices: AAC (Fund of Hedge Funds Benchmark), Altvest/IF (Subindex: FoF), Barclay (Fund of Funds), CISDM (Fund of Funds Median), EDHEC (Funds of Funds) and HFRI (FoF Composite). In order to obtain comparable data samples for these indices we concentrated on index data beginning in January 1997. The results in this section are focused on simple returns but to complete the picture the statistical properties of log returns were also analyzed. As the difference between simple and log returns for the monthly data is rather small the statistical test delivered the same results as for the simple returns.

For this historical track record we first researched into the unconditional return distributions. Here the descriptive statistics for the sample and parameter estimates were reported and compared. Then statistical tests for the unconditional simple returns and the unconditional log returns were carried out. The null hypothesis of normally distributed returns was clearly rejected for all six index series with simple and log returns.

Examining time series properties of the fund of hedge fund index returns discloses significant autocorrelation in the data. Additionally the analysis of stationarity in Section 4.4.3.1 suggests that no trend component is located in fund of hedge fund return series. As a consequence of the significant autocorrelation in the return series a simple unsmoothing approach that is based on the assumption of exponential smoothing of the performance by fund managers was carried out. This yielded a substantially higher standard deviation of returns than we would expect from the original return data and slightly adjusted higher moments.
4.5 Summary

This chapter lays the foundations for the modeling approach described in Chapter 5. At first we analyzed hedge fund data providers, the available hedge fund indices, and potential shortcomings of or problems with hedge fund indices. Based on this background Section 4.2 outlined the problem of biases in hedge fund data. Survivorship bias, selection bias, instant-history bias, stale-price bias and weighting bias were introduced and the consequences of these data problems were quantified if possible. As a result of this analysis fund of hedge fund indices were chosen as proxy for hedge fund investments in general. In Section 4.4 we finally worked out the most important properties of fund of hedge fund index returns based on six major indices. We were able to identify representative ranges for parameter estimates of fund of hedge fund index returns. These characteristics are the rationale for the portfolio modeling approach with hedge fund allocations that is introduced in the next chapter.
5 Portfolio-Selection including Hedge Funds

Over the last years hedge funds gained considerable attention from a wide range of investors. While the focus in the process of selecting individual target hedge funds should be on the qualitative side, quantitative considerations will lead to the actual construction of portfolios with hedge funds in the sense of determining optimal asset weights. In order to attain a favorable or optimal asset allocation, fund of fund managers as well as individual hedge fund investors have to bear their whole portfolio and the corresponding dependencies of returns for different assets or asset classes in mind. A favorable asset allocation should usually include investments with different dependence profiles in order to diversify risk.

The previous chapters pointed out some of the interesting special features of hedge funds or alternative assets in general. These trading strategies usually lead to very different risk profiles\(^1\) and especially the analysis of fund of hedge fund returns in Section 4.4 revealed informative details on the corresponding statistical properties. The resulting univariate and multivariate return distributions are significantly different from the classical model assumptions in a portfolio framework according to Markowitz.\(^2,3\)

\(^1\) See Section 3 on hedge fund strategies.
\(^2\) See the original work of Markowitz (1952).
\(^3\) See the analysis of hedge fund data in Section 4.
In this chapter an alternative approach for portfolio construction is presented which is able to integrate some important characteristics of hedge fund return data. Like Markowitz we will concentrate on the static portfolio construction problem with regard to two points in time without interim portfolio revisions. At first we will briefly illustrate the classical theory of portfolio selection. Then distributional characteristics of return data and preference considerations for investors are presented in Section 5.2 of this chapter. Here we will analyze the different justifications of the traditional portfolio selection methodology. Due to problems with the classical assumptions underlying portfolio selection, an alternative approach to the modeling of multivariate return distributions is outlined in Section 5.3. An empirical analysis of the introduced methodology is finally presented in Section 5.4.

5.1 Traditional Portfolio Selection

The classical portfolio selection framework of Markowitz (1952) regards two points in time. The beliefs on the return distributions of different securities over this single period are the starting point for this approach to portfolio selection. In traditional portfolio theory only the first two moments (mean and variance) of each asset’s return distribution together with coefficients of correlation are deployed to the portfolio selection problem. With these measures of risk and return it is possible to identify the efficient set among all asset combinations. These risk-return efficient portfolios reward a certain amount of risk, which is statistically measured as variance or standard deviation of portfolio returns, with the highest possible expected return. When utility functions, which imply a certain amount of utility to risk-return trade-offs, are evaluated for these efficient sets, it is possible to determine an optimal portfolio allocation in terms of a maximum of expected utility. Then such a maxi-
mum utility portfolio is the optimal investment strategy over the next period for all investors with the same expectations about the first two moments of the relevant return distributions which decide in accordance to the specified utility function.

In order to formalize this approach let vector \( w \) represent the fractions of wealth invested in the available securities. Here as usual the weights should sum for one. The portfolio selection problem of determining the optimal allocations can then be stated as follows\(^5\)

\[
\max_w \mathbb{E}[U(V)]
\]  

(5.1)

with \( U(.) \) as utility function that depends on the end of period wealth \( V = V_0 \cdot (1 + \tilde{R}_P) \), which is a function of the (simple) return \( R_P \) over the period under consideration. \( V_0 \) is the initial wealth that is usually set to unity. Here the random portfolio returns \( \tilde{R}_P \) are obtained from \( \tilde{R}_P = w^T R \), with \( R \) as vector of the random returns for the available securities.

Taking expectations of the utility after expanding \( U(.) \) as a Taylor series around the expected end of period wealth, and assuming that the Taylor series converges gives\(^6\)

\[
\mathbb{E}[U(V)] = U(\mathbb{E}[V]) + \frac{1}{2} \frac{\partial^2 U(V)}{\partial V^2} \mathbb{E}[V] \mathbb{E}[V - \mathbb{E}[V]]^2 + \\
+ \sum_{i=3}^{\infty} \frac{1}{i!} \frac{\partial^i U(V)}{\partial V^i} \mathbb{E}[V] \mathbb{E}[V - \mathbb{E}[V]]^i.
\]  

(5.2)

There are several ways to justify this traditional modeling approach (see Figure 5.1 on the next page). A first reasoning is located in the choice of the utility function \( U(.) \) which is used to identify the optimal portfolio allocation that maximizes expected utility. For arbitrary return distributions the choice of a quadratic utility function leads to the mean-variance model as third and higher order derivatives in Equation

\(^4\) See Huang/Litzenberger (1988) for a detailed discussion of utility theory in the portfolio context. Wilmott (2000a) for example gives a very brief introduction on the most relevant concepts of utility theory, see Wilmott (2000a), pp. 477-483. For details on the Bernoulli principle, see Bamberg/Coenenberg (2004), pp. 81-89.


\(^6\) See Huang/Litzenberger (1988), p. 60. The second term of a regular Taylor series is equal to zero after taking expectations as it is multiplied with \( \mathbb{E}[V - \mathbb{E}[V]] \).
5.2 are equal to zero. The expected utility becomes

\[ E[U(V)] = U(E[V]) + \frac{1}{2} \frac{\partial^2 U(V)}{\partial V^2} E[V] E[V - E[V]]^2. \]  

(5.3)

So when investors decide in accordance to such a quadratic utility function, they do not account for the third and higher central moments of the return distribution when maximizing expected utility. To consider the mean and variance of returns is therefore an adequate approach under the assumption of quadratic utility functions.

Figure 5.1: Assumptions that lead to traditional portfolio selection

Another justification that leads to traditional portfolio selection can be found in the distribution of returns, either on the level of the individual securities or on the portfolio level. If the distribution of portfolio returns is completely determined by its mean and variance, then the higher order terms in Equation 5.2 can be expressed as functions of the first and second moment. In this case the mean-variance model

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8 For the quality of the approximation with Taylor’s series see for example Brockett/Garven (1998), Hlawitschka (1994), Levy/Markowitz (1979) and Loistl (1976).
is valid for arbitrary preference structures.\textsuperscript{10} Examples for portfolio distributions, which are completely described by their first two moments are the normal or the lognormal distribution. But as we are usually interested in a portfolio of securities, this approach based on the distribution of portfolio returns oversimplifies the problem.

When we consider the return distributions of individual securities we have to bear an important requirement in mind. Any linear combination of the random securities returns under consideration has to have a distribution in the same family of distributions.\textsuperscript{11} In the case of the lognormal distribution we can see that this distribution is not closed under the formation of linear combinations.\textsuperscript{12} Therefore, the traditional portfolio selection process starts with the means and variances of different available securities is not valid with the assumption of lognormally distributed asset returns as the portfolio distribution will usually not be lognormal. If we apply the assumption of multivariate normally distributed returns of all risky assets, we again end up with normally distributed portfolio returns that justify the mean-variance portfolio selection according to Markowitz.\textsuperscript{13,14}

If one of these rather strong assumptions about the assets’ return distributions or the investor’s utility function holds true, the traditional mean-variance approach to portfolio optimization is appropriate.\textsuperscript{15}

\textsuperscript{9} The problem that the simple return over one period $R_t$, which is subject to the restriction $-1 \leq R_t < \infty$, is modeled with continuous distributions that allow returns below 100% is usually neglected in this context, see Section 4.4.1 for a brief discussion of this problem.


\textsuperscript{11} See Ingersoll (1987), p. 104.


\textsuperscript{13} See Huang/Litzenberger (1988), pp. 61-62.

\textsuperscript{14} See Appendix B in Ingersoll (1987) for a proof that mean-variance analysis is not only valid for multivariate normal distributions of securities returns but also for the general class of multivariate elliptical distributions, see Ingersoll (1987), pp. 104-106.

\textsuperscript{15} See for example the early discussion on this topic in Borch (1969), Feldstein (1969) and the comment in Tobin (1969).
5.2 Problems with Traditional Portfolio-Selection

The following sections take a closer look at the assumptions underlying the classical theory of portfolio selection from an alternative investments point of view. If the assumptions concerning the investors utility function and the return distribution seem too restrictive and unrealistic, then the traditional way of selecting optimal portfolios does not really lead to a maximization of an investor’s expected utility. In an early publication on conditions under which the traditional mean-variance approach is valid Paul Samuelson already concludes that “in practice, where crude approximations may be better than none, the 2-moment models may be found to have pragmatic usefulness”.\footnote{Samuelson (1967), p. 12.} In order to obtain a more realistic model, we will try to improve the approach that leads to this “crude approximation”.

At first preferences of individual investors are examined in more detail as the classical assumption of quadratic utility functions seems very restrictive. If this assumption is not able to justify the mean-variance approach, the return distributions of the individual asset returns in the portfolio have to be considered. A related and crucial point is the modeling of dependencies between these random asset returns in order to determine multivariate returns and the resulting diversification effect in a portfolio context.

5.2.1 Investor Preferences

When the foundations of modern portfolio selection were laid by Markowitz (1952), the preferences for higher moments than the variance were already addressed.\footnote{See Markowitz (1952), pp. 90-91.} As outlined in Section 5.1, this standard approach for selecting efficient portfolios assumes quadratic utility functions or specific return distributions.\footnote{It therefore}
incorporates only the first and the second moment of return distributions and does not take investor preferences for higher moments like skewness and kurtosis into account.

Such quadratic utility functions are by definition unable to incorporate preference structures for higher moments of wealth or return distributions. When modeled as such a quadratic function of wealth or simple returns, the utility displays the undesirable properties of increasing absolute (and relative) risk aversion and satiation. Increasing absolute risk aversion does not make sense intuitively and implies that risky assets are treated as inferior goods. The property of satiation implies that a further increase in wealth beyond a point of satiation does decrease utility, which is also counter-intuitive and problematic as we usually assume individuals, who prefer more wealth to less.\(^\text{19}\)

Investor preferences for higher moments and the implications for portfolio selection were discussed by many authors.\(^\text{20}\) The risk averse investor is assumed to have a utility function with a preference for the first moment of the return or wealth distribution and an aversion towards the second moment, the variance. But investors with utility functions that exhibit positive marginal utility, consistent risk aversion and consistency of moment preference do care about higher moments. They will have a preference for positively skewed return distributions.\(^\text{21}\) This aversion towards


\(^{19}\) See for example Huang/Litzenberger (1988), p. 61, Tsiang (1972), p. 355, and Copeland et al. (2005), pp. 56-57. For a CAPM-context Rubinstein (1976) and He/Leland (1993) showed that a representative investor must have a power utility function (where higher moments do exist) if the returns on the market portfolio are independently and identically distributed and markets are perfect.


negatively skewed returns can be interpreted as willingness of an investor to trade some of his average return or wealth for a decreased chance that he will experience a large loss.\(^{22}\) Positive preference towards a positive third moment of a return distribution,\(^{23}\) consistent risk aversion and strict consistency of moment preference implies an aversion towards the fourth moment of a return distribution.\(^{24}\) This can be interpreted similar to the aversion towards the second moment, measuring the dispersion of the return distribution. Given these (theoretical) preferences for higher moments of return distributions, the next step will be to consider the complete distribution of asset returns (including higher moments of the return distributions) in the process of portfolio selection in order to obtain a more realistic optimal asset allocation.

### 5.2.2 Return Distributions

As described in Section 5.1, a multivariate normal distribution of asset returns, which is completely characterized by the means and covariances, is able to justify the classical mean-variance approach. This is the standard approach to portfolio modeling.\(^{25}\) For multivariate normal distributions the implication holds that the marginal distributions have to be (univariate) normal. Therefore, if the marginal distributions of asset returns are non-normal, the resulting multivariate distribution cannot be Gaussian.\(^{26}\)

Many researchers examined the return data of hedge funds or alternative investments in general. The results of various studies for different hedge fund strategy data sets are very similar. For the fund of hedge fund return series analyzed in Section 4.4

\(^{22}\) See Harvey et al. (2003), p. 5.

\(^{23}\) See Harvey et al. (2003) for a brief summary of empirical evidence on skewness preference.

\(^{24}\) See the proof in Scott/Horvath (1980), p. 918.

\(^{25}\) Alternative justifications can include the whole class of multivariate elliptical distributions, see Ingersoll (1987), pp. 104-106.

\(^{26}\) See for example Schlittgen/Streitberg (1999), p. 504.
the hypothesis of normally distributed returns is rejected for all of the fund of hedge fund indices. Most of the index returns show significant skewness and all of them exhibit positive excess kurtosis. Such non-normality of (unconditional) hedge fund return distributions is for example also identified in the empirical analysis in Amin/Kat (2003), Brooks/Kat (2001), Geman/Kharoubi (2003), Fung/Hsieh (2001) and Kat/Lu (2002). Therefore, the assumption of normally distributed returns in order to justify the mean-variance approach is usually unrealistic.

5.2.3 Dependence Structures

Besides the preference structure and isolated (marginal) return distributions of potential portfolio assets, the dependence structure of the portfolio assets plays an important or even the major role in modeling portfolio distributions. The dependence structure links the different marginal distribution and therefore determines the diversification effect within a portfolio.

5.2.3.1 Linear Correlation

When dependence between financial assets is measured, the (linear) coefficient of correlation as standardized covariance is a popular but also very often misunderstood measure. Linear correlation to quantify dependence is a cornerstone of classical

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27 In most of these publications log returns are tested for normality. But as pointed out in Section 4.4.1 the differences between the resulting values for the two return definitions are rather small. Therefore, the general test results should not be affected.

28 But in some articles the normality of fund of hedge fund return data is simply assumed. It is expected that higher moments disappear with the diversification of funds even in portfolios with a small number of target funds. See for example Lhabitant/Learned (2002), pp. 32-33, for this problematic assumption that is not based on empirical evidence for fund of hedge fund returns.
mean-variance portfolio theory, and it plays an important role in the equilibrium capital asset pricing model and the arbitrage pricing theory. When the distribution of asset returns in the portfolio is a multivariate normal distribution, the linear correlation (matrix) is a complete summary of the dependence structure. In general when elliptical distributions are employed to model multivariate portfolio returns, the linear correlation coefficient is a natural measure of dependence.

But as stated at the beginning of Embrechts et al. (1999): “Correlation is a minefield for the unwary”. When we leave the class of multivariate elliptical distributions, the use of the correlation coefficient might lead to fallacies about the dependence structure of the risky assets as this (simple) measure is unable to summarize non-linear dependencies.

Some important properties of the coefficient of correlation are presented in the following. First of all the coefficient of correlation is only defined when the variances of the corresponding random variables are finite. The coefficient is calculated as standardized covariance therefore it might be problematic to use correlation with very heavy-tailed distributions. Correlation is invariant under linear transformations of random variables. When the coefficient of correlation is calculated, the random variables are “normalized” by subtracting expected values (when covariances are

\[ \rho(X, Y) = \frac{\text{Cov}[X, Y]}{\sqrt{\text{Var}[X] \cdot \text{Var}[Y]}}. \]

See for example Hilbert (1998) for details on different measures of correlation.

This is a justification for the mean-variance approach, see Section 5.1.

For elliptical distributions the density is constant on ellipsoids. In two dimensions the contour plots of the density surfaces would result in ellipses. See chapters 2 and 3 in Fang et al. (1990) on multivariate elliptical distributions.


Embrechts et al. (1999), p. 69

determined) and divide by the corresponding standard deviations. But the linear coefficient is not invariant under non-linear strictly increasing transformations of the random variables like for example the natural logarithm. Independent random variables show a correlation coefficient of zero but the converse is not true. A correlation coefficient of zero does usually not imply independence. In the case of arbitrary multivariate distributions not all values in the interval \([-1, +1]\) are attainable for the coefficients of correlation. Elliptical distributions are usually able to incorporate correlations for the full range from \(-1\) to \(+1\). But for some multivariate distributions the attainable interval might be quite small. Here the maximum correlation, which is attained when two random variables are perfectly positively dependent or so-called comonotonic, differs from \(+1\). When the dependency of two perfectly negatively dependent random variables (for example when both can be expressed as increasing respectively decreasing deterministic functions of a single random variable) is measured with the correlation coefficient, the value of \(-1\) is also not necessarily within reach.

When we have to leave the class of multivariate normal or in general elliptical distributions, the coefficient of correlation does no longer incorporate the whole information about the dependence structure of the random variables. Even with the standard assumption of normally distributed individual (marginal) asset returns the multivariate distribution is not necessarily a multivariate normal as there are countless multivariate distributions with marginal normal distributions. For four

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36 See Embrechts et al. (1999), pp. 184-185.
37 See for example Haerdle/Simar (2003), pp. 87-88.
38 This interval with the maximum absolute value for the coefficient of correlation of 1 follows from Cauchy’s inequality applied on the random variables minus the corresponding expected values, see Schlittgen/Streitberg (1999), p. 4, and Abramowitz/Stegun (1972), p. 11.
39 See Kat (2002a), p. 4. For an illustration of maximum and minimum attainable correlations with different bivariate distributions see Embrechts et al. (1999), p. 70.
40 See Section 5.3.2.3 for details on these dependence structures.
41 See for example Section 5.3.2, as the copula technique outlined in this section can be employed to construct numerous different multivariate distributions with identical marginal distributions.
Figure 5.2: Bivariate densities with standard normal marginals \(^{40}\)

different dependence structures the contour plots in Figure 5.2 give an impression on the corresponding bivariate density functions with standard normal marginal distributions.

5.2.3.2 Empirical Evidence against Linear Correlation

As the focus of this chapter is on portfolio selection, we are interested in the realistic modeling of multivariate relationships between (random) asset returns. Some of the problems with the linear coefficient of correlation as a measure of dependency have been outlined in 5.2.3.1. Numerous studies on hedge funds and other asset classes find strong evidence for non-linear relationships in different bi- and multivariate (empirical) return distributions, which have to be considered in the modeling
In Favre/Galeano (2001) and Favre/Galeano (2002) the authors employ non-linear local regression analysis with a broad Swiss market index to explain returns of different HFR strategy indices. Here the non-linearities in the relationship between these return series are obvious. The study reveals a concave profile between some arbitrage strategies or the HFR composite index and the Swiss market index. Furthermore the authors find out that with most hedge fund strategies diversification benefits disappear for extremely negative market returns.

In Edwards/Caglayan (2001a) the correlation of hedge fund indices with the S&P 500 index is examined in bull and bear markets over the period from 1990 to 1998. A broad range of equally- and value-weighted strategy and composite indices is calculated from the MAR database for monthly hedge fund returns. The authors report significant differences in (conditional) correlation coefficients for bull markets with a positive S&P500 return and bear markets with a negative index return. The hedge fund returns in bear markets are very often negative and most style indices exhibit higher correlation in such a market environment than in a bull market. Here the authors identify problems with the often stated diversification benefit of hedge funds as the correlation increases when diversification is needed most.

A similar result is presented in Lhabitant (2002). Here the monthly returns for the CSFB/Tremont indices are analyzed over the period from 1994 to 2001. The data is divided into subsets of up- and down-markets for indices that represent US equities, European equities and international bonds. For this sample, the correlation coefficient in down markets is in most cases higher than the coefficient for up markets,

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44 Bull markets in Edwards/Caglayan (2001a) are defined as months with a S&P500 return of 1 percent or more. In bearish months the S&P500 index loses 1 percent or more, see Edwards/Caglayan (2001a), p. 98.
again suggesting problems when investors trust in the diversification effect.\footnote{See Lhabitant (2002), pp. 171-172.}

In Jaeger (2002) the relationship between S&P500 returns and the returns from HFR or MAR hedge fund indices is studied. Between 1990 and 2002, most hedge fund strategies show higher coefficients of correlation in bad months than in good months.\footnote{In this context “bad” and “good” months refer to the development of the S&P500.} But Jaeger (2002) does also identify strategies, where it is the opposite way around. Here a slightly positive correlation of the strategy index Futures Systematic Active with the S&P500 in good months turns into a negative correlation in bad months indicating diversification benefits. This is because investors like positive correlation when equity (or bond) markets advance and negative correlation in declining markets.\footnote{See Jaeger (2002), pp. 123-126.}

The correlation coefficient of EACM indices with a balanced portfolio that consists of the S&P500 index and a bond index for the period from 1990 to 2000 is analyzed in Schneeweis/Spurgin (2000). The difference in the correlation for the top 40 and the bottom 40 months (top and bottom months measured according to the return of the combined stock and bond portfolio) is very high for several hedge fund indices. With most strategies the correlation in top months is higher than in months with poor performance. Like in other studies this is seen as an indication for a correlation breakdown. But like in Jaeger (2002) the authors are also able to identify a strategy with so-called “good correlation”, which stands for a decreasing correlation from top to bottom months.\footnote{See Schneeweis/Spurgin (2000), pp. 3-5.}

In Lo (2001) a so-called “phase-locking” behavior\footnote{“Phase-locking” describes situations, where physical and natural phenomena that are otherwise uncorrelated become synchronized, see Lo (2001), p. 24.} is examined for different hedge fund strategies and the S&P500 index. Lo (2001) determines an asymmetric sensitivity of monthly hedge fund index data to the S&P500 index in form of different
beta coefficients in down-markets and in up-markets. For certain strategy indices, seemingly unrelated returns show very high beta coefficients with the S&P500 index in case of a down-market. The author concludes that the fund returns exhibit non-linearities that are not captured by correlation coefficients and linear factor models.\textsuperscript{51}

The study by Könberg/Lindberg (2001) analyzes the returns of hedge fund indices from the data provider HFR with respect to the performance of the S&P500 index. The authors distinguish the 40 worst, middle, and best months of S&P500 performance, and for every data set the corresponding hedge fund index return correlations are determined. Most of the indices had their lowest correlation in the top months of the S&P500 index, and the highest correlations were calculated in the worst periods for the S&P500 index.\textsuperscript{52}

The results of these and other studies indicate non-linear dependence structures between classical portfolio assets like stocks or bonds and hedge funds. But when we use the correlation coefficient conditional on a return level, we have to bear some theoretical aspects in mind. The analysis of conditional correlations is problematic as outlined for example in Boyer et al. (1997), Longin/Solnik (2001) and Kat (2002a). To estimate correlations conditional on different values of one (or both) variables results in different conditional correlations as the variance of the conditioned variable changes.\textsuperscript{53} Therefore, the absolute values for up- and downside correlations have to be handled with care. But the problem of an asymmetric correlation profile is a generally accepted fact and should be considered in the process of portfolio

\textsuperscript{51} See Lo (2001), pp. 24-27.
\textsuperscript{52} See Könberg/Lindberg (2001), pp. 25-27.
\textsuperscript{53} In Ang/Chen (2002) the authors propose to measure up- and downside-correlations and the corresponding betas relative to a multivariate normal distribution, see Ang/Chen (2002), pp. 460-463. Longin/Solnik (2001) also analyze correlations relative to the correlations of a multivariate normal distribution, see Longin/Solnik (2001), pp. 667-670. Conditional measures of dependence for normally- and Student-distributed variables are derived in Malevergne/Sornette (2002).
5.2.4 Summary

If neither the quadratic utility function nor the multivariate return distributions are able to justify the mean-variance approach, we have to find alternative ways to determine optimal portfolios. Improvements of the classical Markowitz mean-variance approach to portfolio selection were suggested by several authors but only a few of the procedures additionally account for a flexible (non-normal or more general non-elliptical) dependence structure of the relevant assets. In the following sections we will present a modified portfolio selection technique. This technique is flexible enough to integrate different univariate return distributions for the securities to be included in the portfolio and different dependence structures between these potential portfolio constituents in order to determine optimal portfolios.

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54 This asymmetric correlation profile excludes even the more flexible portfolio approach with multivariate elliptical return distributions that was justified in Ingersoll (1987). The class of multivariate elliptical distributions is not able to model asymmetric dependencies.

55 Fung/Hsieh (1999a) state that a mean-variance ranking can be (empirically) justified for a variety of utility functions and return distributions. Rankings with quadratic approximations to power and exponential utility functions are found to be in (close) accordance with the actual ranking that results from the expected utility of historical returns. Therefore, the question if higher moments of return distributions are really important has to be addressed. But as pointed out in the conclusions section of Fung/Hsieh (1999a) the mean-variance approach is not seen as sufficient assess risk in (not-normally distributed) hedge fund returns, see Fung/Hsieh (1999a), p. 57. The researched hedge fund data in Fung/Hsieh (1999a) is equal to the data set in Fung/Hsieh (1997a) but unfortunately no details are available on descriptive statistics of the (unconditional) return distributions.

In this section an alternative approach to model multivariate portfolio returns is presented. Like Markowitz in his seminal article we will begin with the second stage of portfolio selection: starting with relevant beliefs and ending up with the choice of a portfolio.\textsuperscript{57,58} The method we employ to model the multivariate return distributions is visualized in Figure 5.3.

![Figure 5.3: Portfolio modeling approach](image)

We consider a portfolio with a number of \( n \) assets. For these assets the (expected) marginal return distributions are the basis of the portfolio considerations. With historical return data at hand there are different methods for the derivation of such distributions. We will use a non-parametric approach with kernel densities that

\textsuperscript{57} See Markowitz (1952), p. 77.

\textsuperscript{58} Therefore, we will not discuss the problem of how to estimate parameters or distributions in order to get predictive information on the distribution of future returns.
is introduced in Section 5.3.1. Other methods include the fitting of parametric distributions or for example the fitting of an Edgeworth Expansion with estimated moments.

When the marginal distributions are estimated we consider the multivariate dependence structure between the assets. This dependence structure is modeled in Section 5.3.2 with a mathematical tool, the copula. These copula functions associate different random variables with each other by specifying the relation of their behavior. The resulting multivariate return distribution is the starting point for the portfolio selection process.

### 5.3.1 Marginal Distributions

In the portfolio framework we will regard the first four moments of the respective asset distributions as these are of major influence. We will introduce these four moments of a distribution in Section 5.3.1.1. Section 5.3.1.2 provides a non-parametric method to estimate univariate return distributions: the kernel density method.

In our modeling approach the assets’ return distributions are assumed to be continuous. As discrete (or simple) returns are considered this is of course only an approximation. Due to limited liability the support of these returns should be \([-1, \infty)\) but the continuous density functions are usually defined on \(\mathbb{R}\).\(^{59}\) The cumulative density functions for the return distributions of the \(n\) assets under consideration are given by \(F_1, \ldots, F_n\). The corresponding probability density functions of the return distributions are \(f_1, \ldots, f_n\).\(^{60,61}\)

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\(^{59}\) Problems with different return notions and consequences for financial modeling are for example addressed in Bamberg/Dorfleitner (2002) and Dorfleitner (2002). For the distributions under consideration in the empirical analysis the error due to this approximation is negligible, see Table 5.2 in Section 5.4.2.

\(^{60}\) Here we implicitly assume that discrete returns are defined on the real line, see Sections 4.4.1 and 5.4.3 for this problem.
5.3.1.1 Moments of Univariate Distribution Functions

Based on the return distributions the relevant moments of these marginal return distributions are given in general form for all assets $i \in \{1, \ldots, n\}$ in equations 5.4 – 5.7. The first two (central) moments, mean ($\mu_i$) and variance ($\sigma_i^2$), of these return distributions are\(^{62}\)

$$\mu_i = \mathbb{E}^{F_i}[R] = \int R f_i(R) dR$$

(5.4)

and

$$\sigma_i^2 = \mathbb{E}^{F_i}\left[\left(R - \mathbb{E}^{F_i}[R]\right)^2\right] = \int (R - \mu_i)^2 f_i(R) dR.$$  

(5.5)

Skewness, the third moment of a distribution, is calculated as standardized third central moment according to\(^{63}\)

$$SK_i = \mathbb{E}^{F_i}\left[\left(\frac{R - \mathbb{E}^{F_i}[R]}{\sigma_i}\right)^3\right] = \int \left(\frac{R - \mu_i}{\sigma_i}\right)^3 f_i(R) dR.$$  

(5.6)

The kurtosis $KU_i$ of a continuous return distribution $F_i$ is also derived by the standardization of a central moment:\(^{64}\)

$$KU_i = \mathbb{E}^{F_i}\left[\left(\frac{R - \mathbb{E}^{F_i}[R]}{\sigma_i}\right)^4\right] = \int \left(\frac{R - \mu_i}{\sigma_i}\right)^4 f_i(R) dR.$$  

(5.7)

\(^{61}\) The cumulative density function or distribution function of such a random return variable $R$ is given by $F(R)$ and it is defined as $F(R) = P(\tilde{R} \leq R)$. Therefore, with continuous distributions

$$F(R) = \int_{-\infty}^{R} f(x) dx.$$  

Mathematically the probability density function $f(R)$ of the continuous return distribution is defined as derivative of it’s cumulative density function and should fulfill the following two properties:

$$f(R) \geq 0, \forall R \in \mathbb{R}$$

and

$$\int_{-\infty}^{+\infty} f(R) dR = 1,$$

see for example Hartung et al. (2002), p. 106.

\(^{62}\) See for example Abramowitz/Stegun (1972), p. 928.

\(^{63}\) See for example Abramowitz/Stegun (1972), p. 928.

\(^{64}\) See for example Abramowitz/Stegun (1972), p. 928.
5.3.1.2 Kernel Densities

A non-parametric method of modeling the univariate distribution of a random variable is the kernel density estimation. With a pre-defined kernel function the density is determined using univariate observed data for the random variable. The probability density function of a random variable $X_i$ with $T$ realized historical values $x_{i,1}, \ldots, x_{i,T}$ is determined according to

$$\hat{f}_i(x) = \frac{1}{T \cdot h} \sum_{t=1}^{T} K \left( \frac{x - x_{i,t}}{h} \right).$$

(5.8)

The variable $h$ describes the bandwidth or window width of the kernel estimator. This smoothing parameter will be set to the value derived in Silverman (1986) which is

$$h = \left( \frac{4}{3} \right)^{\frac{1}{5}} \sigma T^{-\frac{1}{5}}$$

(5.9)

with $\sigma$ as sample standard deviation of the $x_{i,1}, \ldots, x_{i,T}$. The kernel function $K(\cdot)$ determines the shape of the distribution. This function has to satisfy the non-negativity condition $K(\cdot) > 0$ and has to integrate for one:

$$\int_{-\infty}^{\infty} K(x) \, dx = 1.$$  

(5.10)

When the kernel function $K(\cdot)$ fulfills these conditions then the function $\hat{f}_i(\cdot)$ is a probability density function as it is also non-negative and integrates for one. We will use the Gaussian kernel function

$$K(u) = \frac{1}{\sqrt{2\pi}} e^{-\frac{u^2}{2}}$$

(5.11)

because of the continuity and differentiability properties. As a result of this estimation technique we obtain marginal distributions based on the data $x_{i,1}, \ldots, x_{i,T}$.

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65 See Silverman (1986), pp. 45-46. Alternative bandwidth values can for example be based on the interquartile range of the $x_{i,1}, \ldots, x_{i,T}$.


On one hand the density functions $\hat{f}_i(\cdot)$ are estimated but we can also derive the cumulative density functions $\hat{F}_i(\cdot)$ by integration.

### 5.3.2 Copulas

As the marginal return distributions for the assets can be estimated with the kernel densities we take a look at the dependence structure of returns. In this subsection we want to introduce the copula technique to obtain multivariate distribution functions. This technique is especially appropriate when there are strong views about the univariate distributions of assets and when the dependence structure is no longer linear. Copulas are a very powerful tool for modeling (financial) dependencies. The first and central difficulty when we work with this mathematical tool is the choice of an appropriate copula function in order to link different marginal distributions. The resulting multivariate distribution gives a complete picture of the multivariate dependencies in the model and should therefore be as realistic as possible.

In this section we will first take a look at the mathematical definition of a copula. Afterwards some general properties of copulas are introduced in Section 5.3.2.2 and in Section 5.3.2.3 some important copula functions are described. Section 5.3.2.4 considers the fitting of copula function to given multivariate data and finally Section 5.3.2.5 discusses the selection of the best fitting copula function.

#### 5.3.2.1 Definitions

The copula concept, which was introduced by Sklar (1959), allows to separate the univariate marginal distributions and the multivariate dependence structure for a multivariate distribution. Here the name “copula” emphasizes the way a copula

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69 An overview on applications of copulas in financial modeling is for example given in Bouyé et al. (2000).

function “couples” a joint or multivariate distribution to its univariate margins.\textsuperscript{71} So a copula is a function that is able to link univariate marginal distributions to a joint multivariate distribution.\textsuperscript{72}

A copula is basically defined as multivariate distribution function of random variables with standard-uniform marginal distributions.\textsuperscript{73,74} Three properties such a copula function with standard-uniformly distributed random variables \( u_1, \ldots, u_n \) has to fulfill are\textsuperscript{75}

\begin{equation}
C (u_1, \ldots, u_n) \text{ is increasing in each component } u_i \in [0, 1],
\end{equation}

\begin{align}
C (1, \ldots, 1, u_i, 1, \ldots, 1) &= u_i \text{ and } \\
C (u_1, \ldots, u_i = 0, \ldots, u_n) &= 0, \\
\forall i &\in \{1, \ldots, n\} \text{ with } u_i \in [0, 1],
\end{align}

and \( \forall (a_1, \ldots, a_n), (b_1, \ldots, b_n) \in [0, 1]^n \) with \( a_i \leq b_i \)

\begin{equation}
\sum_{i_1=1}^2 \cdots \sum_{i_n=1}^2 (-1)^{i_1+\cdots+i_n} C (x_{1i_1}, \ldots, x_{ni_n}) \geq 0
\end{equation}

where \( x_{j1} = a_j \) and \( x_{j2} = b_j, \forall j \in \{1, \ldots, n\} \).

These three characteristics of copula functions ensure that the resulting function is a multivariate cumulative probability function.

\textsuperscript{71} See Nelsen (1999), p. 15.

\textsuperscript{72} For simplicity we will consider these marginal distributions to be continuous. If the marginal distributions are not continuous a copula representation of the joint distribution is still possible but it is no longer unique, see Embrechts et al. (2002), p. 5, or Nelsen (1999), p. 15.

\textsuperscript{73} See for example Embrechts et al. (2002), p. 4.

\textsuperscript{74} Standard-uniform distributions are uniform distributions on \([0, 1]\) with cumulative density function

\[ F_U (x) = \begin{cases} 
0, & \text{when } x < 0 \\
x, & \text{when } 0 \leq x \leq 1 \\
1, & \text{when } x > 1
\end{cases} \]

and corresponding probability density function \( f_U (x) = 1 \) on the interval \([0, 1]\). See for example Bamberg/Baur (2002), pp. 106-107.

Cumulative probability density functions of continuous random variables are uniformly distributed on \([0, 1]\). As the marginal distribution functions \(F_1, \ldots, F_n\) for the random variables \(x_1, \ldots, x_n\) are uniformly distributed on \([0, 1]\) the copula associated with the marginal cumulative probability functions is a function on the unit \(n\)-cube \(C: [0, 1]^n \rightarrow [0, 1]\) that satisfies\(^{76}\)

\[
F(x_1, \ldots, x_n) = C(F_1(x_1), \ldots, F_n(x_n)).
\]  

(5.15)

This multivariate extension of the famous result of Sklar (1959) is the foundation of many applications of copula theory. From the theorem of Sklar we see that for continuous multivariate distribution functions, the univariate margins and the multivariate dependence structure represented by \(C\) can be separated.\(^{77}\)

If \(F\) is again an \(n\)-dimensional distribution function with (continuous) marginal distribution functions \(F_1, \ldots, F_n\) and \(u_1, \ldots, u_n \in [0, 1]\) then the copula function \(C\) is given by

\[
C(u_1, \ldots, u_n) = F(F_1^{-1}(u_1), \ldots, F_n^{-1}(u_n))
\]  

(5.16)

with \(F_i^{-1}\) as the quantile or inverse cumulative density function of \(F_i\).\(^{78}\) This relationship also provides a method to construct copulas from multivariate distribution functions.\(^{79}\)

From the multivariate cumulative density function represented by the copula function \(C\) in Equation 5.15 we can easily derive a multivariate probability density function:\(^{80}\)

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\(^{77}\) See Romano (2002a), p. 4.

\(^{78}\) The quantile function or inverse cumulative density function \(F_i^{-1}\) of a distribution function \(F_i\) is given by

\[
F_i^{-1}(\alpha) = \inf \{ x | F_i(x) \geq \alpha \}
\]

with \(\alpha \in [0, 1]\). See for example Joe (1997), p. 10.

\[ f(x_1, \ldots, x_n) = \frac{\partial^n F(x_1, \ldots, x_n)}{\partial x_1 \ldots \partial x_n} = \prod_{i=1}^{n} \left( \frac{\partial F_i(x_i)}{\partial x_i} \right) \frac{\partial^n C(F_1(x_1), \ldots, F_n(x_n))}{\partial x_1 \ldots \partial x_n}. \] (5.17)

This multivariate density function for the \( n \) underlying marginal distributions is the product of the marginal densities (the differentiated cumulative density functions) and the differentiated copula function.

### 5.3.2.2 Important Copula Properties

In this section we will describe two important properties of copula functions. At first upper and lower bounds for multivariate copula functions are presented and then we will focus on the attractive invariance property of copulas under increasing and continuous transformations of the marginals.

Fréchet-Hoeffding bounds for joint distribution functions also apply for copulas.\(^81\) These bounds for any copula function \( C \) with all \( u_i \) in \([0, 1]\) are given by\(^82\)

\[ W(u_1, \ldots, u_n) \leq C(u_1, \ldots, u_n) \leq M(u_1, \ldots, u_n), \] (5.18)

with lower bound

\[ W(u_1, \ldots, u_n) = \max (u_1 + \ldots + u_n - n + 1, 0) \]

and upper bound

\[ M(u_1, \ldots, u_n) = \min (u_1, \ldots, u_n). \]

The invariance under increasing and continuous transformations of the marginal distributions is a very useful copula property. Strictly increasing transformation functions \( T_1, \ldots, T_n \) for the underlying continuous random variables \( X_1, \ldots, X_n \) with copula function \( C \) result in the transformed variables \( T_1(X_1), \ldots, T_n(X_n) \) having the same copula function \( C \).\(^83\) Thus the dependence structure of \( X_1, \ldots, X_n \) is captured

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\(^80\) See for example Romano (2002b), p. 6 for such a density function in two dimensions.

\(^81\) These bounds are also known as Fréchet bounds, see for example De Matteis (2001), p. 13.


by the copula, regardless of the scale in which the variables are measured.\(^8^4\)

### 5.3.2.3 Different Copula Functions

In this section we will discuss different copula functions. At first the product copula is presented for independent random variables. If the random variables are not independent, their dependence structure can be modeled by different families and types of copula functions. In the following we will present several parametric copula functions from the elliptic and Archimedian class of copulas that are very common in financial modeling.\(^8^5\) For these parametric copula functions a vector of copula parameters \(\phi\) determines the copula function \(C(u_1, \ldots, u_n|\phi)\). This section concludes with the introduction of the empirical copula which is a non-parametric copula that is especially useful when we will determine the best fitting copula function for a given data set in Section 5.3.2.5.

#### Product Copula

When the information of independence between the random variables is given, no dependence structure in the form of a special copula function is needed to determine the joint multivariate probability function. This link between univariate and multivariate probability functions which is the simplest example of a copula is usually entitled "product copula".\(^8^6\) For independent random variables \(x_1, \ldots, x_n\) the copula function \(C_P\) equals the product of the marginal cumulative density functions \(F_1, \ldots, F_n\).\(^8^7\)

\[
F(x_1, \ldots, x_n) = C_P(F_1(x_1), \ldots, F_n(x_n)) = F_1(x_1) \cdot \ldots \cdot F_n(x_n). \tag{5.19}
\]

\(^8^4\) See De Matteis (2001), p. 17.
\(^8^5\) For an analysis of non-parametric or empirical copula functions see Durrleman et al. (2000), pp. 7-11, and Bouyé et al. (2000), pp. 22-23.
\(^8^7\) See for example Embrechts et al. (2002), p. 5.
This result is straightforward and the corresponding multivariate probability function $f$ is the product of the given univariate density functions:

$$f(x_1, \ldots, x_n) = \frac{\partial^n F(x_1, \ldots, x_n)}{\partial x_1 \ldots \partial x_n} = \prod_{i=1}^{n} \left( \frac{\partial F_i(x_i)}{\partial x_i} \right).$$  \hspace{1cm} (5.20)

As we pointed out the assumption of independent random variables is usually not met in the case of financial returns. Therefore, we will present further copula functions that are able to produce a realistic dependence structure for the random variables under consideration.

**Elliptical Copulas**

As a starting point into different families of copula functions we will examine so-called elliptical copula functions.\textsuperscript{88} To put it simple elliptical copulas are copulas of elliptical distributions thus sharing many of their tractable properties.\textsuperscript{89} These copulas are derived from multivariate elliptical distributions (the contour plots of these distributions are elliptical) with Sklar’s theorem in Equation 5.15 on page 198. The most prominent representative of this class of copulas is the Normal copula but we will also present the t-copula that is based on the (Student) t-distribution.

The archetype of elliptical distributions is the normal distribution. The corresponding copula is the Normal or Gaussian copula. This copula is parametrical as it involves a correlation matrix $\rho$ with the linear correlation coefficients $\rho_{i,j}$ (see Section 5.2.3) for every pair $i, j$ of random variables. The $n$-variate Gaussian or Normal copula $C_N$ for the quantiles $u_1, \ldots, u_n$ which depends on the matrix $\rho$ of coefficients of correlation\textsuperscript{90} has the general form:\textsuperscript{91}

$$C_N(u_1, \ldots, u_n|\rho) = \Phi_{\rho} \left( \Phi^{-1}(u_1), \ldots, \Phi^{-1}(u_n)|\rho \right)$$  \hspace{1cm} (5.21)

\textsuperscript{88} See Frahm et al. (2002) for a brief introduction to elliptical copulas.

\textsuperscript{89} See for example Embrechts et al. (2001), pp. 22-30.

\textsuperscript{90} The matrix $\rho$ has to be symmetric and positive definite.

where $\Phi_{\rho}$ is the standardized multivariate normal distribution function and $\Phi^{-1}$ is the inverse of the standardized univariate normal distribution function. The Gaussian copula in Equation 5.21 is derived from the multivariate standard normal distribution. The covariance matrix for the multivariate normal distribution is equal to the relevant (linear) correlation matrix $\rho$.92 Another expression for the multivariate Normal copula with $x$ as column vector of the variables $x_1, \ldots, x_n$ is therefore

$$C_N (u_1, \ldots, u_n | \rho) = \int_{-\infty}^{\Phi^{-1}(u_1)} \cdots \int_{-\infty}^{\Phi^{-1}(u_n)} (2\pi)^{-\frac{n}{2}} |\rho|^{-\frac{1}{2}} e^{-\frac{1}{2} x^T \rho^{-1} x} dx. \quad (5.23)$$

The density of the Normal copula with correlation matrix $\rho$ is given by:

$$c_N (u_1, \ldots, u_n | \rho) = |\rho|^{-\frac{1}{2}} e^{-\frac{1}{2} \xi^T (\rho^{-1} - I) \xi} \quad (5.24)$$

with the column vector $\xi$ of the normal inverse $\Phi^{-1} (u_1, \ldots, \Phi^{-1} (u_n)$.

Figure 5.4 on the following page gives an impression on how the structure of the Gaussian copula looks like. For different coefficients of correlation the density function given in Equation 5.24 is plotted in the bivariate case.

The $n$-variate t-copula is derived from the multivariate (Student) t-distribution. Similar to the Normal copula the t-copula $C_t$ depends on the linear correlation matrix $\rho$. The general form of the t-copula with $\nu$ degrees of freedom is

$$C_t (u_1, \ldots, u_n | \rho, \nu) = T^n_{\rho,\nu} (T^{-1}_{\nu} (u_1), \ldots, T^{-1}_{\nu} (u_n) | \rho, \nu) \quad (5.25)$$

where $T^n_{\rho,\nu}$ denotes the cumulative density function of the $n$-variate t-distribution function with correlation matrix $\rho$ and $T^{-1}_{\nu}$ is the inverse of the distribution function of the univariate t-distribution.95

92 The density of the multivariate standard normal or Gaussian distribution $\phi$ with $x$ as column vector of the variables $x_1, \ldots, x_n$ and correlation matrix $\rho$ is:

$$\phi_{\rho} (x_1, \ldots, x_n) = (2\pi)^{-\frac{n}{2}} |\rho|^{-\frac{1}{2}} e^{-\frac{1}{2} x^T \rho^{-1} x}, \quad (5.22)$$

see for example Thode (2002), pp. 182-183. For the multivariate standard normal distribution the mean $\mu_i$ of all random variables $x_i$ is zero and the corresponding variances equal 1.

93 See Bouyé et al. (2000), p. 17.

Figure 5.4: Bivariate densities of Gaussian copulas with different correlation coefficients

The t-copula with $\mathbf{x}$ as column vector of the variables $x_1, \ldots, x_n$ can be written

$$
C_t(u_1, \ldots, u_n | \rho, \nu) = \int_{-\infty}^{u_1^{-1}(u_1)} \cdots \int_{-\infty}^{u_n^{-1}(u_n)} |\rho|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\mathbf{x}^T \rho^{-1} \mathbf{x}}{\nu}\right)^{-\frac{\nu+n}{2}} dx,
$$

with the gamma function $\Gamma(.)$. The corresponding density function for the t-copula

$$
t^n_{\rho, \nu}(x_1, \ldots, x_n) = (\pi \nu)^{-\frac{n}{2}} |\rho|^{-\frac{n}{2}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)} \left(1 + \frac{\mathbf{x}^T \rho^{-1} \mathbf{x}}{\nu}\right)^{-\frac{\nu+n}{2}},
$$

see for example Bouyé et al. (2000), p. 17.

95 The density of the standardized $n$-variate t-distribution $t^n_{R, \nu}$ with $\mathbf{x}$ as column vector of the variables $x_1, \ldots, x_n$ and correlation matrix $\rho$ is given by

is given by

\[ c_t(u_1, \ldots, u_n|\rho, \nu) = |\rho|^{-\frac{1}{2}} \frac{\Gamma\left(\frac{\nu+n}{2}\right)[\Gamma\left(\frac{\nu}{2}\right)]^n(1 + \frac{1}{\nu} \xi^T \rho^{-1} \xi)^{-\frac{\nu+n}{2}}}{\Gamma\left(\frac{\nu+1}{2}\right)^n \Gamma\left(\frac{\nu}{2}\right) \prod_{i=1}^{n} \left(1 + \frac{\xi_i^2}{\nu}\right)^{-\frac{\nu+1}{2}}}, \tag{5.29} \]

with \( \xi_i = t^{-1}_\nu(u_i) \), for all \( i \in \{1, \ldots, n\} \).\(^{98}\)

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**Figure 5.5:** Bivariate densities of t-copulas with different degrees of freedom

In Figure 5.5 the copula density is plotted for different degrees of freedom \( \nu \) with a fixed correlation while Figure 5.6 on the following page shows four t-copula densities.

---

\(^{97}\) The gamma function \( \Gamma(.) \) is given by:

\[ \Gamma(x) = \int_0^\infty h^{x-1} e^{-h} dh, \tag{5.28} \]

see for example Abramowitz/Stegun (1972), p. 255.

with 6 degrees of freedom and different correlations. With more degrees of freedom the probability mass in the tails of the bivariate distribution increases. When we take a look at Figure 5.6 it is obvious that the concentration of the probability increases with higher correlation coefficients.

Figure 5.6: Bivariate densities of t-copulas with different correlation coefficients

While elliptical copulas have many attractive features\(^9\) there are also some drawbacks. These copulas do not have closed form expressions and they are restricted to have radial symmetry.\(^\text{10}\) As in many circumstances it is more appropriate to model

\(^9\) For example tail dependence can be modeled with a t-copula while the Normal copula is not able to produce such extreme dependencies, see for example Bouyé et al. (2000), p. 18, and Embrechts et al. (2001), p. 30.

\(^\text{10}\) See Embrechts et al. (2001), p. 30.
a stronger dependence between financial losses than between gains we will also take other copula functions into consideration.

**Archimedean Copulas**

In the following we present dependence functions from the Archimedean copula family.\(^{101}\) In contrast to elliptical copulas these Archimedian copulas have closed form expressions and they are not derived with the theorem of Sklar in Equation 5.15 on page 198. For Archimedian copulas the so-called generator function \(\varphi\) plays an important role. This is a function \(\varphi(u) : [0, 1] \rightarrow [0, \infty]\) which is continuous, strictly decreasing \((\varphi'(u) < 0 \text{ for all } u \in [0, 1])\), and convex \((\varphi''(u) > 0 \text{ for all } u \in [0, 1])\).\(^{102}\)

With such a generator function \(\varphi\) the function \(C : [0, 1]^n \rightarrow [0, 1]\)

\[
C(u_1, \ldots, u_n) = \varphi^{-1}(\varphi(u_1) + \ldots + \varphi(u_n)) \quad (5.30)
\]

is a copula if the pseudo-inverse \(\varphi^{-1}\) of \(\varphi\) is continuous, decreasing on \([0, \infty]\) and strictly decreasing on \([0, \varphi(0)]\).\(^{103,104}\)

In the following we will introduce three multivariate Archimedean copula functions that are based on different generator functions: the Clayton copula, the Frank copula, and the Gumbel copula.

The \(n\)-variate Clayton copula has a generator \(\varphi(u) = u^{-\beta_C} - 1\) with \(\beta_C > 0\). The corresponding copula function \(C_C\) is given by\(^{105}\)

\(^{101}\) See Nelsen (1999), pp. 89-124, for a discussion of Archimedian one-parameter and two-parameter copula functions.


\(^{104}\) The pseudo-inverse of \(\varphi\) is the function \(\varphi^{-1} : [0, \infty] \rightarrow [0, 1]\) given by

\[
\varphi^{-1}(t) = \begin{cases} 
\varphi^{-1}(t), & 0 \leq t \leq \varphi(0) \\
0, & \varphi(0) \leq t \leq \infty 
\end{cases} \quad (5.31)
\]

see Embrechts et al. (2001), p. 31.
Figure 5.7 gives an impression of the dependence structure imposed by a Clayton copula with different $\beta_C$ coefficients.

The n-variate Frank copula is based on the generator function
\[
\varphi(u) = \ln \left[ \frac{\exp(-\beta_F u_i) - 1}{\exp(-\beta_F) - 1} \right]
\]
with $\beta_F > 0$. The multivariate Frank copula function $C_F$ for $n \geq 3$ is therefore given by
\[
C_F (u_1, \ldots, u_n | \beta_F) = -\frac{1}{\beta_F} \ln \left[ 1 + \prod_{i=1}^{n} \frac{1 - \exp(-\beta_F u_i)}{(e^{-\beta_F} - 1)^{n-1}} \right]. \tag{5.33}
\]

In Figure 5.8 the Frank copula density is plotted for different $\beta$-values.

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$\beta_C$
The generator function for the n-variate Gumbel copula is \( \varphi(u) = (-\ln u)^{\beta_G} \) with \( \beta_G \geq 1 \). The corresponding multivariate Gumbel copula \( C_G \) is given by:

\[
C_G (u_1, \ldots, u_n | \beta_G) = e^{-\left[ \sum_{i=1}^{n} (-\ln u_i)^{\beta_G} \right]^{\frac{1}{\beta_G}}}
\] (5.34)

Figure 5.9 on the following page shows the bivariate Gumbel copula for different parametrizations.

**Empirical Copula Functions**

After introducing different parametric copula functions we will take a look at the non-parametric empirical copula. This method for constructing a copula function from empirical data was introduced by Deheuvels (1979).\(^{108}\) The empirical copula

\(^{107}\) See for example Romano (2002a), p. 4.
function is especially useful when it comes to the selection of a particular parametric copula. In Section 5.3.2.4 we are interested in the best fit copula for particular return data. Here we will make use of the empirical copula as a reference for the parametric copula functions which have been fitted to the data set. We will measure the deviation of the parametric copulas from the empirical copula by different measures of distance.

For a sample of size $T$ with a number of $n$ observable (random) variables the data set for the empirical copula is given by $x_{1,t}, \ldots, x_{n,t}$ with $t$ in $1, \ldots, T$. The corresponding rank statistic for the sample is $\text{rank}_{1,t}, \ldots, \text{rank}_{n,t}$ with $t$ in $1, \ldots, T$.\footnote{Therefore, the empirical copula is often explicitly called Deheuvels copula, see Durrleman et al. (2000), p. 7.}

According to Deheuvels (1981) an empirical copula is characterized as a copula $\hat{C}$\footnote{See for example Hartung et al. (2002), p. 140, on rank statistics.}
that is defined on the lattice \( L \) by

\[
L = \left\{ \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) ; t_i = 1, \ldots, T; 1 \leq i \leq n \right\}
\]

(5.35)

by

\[
\hat{C}\left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) = \frac{1}{T} \sum_{t=1}^{T} \prod_{i=1}^{n} 1_{[\text{rank}_i,t \leq t_i]}.
\]

(5.36)

The corresponding empirical copula density or frequency \( \hat{c} \) is defined in terms of the empirical copula function \( \hat{C} \) by

\[
\hat{c}\left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) = \sum_{j_1=1}^{2} \ldots \sum_{j_n=1}^{2} (-1)^{j_1+\ldots+j_n} \hat{C}\left( \frac{t_1-j_1+1}{T}, \ldots, \frac{t_n-j_n+1}{T} \right).
\]

(5.37)

The straightforward relation between the empirical copula distribution \( \hat{C} \) and the empirical copula density or frequency \( \hat{c} \) is given by

\[
\hat{C}\left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) = \sum_{j_1=1}^{t_1} \ldots \sum_{j_n=1}^{t_n} \hat{c}\left( \frac{j_1}{T}, \ldots, \frac{j_n}{T} \right).
\]

(5.38)

Figure 5.10 illustrates the surface of an empirical copula function from two different perspectives.

\[\text{Figure 5.10: Empirical copula function}^{113}\]

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110 See Nelsen (1999), pp. 176-177, for a bivariate representation.


113 The data for the empirical copula function is taken from the two time series MSCI and HFR FoF, see Section 5.4.
5 Portfolio-Selection including Hedge Funds

5.3.2.4 Fitting Copula Functions

In this section we will introduce a method to estimate the parameters of a given copula function. Different methods to determine copula parameters have been proposed in the literature. The maximum likelihood estimation of the parameters of a multivariate distribution for a given set of (multivariate) realizations is a classical method to fit a distribution and widely used in the copula context.\textsuperscript{114} The main problem with the maximum likelihood method is that it becomes computationally very extensive in the case of increasing dimensions of the multivariate distribution.\textsuperscript{115} Another approach for fitting copulas is to estimate measures of dependence from the data set. According to these estimates the copula parameter is selected. This is usually rather simple as most dependence measures can be represented with a function of the copula under consideration.\textsuperscript{116} A major drawback of this method is that it works only with bivariate one-parameter copula functions.\textsuperscript{117} Non-parametric estimation of a copula function is another approach to the fitting of a multivariate distribution to data. No particular (parametric) copula function is specified in this case, rather a so-called empirical copula is determined.\textsuperscript{118} We will make use of the non-parametric empirical copula that has been introduced in Section 5.3.2.3 when the fit of different copulas is examined in Section 5.3.2.5.\textsuperscript{119}

In Joe/Xu (1996) the authors proposed the method of inference functions for margins. Because of the problems with the joint estimation of the parameters of a multivariate distribution, the copula representation is used when the parameters

\textsuperscript{114} See for example Romano (2002b), p. 6-7.
\textsuperscript{115} See Durrleman et al. (2000), p. 4.
\textsuperscript{116} See for example Embrechts et al. (2001) for measures of dependence for different copula functions.
\textsuperscript{117} See Romano (2002b), p. 9.
\textsuperscript{118} See Durrleman et al. (2000), pp. 7-9.
\textsuperscript{119} The construction or fitting of non-parametric copulas will not be subject of this section. See for example Romano (2002b), p. 10, and De Matteis (2001), pp. 36-37, for different non-parametric approaches.
are split into parameters for the marginal distributions and parameters for the dependence structure.\textsuperscript{120} The estimation process is then conducted in two steps. At first the parameters of the marginal distributions are estimated and then based on the univariate marginal distributions the parameters of the multivariate dependence structure are determined. For the estimation of the parametrical marginals and the dependence structure Joe/Xu (1996) proposed a maximum likelihood approach. We will make use of this idea called method of inference functions for margins as we split the estimation process in two steps. At first the marginal distributions are determined and afterwards the copula is fitted with a maximum likelihood approach.

Therefore, we assume that the multivariate probability function, we would like to identify, has the density\textsuperscript{121}

\[
f(x_1, \ldots, x_n|\phi, \gamma_1, \ldots, \gamma_n) = c(F_1(x_1|\gamma_1), \ldots, F_n(x_n|\gamma_n)|\phi) \prod_{i=1}^{n} (f_i(x_i|\gamma_i)).
\]  

(5.39)

\(c(.)\) is the density of the copula function under consideration with parameter vector \(\phi\). The \(n\) marginal distribution functions \(F_1, \ldots, F_n\) and \(n\) marginal densities \(f_1, \ldots, f_n\) are dependent on the parameter vectors \(\gamma_1, \ldots, \gamma_n\). This \(n\)-variate probability density function can be estimated via maximum likelihood. The log-likelihood function for the joint distribution with a number of \(T\) observed random vectors \(x_{1,t}, \ldots, x_{n,t}\) with \(t\) in \(1, \ldots, T\) is\textsuperscript{122}

\[
L(\phi, \gamma_1, \ldots, \gamma_n) = \sum_{t=1}^{T} \ln f(x_{1,t}, \ldots, x_{n,t}|\phi, \gamma_1, \ldots, \gamma_n).
\]

(5.40)

Using Equation 5.39 the log-likelihood function in Equation 5.40 is given by\textsuperscript{123}

\[
L(\phi, \gamma_1, \ldots, \gamma_n) = \sum_{t=1}^{T} \ln c(F_1(x_{1,t}|\gamma_1), \ldots, F_n(x_{n,t}|\gamma_n)|\phi) + \sum_{t=1}^{T} \sum_{i=1}^{n} \ln f_i(x_{i,t}|\gamma_i).
\]

(5.41)

\textsuperscript{120} See Durrleman et al. (2000), p. 4.
\textsuperscript{121} See Joe/Xu (1996), p. 3.
\textsuperscript{122} See Ané/Kharoubi (2003), p. 422.
\textsuperscript{123} See Geman/Kharoubi (2003), p. 65.
The parameters $\gamma_1, \ldots, \gamma_n$ of the marginal distributions can also be determined by a log-likelihood estimation. If the marginal distributions with the estimated parameter vectors $\hat{\gamma}_1, \ldots, \hat{\gamma}_n$ are given, the vector of copula parameters can be estimated by maximizing the log-likelihood function over $\phi$ in order to obtain $\hat{\phi}$:

$$\hat{\phi} = \max_\phi L (\phi, \hat{\gamma}_1, \ldots, \hat{\gamma}_n) = \sum_{t=1}^T \ln c(F_1(x_{1,t}|\hat{\gamma}_1), \ldots, F_n(x_{n,t}|\hat{\gamma}_n)|\phi). \quad (5.42)$$

We will employ a slightly different approach to estimate the dependence structure for the copula function. We will use the non-parametric kernel density approach outlined in Section 5.3.1.2 on page 195 in order to estimate the marginal densities $\hat{f}_i(.)$ and the corresponding cumulative density functions $\hat{F}_i(.)$. When the marginal distributions are determined, the return data is transformed with the cumulative density function of the marginals. The second step (the maximum likelihood fitting of the copula function) is carried out as described before. The log-likelihood function becomes

$$\hat{\phi} = \max_\phi L \left( \phi, \hat{f}_1, \ldots, \hat{f}_n \right) = \sum_{t=1}^T \ln c \left( \hat{F}_1(x_{1,t}), \ldots, \hat{F}_n(x_{n,t}) | \phi \right). \quad (5.43)$$

This approach is mainly suitable for Archimedean copulas. In case of the Normal and the $t$-copula we will use the correlation matrix to fit the dependence structures. The Gaussian copula is fully specified with the correlation data in a matrix $\hat{\rho}$ but the $t$-copula additionally has the parameter $\nu$. Therefore, in case of the $t$-copula we also make use of a maximum likelihood estimation for the degrees of freedom $\nu$.\footnote{See Demarta/McNeil (2004), p. 9, for this technique.}

With the correlation matrix held fixed we can determine the value for $\nu$ that maximizes the probability of the data sample.

### 5.3.2.5 Selecting Copula Functions

In this section certain criteria for the identification of the copula function that delivers the best fit for a given data sample is outlined. We will make use of the

\footnote{See Joe/Xu (1996), p. 3.}
empirical copula function introduced in Section 5.3.2.3. In order to compare the
difference between the parametric copulas that have been fitted with the maximum
likelihood approach in Section 5.3.2.4 and the empirical copula we can apply different
measures of distance. The empirical copula, which is used as a benchmark, is defined
on a lattice according to formula 5.35 on page 210. Therefore, the distance between
the continuous copula functions and the empirical copula is measured with discrete
norms.\textsuperscript{126}

In the following we employ two measures to determine the goodness of fit, the
Anderson-Darling and the Integrated Anderson-Darling distance statistics. The
Anderson-Darling statistic ($AD$) is given by\textsuperscript{127}

$$AD = \max_{(t_1, \ldots, t_T) \in \mathbb{L}} \left| \frac{\hat{C} \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right) - C \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right)}{\sqrt{C \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right) \left( 1 - C \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right) \right)}} \right|$$

(5.44)

with $t_i = 1, \ldots, T$ for all $i$ in $1, \ldots, n$.

With the Anderson-Darling statistic the maximum relative\textsuperscript{128} difference between
the empirical copula function $\hat{C}$ and the parametric copula under consideration $C$
is determined for all the points $\left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right)$ on the multivariate lattice $\mathbb{L}$. When the
parametric copula function $C$ attains a value of 1 the denominator of Equation 5.44
becomes equal to zero. This is the case for $t_1 = \ldots = t_n = T$. Therefore, we exclude
the point $\left( \frac{T}{T}, \ldots, \frac{T}{T} \right)$ on the lattice $\mathbb{L}$ from the calculation of the Anderson-Darling
statistic. As the value of the empirical copula does by definition also reach 1 at
these coordinates\textsuperscript{129} the difference is still measured correctly.

The second measure for the goodness of fit for the parametric copulas is the Inte-
grated Anderson-Darling statistic ($IAD$) which is given by\textsuperscript{130}

127 See Geman/Kharoubi (2003), p. 67, for the bivariate case.
128 This difference is not determined in absolute terms. The difference of the distribution functions
in Equation 5.44 is calculated relative to the square root of $C \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right) \left( 1 - C \left( \frac{t_1}{T}, \ldots, \frac{t_T}{T} \right) \right)$.
129 See Equation 5.13 on page 197.
\[ IAD = \sum_{t_1=1}^{T} \ldots \sum_{t_n=1}^{T} \frac{\left[ \hat{C} \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) - C \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) \right]^2}{C \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) \left( 1 - C \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) \right)} \] (5.45)

The Integrated Anderson-Darling statistic reflects the sum of the relative differences between the empirical copula function \( \hat{C} \) and the parametric copula under consideration \( C \) over all the points \( \left( \frac{t_1}{T}, \ldots, \frac{t_n}{T} \right) \) on the lattice \( L \). Similar to the Anderson-Darling statistic we have to exclude the coordinate \( \left( \frac{T}{T}, \ldots, \frac{T}{T} \right) \) on the lattice \( L \).

\[ \text{See Ané/Kharoubi (2003), p. 426, for the bivariate case.} \]
5.4 Empirical Analysis

In the first sections of this chapter we discussed the theoretical background of modeling multivariate dependence structures for random variables. This section outlines the introduced copula technique for a given set of asset returns. In this single-period model we determine different optimal asset allocations in the case of three assets, namely bonds, stocks, and hedge funds. We discuss some rather technical matters that concern portfolio optimization and risk measures in Section 5.4.1 and in Section 5.4.2 we take a look at the chosen portfolio constituents. Section 5.4.3 illustrates the modeling approach for the three specific marginal distributions of asset returns and their dependence structure. Especially the technique of fitting copula functions to data and the corresponding results are presented in detail. The portfolio optimization that is based on the scenario approach is conducted in Section 5.4.4. Here optimal portfolio allocations are determined with respect to a variety of risk measures.

5.4.1 Technical Aspects

In our portfolio selection framework we will consider \( n \) investment opportunities which deliver random monthly (simple) returns \( \tilde{R} = (\tilde{R}_1, \ldots, \tilde{R}_n)^T \). The uncertainty including the dependence structure of the random returns is modeled with the copula approach introduced in Section 5.3. At the beginning of the period under consideration the investor has to allocate his funds to \( n \) different assets. As we already outlined in Section 5.1 the specific allocations to these investments are denoted \( w_1, \ldots, w_n \) and aggregated in the weight vector \( \mathbf{w} \). As budget constraint these weights should sum for one. In this framework short sales are not allowed \((w_i \geq 0 \text{ for all } i \text{ in } 1, \ldots, n)\) and no riskless asset is considered.

The resulting random portfolio return is a function of the portfolio weights \( \tilde{R}_P(\mathbf{w}) = \)
As we consider a general optimization in the two dimensions risk and return it is possible to optimize different risk measures for a given (expected) portfolio return. The common problem can be stated for an arbitrary risk measure $\Pi$ which is a function of the portfolio returns that depend on the portfolio weights $w$ as:

$$\begin{align*}
\min & \quad \Pi(w) \\
\text{subject to} & \quad w^T \bar{R} \geq \mu \\
& \quad w^T 1 = 1 \\
& \quad w_i \geq 0
\end{align*}$$

(5.46)

with $\bar{R}$ as vector of the expected asset returns $\bar{R}_1, \ldots, \bar{R}_n$ and an expected portfolio return of $\mu$.

For different risk measures $\Pi$ such an optimization can be conducted. As we set up a probabilistic model for asset returns the optimization in this framework might be very cumbersome and time-consuming. Therefore, we employ a technique called scenario optimization in order to work in a deterministic world where optimization is rather simple. The scenario optimization is based on simulation from specified probability distributions. For the generated scenarios the uncertainty is removed. Then we are left with a deterministic problem where we have to determine an optimal portfolio allocation in order to maximize an objective while certain constraints are observed.\textsuperscript{132,133}

In order to obtain a number of $S$ scenarios, we will simulate from certain multivariate distributions that are constructed with copulas. These return series are obtained by different simulation algorithms that are based on the specific copula function under consideration.\textsuperscript{134} The finite set of scenarios is then represented by the return vectors

\textsuperscript{131} See for example Topaloglou et al. (2002), p. 1539.
\textsuperscript{132} See for example Scherer (2002), pp. 137-159, for a brief introduction to scenario optimization.
\textsuperscript{133} As an alternative we could directly use multivariate distribution functions and integrate over the different dimensions in order to determine a portfolio distribution of returns. The resulting portfolio return distribution would be a function of the portfolio weights for which an optimization is desired.
\( R^s = (R^s_1, \ldots, R^s_n) \) with \( s \) in \( 1, \ldots, S \). Each of the \( S \) vectors comprises a particular realization of the \( n \) asset returns which is of equal probability.\(^{135}\)

As stated in the optimization problem 5.46 on the preceding page, risk measures can be expressed as a function of the portfolio weights. Together with the individual asset returns the weighting vector determines the corresponding portfolio returns. The risk measures we employ for the following portfolio optimization analysis in Section 5.4.4 are the standard deviation, the mean-absolute deviation, the shortfall probability, the minimum regret, the Value-at-Risk, and the Conditional Value-at-Risk.

The standard deviation of returns is the basic risk measure in portfolio theory. This statistical figure gives an impression how tightly the returns are clustered around the mean. The standard deviation (SD) is calculated as square root of the corresponding variance of returns:

\[
SD(w) = \sqrt{E \left[ (R_P(w) - E[R_P(w)])^2 \right]}.
\] (5.47)

In the framework with \( S \) scenarios the common risk measure from Equation 5.47 becomes

\[
SD(w) = \sqrt{\frac{1}{S} \sum_{s=1}^{S} (w^T R^s - w^T \bar{R})^2}.
\] (5.48)

Another risk measure that is often used as an alternative to the classical standard deviation of returns, is the mean-absolute deviation (MAD). The MAD is defined

\(^{134}\) The simulation from the Gaussian copula is similar to drawing a sample from a normal distribution, see for example Romano (2002a), pp. 6-7. Like the simulation from the Gaussian copula to determine random numbers from a multivariate t-copula involves the Cholesky decomposition of the linear correlation matrix and in addition random variates from the \( \chi^2 \) distribution, see Romano (2002b), pp. 12-13. The algorithm for random variate generation from multivariate Archimedian copulas uses the inverse of the generator function \( \varphi \), see Lindskog (2000), pp. 38-43.

\(^{135}\) In a general scenario framework we could also assign different probabilities to certain scenarios, however this is not the case here.
as the mean absolute deviation of a random portfolio return $R_P$ (which is a function of the portfolio weights $w$) from the corresponding expected value:

$$\text{MAD}(w) = \mathbb{E} \left[ |R_P(w) - \mathbb{E}[R_P(w)]| \right].$$

(5.49)

As risk is measured as absolute deviation from the mean, outliers have less influence on the risk measure, compared to the classical risk measure standard deviation where squaring gives more weight to extreme values. The MAD implies that a further unit of underperformance or outperformance relative to the mean does entail the same difference in utility no matter how big the losses or gain already are. \(^{137}\) In our scenario framework the general risk measure MAD from Equation 5.49 becomes

$$\text{MAD}(w) = \frac{1}{S} \sum_{s=1}^{S} \left| w^T R_s - w^T \bar{R} \right|. \quad (5.50)$$

The risk measures standard deviation and mean-absolute deviation might be problematic because of the symmetry that punishes negative and positive deviations from the mean. Therefore, we will also consider risk measures that focus only on the downside of the return distribution.

The shortfall probability (SfP) to some defined target return is a straightforward risk measure. \(^{138}\) Here an investor is directly able to infer the probability of not reaching a specified minimum return $R_{\text{Target}}$. \(^{139}\) This risk figure is of special importance in the context of asset-liability management. In our analysis we will measure the shortfall probability relative to a return of 0\%.

$$\text{SfP}(w, R_{\text{Target}}) = \mathbb{P} \left( R_P(w) \leq R_{\text{Target}} \right).$$

(5.51)

Another rather simple risk measure is the minimum regret (MR). Investors that want to minimize the maximum portfolio loss should decide in accordance to this

\(^{136}\) See for example Topaloglou et al. (2002), p. 1542.


\(^{138}\) This risk measure is equal to the lower partial moment of order 0.

\(^{139}\) See Roy (1952) for the so-called “safety-first” principle.

\(^{140}\) See for example Schubert (2005) for a portfolio analysis with shortfall probabilities.
risk measure or at least take this measure into consideration. Therefore, decisions based on this risk measure are usually suitable for investors with a strong form of risk aversion.\footnote{See Young (1998), p. 673 and pp. 677-678.} The minimum regret measure MR for a set of realized asset returns is the maximal minimum portfolio return that depends on the weight vector $\mathbf{w}$.\footnote{See for example Scherer (2002), p. 146, on the minimum regret measure.}

\[ R_{\text{P}}^{\min}(\mathbf{w}) = \min \{ R_{\text{P}}(\mathbf{w}) \} \]  

(5.52)

For the optimization in Section 5.4.4 we are interested in the maximum of these return minima from Equation 5.52 for all the scenarios under consideration.\footnote{See Young (1998), p. 674.}

The Value-at-Risk (VaR) is widely used in the financial industry to summarize risk in a single number. With a certain level of confidence the VaR measure makes the statement that not more than the VaR (in percent or as absolute value in a specified currency) will be lost within a fixed period of time.\footnote{See Hull (2003), p. 346.} Formally the VaR (relative to the distribution’s expected value) is determined according to Equation 5.53: \footnote{See Zagst (2002), p. 252.}

\[ \text{VaR}(\mathbf{w}, \alpha) = \mathbb{E}[R_{\text{P}}(\mathbf{w})] - \sup \{ x \in \mathbb{R} : P(R_{\text{P}}(\mathbf{w}) \leq x) \leq \alpha \} . \]  

(5.53)

$1 - \alpha$ is the confidence level for the VaR. In the following analysis we will work with a confidence level of 95% which corresponds to a value of $\alpha$ of 5%.\footnote{The shortfall probability in Equation 5.51 with a target return that equals -VaR (measured relative to zero) is $\alpha$, see for example Deutsch (2004), p. 379.}

A problem with the Value-at-Risk measure is that it does not provide detailed information on the negative tail of a return distribution as it is focused on the $\alpha$-quantile of a distribution and does not take the shape of this tail into account.\footnote{In Yamai/Yoshiba (2005) this potential underestimation of portfolio risk with fat tailed distributions is called “tail-risk” of the VaR statistic, see Yamai/Yoshiba (2005), p. 1000.}

Moreover the VaR statistic is difficult to optimize using scenarios as the VaR (like
the shortfall probabilities) is a non-smooth and non-convex function with respect to
the portfolio weights and exhibits multiple local extreme values.\footnote{See Uryasev (2000), p. 15.}

As the VaR measure also lacks the desirable property of subadditivity\footnote{See Artzner et al. (1999) for an introduction of the coherence axioms and an analysis of the VaR.} more so-
phisticated risk measures have been introduced. The coherent risk statistic Con-
ditional Value-at-Risk (CVaR) reports the expected value of all losses that exceed
the corresponding VaR. Therefore, we have to specify a confidence level in order to
determine the VaR. Again we will use the common confidence level of 95%. The
CVaR equals

\[
\text{CVaR}(\mathbf{w}, \alpha) = -\mathbb{E}[\mathbf{R}_P(\mathbf{w})|\mathbf{R}_P(\mathbf{w}) \leq \sup\{x \in \mathbb{R} : P(\mathbf{R}_P(\mathbf{w}) \leq x) \leq \alpha\}]. \tag{5.54}
\]

In order to emphasize the link between VaR and CVaR we can restate Equation
5.54:

\[
\text{CVaR}(\mathbf{w}, \alpha) = -\mathbb{E}[\mathbf{R}_P(\mathbf{w})|\mathbf{R}_P(\mathbf{w}) \leq -(\text{VaR}(\mathbf{w}, \alpha) - \mathbb{E}[\mathbf{R}_P(\mathbf{w})])]. \tag{5.55}
\]

In Equation 5.55 the VaR is measured relative to the expected value.\footnote{If the VaR is measured relative to zero we get a intuitive formula for the CVaR:
\[
\text{CVaR}(\mathbf{w}, \alpha) = -\mathbb{E}[\mathbf{R}_P(\mathbf{w})|\mathbf{R}_P(\mathbf{w}) \leq -\text{VaR}_0(\mathbf{w}, \alpha)]. \tag{5.56}
\]

See for example Zagst (2002), pp. 251-266, for details on the connection between VaR and
CVaR.\footnote{When the distribution of returns is normal, the VaR and the CVaR are equivalent in a sense
that both risk measures provide the same optimal portfolios, see for example Uryasev (2000), p. 15.}
The six risk measures (standard deviation, shortfall probability, minimum regret, mean-absolute deviation, VaR and CVaR) introduced in this section will be considered in the portfolio optimization process in Section 5.4.4.

5.4.2 Portfolio Constituents

The portfolio that is analyzed should consist of three asset classes: stocks, bonds, and hedge funds. The MSCI All Countries World Index (MSCI) is used as proxy for the stock universe, bonds are represented by the Lehman Brothers Government/Credit Bond Index\textsuperscript{152} (LBGC), and the fund of hedge fund index from HFR delivers hedge fund return data. This index is abbreviated HFR FoF.\textsuperscript{153} All three indices are available in US Dollars and the corresponding (discrete) returns are therefore determined in this currency.

The development of the three index return series over the 96 months is illustrated in Figure 5.11 on the next page. A total of 96 monthly index returns is used to fit the marginal distributions and the dependence structures. The first simple returns are from January 1997 and the last returns have been realized in December 2004.

5.4.3 Modeling Multivariate Return Distributions

5.4.3.1 Marginal Distributions

The first step to determine the multivariate distribution of returns is the modeling of the corresponding marginal distributions. The kernel densities for the unconditional return distributions are estimated according to the method described in

\textsuperscript{152} The Lehman Brothers Government/Credit Bond Index was formerly known as Lehman Brothers Government/Corporate Bond Index.

\textsuperscript{153} As a consequence of the data problems outlined in Chapter 4 the annual returns of the HFR FoF index were adjusted (reduced) by 0.80% p.a.
Section 5.3.1. Each marginal distribution is based on 96 monthly returns for the corresponding index. Figure 5.12 on the following page shows the empirical cumulative density functions and the cumulative density functions based on the kernel densities.\footnote{In terms of the root mean squared error the cumulative kernel densities are a reasonable approximation to the empirical distribution functions for all three data sets.}

In order to get an idea of the general shape of the distributions Figure 5.13 illustrates the kernel densities over the [-0.20;0.20] interval and Table 5.1 reports certain descriptive statistics.

The non-parametric estimate of the stock return distribution that is based on the MSCI data is skewed to the left with an excess kurtosis close to zero. For the stocks we would expect the highest standard deviation of returns and this is reflected in the distribution based on the kernel density. The kernel density estimate from the LBGC-data represents the distribution of bond returns. This density is plotted
Figure 5.12: Empirical distribution function (black) and corresponding cumulative kernel densities (grey)

<table>
<thead>
<tr>
<th></th>
<th>Mean p.a.</th>
<th>Mean SD</th>
<th>Skewness</th>
<th>Excess Kurtosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>MSCI</td>
<td>0.61%</td>
<td>7.62%</td>
<td>-0.459</td>
<td>0.151</td>
</tr>
<tr>
<td>LBGC</td>
<td>0.54%</td>
<td>6.61%</td>
<td>-0.484</td>
<td>0.882</td>
</tr>
<tr>
<td>HFRI</td>
<td>0.57%</td>
<td>7.01%</td>
<td>-0.147</td>
<td>3.026</td>
</tr>
</tbody>
</table>

Table 5.1: Descriptive statistics for the estimated kernel densities

in the middle of Figure 5.13. A smaller return is expected while the volatility of returns should be significantly below the volatility for the stocks. This distribution is also skewed to the left and shows slightly more excess kurtosis compared to the stock return distribution. The marginal distribution of the hedge fund component is estimated with HFR FoF data. The expected monthly return on a fund of hedge funds investment that is modeled with this kernel density is below the expected return on stocks and above the corresponding expectation for bonds. The monthly hedge fund return distribution has a negative skewness that is higher than the values
for stocks and bonds. While the skewness statistics favors hedge funds, the fat tails of the estimated distribution are much more pronounced than those of bonds and stocks. The excess kurtosis of the fund of hedge fund distribution is significantly higher than the statistics for the other potential portfolio constituents.\textsuperscript{155}

The problem with a continuous return distribution that is defined on $\mathbb{R}$ instead of starting with a minimum return of $-1$ or $-100\%$ has to be analyzed for the fitted kernel densities. Table 5.2 on the next page reports the cumulative probabilities for certain return levels. All three kernel density functions assign no probability to returns below $-100\%$.\textsuperscript{156}

\textsuperscript{155} In Table 5.1 a problem of the kernel density method becomes obvious as the moments from the data set are not exactly met by the kernel density estimates, see Section 4.4 on the comparable descriptive statistics of the HFR FoF return series.

\textsuperscript{156} The cumulative probabilities of the three kernel distributions have been calculated at a maximum precision level of 4.9E-324.
5 Portfolio-Selection including Hedge Funds

### Table 5.2: Cumulative probabilities for the estimated kernel densities

<table>
<thead>
<tr>
<th>Probabilities at</th>
<th>Stocks (MSCI)</th>
<th>Bonds (LBGC)</th>
<th>Hedge Funds (HFR FoF)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-100%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-90%</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-80%</td>
<td>4.9621E-261</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-70%</td>
<td>8.5618E-192</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-60%</td>
<td>5.3743E-134</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>-50%</td>
<td>1.2273E-87</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

### 5.4.3.2 Multivariate Distributions

When the marginal distributions are determined we can proceed with the dependence structure. The copula method to construct multivariate return distributions that is described in Section 5.3 makes use of the dependence information in the return series to fit a multivariate dependence structure.

### Table 5.3: Linear correlation matrix of historical returns

<table>
<thead>
<tr>
<th></th>
<th>MSCI (Stocks)</th>
<th>LBGC (Bonds)</th>
<th>HFR FoF (Hedge Funds)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stocks</td>
<td>1.00</td>
<td>-0.19</td>
<td>0.59</td>
</tr>
<tr>
<td>Bonds</td>
<td>-0.19</td>
<td>1.00</td>
<td>-0.05</td>
</tr>
<tr>
<td>Hedge Funds</td>
<td>0.59</td>
<td>-0.05</td>
<td>1.00</td>
</tr>
</tbody>
</table>

As a brief summary of the dependencies the correlation matrix which is determined from the historical return data is given in Table 5.3.\textsuperscript{157}
This correlation information is used with the Gaussian copula. For these linear correlations we can furthermore determine the best fitting t-copula by maximum likelihood.\textsuperscript{158} The results for different degrees of freedom $\nu$ are given in Figure 5.14. We obtain a t-copula with 6 degrees of freedom as the best fit.\textsuperscript{159}

For the Archimedean copula functions the copula parameters are also estimated with the maximum likelihood approach. The estimated parameters $\hat{\beta}_C$, $\hat{\beta}_F$ and $\hat{\beta}_G$ for the elliptical copulas are found in Table 5.4.

<table>
<thead>
<tr>
<th></th>
<th>Clayton</th>
<th>Frank</th>
<th>Gumbel</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\beta}$</td>
<td>0.2408</td>
<td>1.0975</td>
<td>1.1381</td>
</tr>
</tbody>
</table>

Table 5.4: Estimates for $\beta_C$, $\beta_F$, and $\beta_G$

For the copulas introduced in Section 5.3.2 the fit relative to the empirical copula is determined. The information on the fit of the whole dependence structure is

\textsuperscript{157}When the return data is mapped on (0,1) with the estimated cumulative kernel densities the result is a slightly adjusted linear correlation matrix. This difference is caused by the properties of the linear correlation coefficient, as the linear correlation is not-invariant under non-linear transformations, see Section 5.2.3.1 for more details.

\textsuperscript{158}The dependence information for elliptical copulas can also be extracted with the rank correlation measured by Kendall’s tau, see for example Lindskog (2000) or Lindskog et al. (2001).

\textsuperscript{159}The same result is obtained with Kendall’s tau as relevant correlation measure.
aggregated in Table 5.5. The maximum absolute difference between the empirical copula and the different fitted copulas is reported in the second column. In the other columns the Anderson-Darling (AD) and the Integrated Anderson-Darling (IAD) measure of difference are given. For all measures the Gumbel copula is the best fitted Archimedian dependence structure. This result is really impressive, as the gumbel copula is fitted with one parameter while for example the t-Copula offers four parameters (three correlation coefficients and the degrees of freedom) to fit. While the Gumbel copula is the best overall fit in terms of the maximum difference and the Anderson-Darling measure, the Gaussian copula gives the best results for the Integrated Anderson-Darling measure. Furthermore the differences between the two other goodness-of-fit measures for the elliptical copulas are rather small. Therefore, Section 5.4.4.1 will take a closer look at the simulation results for the Gumbel copula as an Archimedian copula function and the Gaussian copula for the class of elliptical copulas.

<table>
<thead>
<tr>
<th></th>
<th>Maximum Difference</th>
<th>AD</th>
<th>IAD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gaussian</td>
<td>0.1294</td>
<td>1.3257</td>
<td>7216.5</td>
</tr>
<tr>
<td>t-Copula ((\nu = 6))</td>
<td>0.1290</td>
<td>1.3256</td>
<td>7359.3</td>
</tr>
<tr>
<td>Gumbel</td>
<td>0.1248</td>
<td>0.2758</td>
<td>7335.5</td>
</tr>
<tr>
<td>Frank</td>
<td>0.6565</td>
<td>1.3928</td>
<td>50252</td>
</tr>
<tr>
<td>Clayton</td>
<td>0.9337</td>
<td>4.0647</td>
<td>840830</td>
</tr>
</tbody>
</table>

Table 5.5: Goodness-of-fit for the different copulas

The Anderson-Darling difference measure is illustrated in Figure 5.15. In the space in which stocks and hedge funds are located the value of the fraction in equations

---

\(^{160}\) See Section 5.3.2.5.

\(^{161}\) The third dimension in the dataset (bonds) is held constant at a value of 0.5 to plot Figure 5.15.
5.44 and 5.45 is plotted.

![Figure 5.15: Illustration of the AD measure for a Gumbel copula](image)

Brighter squares in the stocks-hedge funds space indicate larger differences as determined by the fraction in the Anderson-Darling measure. While the Anderson-Darling measure reports the maximum over all (in this case three) dimensions, the Integrated Anderson-Darling measure reports the sum of all these deviations.

For the fitted Gumbel copula with the parameter $\beta_G$ equal to 1.1381 Figure 5.16 displays the dependence structure in the stocks/hedge fund space.

![Figure 5.16: Dependence structure from a Gumbel copula](image)
5.4.4 Portfolio Analysis

In order to determine optimal portfolio allocations the scenario optimization technique is employed for a set of 10,000 simulated random variates from two different copula functions. As the fit for the dataset is best with the Gaussian copula (in terms of the Integrated Anderson-Darling measure) and the Gumbel copula (in terms of the maximum distance from the empirical copula and the Anderson-Darling measure)\textsuperscript{162} we will emphasize the simulation results for the two multivariate modeling approaches with these copulas in the following.

After a short illustration of the scenario simulation in Section 5.4.4.1 we compare the minimum risk portfolios for the three asset portfolios. For the six risk measures introduced in Section 5.4.1, namely standard deviation, shortfall probability, minimum regret, mean-absolute deviation, VaR, and CVaR, the minimum risk portfolios are determined in Section 5.4.4.2. These results shall give a first impression on the risk diversification benefits of hedge funds.

Section 5.4.4.3 analyzes the different efficient frontiers for all the risk measures and determines the impact of the two different modeling approaches. We compare the efficient sets for the Gaussian and the Gumbel dependence structures with respect to the resulting risk statistics and the resulting portfolio allocations in bonds, stocks, and hedge funds.

The benefits of hedge funds are focused in Section 5.4.4.5. In order to quantify the diversification benefits and the return enhancement we compare the minimum risk portfolios and additionally the complete efficient frontier with and without hedge funds as potential portfolio ingredients.

\textsuperscript{162} See Table 5.5 on page 228 for the different goodness-of-fit measures.
5 Portfolio-Selection including Hedge Funds

5.4.4.1 Simulation Approach

To demonstrate the scenario simulation from the different copula functions and the resulting multivariate distributions, the figures 5.17-5.19 visualize 1,000 draws from a three-dimensional Gaussian copula for different bivariate dimensions. The top left graph in Figure 5.17 shows a scatter plot for the returns of the MSCI and the LBGC. In the top right graph the returns are mapped on the interval (0; 1) with the cumulative density functions for the marginal kernel density estimates. The bottom right graph shows 1000 random variates from the Gaussian copula. The bivariate data for stocks and bonds in the bottom left plot is determined with the inverses of the marginal cumulative density functions. The Figures 5.18 and 5.19 give further information on the multivariate dependence structure as they focus on the stock-hedge funds and the bonds-hedge funds space. Figure 5.18 illustrates the MSCI and HFR FoF dependence structure and the corresponding random variates that are representative for stocks and bonds. In Figure 5.19 the same information for the LBGC and HFR FoF is given.

![Figure 5.17: Bivariate scatter plots for stock and bond returns](image)

Figure 5.17: Bivariate scatter plots for stock and bond returns
Figure 5.18: Bivariate scatter plots for stock and hedge fund returns

Figure 5.19: Bivariate scatter plots for bond and hedge fund returns
5.4.4.2 Minimum Risk Portfolios

For the six risk measures under consideration we can determine the corresponding minimum risk portfolios. Table 5.6 reports these portfolio allocations in stocks, bonds, and hedge funds for the minimal risk portfolios in terms of standard deviation, mean-absolute deviation, shortfall probability, minimum regret, VaR, and CVaR.

<table>
<thead>
<tr>
<th>Copula</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Hedge Funds</th>
<th>Risk Measure</th>
<th>Expected Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>$C_N$</td>
<td>2.2%</td>
<td>79.4%</td>
<td>18.4%</td>
<td>0.89%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>82.3%</td>
<td>17.7%</td>
<td>0.95%</td>
</tr>
<tr>
<td>MAD</td>
<td>$C_N$</td>
<td>0.0%</td>
<td>78.6%</td>
<td>21.4%</td>
<td>0.69%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>81.1%</td>
<td>18.9%</td>
<td>0.73%</td>
</tr>
<tr>
<td>SfP</td>
<td>$C_N$</td>
<td>0.0%</td>
<td>81.1%</td>
<td>18.9%</td>
<td>24.5%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>84.8%</td>
<td>15.2%</td>
<td>25.6%</td>
</tr>
<tr>
<td>MR</td>
<td>$C_N$</td>
<td>5.3%</td>
<td>88.0%</td>
<td>6.7%</td>
<td>-3.26%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.1%</td>
<td>90.1%</td>
<td>9.8%</td>
<td>-3.52%</td>
</tr>
<tr>
<td>VaR</td>
<td>$C_N$</td>
<td>5.1%</td>
<td>88.8%</td>
<td>6.1%</td>
<td>1.52%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.2%</td>
<td>87.6%</td>
<td>12.2%</td>
<td>1.61%</td>
</tr>
<tr>
<td>CVaR</td>
<td>$C_N$</td>
<td>2.7%</td>
<td>76.4%</td>
<td>20.9%</td>
<td>1.56%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>2.4%</td>
<td>74.6%</td>
<td>23.1%</td>
<td>1.71%</td>
</tr>
</tbody>
</table>

Table 5.6: Minimum risk portfolios for different risk measures

The expected returns for the minimum risk portfolios are very similar. The return level is about 0.54%, only the expected returns for the minimum CVaR portfolios are slightly higher with 0.544%. As the Gumbel copula is able to model more extreme dependencies than the Gaussian copulas the reported risk measures are in all cases higher for this Archimedean dependence structure.
When we compare the results for the different risk measures we obtain related portfolio weights in case of the minimum risk portfolios for the standard deviation, the shortfall probability, and the mean-absolute deviation. The reported portfolios for these risk measures comprise a maximum allocation of 2% in stocks and a 79% to 85% investment in bonds. The remaining funds for hedge funds are about 15% to 21%.

The net effect of extreme returns can be evaluated when we compare the standard deviation and the mean-absolute deviation. As the standard deviation punishes extreme returns as the differences to the mean are squared, the minimum mean-absolute deviation portfolios show a slightly higher concentration in the asset with the lowest risk, namely bonds. The hedge fund quota for the mean-absolute deviation portfolio is also higher than in the minimum standard deviation case.

The analysis of the minimum risk portfolios for the minimum regret and the VaR as risk measure yields related results as both risk measures focus on the tails of the return distributions. In case of the Gaussian dependence structure the minimum risk investment is a 5% allocation to stocks and a 88% to 89% investment in bonds. The remaining 6% to 7% are invested in hedge funds. For the Gumbel dependence structure the minimum risk portfolio consist of a 10% and 12% allocation to hedge funds and a 90% and 88% investment in bonds.

The CVaR risk measure delivers similar minimum risk portfolios in the case of the Gumbel and the Gaussian copula. For an investor a minimum risk allocation in terms of the CVaR an optimal allocation in our model offers the highest expected return of all minimum risk portfolios. The allocation has a high proportion of hedge funds in the portfolio. 21% to 23% are invested in this asset class while bonds have a weight of 75% to 76%. The remainder of the portfolio volume is allocated to stocks. The remaining portfolio In comparison with the minimum regret criterium or the VaR which assign substantially lower weights to hedge funds, the CVaR does not
only focus on one extreme outlier in the portfolio performance or a single quantile value but on the complete tail of the distribution.

In Figure 5.20 the different minimum risk portfolios of the multivariate models with the Gaussian and the Gumbel copula are plotted in the standard deviation-expected return space.

Figure 5.20: Standard deviations of all minimum risk portfolios

The standard deviations for the risk statistics MAD and SD are close in both multivariate return models as these risk measures are highly correlated.\(^{163}\) In terms of the classical mean-variance portfolio selection framework most of the minimum risk portfolios are dominated by the minimum variance (or minimum standard deviation) portfolio.

Only the minimum CVaR portfolio offers a higher return for the higher standard deviation.

\(^{163}\) Only in case of a high proportion of extreme returns the optimal portfolios for standard deviation and the mean-absolute deviation are substantially different.
deviation in both modeling approaches. Therefore, a classical portfolio selection model would exclude the other minimum risk portfolios as they are not efficient in a standard deviation-expected return sense, although these portfolios might be more appropriate for a specific investor that should optimize one of these alternative risk statistics.

### 5.4.4.3 Efficient Portfolios

The Figures 5.21-5.30 report risk-return efficient sets for the Gaussian and Gumbel dependence structures for all of the six risk measures. Besides the risk measures and the corresponding expected returns the information on the asset weights is also included in these graphics.\(^{164}\)

In case of the VaR statistic the problems addressed in Section 5.4.1 become obvious when the efficient sets are analyzed. We obtain a non-smooth and non-convex function with respect to the portfolio weights. Therefore, the VaR efficient sets will not be discussed in detail and we will prefer to take a closer look at the results for the related CVaR.

Two general structures of the weights characteristics can be identified. Both structures at first substitute bonds with hedge funds when a higher level of expected return is targeted. The prevalent structure which results for all the risk measures except the shortfall probability and the minimum regret further increases the exposure to hedge funds and subsequently adds stocks for higher expected returns. Therefore, the bond allocation shrinks to zero at an expected return level of approximately 0.57% to 0.585%. Then the portfolio allocation is completely determined by stocks and hedge funds according to the expected return level of the investment alternatives.

\(^{164}\) The allocations are plotted in three different colors: bonds (black), stocks (light grey), and hedge funds (dark grey).
In case of the shortfall probability and the minimum regret\textsuperscript{165} the weights structure is slightly different as the allocation to bonds decreases more slowly. Because of the risk characteristics of stocks and hedge funds the minimization of these risk measures delivers substantially higher allocations for bonds. The hedge fund quota is at first increased but the portfolios on the efficient set do not exhibit higher allocations than 40%. When the portfolio optimization reaches a return level of approximately 0.6% the expected portfolio return is again completely determined by stocks and hedge funds.

The hedge fund proportion in the portfolios is rather different for these two weighting structures. While an investor that considers the shortfall probability as relevant risk measure would never allocate more than 40% to hedge funds no matter what expected return is targeted, a decision according to other risk measures could result in a hedge fund quota of more than 80%.

The differences in the two modeling approaches are examined in Section 5.4.4.4. The differences in the two copula approaches are evaluated and especially the weight differences in the efficient frontier are studied. In order to isolate the diversification benefits of hedge funds, Section 5.4.4.5 takes a look at portfolios with and without hedge funds.

\textsuperscript{165} This structure for the minimum regret risk statistic is obtained when the uncertainty is modeled with the Gaussian copula.
Figure 5.21: Standard deviation - efficient frontier for Gaussian copula

Figure 5.22: Standard deviation - efficient frontier for Gumbel copula
Figure 5.23: MAD - efficient frontier for Gaussian copula

Figure 5.24: MAD - efficient frontier for Gumbel copula
Figure 5.25: Shortfall probability - efficient frontier for Gaussian copula

Figure 5.26: Shortfall probability - efficient frontier for Gumbel copula
Figure 5.27: Minimum Regret - efficient frontier for Gaussian copula

Figure 5.28: Minimum Regret - efficient frontier for Gumbel copula
5 Portfolio-Selection including Hedge Funds

Figure 5.29: CVaR - efficient frontier for Gaussian copula

Figure 5.30: CVaR - efficient frontier for Gumbel copula
5.4.4.4 Impact of Modeling

As mentioned before in the case of minimum risk portfolios, the Gumbel copula delivers higher risk figures for the minimum risk portfolios. This pattern can be revealed from figures 5.31, 5.35, 5.37, 5.33, and 5.39 that report the differences of the risk measures for the two copula functions. But starting with an expected return of approximately 0.55% to 0.56% most of the risk measures report higher values for the efficient sets determined with the Gaussian copula.

As mentioned before, the Gumbel copula, which is the best fit to our three-dimensional data set in terms of the maximum difference from the empirical copula and the Anderson-Darling measure, is able to model extreme dependencies in a more realistic way than the Gaussian copula. The dependence structure is not focused on linear correlations and determines rather complex dependence structures. But the copula depends on only one parameter to incorporate the dependence structure, while the Gaussian copula is able to model the dependence in all dimensions separately with certain correlation parameters. When several dimensions are considered and the Gaussian copula assigns a relatively low dependence, the Gumbel copula\textsuperscript{166} models a stronger dependence structure. This indicates less dependence in case of regular movements for bonds and hedge funds in a Gaussian copula model, while stocks and hedge funds or stocks and bonds exhibit less dependence in the Gumbel model.\textsuperscript{167}

The weighting differences between the two modeling approaches are visualized in figures 5.32, 5.36, 5.38, 5.34, and 5.40.\textsuperscript{168} In general the Gaussian copula model assigns more weight to bonds and stocks than the Gumbel copula. The differences

\textsuperscript{166}As the Gumbel copula is only fitted with one parameter this dependence structure reflects an “average” dependence on all dimensions.

\textsuperscript{167}See Table 5.3 on page 226 for the correlation coefficients of the portfolio constituents.

\textsuperscript{168}In order to calculate these differences the risk figures and weights for the Gumbel copula are subtracted from the corresponding values for the Gaussian copula. The differences in the weights are only plotted for bonds (black), and stocks (light grey). The third asset (hedge funds) in the portfolios will change in accordance to the sum of the changes in bonds and stocks.
for the allocations are rather small when the standard deviation, the mean-absolute deviation or the CVaR is considered. But for the minimum regret sizeable differences of up to 40% for bonds and 20% for stocks are reported. This leads to efficient hedge fund allocations with a difference of about 60% in hedge funds for the same expected return. Again this is a result of the two different parametrization techniques of the copulas used in the modeling approach. As the minimum regret tries to minimize the maximum portfolio loss this risk figure is especially sensitive to the modeling of extreme events. While the Gaussian copula is fitted with dependence information from the linear correlation matrix the dependence of hedge funds and stocks is rather strong. Therefore, compared to the Gumbel copula, the optimization delivers a higher proportion of bonds in order to minimize the extreme portfolio results.\textsuperscript{169}

When the risk measure shortfall probability is analyzed the resulting weights from the two modeling approaches do not deviate substantially. The weight differences are zero for almost the complete efficient set. As the shortfall probabilities take all returns below a target return of zero into account, to minimize these shortfall probabilities does not put weight on the size of extreme portfolio returns. When the two multivariate return distribution that are based on the Gumbel and the Gaussian copula deliver rather similar results in the “middle” of the three dimensional return-probability space this result is no surprise. For a lower target return the differences between the two efficient sets increase because of the deviations in the dependence structures.

As outlined in Section 5.4.4.3 the portfolio allocations in stocks and hedge funds above a certain expected return is completely determined by the corresponding return expectations. Therefore, the weighting of stocks and hedge funds in the two different copulas frameworks is equal for higher return expectations.

\textsuperscript{169} When extreme (positive or negative) returns are considered the effect of the conditional dependence structure in the Gumbel copula at least partially compensates for the problem of averaging multivariate dependencies.
Figure 5.31: Differences in standard deviation for the efficient frontiers (Gaussian copula minus Gumbel copula results)

Figure 5.32: Differences in weights for the standard deviation - efficient frontiers (Gaussian copula minus Gumbel copula results)
Figure 5.33: Differences in MAD for the efficient frontiers (Gaussian copula minus Gumbel copula results)

Figure 5.34: Differences in weights for the MAD - efficient frontiers (Gaussian copula minus Gumbel copula results)
Figure 5.35: Differences in shortfall probability for the efficient frontiers (Gaussian copula minus Gumbel copula results)

Figure 5.36: Differences in weights for the shortfall probability - efficient frontiers (Gaussian copula minus Gumbel copula results)
Figure 5.37: Differences in minimum regret for the efficient frontiers (Gaussian copula minus Gumbel copula results)

Figure 5.38: Differences in weights for the minimum regret - efficient frontiers (Gaussian copula minus Gumbel copula results)
Figure 5.39: Differences in CVaR for the efficient frontiers (Gaussian copula minus Gumbel copula results)

Figure 5.40: Differences in weights for the CVaR - efficient frontiers (Gaussian copula minus Gumbel copula results)
5.4.4.5 Benefits of Hedge Funds

After we examined the efficient sets for the three asset portfolios we analyze the benefits of hedge fund investments. Therefore, we take a look at the risk-return efficient sets when hedge funds are excluded from the portfolio selection process. The resulting portfolios are presented in figures 5.41-5.45 together with the efficient sets from Section 5.4.4.3 where investments in hedge funds were allowed.

For the minimum risk portfolios without hedge funds Table 5.7 reports the minimum risk figures and the portfolio allocations.

<table>
<thead>
<tr>
<th>Copula</th>
<th>Stocks</th>
<th>Bonds</th>
<th>Risk Measure</th>
<th>Expected Return</th>
<th>Risk Difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>SD</td>
<td>$C_N$</td>
<td>7.5%</td>
<td>92.5%</td>
<td>0.94%</td>
<td>0.541%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>100.0%</td>
<td>1.02%</td>
<td>0.535%</td>
</tr>
<tr>
<td>MAD</td>
<td>$C_N$</td>
<td>7.0%</td>
<td>93.0%</td>
<td>0.73%</td>
<td>0.541%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>100.0%</td>
<td>0.78%</td>
<td>0.535%</td>
</tr>
<tr>
<td>SfP</td>
<td>$C_N$</td>
<td>6.5%</td>
<td>93.5%</td>
<td>25.4%</td>
<td>0.540%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.5%</td>
<td>99.5%</td>
<td>26.8%</td>
<td>0.536%</td>
</tr>
<tr>
<td>MR</td>
<td>$C_N$</td>
<td>10.5%</td>
<td>89.5%</td>
<td>-3.42%</td>
<td>0.543%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>0.0%</td>
<td>100.0%</td>
<td>-3.83%</td>
<td>0.535%</td>
</tr>
<tr>
<td>VaR</td>
<td>$C_N$</td>
<td>7.0%</td>
<td>93.0%</td>
<td>1.57%</td>
<td>0.541%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>1.0%</td>
<td>99.0%</td>
<td>1.72%</td>
<td>0.536%</td>
</tr>
<tr>
<td>CVaR</td>
<td>$C_N$</td>
<td>10.0%</td>
<td>90.0%</td>
<td>1.72%</td>
<td>0.543%</td>
</tr>
<tr>
<td></td>
<td>$C_G$</td>
<td>3.0%</td>
<td>97.0%</td>
<td>1.98%</td>
<td>0.538%</td>
</tr>
</tbody>
</table>

Table 5.7: Minimum risk portfolios without hedge funds

The column “risk difference” in Table 5.7 summarizes the net effect in terms of the risk statistic under consideration. The negative sign indicates that hedge funds in
the portfolio decrease the risk measure under consideration. This is the case for all six risk measures and both dependence structures. The effect is more distinctive with the Gumbel copula. As the Gaussian copula assigns a high correlation to stocks and hedge funds this is no surprise.

The highest diversification benefit in terms of the minimum risk portfolio is reported for the CVaR. The minimum risk measure is decreased by 9.3% and 13.5%, respectively. To add hedge funds to the portfolio therefore decreases the tail risk substantially. Even the classical risk measure standard deviation is decreased by a relative amount of 5.2% and 6.5% when hedge funds are added to the minimum variance portfolio.

In all but one cases the return of the minimum risk portfolio is also enhanced when hedge funds are added. This effect is small (up to 1.2% of the return without hedge funds) but apart from the minimum regret portfolio for the Gaussian dependence structure this result is derived for all the risk measures.

When the complete efficient sets are analyzed the impact of hedge funds on the reduction of risk and the enhancement of expected return is again more significant when the Gumbel model is considered. For both dependence structures the efficient sets in terms of standard deviation, mean-absolute deviation, and CVaR are affected most. The minimum regret efficient frontier is not that sensitive to the hedge fund allocation although the effect in the minimum risk portfolio is substantial. When shortfall probabilities are considered the difference in the efficient sets is also not very severe. The reason is located in the small hedge fund allocation along the efficient sets.\textsuperscript{170}

\textsuperscript{170} See figures 5.25 and 5.26 on page 240.
Figure 5.41: Standard deviation - efficient frontier with (black) and without (grey) hedge funds

Figure 5.42: MAD - efficient frontier with (black) and without (grey) hedge funds
5 Portfolio-Selection including Hedge Funds

Figure 5.43: Shortfall probabilities - efficient frontier with (black) and without (grey) hedge funds

Figure 5.44: Minimum regret - efficient frontier with (black) and without (grey) hedge funds
Figure 5.45: CVaR - efficient frontier with (black) and without (grey) hedge funds

5.5 Summary

In this chapter we derived an alternative portfolio selection framework. This model is able to take the distributional characteristics of hedge funds or other non-normal investments into consideration. Because of problems with the classical portfolio selection approach that are outlined in Section 5.2 of this chapter, we model the multivariate distribution of asset returns with very flexible copula functions. The method to obtain these multivariate return distributions was described in Section 5.3. The concept of copula functions we introduced allows to separate the marginal distributions and the dependence structure for a multivariate distribution. With this mathematical tool at hand multivariate return distributions can be modeled with different dependence concepts. As we are interested in fitting the multivariate distributions to data the sections 5.3.2.4 and 5.3.2.5 focus on fitting and selecting copula functions.
The last section of this chapter presented an empirical example of the modeling technique. For a portfolio with stocks, bonds and hedge funds the multivariate return distributions were fitted to data. Based on these return distributions we derived optimal portfolios in terms of different risk measures.

We determined minimum risk portfolios and the complete efficient frontiers for the different risk statistics. Some of these efficient portfolios include rather high hedge fund allocations. For the different risk measures and the different expected portfolio returns optimal hedge fund quotas of 10% to 80% are calculated. These portfolio weights might be too high for a practical implementation especially when regulatory requirements for investors are considered. But as we are interested in theoretical optimality, these results offer an indication for portfolio construction.

We analyzed the impact of two modeling approaches with the elliptical Gaussian copula and the Archimedean Gumbel copula. The Gumbel copula is able to model more extreme dependencies and also non-symmetrical dependencies for two random variables. In a multidimensional setting it became obvious that the parameter of the Archimedean copulas averages the dependencies over the different dimensions. Here the Gaussian copula was able to model various dependence structures in different dimensions while the single parameter of the Gumbel copula usually over- or underestimates bivariate dependencies. Therefore, the next step in the development of this portfolio selection technique should be to analyze Archimedean copulas with more parameters that are able to model extreme dependencies and different dependence structures for different bivariate dimensions.

In Section 5.4.4.5 we finally analyzed the benefits of hedge funds in a classical bond and stock portfolio. Therefore, we compared risk-return efficient portfolios with and without hedge funds. As a result hedge funds enhance the portfolio return and diversify portfolio risk for all of the examined risk statistics. The highest diversification benefits are achieved with the CVaR as relevant risk measure. Therefore,
especially investors that measure risk with the CVaR statistic should think about allocating substantial proportions of their portfolio to hedge funds.
6 Summary and Conclusion

Hedge funds are not really a new capital market phenomenon but have attracted considerable attention over the last few years. Although there are several risks associated with hedge fund investments, even the chairman of the US Federal Reserve Board has recently come to a positive conclusion and outlook concerning them, having emphasized the important contributions hedge funds have made to market efficiency and financial stability by increasing market liquidity and enhancing economic flexibility and resilience.¹

Over the years there has been growing interest from investors in more complex hedge fund strategies and reliable hedge fund data. These investor issues and the question how hedge fund allocations can be modeled in a portfolio context form the focus of this work.

The second chapter discussed some important hedge fund characteristics. It is clear that the hedge fund approach to asset management is a broader and perhaps more intuitive investment process. Therefore, hedge funds or alternative investments in general are defined as the whole investment universe minus a small slice, namely the traditional investments. In general the term “hedge fund” implies the full spectrum of investment strategies. Such a broad definition indicates that hedge funds are not homogeneous and this is the reason for the in-depth strategy analysis in this work. The second chapter also provides an introduction to the key characteristics of hedge

¹ See Greenspan (2005).
6 Summary and Conclusion

funds. Many of these characteristics are fundamental to an understanding of the investment opportunity offered by such funds. The conclusion is drawn that the very flexibility of hedge funds might result in a change in the investment industry whereby what is today referred to as “alternative” will become the norm.\(^2\)

Chapter 3 provides information on hedge fund trading strategies. We identify two categories of risk, industry-inherent risk and strategy-specific risk. Using this classification we are able to separate the risk associated with the characteristics of hedge funds, which can be seen as a systematic risk for general hedge fund investments, from the special risk associated with a particular hedge fund trading strategies. Based on this understanding of risk, the specific risk of different hedge fund strategies is analyzed. We describe the most important trading strategies and provide a comprehensive and detailed analysis of these strategies and the corresponding trades. Furthermore, Chapter 3 includes a section on funds of hedge funds in which we introduce the most important advantages and disadvantages of these investment vehicles. In this context we also describe the key fund selection process for funds of hedge funds. Taken as a whole, the third chapter provides an exhaustive analysis of hedge fund strategies. Apart from legal considerations, detailed strategy analysis is probably the most important form of information for investors willing to allocate money to hedge funds.

The current spectrum of hedge fund indices is described in Chapter 4 where we also identify desirable properties for hedge fund benchmarks, namely representativeness, accuracy, transparency, and investability. Major hedge fund index providers, proprietary databases, and calculated indices are analyzed in order to provide an overview of the available hedge fund data. In addition, the different biases in hedge fund databases and indices are described, and their extent is estimated. We also present the results of a broad range of academic studies on different databases.

As a consequence of data biases we suggest using fund of hedge funds data when the performance of hedge fund investments in general is of interest. A detailed data analysis of monthly returns is provided for six major fund of hedge funds indices. We found very similar results for all indices. When the unconditional return distributions are considered, all the historical index return distributions are skewed and exhibit significant excess kurtosis. Based on historical data covering an eight year period, different statistical tests reject the hypothesis of normally distributed returns. In a time series context we find that the return series are significantly autocorrelated. When the autocorrelation is removed from the historical return data, we are left with a higher standard deviation of the resulting unconditional return distribution. All in all the analyzed return distributions are highly non-normal.

The distributional characteristics of hedge funds, the non-linear correlation structures observed in practice, and the unrealistic quadratic utility assumption lead to a rejection of the classical Markowitz framework that is based on the first two moments of return distributions. The alternative framework that we present in Chapter 5 is based on copula theory. The chosen approach is very flexible as it allows the modeling of marginal distributions to be separated from the corresponding dependence structure. Furthermore, this approach is able to include a variety of non-linear dependence structures. Above all, extreme dependencies between random variables can be modeled explicitly. We present different copula functions from the elliptical and the Archimedian class to model the multivariate dependencies.

In an empirical example we show the usefulness of the approach in practice. For a given set of return data for bonds, stocks, and hedge funds, we apply the introduced technique to model multivariate return distributions including hedge funds. The marginal distributions are modeled with kernel densities and the dependence structure is assumed to be Gaussian or Gumbel. To determine optimal portfolio allocations we employ scenario optimization and a variety of risk measures.
All the resulting minimum risk portfolios include reasonable hedge fund allocations. Given these optimal allocations, an implementation in a real world portfolio might violate regulatory restrictions in some cases. However, we are primarily interested in theoretically optimal portfolios, and the results do in any case offer indications for portfolio construction. The optimal hedge fund allocations in these minimum risk portfolios range from 6% to 23%. When the impact of hedge funds on the portfolio risk and return is determined, the net effect of hedge funds is to enhance the portfolio return and to diversify portfolio risk, no matter which dependence structure is used. When we consider different risk measures, we obtain the best diversification benefits of hedge funds with the CVaR statistic. Consequently, in particular those investors who determine risk according to this risk measure should consider investing a substantial amount in hedge funds.

Hedge funds provide a very broad field for researchers and the research presented in this work and related publications could be extended in several directions. Hedge funds are a very dynamic part of the investment universe, and new strategies are constantly emerging, whether due to special market situations, research results, or new financial products. Furthermore, new index providers and new indices or index families are making more and more data on hedge funds available. The index universe introduced in Chapter 4 is therefore also subject to ongoing change.

As regards the modeling approach described in Chapter 5, we could envisage other techniques for determining the marginal distributions or the use of different copula functions. As the fitting results for the one-parameter Archimedean copula were good, an Archimedean copula with more than one parameter might possibly provide an even better fit to a multidimensional data set. Another extension of our approach might be a multi-period model.

All in all, the hedge fund industry is very dynamic. According to research institutes, hedge funds have reached the “end of their beginning” as the industry consolidates
and many hedge fund managers have come to resemble traditional asset managers as regards their organizational structures. Furthermore, several derivatives on hedge funds have been introduced. Options on hedge funds are available in the OTC-market and securitization is already on the way as a next step for this asset class. Given the attractive return characteristics of hedge funds, portfolios of hedge funds can serve as efficient collateral platforms for structured products and investors will soon be able to benefit from a tranching approach to investing with a position in a collateralized fund obligation (CFO). This development towards customizing asset-backed tranches may stimulate yet further inflows of capital into the hedge fund industry.

4 See for example Cheng (2002) and Mahadevan/Schwartz (2002) on this investment alternative.
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