INFLUENCE OF DISSIPATION ON THE FINITE TEMPERATURE PHASE TRANSITION IN JOSEPHSON JUNCTION ARRAYS

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We investigate the phase diagram of an array of Josephson junctions at finite temperatures. The capacitive interactions of charges on the superconducting islands and the associated quantum mechanical effects, as well as the flow of normal currents due to single electron tunneling, are taken into account. Using a quantum Ginzburg–Landau theory we derive the fluctuation conductivity above the critical temperature. Depending on junction parameters we find fluctuation broadened superconducting transitions, reentrant, or quasireentrant behavior. This may be related to the resistivity minima seen in thin granular films.

1. Introduction

Recent experiments on granular thin films [1, 2] and three-dimensional granular samples [3] have increased the interest in superconducting arrays or networks. These experiments have demonstrated the fundamental role of quantum effects and dissipation in thin films characterized by the normal state sheet resistance R_n. Due to highly developed submicron techniques it is now also possible to fabricate regular arrays of junctions with parameters such that the quantum effects are significant. In square regular arrays the normal state resistance R_n can directly be identified with the normal state resistance of a single junction as a simple consequence of Kirchhoff's laws. The most remarkable result of the experiments of the Minnesota group [1] on thin films is the existence of a threshold value R_c for R_n, close to the quantum of resistance $R_0 = h/4e^2 = 6.45\,\mathrm{k}\Omega$. Films with $R_n < R_c$ establish global superconductivity at low temperatures, while the resistance of films with $R_n > R_c$ remains finite at low temperatures, showing quasireentrant or semiconducting behavior. The fact that the same threshold was found for Sn, Pb and (amorphous) Ga films with presumably different microstructures has raised the speculation of a universal threshold resistance, although more recent experiments seem to indicate a lower critical sheet resistance for granular aluminum films [4].

2. The model

In this paper we consider a regular cubic array of superconducting islands. This may serve as a model for the granular disconnected films. We neglect fluctuations in the modulus of the superconducting order parameters of the individual islands, assuming that we are well below the single grain transition temperature (but see also the article by G. Giaquinta [5]). The relevant degrees of freedom are the phase differences of the order parameters on neighboring islands and the charges accumulating

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on the electrodes. The quantum statistical properties of this system are characterized by an effective action in imaginary times $0 \leq \tau \leq \hbar\beta$:

$$S[\varphi] = \int_0^{\hbar\beta} \mathrm{d}\tau \sum_{\langle ij \rangle} \left\{ \left(\frac{\hbar}{2e}\right)^2 \frac{\partial \varphi_i}{\partial \tau} \frac{C_{ij}}{2} \frac{\partial \varphi_j}{\partial \tau} + E_\mathrm{J}(1 - \cos(\varphi_{ij}(\tau)) \right\} + S_\mathrm{D}[\varphi] \, , \tag{1}$$

where $\varphi_{ij} = \varphi_i - \varphi_j$. For example, the partition function,

$$Z = \prod_i \int D\varphi_i \exp(-S[\varphi]/\hbar) \, , \tag{2}$$

or expectation values can be expressed by Feynman path integrals, weighted by $\exp(-S[\varphi]/\hbar)$. C_{ij} is the capacitance matrix describing the Coulomb energies of the charges on the islands and E_J is the Josephson coupling energy arising from Cooper pair tunneling between nearest neighbor grains. The diagonal matrix elements of C_{ij} correspond to a capacitive coupling to the ground plane, while the off-diagonal elements describe the Coulomb interaction between charges on different islands, which is long ranged if no screening effects are incorporated. Two limiting versions of the charging energy are commonly used: the self-charging model (sc) considers only diagonal elements, i.e. $C_{ij} = C\delta_{ij}$, and the nearest neighbor model (nn) considers only the interaction of charges on neighboring grains. The corresponding charging energies may be equally well important depending on the physical situation [6, 7]. For definiteness in what follows we use the sc model; the extension to nn charging energies is straightforward.

A complete description of the junction array also has to take into account the flow of normal currents, which gives rise to dissipation. In a microscopic treatment of a single junction between two BCS superconductors all the microscopic electronic degrees of freedom can be integrated out [8], leading to an effective action of the form (1) with

$$S_\mathrm{D}[\varphi] = \int_0^{\hbar\beta} \mathrm{d}\tau \int_0^{\hbar\beta} \mathrm{d}\tau' \sum_{\langle ij \rangle} \alpha_\mathrm{qp}(\tau - \tau')\left(1 - \cos \frac{\varphi_{ij}(\tau) - \varphi_{ij}(\tau')}{2}\right) . \tag{3}$$

This nonlocal interaction term expresses in a fully quantum mechanical way the dissipation due to the tunneling of single quasiparticles. The kernel $\alpha_\mathrm{qp}(\tau)$ is given by [8]:

$$\alpha_\mathrm{qp}(\tau) = -\frac{2|T|^2}{\hbar^2} \int \frac{\mathrm{d}^3 p_\mathrm{L}}{(2\pi\hbar)^3} \int \frac{\mathrm{d}^3 p_\mathrm{R}}{(2\pi\hbar)^3} \, G(\tau, p_\mathrm{L}) G(-\tau, p_\mathrm{R}) \, , \tag{4}$$

where $G(\tau, p)$ is the diagonal component of the Nambu Green's function with Fourier transform:

$$G(\omega_\mathrm{m}, p) = -\hbar \frac{\mathrm{i}\hbar\omega_\mathrm{m} + \varepsilon_p}{(\hbar\omega_\mathrm{m})^2 + E_p^2} \, , \quad E_p^2 = \varepsilon_p + \Delta^2 \, . \tag{5}$$

Here, ω_m are the fermionic Matsubara frequencies $\omega_\mathrm{m} = (2m+1)\pi/\hbar\beta$, T is the tunnel matrix element and the subscripts R and L refer to the two (identical) BCS electrodes of the junction. Using the ideal BCS density of states, corresponding to vanishing subgap conductance, the momentum integrals in (4) can readily be performed. For temperatures low compared to the energy gap Δ the Fourier transformed kernel has the limiting forms [8] (with the bosonic Matsubara frequency $\omega_\mu = 2\pi\mu/\hbar\beta$):

$$\alpha_{qp}(\omega_\mu) = \begin{cases} -\dfrac{3\alpha\hbar\omega_\mu^2}{32\Delta} & \text{for } \hbar\omega_\mu \ll 2\Delta\,, \\ -\dfrac{\alpha|\omega_\mu|}{\pi} & \text{for } \hbar\omega_\mu \gg 2\Delta\,. \end{cases} \tag{6}$$

Here we have defined $\alpha = h/4e^2R_n$ with normal state conductance $1/R_n = 4\pi e^2|T|^2N^2(0)/\hbar$, $N(0)$ is the density of states at the Fermi level. The low-frequency part leads to a renormalization of the nearest neighbor capacitance which, for ideal tunnel junctions at zero temperature, is given by:

$$\frac{\delta C}{C} = \frac{3\pi\hbar}{32\Delta R_n C}\,. \tag{7}$$

Depending on the spectrum of the quasiparticles (especially on the existence of a well-defined energy gap) or the dissipative mechanism it may be appropriate to use different kernels $\alpha(\tau)$ to account for the given physical situation.

(i) If the subgap conductance is nonzero, the kernel $\alpha_{qp}(\omega_\mu)$ acquires an Ohmic contribution linear in $|\omega_\mu|$ also at small frequencies:

$$\alpha_{qp}(\omega_\mu) = -\frac{\hbar}{2e^2R_{qp}}\,|\omega_\mu|\,. \tag{8}$$

Here, $1/R_{qp}$ is the subgap conductance for small voltages $V \to 0$. In order to describe both the ideal energy-gap-dependent nonlinear conductance and the nonideal subgap conductance of the quasiparticle current, we have to add the kernel (8) to the ideal form mentioned above. This contribution linear in $|\omega_\mu|$ gives rise to infrared singularities and can induce (dissipative) phase transitions by itself. The phase diagram for this situation has been studied by means of a variational calculation in ref. [9].

(ii) For an ideal superconducting tunnel junction with vanishing subgap conductance a gradient expansion (in imaginary times) may be performed provided the phases vary slowly on the time scale $\hbar/\Delta$ [10]. This approximation leads to

$$S_D[\varphi] = \frac{1}{2}\,\frac{\hbar}{4e^2}\,\frac{3\pi\hbar}{32\Delta R_n}\int_0^{\hbar\beta} d\tau \sum_{\langle ij\rangle}\left(\frac{\partial\varphi_{ij}(\tau)}{\partial\tau}\right)^2. \tag{9}$$

In contrast to (i) this quadratic low-frequency form cannot induce phase transitions by itself but instead yields only quantitative corrections to existing transitions.

(iii) If the flow of normal currents is not due to the tunneling of quasiparticles, but is due to shunt resistors between the islands, the trigonometric function in the dissipative term $S_D[\varphi]$ is replaced by its quadratic expansion and the kernel is of the form of eq. (8) with R_{qp} replaced by the shunt resistor R_s [11]. This Ohmic dissipation is equivalent to the phenomenological model of Caldeira and Leggett [12], where the phase degrees of freedom are linearly coupled to a bath of harmonic oscillators with a suitably chosen spectral density.

In this paper we now use the full microscopic quasiparticle kernel as given in (4). But instead of using the ideal equilibrium Green's function $G(\omega_m, p)$ we explicitly incorporate the effects of temperature-dependent and temperature-independent scattering mechanisms and replace G by [13]:

$$G_s(\omega_m, p) = -\hbar \frac{i\hbar\Omega_m + \varepsilon_p}{(\hbar\Omega_m)^2 + E_p^2}\,,$$
$$\Omega_m = \omega_m + \Gamma\,\mathrm{sign}(\omega_m)\,. \tag{10}$$

The pair-breaking parameter Γ can be represented by the sum of inverse scattering times corresponding to electron–phonon or electron–electron scattering and other pair-breaking terms [13, 14]:

$$\Gamma = \frac{1}{2\tau_{ep}} + \frac{1}{2\tau_{ee}} + \cdots \text{“pair breaking”}\,. \tag{11}$$

In general, Γ depends on the temperature. It may, for example, display the temperature dependence of the inverse electron–phonon scattering time (see, for example, ref. [15]). In the presence of (magnetic) impurity scattering, Γ will have a finite value even at zero temperature. Γ is directly related to the broadening of the BCS density of states which can be expressed as [13, 16]:

$$N(E) = Re\,\frac{E + i\Gamma}{((E + i\Gamma)^2 - \Delta^2)^{1/2}}\,. \tag{12}$$

As is obvious from (12), pair-breaking mechanisms lead to a finite subgap conductance.

Using the Green's function G_s we obtain for the quasiparticle damping kernel:

$$\alpha_{qp}(\omega_\mu) = \frac{\pi}{2e^2R_n\beta}\sum_m \left\{ \frac{\hbar\Omega_m(\hbar\Omega_m - \hbar\omega_\mu)}{[((\hbar\Omega_m)^2 + \Delta^2)((\hbar\Omega_m - \hbar\omega_\mu)^2 + \Delta^2)]^{1/2}} - \frac{(\hbar\Omega_m)^2}{(\hbar\Omega_m)^2 + \Delta^2} \right\}, \tag{13}$$

which is now explicitly temperature dependent. In fig. 1 this kernel is plotted for different values of Γ.

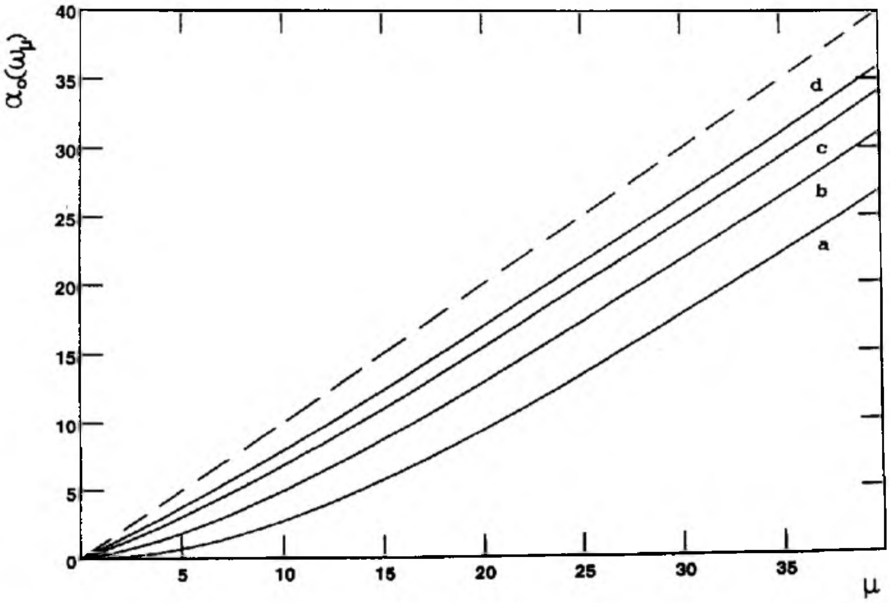


Fig. 1. Plotted is the quasiparticle damping kernel $\alpha_{qp}(\omega_\mu) = -(1/2e^2R_n)2\pi kT\alpha_0(\omega_\mu)$ as a function of $\mu = \hbar\omega_\mu/2\pi kT$ for $\Delta/\pi kT = 20$, and different (fixed) values of $\Gamma_0 = \Gamma/\pi kT$: (a) $\Gamma_0 = 0$; (b) $\Gamma_0 = 10$; (c) $\Gamma_0 = 20$; (d) $\Gamma_0 = 30$. The dashed line is the Ohmic result $\alpha_0 = \hbar|\omega_\mu|/2\pi kT$.

If $\alpha_{qp}(\tau)$ is short ranged, i.e. if the subgap conductance vanishes, one can approximate the trigonometric function in $S_D[\varphi]$ by its quadratic expansion, provided the phases vary slowly on the time scale $\Delta/\hbar$ (see ref. [10]). Furthermore, in our previous analysis [9] we found that even in the case where $\alpha_{qp}(\tau)$ is long ranged, i.e. if the subgap conductance $1/R_{qp}$ is finite, the effect of dissipation by quasiparticle tunneling and of an equivalent Ohmic shunt resistor with $R_s \approx R_{qp}$ on the phase diagram differ only quantitatively. We therefore replace in the following the dissipative part of the effective action by:

$$S_D[\varphi] = -\frac{\hbar}{4\beta} \sum_{k,\mu} \alpha_{qp}(\omega_\mu)(z - \gamma_k)|\varphi(k, \omega_\mu)|^2 . \tag{14}$$

The quantity

$$z - \gamma_k = \sum_{i=1}^{z} [1 - \cos(ke_i a)] \tag{15}$$

is the usual dispersion for nearest neighbor couplings on a lattice with coordination number z and lattice spacing a. In what follows we consider a simple-cubic lattice in two dimensions where $z = 4$.

3. Mean field phase transition

Analogous to the coarse graining approach [17] we now introduce local field variables ψ_i which couple to $\zeta_i = \exp(i\varphi_i)$ by means of the conventional Hubbard–Stratonovich procedure. In this way the partition function can be rewritten as:

$$Z = Z_0 \prod_k \left(\frac{2\beta}{\pi E_J \gamma_k}\right) \int D\psi_k(\tau) \exp(-F[\psi]) , \tag{16}$$

where

$$F[\psi] = \frac{1}{\hbar} \int_0^{\hbar\beta} d\tau \sum_k (E_J \gamma_k/2)^{-1} \psi_k^*(\tau)\psi_k(\tau) - \ln\left\langle \exp\left(-\frac{1}{\hbar} \sum_k \int_0^{\hbar\beta} d\tau [\zeta_k^*(\tau)\psi_k(\tau) + \text{h.c.}]\right)\right\rangle_0 . \tag{17}$$

The zero order part Z_0 is expressed by the quadratic action:

$$S_0[\varphi] = \frac{\hbar}{2\beta} \sum_{k,\mu} \left(\frac{(\hbar\omega_\mu)^2}{8E_c} - \tfrac{1}{2}\alpha_{qp}(\omega_\mu)(z - \gamma_k)\right)|\varphi(k, \omega_\mu)|^2 , \tag{18}$$

which contains the charging energy $E_c = e^2/2C$ in the self-charging limit and the dissipation. The expectation value in the logarithm is taken with S_0. Since the expectation values of ψ_i and $\exp(i\varphi_i)$ are closely related, we can identify ψ_i as an order parameter for the phase coherence and perform a cumulant expansion of the free energy functional (17) close to the phase boundary. Above the transition temperature a second-order expansion is sufficient. For a slowly varying order parameter field $\psi_i(\tau)$ we can further expand to lowest order in k and ω_μ and obtain [18, 19]:

$$F[\psi] = \sum_{k,\mu} (r + (ak)^2/z + c_0\omega_\mu^2)|\psi(k, \omega_\mu)|^2 , \tag{19}$$

with the coefficients:

$$r = 1 - \frac{zE_J}{2\hbar}\int_0^{\hbar\beta} d\tau\, g(\tau) , \quad c_0 = \frac{\int_0^{\hbar\beta} d\tau\, \tau^2 g(\tau)}{2\int_0^{\hbar\beta} d\tau\, g(\tau)} . \tag{20}$$

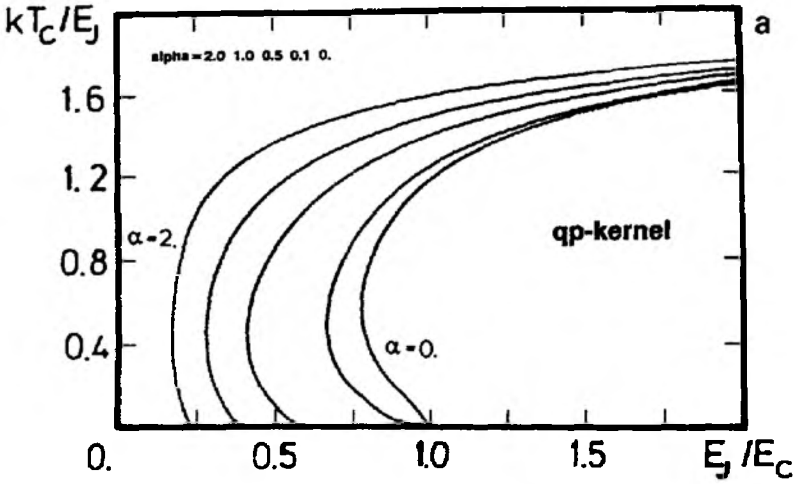
All the information about quantum effects and dissipation is contained in the correlation function:

$$g(\tau) = \langle \exp(\mathrm{i}[\varphi_i(\tau) - \varphi_i(0)])\rangle_0 = \exp\left(-\frac{1}{\hbar^2\beta N}\sum_{k,\mu}\frac{1-\cos(\omega_\mu\tau)}{\frac{\omega_\mu^2}{8E_c} - \frac{1}{2}\alpha_{qp}(\omega_\mu)(z-\gamma_k)}\right) . \tag{21}$$

The mean field phase diagram is determined by the condition $r = 0$. In fig. 2(a) the critical temperature is plotted as a function of the ratio E_J/E_c for different strengths of the dissipation. Here we have used for simplicity an exponential form for $\Gamma(T) = \Gamma(T=0) + \Gamma_1 \exp(-\Delta/kT)$ and assumed a finite value for $\Gamma(T=0)$ of $0.1\Delta(T=0)$. The same value is chosen for Γ_1, which is a characteristic scale for Γ according to the experimental data of Dynes et al. [16]. The temperature dependence of the energy gap is neglected and Δ is fixed at its zero temperature value. In any case the position of the phase boundary line is not crucially sensitive to different choices of Γ. An important feature of the phase boundary is the reentrant-type behavior at low temperatures for all values of α. For comparison, fig. 2(b) shows the corresponding curves for strictly Ohmic damping. For $\alpha = 0$, of course, the phase boundaries are identical and in this case of vanishing dissipation we recover the reentrant behavior recently found in a quantum Monte Carlo simulation [20] for a square array of Josephson junctions, including self-charging Coulomb effects. For the Ohmic damping case the reentrance rapidly vanishes with increasing α, while the reentrance is still present even for large values of α, if the dissipation is due to quasiparticle tunneling. This difference between the two dissipative mechanisms arises from the different low-frequency behavior of the damping kernels, which becomes most important at low temperatures. Furthermore, the pair-breaking parameter Γ monotonously decreases with decreasing temperature. The density of states therefore approaches its ideal form, and the quasiparticle tunneling and thus the subgap conductance is considerably reduced. (For further discussion of the phase diagram see also ref. [9].)

In the experiments by Jaeger et al. [1] a logarithmic dependence of the superconducting transition temperature on the normal state sheet resistance was found. For comparison with the experimental result T_c is plotted as a function of $\ln(1/\alpha) = \ln(R_n/R_0)$ in fig. 3. An approximate logarithmic behavior is realized for a broad range of the ratio Δ/E_C.

Both finite temperature phase diagrams, fig. 2 and fig. 3, show no evidence of a universal threshold value for α. Only for a limited range of the ratio E_J/E_C the criterion whether the film or the array becomes superconducting at low temperatures is fixed by $\alpha > 1$ or $\alpha < 1$. Since the microstructure of the thin films of ref. [1] could not be studied in situ, very little is known about the parameters E_J/E_C in these samples. Therefore the above results do not necessarily contradict the experimental findings.



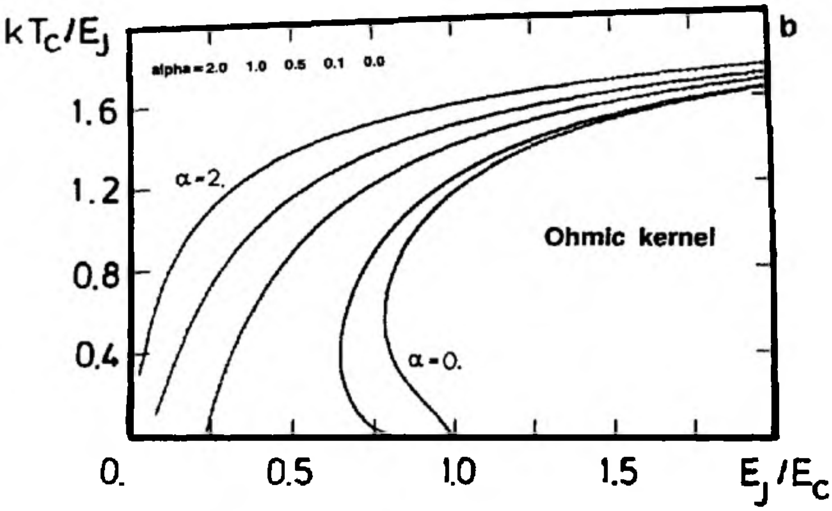


Fig. 2. (a) Mean field phase boundary for quasiparticle dissipation for different strengths of the dissipation $\alpha = h/4e^2R_n$. (b) Same as in (a) for Ohmic dissipation.

4. Fluctuation conductivity

The free energy functional (19) can be used to derive the fluctuation conductivity above the critical temperature. Performing the analytical continuation to real times, a time-dependent (quantum) Ginzburg–Landau (TDQGL) equation can be deduced from (19):

$$-\tau_0 \frac{\partial \psi_k}{\partial t} = r(1 + [\xi k]^2)\psi_k + c_0 \frac{\partial^2 \psi_k}{\partial t^2} , \tag{22}$$

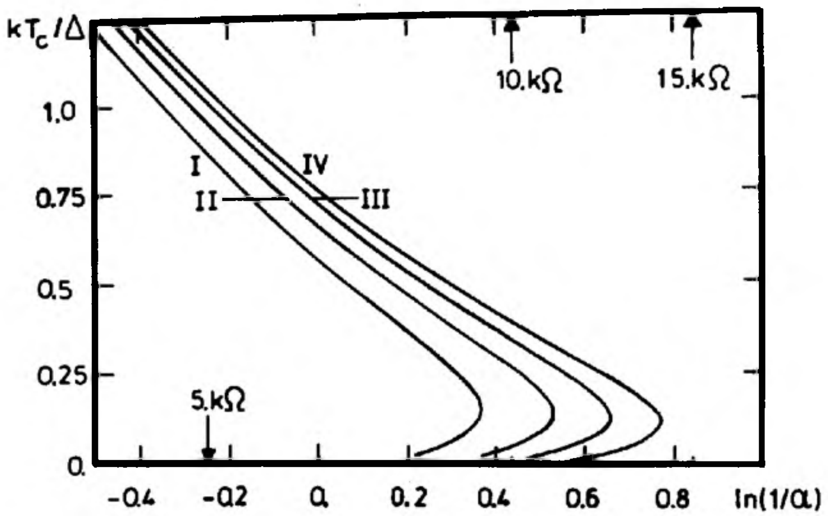


Fig. 3. Mean field phase boundary for quasiparticle dissipation for different values of the ratio $\Delta/E_C \cdot \Delta/E_C = 1.0$ (I), 1.4 (II), 1.8 (III), 2.2 (IV).

with a correlation length $\xi^2 = a^2/zr$. A phenomenological relaxation term has been added on the left-hand side in order to account for the decay of order parameter fluctuations around the absolute minimum $\psi \equiv 0$ of the free energy functional $F[\psi]$. Since a microscopic derivation of the relaxation time is beyond the scope of the present paper, we choose for τ_0 the same characteristic time scale of relaxation as for ordinary homogeneous (BCS) films, $\tau_0 = \pi\hbar/8kT$ [21], which is determined by the temperature only. Contrary to the conventional TDGL equation, the equation of motion (22) also involves the second derivative in time. This is reminiscent of the quantum fluctuations of $\psi_i(\tau)$ and leads, for example, to small oscillations around the stable minimum of the free energy functional below T_c with a frequency of the order of the Josephson plasma frequency $(E_J E_C/\hbar^2)^{1/2}$.

We may now follow the standard procedure [22] and consider only fluctuations of the supercurrent:

$$j_s = \frac{2ec_0}{\hbar\beta i}\,(\psi^*\nabla\psi - \psi\nabla\psi^*)\,. \tag{23}$$

The longitudinal fluctuation conductivity is given by the Fourier transform of the supercurrent correlation function:

$$\sigma_{xx}(\omega) = \beta \int_0^\infty \mathrm{d}t \langle j_{xs}(0) j_{xs}(t)\rangle \cos(\omega t)\,. \tag{24}$$

In the overdamped limit $\tau_0/c_0 \gg (ak)^2$ of the equation of motion (22) we obtain for the static fluctuation conductivity:

$$\sigma(\omega = 0) = \frac{\beta^2\pi\hbar e^2}{zc_0^2} \sum_k \frac{(\xi ka)^2}{1 + (\xi k)^2}\left[\frac{1}{\Omega_k}\coth\left(\frac{\beta\Omega_k}{2}\right)\right]^2, \tag{25}$$

with Doniach's plasmon mode frequency $\Omega_k^2 = (r/c_0)(1 + [\xi k]^2)$. The corresponding conductance for two-dimensional arrays close to the transition ($r \ll 1$) is thus given by:

$$\sigma^{2D}(\omega = 0) = \frac{e^2}{16\hbar} \frac{1}{r}, \tag{26}$$

and diverges as $1/r$ as the phase boundary is approached. (Note that the inclusion of the oscillatory behavior in the underdamped, limit gives rise to another weakly diverging contribution ~$\ln(1/r)$.) For large capacitance junctions $E_J/E_C \ll 1$ quantum effects can be neglected and $r \approx T/(T - T_c)$ with $T_c \approx zE_J/2$. In this limit we recover the temperature dependence of the Aslamasov–Larkin result for homogeneous (BCS) superconducting films [23].

Adding the fluctuation contribution to the normal conductance $1/R_{qp}$ arising from the tunneling of quasiparticles between the superconducting islands, we obtain for the resistivity of the array:

$$R(T) = \frac{R_{qp}}{1 + \frac{\pi}{32\alpha} \frac{R_{qp}}{R_n} f\{r; (\hbar\beta)^2/c_0)\}}, \tag{27}$$

with the dimensionless function:

$$f(r; \delta^2) = \frac{\delta^2}{2} \int_0^{\pi} \frac{x\,dx}{(r + x)^2} \coth^2\left(\frac{\delta}{2} [r + x]^{1/2}\right),$$

which in the limit $r \ll 1$ simply reduces to $f(r, \delta^2) \approx 1/r$.

The resistivity R is plotted as a function of temperature in fig. 4 for a given ratio E_J/E_C and different values of α. For simplicity R_{qp} is modeled by:

$$R_{qp}^{-1} = \left(\frac{\Gamma(T=0)}{\Delta}\right)^2 R_n^{-1} + \exp\left(\frac{-\Delta}{kT}\right)\left(R_n^{-1} - \left(\frac{\Gamma(T=0)}{\Delta}\right)^2\right)$$

to interpolate between low temperatures where R_{qp} is determined by $R_n(\Delta/\Gamma)^2$, and high temperatures where R_{qp} approaches the normal state sheet resistance when the local superconductivity in the islands vanishes. The remarkable feature is that the experimental scenario is qualitatively reproduced, showing ordinary fluctuation broadened superconducting transitions for films or arrays with $\alpha \geq 1.0$, quasireentrant behavior with a resistivity minimum for $\alpha \approx 0.5$ and semiconducting behavior for $\alpha \leq 0.4$. True reentrant behavior is realized for intermediate values $0.6 \leq \alpha \leq 0.8$. But any kind of disorder will presumably wash out the true reentrance, restoring a quasireentrant behavior and percolation type arguments will become important. Again we have to stress that no evidence exists for a universal threshold. For a different value of E_J/E_C the same qualitative behavior as described above appears, however, for different values of α.

The low-temperature resistance of those films which do not become superconducting is given by the subgap conductance R_{qp}^{-1} to be taken from the I–V characteristics. This explains the rapid increase of the resistance by orders of magnitude at low temperatures (disregarding the observed flattening off).

The Josephson coupling energy E_J depends explicitly on α and the temperature, as given by the Ambegaokar–Baratoff formula [24]:

$$E_J = \frac{\alpha\Delta}{2} \tanh\left(\frac{\Delta}{2kT}\right). \tag{28}$$

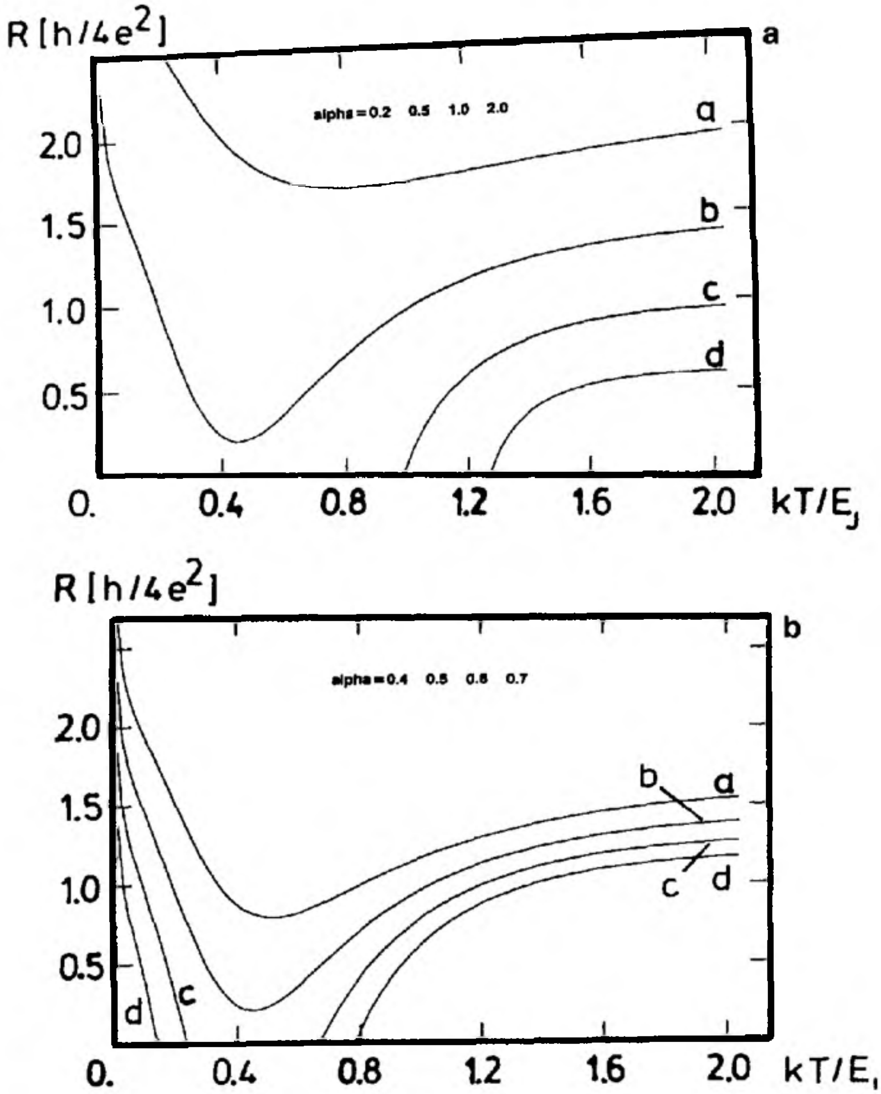


Fig. 4. (a) Temperature dependence of the resistivity, measured in units of $h/4e^2R_n$, for $E_J/E_C = 0.4$ and different values of α: (a) $\alpha = 0.2$; (b) $\alpha = 0.5$; (c) $\alpha = 1.0$; (d) $\alpha = 2.0$. (b) Same as in (a) for different values of α: (a) $\alpha = 0.4$; (b) $\alpha = 0.5$; (c) $\alpha = 0.6$; (d) $\alpha = 0.7$.

The resistivity is plotted again in fig. 5 on a logarithmic scale as a function of temperature, measured in units of the energy gap, for a constant value of $\Delta/E_C = 1.0$. Here, the formula (28) has been used in its low-temperature approximation $E_J \approx \alpha\Delta/2$.

5. Summary and concluding remarks

In this paper we have studied the influence of the quasiparticle dissipation on the finite temperature phase transition in Josephson junction arrays. The full microscopic expression for the damping kernel

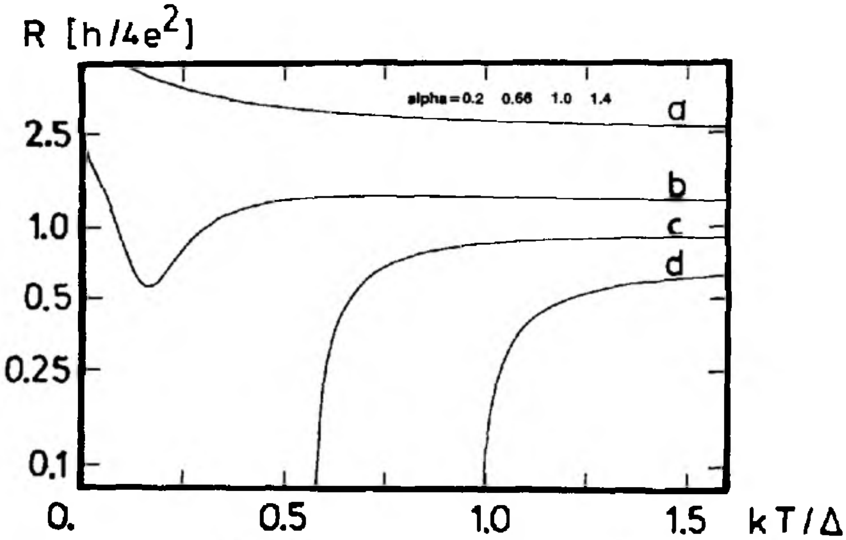


Fig. 5. Temperature dependence of the resistivity for $\Delta/E_C = 1.0$ and different values of α: (a) $\alpha = 0.2$; (b) $\alpha = 0.66$; (c) $\alpha = 1.0$; (d) $\alpha = 1.4$.

has been used including also the effects of scattering mechanisms. The mean field phase diagrams show reentrant-type behavior at low temperatures which, for large $\alpha = h/4e^2 R_n$, is unique to the quasiparticle damping mechanism. As a consequence, the fluctuation conductivity may have a nonmonotonic temperature dependence, depending on the junction parameters E_J/E_C and α. This offers a possible explanation for the experimentally observed resistivity minima in granular samples or thin disconnected films and further supports the relevance of dissipative junction array models for these systems.

The relaxation mechanisms of order parameter fluctuations requires further study to remove the ambiguity for the choice of the characteristic relaxation time. A real-time analysis is in order and will be the subject of future work.

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