

# Phase Transition and Dissipation by Quasiparticle Tunneling in Arrays of Josephson Junctions

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Arrays of Josephson junctions show different phase transitions depending e.g. on the dimensionality of the network. The dynamics associated with the charging of the superconducting islands introduces quantum mechanical effects. We investigate the influence of the dissipative quasiparticle tunneling currents on the quantum effects and the phase transitions.

Arrays of Josephson junctions are realizations of X-Y models. They exhibit different phase transitions, e.g. in two dimensions the Kosterlitz-Thouless transition. Quantum fluctuations associated with the charging energy inhibit the order and provide the possibility for a fluctuation driven phase transition at zero temperature<sup>1)</sup>. On the other hand, the dissipation due to the flow of normal currents tends to suppress the quantum effects<sup>2)</sup>. Here we show how the dissipation caused by quasiparticle tunneling influences the phase transitions of the arrays<sup>3)</sup>.

We consider a regular d-dimensional arrays of superconducting islands coupled by the Josephson energy  $E_J$  ( $1 - \cos \varphi_{ij}$ ). Here  $\varphi_{ij} = \varphi_i - \varphi_j$  is the difference of the phases on neighboring islands. We assume that the Coulomb energy of charges  $Q_i$  on the islands is dominated by the self-charging energy  $Q_i^2/2C$ . This introduces an energy scale  $E_C = e^2/2C$ . A quantum mechanical description, which includes the dissipation due to the quasi-particle tunneling currents is given by the action<sup>4)</sup>

$$S(\varphi) = \int_0^{\hbar\beta} d\tau \left[ \sum_i \frac{C}{2} \left( \frac{\hbar\dot{\varphi}_i}{2e} \right)^2 + \sum_{\langle ij \rangle} E_J (1 - \cos \varphi_{ij}) \right] + \frac{1}{2} \int_0^{\hbar\beta} d\tau \int_0^{\hbar\beta} d\tau' \alpha_{qp}(\tau - \tau') \left[ 1 - \cos \frac{\varphi_{ij}(\tau) - \varphi_{ij}(\tau')}{2} \right] \quad (1)$$

If the conductance  $1/R_{qp}$  of the junctions at small voltages is finite - we remark that inelastic scattering processes or other pairbreaking mechanisms give rise to such a non-BCS type behavior - the kernel  $\alpha_{qp}(\tau)$  has a Fourier transformed

$$\alpha_{qp}(\omega) = -(\hbar/2e^2 R_{qp}) |\omega| \equiv -(\alpha/\pi) |\omega| \quad (2)$$

The coupling between the islands is short range. If the dynamics can be neglected the 2- or 3-dimensional array undergoes a phase transition at a critical temperature  $kT_c$  of the order of  $E_J$ . From the action (1) it is clear that the dynamics associated with the charging energy and the dissipation introduces short and long range couplings in time direction, respectively. As a result a 1-dimensional array can undergo a fluctuation driven phase transition at zero

temperature<sup>1)</sup>. Moreover, the long range interaction representing the dissipation give rise to a  $T = 0$  phase transition even in a single junction at a critical strength of the dissipation  $\alpha_c \approx 1$ .

We analyze the array described by the action (1) by means of a variational calculation, which extends the one employed for a model with ohmic dissipation<sup>5)</sup>. The Gibbs-Bogoliubov inequality provides a bound  $F^*$  for the free energy  $F$ , namely  $F < F^* = F_{var} + \langle S - S_{var} \rangle_{var}$ . The expectation value and  $F_{var}$  are evaluated with a trial action  $S_{var}$ . We chose  $S_{var}$  to be the quadratic expansion of  $S$  in  $\varphi_{ij}$ , but allow the coefficient of the coupling in space  $(1/2) m \varphi_{ij}^2$  and the strength  $\alpha_{eff}$  of the effective quadratic dissipation to be variational parameters. We determine these parameters

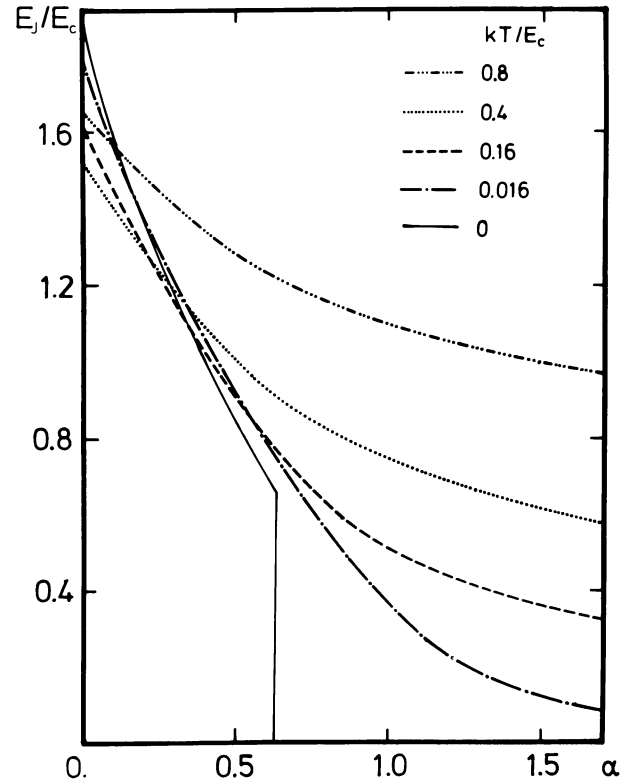


Fig. 1: Phase diagram of a 2-dimensional array.

such as to make  $F^*$  a global minimum for given junction parameters  $\alpha$ ,  $E_J/E_C$ , and  $T$ .

Superconductivity in the array requires phase coherence. We, therefore, consider the correlation function  $\langle \cos \phi_{ij} \rangle$ . If the value of  $m$  which makes the bound  $F^*$  a minimum is nonzero the correlation function calculated with  $S_{\text{var}}(m>0)$  shows one of the following behaviors (depending on  $d$  and  $T$ ): (i) it approaches a constant for large distances, (ii) it decays like a power law, or (iii) it decays exponentially. The correlation function vanishes if  $m = 0$ . We interpret the behavior (i) and (ii) as criterion that the array is in the superconducting state. In contrast in case (iii) or if  $m = 0$ , it is in a disordered state. This means, that the array shows a resistive response although the individual islands are superconducting.

In Fig.1 we show the resulting phase diagram of a 2-dimensional array at different temperatures. For large values of  $E_J/E_C$  or strong dissipation  $\alpha$  the array is in a superconducting state, otherwise it is in a disordered (resistive) state. At  $T = 0$ , for small values of  $E_J/E_C$  the phase diagram depends only on the dissipation  $\alpha$ . At  $T \neq 0$  the transition occurs at roughly the same values of  $\alpha$  but it also depends on the ratio  $E_J/E_C$ .

In Fig.2 we plot  $T_c$  of a 2-dimensional array as a function of  $E_J/E_C$  for different strength of the dissipation. For large values of the capacitance we recover the Kosterlitz-Thouless transition. In fact our variational procedure yields a good value for  $T_c$ . For small values of  $C$  the thermal or quantum fluctuations become stronger and for weak dissipation suppress the transition to an ordered state. In a narrow regime of values of  $E_J/E_C$  and small  $\alpha$  we find a reentrant behavior: upon cooling we enter a superconducting state but at lower  $T$  enter into a resistive state again.

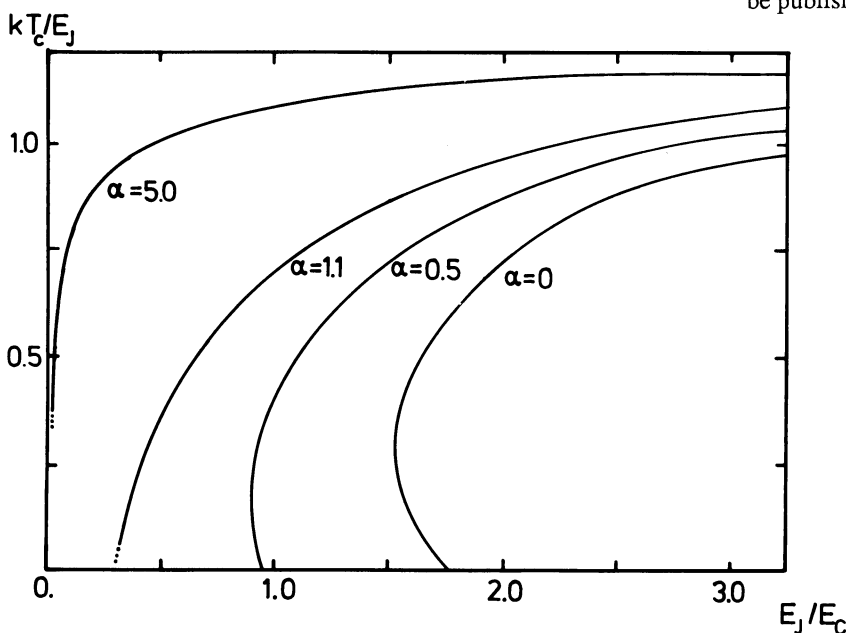


Fig.2: Transition temperature  $T_c$  of a 2-dimensional array.

Although we considered an ideal array our results have implications for the transition between a superconducting and a resistive state observed in experiments on granular films<sup>6)</sup> at a critical strength of the dissipation  $\alpha_c \approx 1$ . We observe a transition at roughly the same  $\alpha$ . However, at  $T \neq 0$  we find that the critical  $\alpha$  is not as universal as suggested by the experiments, but depends on  $E_J/E_C$ . We stress again that the transition between a superconducting and a resistive state of the array occurs at a  $T_c$  which is lower than the superconducting  $T_{BCS}$  of the individual islands. For weak dissipation the transition temperatures are well separated. This is relevant for the interpretation of the transition temperature of granular materials including the new high  $T_c$  materials. Regular arrays of Josephson junctions have been fabricated and the Kosterlitz-Thouless transition has been observed<sup>7)</sup>. So far the parameters allowed us to ignore the charging effects and the dissipation. However, it is possible to fabricate junction arrays with parameters where the quantum fluctuations and the suppression of  $T_c$  can be observed.

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