

# INTRODUCTION TO THE PAPERS AND POSTERS OF WG3: ALGEBRAIC THINKING

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In CERME-8, as a long-standing group, Working Group 3 “Algebraic thinking” continued the work carried out in previous CERME conferences (Cañadas, Dooley, Hodgen & Oldenburg, 2011).

## THE PAPERS AND THEMES

A total of 16 papers and 5 posters with a total of 25 group participants representing 14 countries, Canada, France, Germany, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Tunisia, Turkey, UK, USA / Romania. We considered the papers in the following themes (although, of course, there are many overlaps across the themes):

### Entry to algebra

Several studies focused on the relation between algebra and arithmetic. These continue and extend discussions from previous CERME conferences (e.g., Ainley, Bagni, Hefendehl-Hebeker, & Lagrange, 2009). We note that the focus is on ‘early algebra’ in both primary and secondary education. Working in Portugal, where an ‘early algebra’ focused curriculum has been introduced, Mestre and Oliveira describe and analyse a teaching experiment in which Grade 4 children are introduced to the use of informal symbols as quasi-variables. They find that such an approach has benefits for the development of algebraic thinking, particularly in moving from equations involving specific unknowns to equations expressing generalisations about arithmetic. In contrast, Gerhard examines how a generalised number approach may hinder the development of algebraic thinking. Using her own interdependence analysis tool she examines how new patterns of action are related to old patterns of action. In particular, she argues that poorly developed patterns of action relating to multiplication (such as an over-reliance on repeated addition) may be overcome by Davydov’s more geometrically-based approach. Mellone, Romano & Tortora report research with older students (Grade 10) and examine how the students tackle the problem  $10^{38}+10^{37}$ . They argue that, in making sense of the problem, students draw on a ‘social’ rationality in addition to purely mathematical reasoning and that this has implications for the relationship between arithmetic and algebra. Mata-Pereira & da Ponte’s poster examines how Portuguese secondary students’ reasoning about real numbers and generalized arithmetic.

## **Equivalence**

Several papers addressed issues relating to equivalence, perhaps reflected a general resurgence of interest in the field. Drawing on Frege, Pilet argues that the French curriculum places little explicit attention to the issues of equivalence and that this exacerbates students' conceptual difficulties. She analyses a learning situation designed to do this. Zwetschler & Prediger argue that, whilst previous research has examined the learning of equivalence in transformational situations, little attention has been devoted to the equivalence of expressions in generational activities. In a case study designed to address this gap, they find two conceptual barriers to understanding: an over-emphasis on operational rather than relational meanings, and difficulties relating to the generality of variables and figures. Vieira, Palhares & Gimenez report the findings of a pilot study that investigated Portuguese primary children's understanding of equality and the equal sign following the introduction of a new curriculum. Although the children in their study performed better than in previous studies of the equal sign, they suggest that their understanding of relational meanings and equivalence are nevertheless poor.

## **Structural generalisation**

Unsurprisingly, generalization continues to be a theme of interest for WG3 participants. Måsøval analyses how two student teachers struggle to find an explicit general formula for a sequence. Using Brousseau, she shows how the milieu for algebraic generalisation is constrained both by the design of the task and by the teacher educator's (incorrect) presupposition of prior knowledge. Dooley examines the strategies that primary children (aged 9-10) in Ireland used when solving a quadratic problem and analyses how mediation by the teacher (and the children themselves) can enable young children to tackle relatively sophisticated algebraic problems. Postelnicu reports the results of two surveys examining secondary school and college students' difficulties with Cartesian representations of linear functions. She argues that the difficulties she finds are due to epistemological, rather than simply procedural, obstacles and these in turn relate to an overemphasis on geometric rather than analytic perspectives on slope and gradient. Rolfes, Roth & Schnotz's poster presents the early results of a teaching experiment aimed at improving secondary understanding of covariation of functions.

## **The relationship between semantic and syntactic understandings**

Two papers focus on the relationship between syntactic and semantic understandings of algebra. Focusing on equations and inequations, Kouki & Chellougui discuss the difficulties that students encounter in mastering the syntactic rules of algebra and argue that both syntactic and semantic understanding is necessary for such mastery. Oldenburg, Hodgen & Küchemann examine whether these two aspects of algebraic thinking, syntactic and semantic, can be distinguished empirically using test items. Although there are considerable difficulties in operationalizing the distinction using test items, their exploratory analysis suggests a fruitful line of analysis may be

to treat the semantic aspect as consisting of two sub-dimensions, based on whether one or many meanings or interpretations appear to be required.

### **Teachers and teaching**

Several papers and posters address issues relating to teachers and the teaching of algebra. Kilhamn examines how two teachers enact the same teaching materials and finds that outwardly similar content is addressed differently. Although these differences at first appear subtle, the two teachers approaches tackle issues such as variable in radically different ways. Çelikdemir & Erbaş analyse TIMSS 2007 data on Turkish mathematics teachers' self-reported preparedness to teach school algebra, finding that they reported that they were less well prepared to teach algebra than their counterparts around the world. Sari & Özdemir report on the effects of a teaching intervention supporting metacognitive strategy use on improving Turkish seventh grade students' conceptual and procedural knowledge on algebraic expressions and equations. They find a significant difference between experimental and control groups in terms of gain scores on conceptual and procedural knowledge in favor of experimental group. Two posters, by Wathne and Røj-Lindberg, present the methodology and early findings of the VIDEOMAT, a video study of mathematics lessons in Finland, Norway, Sweden and the USA on the introduction to algebra.

### **New directions**

Several papers address new directions for the Algebraic Thinking Working Group at CERME. Proulx reports on an exploratory study examining mental algebra. He argues that too little attention has been devoted to understanding mental mathematics activities with objects other than numbers and discusses its potential for algebra teaching and learning. In the only paper to directly address issues involving technology, Lagrange discusses the limitations of an exclusively functional approach to algebra. He argues that both experiencing covariation and bodily activity are crucial for students' understanding of functions, showing how this can be facilitated through the Casyopée environment. Kop's poster presents early data from a study of how expert mathematicians sketch graphs of functions.

### **GENERAL REFLECTIONS**

As we observed in our CERME7 report, algebraic thinking is a “mature” sub-domain within mathematics education research (Cañadas et al., 2011). As a result, our group discussions touched on many familiar themes, including the recontextualisation of existing research, the need for both ‘young’ researchers and less developed communities to ‘make it their own’, the relationship between different theoretical approaches and the nature of algebra / algebraic thinking / algebra as an activity. But it was also exciting to revisit epistemological debates and the implications of these for the teaching and learning of algebra.

## LOOKING FORWARD TO CERME-9

Finally, in looking forward to CERME-9, the group hope that future research will address advanced as well as early algebra, the nature and design of paradigmatic tasks, the extent to which (research on) algebraic thinking is influenced by context and culture and the tensions between rigorous research and classroom practice and students' difficulties and misconceptions. We also hope to engage in continuing discussions about the nature and utility of the familiar dualities in algebra (syntactic / semantic, procedural / conceptual etc). We were surprised to have only one paper addressing digital technologies and hope that this omission will be rectified at CERME9.

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