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**Putting Citizens First: Representation and Power in the European Union**

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# Putting Citizens First: Representation and Power in the European Union

*The European Union's 2007 Intergovernmental Conferences in Brussels and Lisbon agreed on a new composition of the European Parliament, and on a new voting system for the Council of Ministers. For the legislative period 2009–2014, the seats of the European Parliament are assigned to the 27 Member States following a proposal of the Committee on Constitutional Affairs, plus one additional seat for Italy, here referred to as the “AFCO + 1” seat allocation.<sup>1</sup> Starting in 2014, the Council of Ministers will use a “Double Majority” voting system, whereby an act is adopted if carried by at least 55 percent of the Member States representing 65 percent of the Union's population.*

*The resolution of the European Parliament (2007) draws attention to the overall reform package for the Union's institutions and demands that a future reform should at all events correct any inequalities which have arisen for historical reasons. As a contribution to the prospective debate we here discuss two citizen-based procedures, a “Fix + Prop.” seat apportionment for the European Parliament, and the “Jagiellonian Compromise” voting system for the Council of Ministers. A shift to the envisioned citizen-based procedures turns out to go along with a surprisingly balanced compensation of weights, between the European Parliament and the Council of Ministers.*

## 1. Introduction

Parliamentary representation systems and governmental decision schemes are always subject to debate and reform. They should permit efficient political operations and, in modern democracies, they should stay close to the citizens. As a recent review of the new Scottish Parliament puts it, the issue is to place the citizens at the center of concerns (Arbuthnott, 2006).

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<sup>1</sup> In EU parlance, the Committee is called the AFCO (affaires constitutionnelles) Committee.

The present paper is written under the premise that in the European Union, too, citizens should come first. For the European Union this is more of a daring assumption than an undisputed fact (Moberg, 2007). After all, the European Community and its forerunners started out as a union of states which, in turn, were represented by their governments, without assigning an institutional role to citizens. We justify our premise by the belief that this has changed. The 2007 Intergovernmental Conferences, in conjunction with the actions of the European Parliament, have made considerable progress in incorporating citizens into the political process of the Union. Yet, as we aim to show in this paper, there is room for further improvement.

When the citizens are taken to be the reference set, a central question is which data base to use. The Committee on Constitutional Affairs emphasized that this poses a problem urgently needing attention (Section 2). In proposing its seat allocation, the Committee relied on a “degressive proportionality” concept that, unfortunately, does not stand up to mathematical scrutiny. The concept happens to apply to the population data the Committee chose to work with, but fails with other data. Degressive proportionality, as defined by the Committee and Parliament, is an inept concept that is inappropriate to serve as a court-proof standard of orientation (Section 3).

Section 4 shows that such notions as degressive proportionality, when interpreted in a broad sense, can be served better. We favor an allocation of parliamentary seats that guarantees each Member State’s citizenry a fixed base of seats and apportions the remaining seats by applying the one-person-one-vote principle to the Union’s citizenry at large. A particular such method, called the Fix + Prop. apportionment, is described in greater detail. It assigns 6 fixed seats to each Member State’s citizenry, and apportions the remaining 589 seats in proportion to populations subject to a ceiling of 96 seats. Alternative variants are outlined in the Appendix. We also record how the AFCO + 1 seat allocation deviates from the citizen-based Fix + Prop. apportionment.

Section 5 deals with voting systems for the Council of Ministers. We first explain the workings of the distinguished citizen-based system that is known as the Jagiellonian Compromise. Then we calculate the deviation of each Member State’s decision power, between the decreed Double Majority voting system and the envisioned citizen-based Jagiellonian Compromise. It so happens that, for a vast majority of Member States, the seat deviations in Parliament outweigh fairly well the power deviations in the Council.

We conclude (Section 6) that a shift to citizen-based procedures, besides accommodating the democratic motto of putting citizens first, corrects some of the inequalities which have arisen for historical reasons, and does so in a surprisingly balanced fashion. Admittedly the details hinge on exactly which constraints the Union’s institutions will set: Which population data will be used? Is the European Parliament’s house size to stay at 751, or will it revert to 750? Does the above Fix + Prop. apportionment method meet with approval, or is a variant to be preferred?

## 2. The inception of an apportionment population

In the document submitted to the plenary session (Lamassoure/Severin, 2007a), the Committee on Constitutional Affairs stressed the currently insufficient harmonization of the concept of citizenship between the Member States. As time was too short to solve the problem, the Committee agreed to refer for once to the population figures used for qualified majority decisions in the Council of Ministers (Steinmeier, 2007). In the long run the apportionment population needs to be reconsidered and properly defined, whether derived from European citizens, nationals, residents, or voters.

In the United States of America, with more than two centuries of constitutional history, plenty of cases have been brought before the Supreme Court to decide whom to count, or not to count, among the “apportionment population” used to apportion the 435 seats of the House of Representatives among the 50 US States. No matter which definition the European Union chooses to adopt, time is needed for EuroStat and 27 national statistical offices to provide the relevant figures. Therefore the Committee on Constitutional Affairs is to be commended that it acted instantly by initiating a report to address the question, with Andrew Duff MEP appointed rapporteur, rather than letting the issue rest until a new Parliament will convene in 2009.

Indeed, we would like to voice some concern about the use of the population figures from Steinmeier (2007). They are given in multiples of a hundred, which the Committee on Constitutional Affairs rounds yet further to the nearest thousand. These aggregation steps deprive citizens of their individuality. The legislator views the numbers with the contemptuous eyes of a field marshal who counts his troops only in cohorts of a hundred (for the Council), or in legions of a thousand (for Parliament). The question is not whether 404 346 is a more accurate count for Malta than 404 000, but which figure sends the enlightening message to citizens that, as individual human beings, they are counted one by one.

Moreover, a source of error mentioned in passing only are *double counts* of citizens who are voting in one state while simultaneously being counted into the population of another. When the rapporteurs Alain Lamassoure MEP and Adrian Severin MEP (2007b) present the Committee’s report to the press, the final question is posed by an Italian journalist asking whether the numbers of people eligible to vote in the 2004 European Parliament election would have provided a more appropriate basis. The rapporteurs’ impatient, if not unwilling answer is that Parliament represents not only the electorate, but also children. Every community treasures children, of course, but they hardly explain the visible statistical inconsistencies.

Luxembourg provides a telling example. In 2004 they had an electorate of 214 318, while EuroStat gives a population of 304 283 for that year. The rapporteurs’ reasoning would imply that close to a third of Luxembourg’s population are children and minors. Plenty of double counts may explain the discrepancy more plausibly (Hovehne, 1999, p. 310). Part of the rapporteurs’ unwillingness may have been fueled by the fact that France is ranked second and Italy fourth using the Council’s population data (see Exhibit 1) whereas, on the basis of the 2004 electorate (Scheffler, 2005), France drops to rank four and Italy advances to rank two. This only indicates in yet another way how sensitive an issue the definition of an apportionment population will be.

### 3. The sudden death of AFCO-degressivity

Degressivity of some sort or other, of weights, power, and representation, has been around since the early days of the Union, without a general and lasting definition (Moberg, 2002, 2007). The European Parliament now sees itself called upon to spell out the principle of degressive proportionality clearly and objectively. Since the term has become a principle of primary law, violation of this principle in secondary legislation might even result in penalization by the Court of Justice of the European Communities. The Committee on Constitutional Affairs proposes a definition that dissects degressive proportionality into three items, forming Article 1 of the draft decision of the European Council (European Parliament, 2007), as follows.

- (1) The minimum threshold of 6 seats and the maximum allocation of 96 seats per Member State must be fully utilized to ensure that the seat apportionment reflects as closely as possible the range of populations of the Member States.
- (2) The larger the population of a Member State, the greater its entitlement to a large number of seats.
- (3) The larger the population of a Member State, the more inhabitants are represented by each of its Members of the European Parliament.

Item (1) is stronger than what is demanded by Article I-20 of the Treaty establishing a Constitution for Europe (Pukelsheim, 2007). There, minimum and maximum restrictions are taken to specify a range from 6 to 96 seats wherein the seat numbers have to come to lie. Here, the limits must be utilized. The justification given is void. Utilization of the limits does not ensure, in any provable sense, that the apportionment reflects the populations more closely than when not using the limits. Actually, with the accession of a big country like Turkey any sensible apportionment of 751 seats stays well below the maximum of 96 should a minimum of six persist. For the present Union, the weak version of the old Article I-20 performs just as well as the new item (1); the demand that limits be utilized is superfluous, but does not do any harm either.

Item (2) is called *weak population monotonicity* in the literature (Balinski/Young, 1982a, p. 147). The property is so self-evident that a seat allocation that does not comply is called *absurd* (Kopfermann, 1991, p. 95). Nobody would want to seriously propose a proportional representation scheme for the European Parliament whereby of two states the less populous state is awarded more seats than the more populous state.

With item (1) superfluous and item (2) self-evident, the heart of the definition is item (3) which we name “AFCO-degressivity”. It stipulates that the population-per-seat ratios must be strictly decreasing when passing from larger to smaller Member States. Annex 1 of the Committee’s report shows that the AFCO allocation of the originally 750 seats verifies the criterion, as do other propositions (Chopin/Jamet, 2007).

However, during the European Council in Lisbon the Heads of State and Government granted an extra seat to Italy, thus putting a premature end to AFCO-degressivity. Any Italian deputy now represents 804 818 citizens among 59 million, while Spanish deputies stand for more citizens, 810 339, out of only 44 million; see Exhibit 1. The sudden death of AFCO-degressivity through Council’s action resembles a soccer game where the other team scores the deciding goal right in the first few seconds of overtime.

Even more fatal is the finding that there exist situations where AFCO-degressivity makes no headway at all. At times there simply do not exist any seat apportionments satisfying the criterion. This was brought to light by Victoriano Ramírez-González, and he illustrates the insufficiency by example in his contribution to this Proceedings volume. The reason for the non-existence degeneration is that items (1)–(3) may be incompatible. For instance, 163 seats must be apportioned in such a way that every state gets a minimum of six seats—by (1), thus taking care of 162 seats—and that the 163rd seat is assigned to the largest state, Germany, by (2). The population-per-seat ratio of Germany stays above that of France, so AFCO-degressivity (3) comes for free. However, there is no room to allocate a 164th seat. By (2), it could go only to Germany or else to France. Either way AFCO-degressivity is violated.<sup>2</sup>

How can it happen for the Committee to maneuver itself into a dead-end road? It may come along with the self-satisfaction to steer clear of any mathematical formula. The fact that AFCO-degressivity cannot be found anywhere in the scientific literature should have been a cause for suspicion, not pride. As Parliament comprises only whole seats and no seat fractions, final calculations must always be rounded to whole numbers. This rounding step needs to be accounted for, whether proportionality is strict or degressive. Failing to do so, AFCO-degressivity must be dismissed.

The Committee pays attention when describing *full* proportionality (Lamassoure/Severin, 2007a, Explanatory Statement, no. 13): The population-per-seat ratios should be *the same (or very close)* in all Member States, whence any seat represents *more or less the same* number of inhabitants. These caveats are to the point. Intermediate results must inevitably be rounded to obtain whole seat numbers, and for this reason the ratios cannot be exactly the same, but only more or less so. The Committee admits no provisos at all, however, when turning to *degressive* proportionality. Rather than admitting population-per-seat ratios to be decreasing “more or less” or the like, any non-decreasing instance is considered a clear breach of degressive proportionality. This rigidity causes AFCO-degressivity on occasion to become non-workable.

For the electoral debate to acquire a genuine European dimension, the resolution (European Parliament, 2007, no. 17) proposes to encourage the formation of a European party system. But political parties are formed by citizens, not by Member States. This author fears that any degressivity of whatever sort must be counterproductive, by breaking the European dimension into a spectrum of 27 lines of degressive national valencies. If Parliament demonstrates to the citizens that it considers them not to be equal, how could this be an incentive for them to collectively campaign for political goals?

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<sup>2</sup> If Germany gets the 164th seat, then she has eight seats and her population-per-seat ratio ( $82\,438\,000/8 = 10\,304\,750$ ) falls below that of France ( $62\,886\,200/6 = 10\,481\,033$ ). If France gets the 164th seat, then she has seven seats and her population-per-seat ratio ( $62\,886\,200/7 = 8\,983\,743$ ) falls below that of the United Kingdom ( $60\,421\,900/6 = 10\,070\,316$ ).

For lack of precedence we can only speculate which stance the Court of Justice would take. It seems unlikely that AFCO-degressivity, applying once but not for all, will acquire a lasting legal status. Presumably the Court would not completely run counter to how its fellow courts in the Member States deal with electoral matters. The German Federal Constitutional Court requires an apportionment method to be transparent, calculable, and abstract-general (Pukelsheim/Maier, 2006). The AFCO + 1 allocation misses out on all three requirements, or so we believe. It is not transparent, since it starts out from the Nice negotiations where in the final hours *the Presidency handed out seats like loose change* (Gray and Stubb, 2001) to get a deal. It is not calculable since, besides securing AFCO-degressivity, the sixteen seats beyond Nice were allocated by higher insight of a Committee majority. And it is concrete-specific and not abstract-general, since it applies to the data at hand and not in general.

But then there is also a *continuity principle* in electoral matters that shields the legislator from having to blindly follow an abstract rule when the concrete situation calls for a more sensitive action. In a period where its institutional role is changing, the European Parliament used its margin of discretion to adopt a system that, while debatable, certainly goes in the right direction. Therefore, if the Court of Justice were to declare the AFCO + 1 seat allocation to be unlawful, it would do so presumably not *ex tunc* (since inception), but *ex nunc* (from now on) calling upon Parliament to ameliorate their system before the next election. To this end, however, no court is needed. All speakers in the debate emphasized that the present resolution needs reconsideration during the next legislative period (European Parliament, 2007).

#### **4. The Fix + Prop. seat apportionment**

There are plenty of apportionment formulas reconciling the implicit goals lurking behind the composition of the European Parliament. All of them put a strain of some sort or other on the *status quo*, since none seems to reproduce the AFCO + 1 allocation or the Nice or other former seat assignments. In time, when the goals will become more explicit, the abundance of formulas will undoubtedly narrow down. Here we present what we consider a prototype method, the Fix + Prop. apportionment.

Article I-1 of the Treaty establishing a Constitution for Europe introduces two groups of constitutional subjects of the Union, the citizens and the States of Europe. The question is whether, and how, the two groups are to be represented in Parliament. When the rapporteur Adrian Severin MEP remarks that *we, the European Parliament, are representative of citizens and of the states at the same time*, the minutes record murmurs of dissent, the only emotional reaction during the plenum debate other than applause. Perhaps the dissent questions whether or not Parliament is representative of the *governments* of the states. In contrast, the debate clearly indicates that the Members of the European Parliament see themselves representing the *citizenry*, as a whole, of their home state. In fact, this is a strong point in each speech.



To start, the Fix + Prop. apportionment assigns six seats to the citizenry of each of the 27 Member States, thus allocating 162 seats. This implements the principle of plurality, by allowing the main constituents of the spectrum of the political opinion in each Member State—particularly the majority and the opposition—to be represented (European Parliament, 2007, no. 5). There are many electoral systems guaranteeing a certain number of seats in order to secure a minimum representation. France has a one seat minimum per Département in the Assemblée nationale. Spanish provinces send at least two deputies to the Congreso de los Diputados. Two is a frequent minimum since, in addition to a first seat for the government majority, a sufficiently strong opposition minority might win the second seat. By way of comparison a minimum of six seats sounds excessive, to this author, but seems generally accepted in Parliament.

To finish, the Fix + Prop. method apportions the remaining seats (589) observing the one-person-one-vote principle. In our prototype version we use the divisor method with standard rounding (Webster/Sainte-Laguë) which, as is shown by the seminal research of Balinski/Young (1982a, 1982b), excellently conforms with the philosophy of proportional representation. With the current data, population figures are divided by the common divisor 822 000, and then standard rounding turns the resulting quotients into the desired seat numbers. This is to say that each 822 000 citizens of the Union are allotted about one seat. No quotient must be rounded beyond 90, though, so that together with the six fixed seats the end result complies with the prespecified ceiling of 96 seats. The sole exception of this type is Germany.<sup>3</sup>

Exhibit 1 summarizes the results. The population-per-seat ratios turn out to be of an over-all degressive nature, though the rigid notion of AFCO-degressivity is not met. The last column displays the deviations, per Member State, of the decreed AFCO + 1 seat allocation relative to the citizen-based Fix + Prop. apportionment. Except for Germany and Malta who are curtailed by the maximum and minimum requirements, there emerges a noticeable systematic tendency. Larger and smaller states incur a deficit, while medium-sized states enjoy a surplus. For instance, France loses 11 percent of their Fix + Prop.-seats, Sweden gains 18 percent, Luxembourg loses 14 percent.<sup>4</sup>

The Fix + Prop. apportionment heeds the democratic principle of electoral equality, separately for the two groups of subjects that constitute the Union. Citizens are treated equally by granting about one seat per each 822 000 citizens, and states are treated equally through guaranteeing six seats per citizenry. The one-person-one-vote principle underlying proportional representation is, of course, rather abstract. After all, we are not all equal. In fact, we are proud to be unequal, as is nowhere testified more convincingly than in the debates of the European Parliament. Yet the ideal of equality, as a *principium*, a first and guiding democratic element, has stood the test of time.

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<sup>3</sup> The divisor 822 000 ensures that the resulting seat numbers exhaust the target size 589. Any other value between 821 703 and 822 041 would do just well. — Standard rounding rounds to the nearest whole number, that is, quotients are rounded up (or down) according as their fractional parts are larger (or smaller) than one half. — For Germany, we get  $82\,438\,000/822\,000 = 100.3 \rightarrow 90$ .

<sup>4</sup> France: 83 Fix+Prop.-seats minus 11 percent (9) yield 74 AFCO+1-seats.

Sweden: 17 Fix+Prop.-seats plus 18 percent (3) yield 20 AFCO+1-seats.

Luxembourg: 7 Fix+Prop.-seats minus 14 percent (1) yield 6 AFCO+1-seats.

## 5. The Jagiellonian Compromise

The ideal of equality is also the guiding principle for the Jagiellonian Compromise voting system in the Council of Ministers. Since the Council comprises delegates from governments that, in turn, derive their power from the people, citizens take part in Council decisions only indirectly. Moreover, decision making in the Council concerns not just one decision, but many. The process thus differs from plain proportional representation, yet it, too, must answer to democratic ideals. The Jagiellonian Compromise is distinguished by being citizen-based, in that its derivation starts out from the ideal of equality among citizens, to then trace their indirect contributions to frequent decision making by their government delegates in the Council.

The transition from qualitative-normative ideals to quantitative-operational rules is always a challenge. The goal cannot be but modest, to investigate and classify procedures how close they come to meet ideals. As a result we find that the procedures in use have a top-down format, being imposed on the people with plenty of *ad hoc* components, so characteristic of the AFCO + 1 seat allocation for Parliament as well as of the Double Majority voting system for the Council. In contrast, the Fix + Prop. seat apportionment as well as the Jagiellonian Compromise demonstrably flow from a bottom-up design, justifying political power from a citizen-based starting point.

In the Jagiellonian Compromise voting system every Member State is assigned a *voting weight*, the root of its population figure (rounded to the nearest whole number). For an act to be adopted, the total of the voting weights of the supporting states must reach or surpass a certain *quota*. The Jagiellonian Compromise comes with a simple quota formula, the quota is half of the root of the Union's population plus half of the sum of all voting weights (rounded to the nearest whole number). The essential characteristic is a state's *decision power*, giving the share of constellations where the vote is decisive. In the Jagiellonian Compromise these relative decision powers happen to coincide with the percentage voting weights and thus are found very easily.

Exhibit 2 summarizes the results, using the same population figures as Exhibit 1. For instance, the Jagiellonian Compromise assigns Sweden a voting weight of 3 008, which gives her a relative decision power of 3.14 percent. This indicates that Sweden's vote will be decisive about twice as often as that of Latvia (1.58), and almost half as often as that of Poland (6.44). For the Double Majority voting system, whereby an act is adopted if carried by at least 55 percent of the Member States representing 65 percent of the Union's population, the decision powers are more cumbersome to calculate and are taken from Słomczyński and Życzkowski (2007). The last column displays the deviations of the Double Majority system relative to the citizen-based Jagiellonian Compromise. Again we recognize a systematic tendency. Now it is the medium-sized states that incur a deficit, while the larger and the smaller states enjoy a surplus. For instance, France has 9 percent more power than under the Jagiellonian Compromise, Sweden 16 percent less, Luxembourg 124 percent more.<sup>5</sup>

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<sup>5</sup> France: 8.27 JC power plus 9 percent (0.74) gives 9.02 DM power.

Sweden: 3.14 JC power minus 16 percent (0.50) gives 2.63 DM power.

Luxembourg: 0.71 JC power plus 124 percent (0.88) gives 1.58 DM power.

The inaccuracies are due to rounding the percentage deviations in Exhibit 2 to whole numbers.

The Jagiellonian Compromise visibly starts out from the citizens, by deriving from the population figures via their roots the voting weights and decision powers of the Member States. The exceptional value of the system comes to bear only when returning to citizens and asking, what the decision power share *of a citizen* amounts to. According to a famous result of Penrose (1946), the individual power share is obtained by dividing the decision power of a state by the root of its population. For the Jagiellonian Compromise these individual shares are all identical. Therefore the system is such that it offers all citizens in the Union the same power share to (indirectly) contribute to the Council’s decisions. Thus the Jagiellonian Compromise realizes a powerful, but at the same time rather sophisticated idealization of democratic equality.

The derivation of these results may be found in the seminal monograph of Felsenthal/Machover (1998), a quick outline is given by Kirsch (2001). Felsenthal/Machover (2001) proposed a quota in the vicinity of 60 percent, later confirmed by a simulation study of Chang et al. (2006). The quota formula is due to Życzkowski/Słomczyński (2004), Słomczyński/Życzkowski (2006). The idealizations underlying this approach may of course be questioned, as in the informative succession of the Moberg (2002), Hosli/Machover (2004), Moberg (2007) papers. Our answer would be, not that the ideal of democratic equality must necessarily lead to the Jagiellonian Compromise, but that this system has sufficiently many virtues to match the ideal remarkably well.

## 6. Conclusion

The European Parliament and the Council of Ministers count among the core institutions of the European Union. As of now, the AFCO + 1 seat allocation in Parliament and the Double Majority voting system in the Council are negotiated *ad hoc* procedures. For both institutions there are citizen-based alternatives, the Fix + Prop. seat apportionment for Parliament, and the Jagiellonian Compromise system for the Council. A replacement of the *ad hoc* procedures by citizen-based methods naturally entails some shift of parliamentary seats, and some shift of decision power.

By calculating the percentage deviations of the enacted old procedures relative to the envisioned new methods, as in the last columns in Exhibits 1 and 2, changes are put on the same scale and become comparable. Large and small states lose in the first setting and gain in the second, and medium-sized states gain in the first and lose in the second. Surprisingly, the changes balance out almost perfectly. See Exhibit 3.

For those who do want to stay put this provides a striking argument to defend the *status quo*. Why should dedicated parliamentarists comfort themselves that modifications they endure are compensated by changes in the Council? Why should staunch governmentalsists commit themselves to alterations that are being outweighed in Parliament with whom they have few dealings?

Fortunately, the argument is even more striking for the political elite who wants to move ahead. A transition to citizen-based procedures would relegate any purported old rivalries between European institutions in favor of a new alliance, around the Union’s citizens as the common denominator. The shifts of seats and power would seem to fade away compared to the significant gain in democratic substance, of putting citizens first.

## Appendix. Alternative apportionment methods

The envisioned Fix + Prop. apportionment for the European Parliament and the Jagiellonian Compromise for the Council of Ministers are just two prototypes, out of infinitely many citizen-based representation schemes and voting systems. In this appendix we delineate a few alternatives, with a particular view to what has accumulated in the literature. Reviews how the European Parliament came into being and which problems had to be dealt with are provided, for example, by Silvestro (1990), Bocklet (1992), Hovehne (1999), and Puntscher-Rieckmann et al. (2003).

For voting systems, variations around the quorum formula might be a possibility to adjust certain characteristics such as blocking power or the like, if so desired. Much of this is discussed elsewhere in this Proceedings volume. A continuous transition from full proportionality in Parliament to the square root weighting for the Council, is accomplished by noticing that population figures enter into the calculations just with different exponents. In the first instance the exponent is one, in the second, one half. To our knowledge Theil/Schrage (1977) are the first to consider exponents ranging from zero (equal weights) through one half (root weights) and one (full proportionality) to three (cube law). The transformation was re-discovered by Anders Hagelberg during the 2000 Nice summit (Moberg, 2007), and Ramírez-González et al. (2006).

We believe that for the purpose of representing citizens in parliament no exponent distinct from one is acceptable. Otherwise the system re-weighs citizens, and any weighting of citizens conflicts with the democratic one-person-one-vote principle. For this reason we have concentrated on such apportionment methods as Fix + Prop. that stay within the realm of equity, by safeguarding the identities of the citizenries of every Member States and by treating all citizens as being equal. This Janus-faced approach has a long tradition, earlier references being Kundoch (1976) and Wessels (1990) for its application to the European Parliament, or Moberg (1998) for its use in the Council.

Scheffler (2005, p. 80) discusses two variants of Fix + Prop. type systems. He recommends the divisor method with standard rounding (Webster/Sainte-Laguë), and mentions the divisor method with rounding down (Jefferson/D’Hondt/Hagenbach-Bischoff) as an alternative. However, if anything the D’Hondt method is not degressive but progressive, in the sense that it is calculably biased in favor of larger and at the expense of smaller participants. For the current data it would transfer seven seats from smaller to larger Member States as compared to the results in Exhibit 1. During the deliberations of the Committee on Constitutional Affairs, the D’Hondt variant was tabled by the German CDU delegation, but eventually withdrawn. In the plenum speakers took pleasure to teach the Germans a lesson that when degressivity is sought, the answer cannot be D’Hondt.<sup>6</sup>

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<sup>6</sup> The German delegates presumably acted in good faith by trusting their Federal Constitutional Court, the only institution of any renown unable to recognize any systematic difference between the divisor methods with rounding down (Jefferson/D’Hondt/Hagenbach-Bischoff) and with standard rounding (Webster/Sainte-Laguë). Other German courts do declare the D’Hondt apportionments to be unlawfully non-proportional when its apportionment deviates from the Sainte-Laguë result (Pukelsheim/Maier, 2006, fn. 18). The heterogeneity in just one Member State may serve as an indication how big a challenge it is for all 27 to move towards electoral homogeneity.

The divisor method with standard rounding (Webster/Sainte-Laguë) comes closest to meeting the one-person-one-vote principle, in view of its many virtues distinguishing it over alternative procedures (Balinski/Young, 1982b). Amongst others it is unbiased, so that on average every citizenry receives the share of seats to which it is entitled by the size of its population.

In case the European Parliament wants to move away from unbiasedness in order to push degressivity, the divisor method with rounding up (Adams) may be invoked. This method is biased in the opposite direction of D’Hondt, favoring smaller Member States at the expense of larger states. This is plainly visible when the Sainte-Laguë results are compared with the Adams apportionment. For instance, between the third and fourth column in Exhibit 4, five seats are transferred from larger to smaller Member States, between the next two columns, six. The divisor method with rounding up (Adams) has been used by the French legislator to apportion the seats of the Assemblée Nationale among the Départements (Balinski, 2004, p. 190).

In Exhibit 4, the columns labeled “6+Std” and “6+Up” show the apportionments of the (unbiased) divisor method with standard rounding (Webster/Sainte-Laguë), and the (degressively biased) divisor method with rounding up (Adams), with a base of six fixed seats. With the Adams method, every citizenry gets always at least one seat automatically, whence the fixed base seats could be lowered from six to five. The results are given in the columns with headers “5+Std” and “5+Up” of Exhibit 4. The four apportionments of the 751 seats display a clear trend. From left to right, larger Member States lose seats and smaller Member States gain seats.

Among the apportionments of Exhibit 4 the one that is closest to the AFCO + 1 allocation is the “parabolic” allotment (Ramírez-González et al. 2006; Ramírez-González 2007). The parabolic method is a workable operational approach to the normative idea of degressive proportionality. The functioning and the implementation of the method requires a more elaborate mathematical apparatus, with the implied challenge of communicating parabolicity to the wider public. These examples are not an exhaustive enumeration, many other mathematically sound procedures are feasible (Maier/Pukelsheim, 2007). The decision which to implement is up to the lawmaker.

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**Allocation of European Parliament seats to Member States:  
Citizen-based apportionment method “Fix + Prop.” and  
negotiated *ad hoc* allocation “AFCO + 1”**

Member States	Population	Fix + Prop.	Popul. per	AFCO + 1	Popul. per	Deviation
EU-27	2007	[Divisor 822 000]	Fix+Prop.	2009–2014	AFCO+1	[in %]
Germany	82 438 000	6 + 90 = 96	858 729	96	858 729	0
France	62 886 200	6 + 77 = 83	757 665	74	849 814	–11
United Kingdom	60 421 900	6 + 74 = 80	755 274	73	827 697	–9
Italy	58 751 700	6 + 71 = 77	*763 009	73	804 818	–5
Spain	43 758 300	6 + 53 = 59	741 666	54	*810 339	–8
Poland	38 157 100	6 + 46 = 52	733 790	51	748 178	–2
Romania	21 610 200	6 + 26 = 32	675 319	33	654 855	+3
Netherlands	16 334 200	6 + 20 = 26	628 238	26	628 238	0
Greece	11 125 200	6 + 14 = 20	556 260	22	505 691	+10
Portugal	10 569 600	6 + 13 = 19	*556 295	22	480 436	+16
Belgium	10 511 400	6 + 13 = 19	553 232	22	477 791	+16
Czech Republic	10 251 100	6 + 12 = 18	*569 506	22	465 959	+22
Hungary	10 076 600	6 + 12 = 18	559 811	22	458 027	+22
Sweden	9 047 800	6 + 11 = 17	532 224	20	452 390	+18
Austria	8 265 900	6 + 10 = 16	516 619	19	435 047	+19
Bulgaria	7 718 800	6 + 9 = 15	514 587	18	428 822	+20
Denmark	5 427 500	6 + 7 = 13	417 500	13	417 500	0
Slovak Republic	5 389 200	6 + 7 = 13	414 554	13	414 554	0
Finland	5 255 600	6 + 6 = 12	*437 967	13	404 277	+8
Ireland	4 209 000	6 + 5 = 11	382 636	12	350 750	+9
Lithuania	3 403 300	6 + 4 = 10	340 330	12	283 608	+20
Latvia	2 294 600	6 + 3 = 9	254 956	9	254 956	0
Slovenia	2 003 400	6 + 2 = 8	250 425	8	250 425	0
Estonia	1 344 700	6 + 2 = 8	168 088	6	224 117	–25
Cyprus	766 400	6 + 1 = 7	109 486	6	127 733	–14
Luxembourg	459 500	6 + 1 = 7	65 643	6	76 583	–14
Malta	404 300	6 + 0 = 6	67 383	6	67 383	0
<b>Total</b>	<b>492 881 500</b>	<b>162+589 = 751</b>		<b>751</b>		

Exhibit 1. The population data are those underlying the European Council’s qualified majority voting system in 2007, see Steinmeier (2007). The Fix + Prop. apportionment gives 6 seats to the citizenry of each Member State, and assigns the remaining 589 seats in proportion to populations. To this end the population figures are divided by a common divisor, 822 000, and the resulting fractional numbers are rounded to the nearest whole number. For instance, France receives  $62\,886\,200/822\,000 = 76.504 \rightarrow 77 + 6 = 83$  seats. The AFCO + 1 column displays the seat allocation enacted for the 2009–2014 legislative period. Its deviation from the Fix + Prop. apportionment is found, for instance for France, to be  $(74 - 83)/83 = -0.1084 \rightarrow -11$  percent. Population-per-seat ratios are decreasing except for slight disturbances marked \*.



**Qualified majority voting systems for the Council of Ministers:  
Citizen-based Jagiellonian Compromise “JC” and  
negotiated *ad hoc* Double Majority “DM”**

Member States	Population	JC	JC	DM	Deviation
EU-27	2007	Weight	Power	Power	[in %]
Germany	82 438 000	9 080	9.46	11.66	+23
France	62 886 200	7 930	8.27	9.02	+9
United Kingdom	60 421 900	7 773	8.10	8.69	+7
Italy	58 751 700	7 665	7.99	8.49	+6
Spain	43 758 300	6 615	6.90	6.55	–5
Poland	38 157 100	6 177	6.44	5.71	–11
Romania	21 610 200	4 649	4.85	4.15	–14
Netherlands	16 334 200	4 042	4.21	3.50	–17
Greece	11 125 200	3 335	3.48	2.88	–17
Portugal	10 569 600	3 251	3.39	2.80	–17
Belgium	10 511 400	3 242	3.38	2.80	–17
Czech Republic	10 251 100	3 202	3.34	2.77	–17
Hungary	10 076 600	3 174	3.31	2.74	–17
Sweden	9 047 800	3 008	3.14	2.63	–16
Austria	8 265 900	2 875	3.00	2.53	–16
Bulgaria	7 718 800	2 778	2.90	2.47	–15
Denmark	5 427 500	2 330	2.43	2.19	–10
Slovak Republic	5 389 200	2 321	2.42	2.18	–10
Finland	5 255 600	2 293	2.39	2.17	–9
Ireland	4 209 000	2 052	2.14	2.04	–5
Lithuania	3 403 300	1 845	1.92	1.95	+2
Latvia	2 294 600	1 515	1.58	1.81	+15
Slovenia	2 003 400	1 415	1.47	1.78	+21
Estonia	1 344 700	1 160	1.21	1.69	+40
Cyprus	766 400	875	0.91	1.63	+79
Luxembourg	459 500	678	0.71	1.59	+124
Malta	404 300	636	0.66	1.58	+139
Total	492 881 500	95 916	100.00	100.00	
Quota		59 058	61.57		

Exhibit 2. The voting weights of the Jagiellonian Compromise are the roots of the population figures, rounded to the nearest whole number. Thus Malta enjoys weight  $\sqrt{404\,300} = 635.9 \rightarrow 636$ . The quota for a qualified majority decision obeys the formula  $(\sqrt{492\,881\,500} + 95\,916)/2 = 59\,058.47 \rightarrow 59\,058$  (bottom line). This particular rule is such that percentage weights and decision powers coincide. Since Malta’s weight amounts to  $636/95\,916 = 0.006631 \rightarrow 0.66$  percent, its relative Penrose/Banzhaf power index is the same, 0.66. The power indices for the Double Majority rule, of 55 percent of the Member States and 65 percent of the Union’s population, are taken from Słomczyński and Życzkowski (2007). Its deviation from the Jagiellonian Compromise is found, for instance for Malta, to be  $(1.58 - 0.66)/0.66 = +1.3939 \rightarrow +139$  percent.

**Compensating balance of citizen-based procedures:  
Seat deviation of “AFCO + 1” from “Fix + Prop.”,  
and power deviation of “DM” from “JC”**

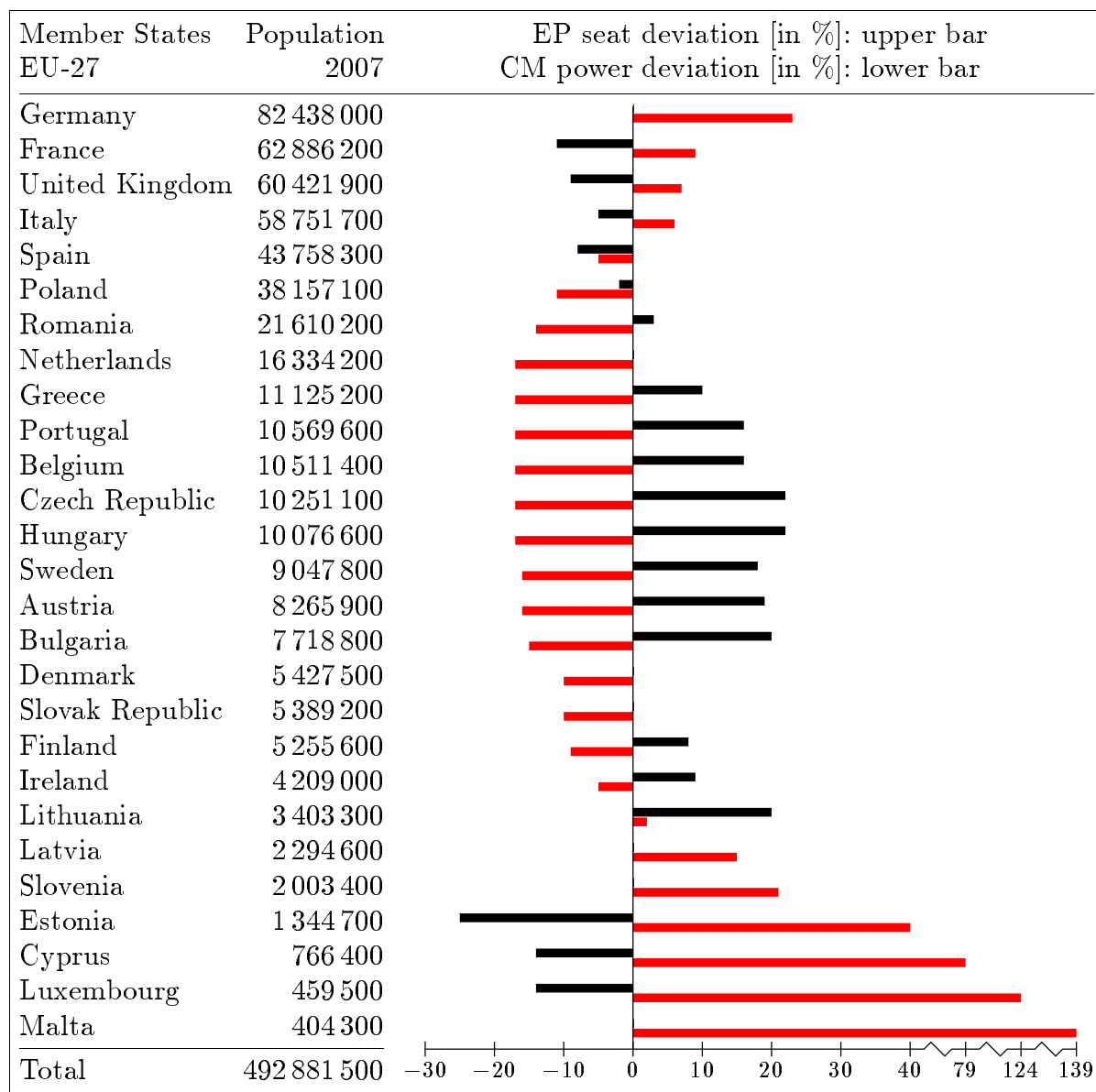


Exhibit 3. The deviations in popular representation and in decision power, of present negotiated *ad hoc* procedures from envisioned citizen-based procedures, balance almost perfectly. A simultaneous adoption of the citizen-based Fix + Prop. seat apportionment in the European Parliament, and of the citizen-based Jagiellonian Compromise voting system in the Council of Ministers would go along with shifts of weight that are mutually compensating, for almost all member states.

**Allocation of European Parliament seats to Member States:  
Apportionments of four variants “base + divide & round”,  
of the parabolic allotment, and of “AFCO + 1”**

Member States	Population	5+Std	5+Up	6+Std	6+Up	para-	AFCO
EU-27	2007	[786 000]	[800 000]	[822 000]	[845 000]	bolic	+1
Germany	82 438 000	96	96	96	96	96	96
France	62 886 200	85	84	83	81	79	74
United Kingdom	60 421 900	82	81	80	78	76	73
Italy	58 751 700	80	79	77	76	75	73
Spain	43 758 300	61	60	59	58	59	54
Poland	38 157 100	54	53	52	52	53	51
Romania	21 610 200	32	33	32	32	34	33
Netherlands	16 334 200	26	26	26	26	27	26
Greece	11 125 200	19	19	20	20	20	22
Portugal	10 569 600	18	19	19	19	20	22
Belgium	10 511 400	18	19	19	19	20	22
Czech Republic	10 251 100	18	18	18	19	19	22
Hungary	10 076 600	18	18	18	18	19	22
Sweden	9 047 800	17	17	17	17	18	20
Austria	8 265 900	16	16	16	16	17	19
Bulgaria	7 718 800	15	15	15	16	16	18
Denmark	5 427 500	12	12	13	13	13	13
Slovak Republic	5 389 200	12	12	13	13	13	13
Finland	5 255 600	12	12	12	13	13	13
Ireland	4 209 000	10	11	11	11	11	12
Lithuania	3 403 300	9	10	10	11	10	12
Latvia	2 294 600	8	8	9	9	9	9
Slovenia	2 003 400	8	8	8	9	8	8
Estonia	1 344 700	7	7	8	8	7	6
Cyprus	766 400	6	6	7	7	7	6
Luxembourg	459 500	6	6	7	7	6	6
Malta	404 300	6	6	6	7	6	6
<b>Total</b>	<b>492 881 500</b>	<b>135 + 616 = 751</b>	<b>162 + 589 = 751</b>			<b>751</b>	<b>751</b>

Exhibit 4. The method 5+Std guarantees each Member State’s citizenry 5 seats, and assigns the remaining 616 seats using the divisor method with standard rounding. Thus, with divisor 786 000 as in the header, France receives  $62\,886\,000/786\,000 = 80.01 \rightarrow 80 + 5 = 85$  seats. The method 5+Up is similar, but always rounds up. Now France gets  $62\,886\,000/800\,000 = 78.61 \uparrow 79 + 5 = 84$  seats. The methods 6+Std (elsewhere called Fix + Prop.) and 6+Up use a base of 6 seats per citizenry, with 589 seats for proportional apportionment. The parabolic allotment is from Ramírez-González (2007). The AFCO+1 allocation has been adopted for the period 2009–2014. In essence, from left to right, larger states lose seats and smaller states gain seats.