

# Spin phenomena in intersubband transitions

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Intersubband transitions have been studied in InAs/AlSb multi quantum wells subjected to a parallel magnetic field. In this Voigt configuration we observe two resonances whose oscillator strengths increase with magnetic field. The high energy line is interpreted as a superposition of two spin-conserving intersubband transitions while the low energy resonance corresponds to two combined spin-flip intersubband transitions. The lines are separated in energy by the depolarization shift which influences only the spin-conserving excitations. The spin-conserving transitions are induced by the parallel magnetic field although the incident light is polarized in the plane of the quantum well. The combined resonances, on the other hand, are made possible by a combination of the InAs bulk inversion asymmetry in conjunction with its narrow band gap and the high carrier density as present in our samples. In the multiple reflection path geometry the spin-flip lines can be resolved and we observe a zero-field spin-splitting of approximately  $\Delta E = 17$  meV which is also a result of the bulk inversion asymmetry.

## 1. Introduction

The electronic spin has profound influence on the physical properties of 2D electron gases (2DEG) as magnetic fields, lifting the spin-degeneracy, have always played an important role in the experimental investigations of 2DEGs [1]. Magnetotransport experiments such as the Shubnikov–de-Haas or quantum Hall effect reveal the spin resolved Landau levels [2]. In optical experiments, however, selection rules make the direct spectroscopic observation of the spin levels more difficult. For absorption experiments the selection rules state that the spin quantum number remains constant [3]. Therefore transitions measured in conventional absorption spectroscopy are all spin-conserving. As a result these experiments are only sensitive to the energetic difference between two adjacent spin-up and spin-down levels which is in most cases very small.

Strong spin-orbit coupling can induce a new class of optical excitations involving a spin-flip. In the investigation of intersubband transitions (IST) inelastic light scattering has been proven to be a powerful method [4] as the excitation of an electron from the valence band—with strong spin-orbit coupling—allows the observation of these spin-flip transitions for many different material systems. Furthermore, both charge density (spin-conserving) and spin density (spin-flip) excitations can be observed simultaneously. These resonances are separated in energy by the depolarization shift acting only upon the spin-conserving transitions [5] as they give rise to a macroscopic electric field. There-

fore the investigation of spin-flip excitations contributes to the understanding of many-body effects in the optical response of 2DEGs [4]. In contrast to inelastic light scattering, the intersubband absorption is a pure intraband phenomenon. Thus spin-flips were only observed for hole systems in the valence band [6] or for electronic systems with strong spin-orbit interaction [7] such as narrow gap semiconductors.

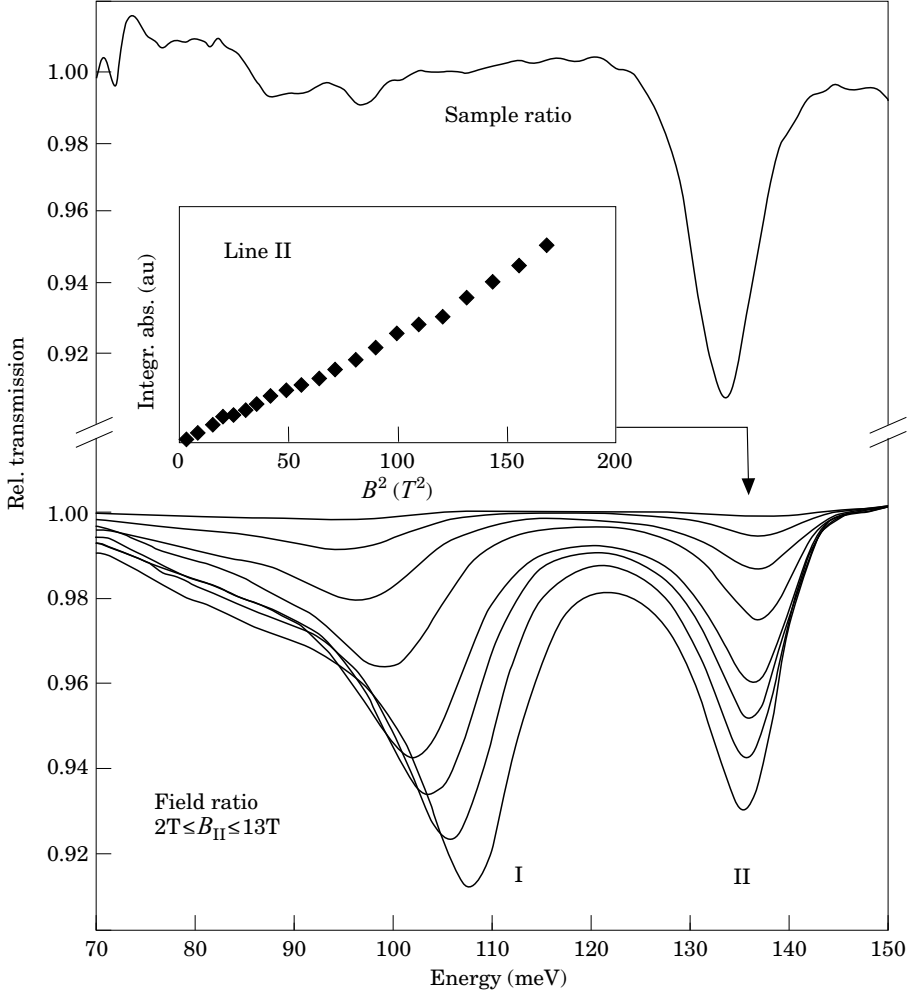
But even without an external magnetic field the spin-degeneracy can be lifted in a structure which is not symmetric under inversion. This inversion asymmetry can be caused by the so-called bulk inversion asymmetry [8] as present in all III–V semiconductors or induced by an external electric field [9]. Physically, this effect can be understood as an electric field which is Lorentz-transformed into the frame of a moving electron resulting in an internal magnetic field  $B_{int}$ . Therefore, the strength of  $B_{int}$  increases with the strength of the electric field and the Fermi velocity. Transport measurements [10] and optical experiments [11] have unambiguously shown the existence of such an effective field  $B_{int}$  in 2DEGs. However, in many cases some doubts remained whether the observed effects were caused by external electric fields, the bulk inversion asymmetry or a combination of both [12].

Here, we wish to review intersubband optical experiments on InAs/AlSb multi-quantum wells elucidating different spin phenomena in 2DEGs. This relatively new material combination provides high mobility electron gases with strong spin-orbit interaction. We chose the Voigt geometry [13,14] or the multiple reflection path geometry [3] as our experimental configurations with a magnetic field oriented parallel to the layer of the 2DEG. At finite magnetic fields we observe both spin-conserving and combined spin-flip intersubband transitions simultaneously [15]. Here, the in-plane magnetic field couples the motion perpendicular and parallel to the interface and thus breaks the polarization selection rule for the spin-conserving IST. The inherent bulk inversion asymmetry and the narrow band gap of our quantum well material in combination with a large wavevector  $k$  gives rise to an excitation mechanism for spin-flip IST. Similar to the case of inelastic light scattering, we can deduce the depolarization shift of the spin-conserving transition from its energetic difference to the spin-flip IST. Furthermore, our experiments allow us to measure the transition matrix elements in a straightforward manner. We also find that the depolarization drastically reduces the nonparabolicity-induced line-broadening of the spin-conserving resonance. In the multiple reflection path configuration both spin-flip lines are well separated in energy at low magnetic fields while the resonances merge at higher fields and eventually cross at  $B=12$ T. This magnetic field dependence gives clear evidence for a zero-field spin-splitting in our material system [16] which is also caused by the InAs bulk inversion asymmetry.

## 2. Experiment

The samples described here were grown on a GaAs substrate followed by a sequence of buffer layers [17] to accommodate a lattice mismatch of roughly 7% between the substrate and the active layers. The 20 period multi-quantum well consists of 15 nm InAs layers sandwiched between 10 nm wide  $\delta$ -doped Te:AlSb barriers. To minimize band-bending effects [18] we chose a 55 nm thick topmost AlSb barrier capped by a 5 nm wide layer of GaSb. Hall effect measurements at low temperatures revealed a carrier density of  $N_s=2.5 \times 10^{12} \text{ cm}^{-2}$  per well. The mid-infrared spectra were taken using a rapid-scan Fourier transform spectrometer with the samples mounted in either Voigt geometry or in multiple reflection path geometry. Experimentally, we determined the relative change in transmission  $T(B \neq 0)/T(B=0)$  for different in-plane magnetic fields  $B$ . Alternatively, we could ratio the spectra against those of a reference substrate.

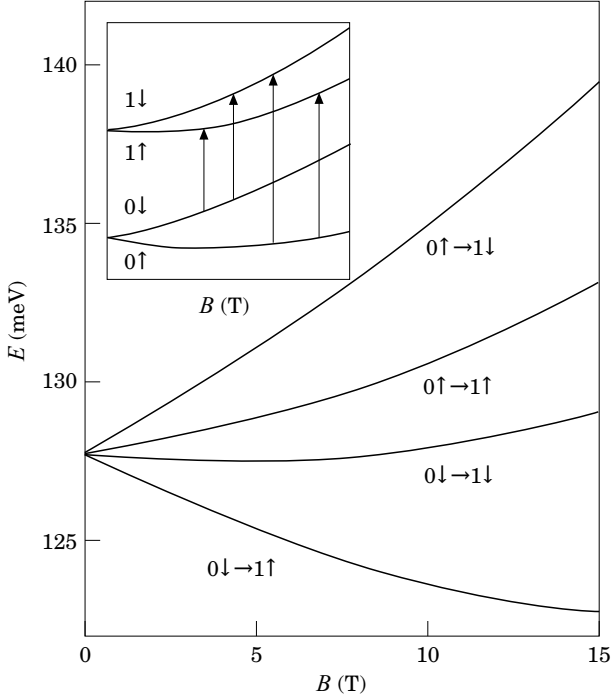
We obtained the lower traces of Fig. 1 in the Voigt configuration with in-plane magnetic fields between  $B=2$ T and  $B=13$ T. As a reference we chose a zero magnetic field spectrum of the sample since under this condition our structure showed no resonant absorption for the in-plane light polarization. Surprisingly, we observe two well separated resonances—one at around  $E=100$  meV (I) and



**Fig. 1.** The upper trace shows a transmission spectrum of our 150 Å InAs/AlSb multi-quantum well ratioed against a reference substrate with the unpolarized radiation incident under an oblique angle of  $57^\circ$ . The resonance strength follows the same polarization dependence as expected for intersubband transitions. In the magnetic field ratio of the same sample we show spectra  $T(B \neq 0)/T(B = 0)$  for in-plane magnetic fields  $B = 2\text{T}, 4\text{T}, 6\text{T}, 8\text{T}, 10\text{T}, 11\text{T}, 12\text{T}$  and,  $13\text{T}$ . Both resonances I and II vanish at zero magnetic field and their absorption increases at higher fields. The inset shows that the integrated absorption of line II depends quadratically on the magnetic field. (Reproduced from reference [15] with permission.)

a second at around  $E = 135$  meV (II) both of which gain strength at high fields. With increasing field, the position of line I noticeably shifts to higher energies whereas the position of line II remains almost constant. As compared to resonance I, line II is narrow and symmetric and its width is independent on the magnetic field. Its integrated absorption, i.e. oscillator strength, increases quadratically with magnetic field as shown in the inset of Fig. 1.

The upper trace shows a transmission spectrum of the same sample taken under an oblique angle of  $57^\circ$  with respect to the direction of light propagation ratioed against a reference substrate. In this configuration the refraction of the incident light results in a finite electric field component perpendicular to the 2DEG as required to excite an IST at  $B = 0\text{T}$  [3]. Both the resonance position



**Fig. 2.** Transition energies between the two lowest subbands in the single particle picture. The spin-conserving transitions differ only very little in energy while the spin-flip excitations are energetically well separated. For  $B \neq 0$  T all subbands are spin-split as shown in the inset and a total of four transitions are possible. The energy levels rise at high fields since the diamagnetic shift dominates the Zeeman effect. (Reproduced from reference [15] with permission.)

and the oscillator strength agree quite well with our theoretical estimates for an IST so that we interpret this line as an intersubband resonance.

From a comparison with the tilted angle experiment, we identify resonance II as the depolarization shifted IST. This conclusion is supported by the fact that the energetic position and lineshape are almost identical in both spectra. The low-energy resonance II, however, cannot be excited by the presence of a perpendicular electric field component alone as present in the tilted angle configuration.

### 3. Interpretation

As a first step to understand the spectra we calculate the magnetic field dependence of the single particle subband energies within the framework of an 8-band model  $\vec{k} \cdot \vec{p}$  [19]. For InAs, the conduction band is separated in energy from the heavy- and light-hole band by  $E_g = 0.42$  eV, the split-off energy is taken to be  $\Delta = 0.38$  eV and the Kane-energy is  $2m_0k^2 = 22.9$  eV. In this approach the electronic wave function is represented by a vector whose components are given by the Bloch functions. For finite wavevectors  $k$  the electronic wavefunction ceases to be purely s-like and mixes with the valence band levels [19]. As a consequence the electronic mass becomes now energy dependent. The band bending induced by the doping has been taken into account by first order perturbation theory. Throughout this paper we choose the direction of growth and the electric confinement to be  $y$  while the magnetic field is applied in the  $z$ -direction  $\vec{B} \parallel z$ .

In the inset of Fig. 2 we schematically plot the energy levels as a function of the magnetic field.

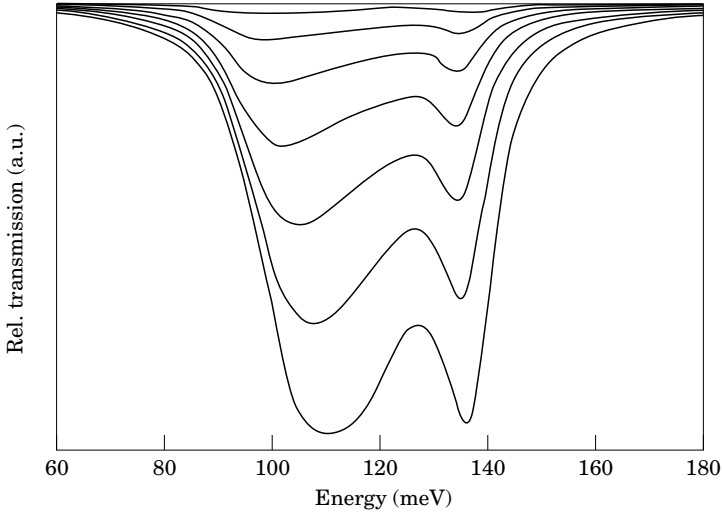
The electric subbands are spin-split for finite fields and four intersubband excitations become possible: two are spin-conserving while the two others involve a spin-flip. At low fields  $B$  the negative  $g$ -factor of InAs results in an energetic decrease of the  $|↑\rangle$ -levels and in an increase of the  $|↓\rangle$ -levels (Zeeman regime). At higher fields the diamagnetic shift as described in [20] dominates and raises the eigenenergies irrespective of the spin-orientation. The calculation of the single-particle transition energies gives the following results for  $k=0$  (Fig. 2): The transition energies for  $|0↑\rangle \rightarrow |1↑\rangle$  and  $|0↓\rangle \rightarrow |1↓\rangle$  differ only very little even at high magnetic fields. The spin-flips  $|0↑\rangle \rightarrow |1↓\rangle$  and  $|0↓\rangle \rightarrow |1↑\rangle$ , on the other hand, are more widely separated. However, all of these splittings are much smaller than the separation of lines I and II observed in the experiment.

It is well known from inelastic light scattering that—in contrast to spin-flip excitations—spin-conserving IST are subject to the depolarization field [4] shifting the resonance to higher energies. Moreover, the depolarization field is known to reduce the nonparabolicity induced line-broadening of IST [21,22] which is a result of the non-parallel subband dispersions. The calculated single particle energies (Fig. 2) correspond roughly to the position of line I for  $B > 0T$  and the separation of lines I and II in the same limit is approximately given by the computed depolarization shift. Therefore we interpret line I as a superposition of the spin-flip transitions and line II as a superposition of the spin-conserving resonances. In this sense the unexpected narrowness of resonance II with a much smaller linewidth than expected from the  $k$ -dependence of the transitions energies is a consequence of the depolarization field. Thus the compensation of the nonparabolicity induced line-broadening by many-body effects can be directly verified in our experiment. However, the spin-splittings as calculated in Fig. 2 are considerably smaller than the depolarization shift and the experimental linewidths of Fig. 1. Therefore we would not expect to resolve all four possible resonances.

We now address the question how the spin-conserving transitions are excited even with in-plane polarized light for  $B \neq 0T$ . At finite magnetic fields the light wave polarized in the  $x$ -direction (the ‘cyclotron resonance active’ mode) is no longer transverse [23] but acquires an electric field component in the growth direction. Physically, this can also be interpreted as a coupling of the cyclotron motion to the intersubband resonance. The component  $E_y^{ind}$  is proportional to the magnetic field  $B$  so that the intensity of the depolarization shifted IST (I) is proportional to  $B^2$  [14] as found experimentally. This excitation mechanism is independent of the underlying bandstructure and has also been demonstrated for the GaAs-system [24]. For detector structures the magnetic field-induced intersubband resonance is a highly accurate method for the determination of intersubband matrix elements [24,25]. Other than in conventional experimental configurations [3] the magnitude of the perpendicular electric field component is well defined in this geometry and thus one source of inaccuracies can be eliminated.

The excitation mechanism of the combined resonances, on the other hand, can be understood only if the bandstructure of our material system is properly taken into account. It has been shown for bulk InSb [26] that the band nonparabolicity in narrow gap semiconductors may induce combined spin-flip and cyclotron resonance excitations. In our case, however, this effect results in an oscillator strength which is too small to explain the observed intensities. Inversion asymmetry induced by external electric fields [9] also leads to the possibility of spin-flip excitations but as our structures are doped symmetrically this mechanism should be very weak. We thus conclude that here the inherent bulk inversion asymmetry is responsible for the spin-flip excitations [27] at high  $k_{||}$ -values.

The dominant inversion asymmetry terms in the matrix Hamiltonian have the schematic form  $G(P_i P_j + P_j P_i)$ , with the material constant  $G$  and the canonical momentum  $P_{i,j}$  [28]. We neglect these terms in the computation of the eigenenergies but we do include them in the Hamiltonian of electron-photon interaction since we are mainly interested in the optical matrix elements. As an approximation we take  $G = 3\hbar^2/2m_0$  for InAs which is the same value as in InSb [28]. To calculate the transmission spectra we assume a level broadening of  $\hbar/\tau = 5$  meV.

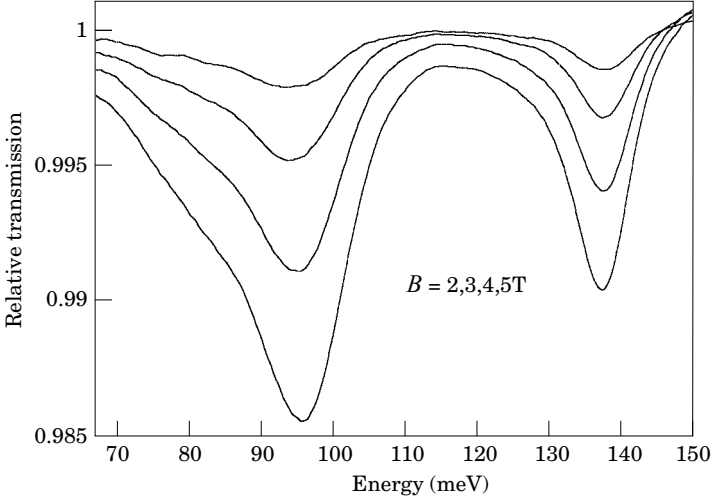


**Fig. 3.** Calculated transmission spectrum for magnetic fields between  $B=2\text{T}$  and  $B=14\text{T}$  with  $\Delta B=2\text{T}$ . Good agreement is reached between our theoretical description and the experimental data. (Reproduced from reference [15] with permission.)

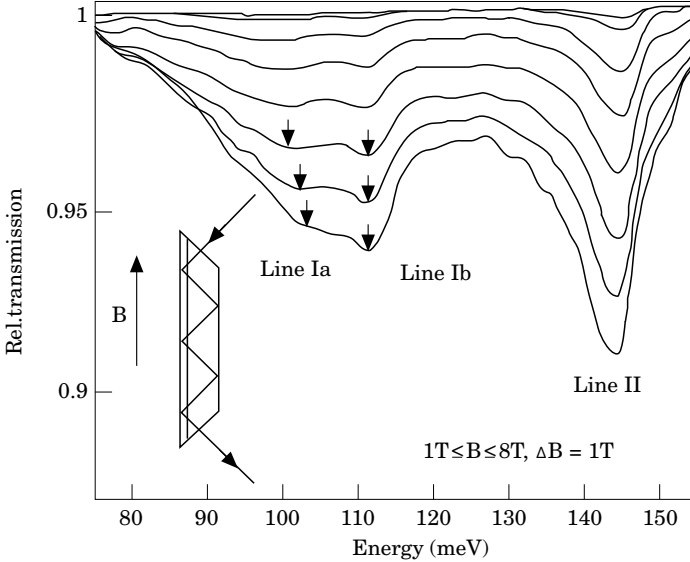
The results of our computation are shown in Fig. 3. Only two resonances can be resolved, although we deal with a total of four transitions. The peak at  $\hbar\omega=135\text{ meV}$  represents the two superposed depolarization shifted spin-conserving transitions with almost equal transition energies and oscillator strengths. Their linewidths are drastically reduced by the depolarization field and not determined by the nonparabolicity broadening. The resonance position remains almost constant as the diamagnetic shift leads to higher transition energies with increasing field which, however, is compensated by the magnetic field dependence of the depolarization integral. The peak at lower energies results from a superposition of both spin-flip transitions which contribute with roughly the same oscillator strength to the absorption. Here, the linewidth is partly determined by the nonparabolicity broadening and thus reflects the subband dispersion. All possible excitations are induced by the same radiation component  $\alpha_x$  whereas the other light component  $\alpha_z$  can be shown to give a negligibly small contribution to the absorption. This theoretical result has been confirmed in an additional experiment with predominantly  $x$ -polarized light. Comparing Fig. 1 and Fig. 3, the oscillator strengths and the energetic positions of our calculated spectra agree very well with the experimental observations. However, resonance I is strongly asymmetric even at low magnetic fields (Fig. 4) whereas our  $\vec{k} \cdot \vec{p}$  calculation predicts a rather symmetric lineshape. If the Zeeman effect were the only source of the splitting between  $|0\uparrow\rangle \rightarrow |1\downarrow\rangle$  and  $|0\downarrow\rangle \rightarrow |1\uparrow\rangle$ —as assumed in the above calculation—then resonance I should become symmetric for  $B \rightarrow 0\text{T}$  which is in clear contrast to the experiment.

In a somewhat different experimental configuration we are able to resolve the splitting of  $|0\uparrow\rangle \rightarrow |1\downarrow\rangle$  and  $|0\downarrow\rangle \rightarrow |1\uparrow\rangle$  [16]. Here, we mounted our sample in the multiple reflection path geometry as sketched in the inset of Fig. 5 [30] with the magnetic field oriented parallel to the 2DEG. The transmission spectra ratioed against  $B=0\text{T}$  are shown in Fig. 5 for  $B \leq 8\text{T}$  with  $\Delta B=1\text{T}$ . Analogous to the Voigt configuration we explain line II as a superposition of the spin-conserving ISTs. In contrast to the previous experiment, however, line I is now clearly split into two resonances Ia and Ib corresponding to the two spin-flip transitions. This splitting increases with decreasing magnetic fields and reaches approximately  $\Delta E=17\text{ meV}$  for  $B \rightarrow 0\text{T}$ . This experimental result is diametrically opposed to our calculations shown in Fig. 2.

The reason for the discrepancy between theory and experiment is the modification of the



**Fig. 4.** The asymmetry of the low energy spin-flip resonance I does not vanish for  $B > 0$ T as expected in the simple 8-band model without the inversion asymmetry terms. Magnetic fields of  $B = 2, 3, 4, 5$ T are shown ratioed against  $B = 0$ T. (Reproduced from reference [16] with permission.)



**Fig. 5.** Transmission spectra of the  $150 \text{ \AA}$  InAs/AlSb multi-quantum well in the multiple reflection path geometry for  $B \leq 8$ T with  $\Delta B = 1$ T. Line II is interpreted as a superposition of the spin-conserving transitions while resonance I is now resolved into the two possible spin-flip excitations denoted Ia and Ib. The inset is a sketch of the experimental configuration. (Reproduced from reference [16] with permission.)

energy levels by the bulk-inversion asymmetry which we have neglected so far. As the inversion asymmetry gives rise to an internal magnetic field for finite values of the wavevector  $k$  the spin degeneracy is lifted even at  $B = 0$ T [31]. This is accounted for by an additional term proportional  $k^3$  in the electronic dispersion with the parameter  $\gamma$  describing the strength of the inversion asymmetry [32]. Hence, the splitting is significant in our sample because of the very high electron density and

the large value of  $k_{\text{Fermi}}$ . The spin-dependent  $k^3$ -term of the electronic energy leads to an energetic separation [33] given by

$$\Delta E = 2\gamma \sqrt{k_x^2(k_y^2 - k_z^2)^2 + k_z^2(k_x^2 - k_y^2)^2} \quad (1)$$

As the penetration of the wavefunction into the barrier is very small in our structure we take  $k_y = \pi/L$  for  $|0\rangle$  and  $k_y = 2\pi/L$  for  $|1\rangle$  [34] as for an infinitely high quantum well. For InAs, the parameter  $\gamma$  has been calculated to be  $130 \text{ eV } \text{\AA}^{-3}$  [35]. Taking this value we find reasonable agreement with the experimental splitting of  $\Delta E = 17 \text{ meV}$  for  $k \approx k_{\text{Fermi}}$  while the calculated spin-splitting decreases rapidly as  $k \rightarrow 0$ . This, however, is to be expected since the oscillator strength of the spin-flip resonance is strongly  $k$ -dependent and only states near  $k \approx k_{\text{Fermi}}$  contribute to the optical matrix elements [15]. We can also calculate  $\gamma$  from the oscillator strength of the spin-flip transitions given by the parameter  $G = 3\hbar^2/2m_0$ . Following Cardona *et al.* [32] we approximate

$$\gamma = \frac{4}{3} G\hbar\kappa \frac{\Delta}{E_g(E_g + \Delta)} \quad (2)$$

and find  $\gamma = 150 \text{ eV } \text{\AA}^{-3}$  which is in good agreement with both the theory and the value deduced from the zero-field spin-splitting.

At finite magnetic fields the total spin-splitting is a combination of the Zeeman effect and the internal magnetic field induced by the bulk inversion asymmetry. For an AlGaAs/GaAs quantum well in the Faraday configuration it has been predicted that the total splitting vanishes at a finite external magnetic field [31]. Our experiment seems to suggest that also a parallel magnetic field can compensate the internal field since the separation of lines Ia and Ib diminishes for high  $B$ . In our case the large value of  $\gamma$  and the high  $k_{\text{Fermi}}$  results in a compensating external field of roughly  $B = 12 \text{ T}$  as lines Ia and Ib cross at this point. However, a detailed description of this effect is rather involved and will be published elsewhere.

## 4. Conclusion

To summarize, we have investigated the magnetic field dependence of intersubband optical transitions in the Voigt geometry. In our InAs/AlSb quantum wells both spin-conserving and spin-flip transitions can be observed. The spin-conserving excitations are induced by an in-plane magnetic field although the incident light is polarized in the plane of the 2D electron gas. Their spin-flip counterparts have a non-zero oscillator strength as a result of the bulk inversion asymmetry of InAs in combination with its narrow band gap and a high carrier density. The simultaneous observation of spin-flip and spin-conserving transitions allows us to measure directly the depolarization shift and to investigate its influence on the lineshape of the resonance. In the multiple reflection path geometry the spin-flip lines can be clearly resolved at low magnetic fields whereas both lines merge at higher fields. From this we conclude that a strong zero-field spin-splitting must exist in our sample lifting the spin-degeneracy at  $B = 0 \text{ T}$ . The magnetic field dependence of the resonance positions further suggests that the effective  $g$ -factor goes through zero at a finite magnetic field.

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