

NON-LINEAR CURRENT THROUGH A BARRIER IN 1D WIRES WITH FINITE-RANGE INTERACTIONS

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Since the discovery of the vanishing transmittivity of a tunnel barrier in a one-dimensional (1D) wire due to the repulsive electron–electron interaction [1] new interest has emerged in the transport properties of 1D electron systems [2–10]. Indeed, the influence of the electron correlations shows up strikingly in non-linear current voltage relations which are investigated experimentally in narrow, semiconducting wires [11]. Local interactions, $v(x - x') = v_0 \delta(x - x')$ are described within the Luttinger model for which the power-law [1–5]

$$I(V) = \frac{\omega_c}{eR_T} \left(\frac{eV}{\omega_c} \right)^{2/g-1} \quad (1)$$

has been predicted for the current–voltage relation through a tunnel barrier with tunnel resistance R_T , at zero temperature $T = 0$. A similar behaviour has been found for the linear conductance $\sim T^{2/g-2}$ as a function of temperature [1, 2, 5]. The exponent depends on the strength v_0 of the interaction, $g \equiv (1 + v_0/\pi v_F)^{-1/2}$ (v_F : Fermi velocity). Repulsion corresponds to $g < 1$.

The Luttinger model limits the energies to values well below the upper cut-off ω_c which can be identi-

fied with the Fermi energy and can be small in semiconducting devices at low electron densities. At larger voltages, equation (1) formally describes currents that exceed even the value V/R_T for non-interacting electrons. This indicates that some new energy or length scale must become important at higher energies. Here we shall identify the finite range of the $e-e$ -interaction to cause an asymptotic approach of the current towards V/R_T at large voltages. In the case of very strong interaction the cross-over can occur in an oscillatory manner while for more realistic interaction strengths the current stays below V/R_T for all voltages. The low energy, and hence long wave length properties, are well described within the Luttinger model where the range of the interaction $1/\alpha$ is assumed to be shorter than even the inter-electron spacing $a \equiv \pi v_F/2\omega_c$ for spinless electrons. Finite voltages, however, introduce a wave length $1/\Delta k = v_F/eV$ on which eventually a finite value for $1/\alpha$ can be experienced. Large voltages change the momenta at the Fermi points by $|\Delta k|$ that exceeds the scale α on which the Fourier components of the interaction vanish, $\hat{v}(\Delta k > \alpha) \approx 0$ so that $V > \alpha v_F/e$ suppress the effect of interactions.

Accordingly, at high temperatures $T > \alpha v_F$, $I(V)$ becomes independent of T so that the differential conductance approaches the constant $1/R_T$, like in the non-interacting case (we do not consider the effect of lattice vibrations here [12]). Finite temperatures may

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even reduce the current. Both the voltage and the temperature dependencies of the current show the important common feature of a crossover which in principle allows to extract the range $1/\alpha$ of the interaction.

The importance of finite range interactions has been found already in zero dimensional systems, quantum dots, where 'crystallisation' of the charge density distribution can occur leading to qualitative changes in the low energy excitation spectra [13–15] as compared to what is expected for a contact interaction [16].

For convenience we concentrate on the interaction [17]

$$v(x-x') = v_0 \frac{\alpha}{2} e^{-\alpha|x-x'|}. \quad (2)$$

In the presence of metallic gates close to the 1D channel a more realistic form would vary $\sim |x-x'|^{-3}$ at large distances, however, we expect qualitatively the same results for the latter finite range interaction as for (2), cf. below. The non-Fermi liquid behaviour of the charge excitations in a 1D wire is expressed most conveniently by the Hamiltonian [9, 18–20]

$$H_w = \frac{v_F}{2} \int dx [(\Pi(x))^2 + (\partial_x \theta(x))^2] + \quad (3)$$

$$\frac{1}{2\pi} \int dx dx' (\partial_x \theta(x)) v(x-x') (\partial_{x'} \theta(x'))$$

where the Fermi-fields are expressed [1, 18] through Bose fields, $\Pi(x) \equiv \partial_x \phi(x)$ and $\theta(x)$, with $[\phi(x), \theta(x')] = -(i/2) \text{sgn}(x-x')$. The spatial derivative, $\partial_x \theta$, measures the fluctuations of the charge density, and the time derivative, $\partial_t \theta$, is proportional to the current.

Here, we account for the dispersion relation of the charged modes in the wire, as it can be obtained from (3) by spatial Fourier transform

$$\omega(k) = v_F |k| \sqrt{1 + \frac{\hat{v}(k)}{\pi v_F}}. \quad (4)$$

The Fourier transform of the interaction potential (2)

$$\hat{v}(k) = v_0 \frac{\alpha^2}{k^2 + \alpha^2} \quad (5)$$

is constant $\hat{v}(k) = v_0$ in the limit $\alpha \rightarrow \infty$ of the Luttinger liquid used in previous calculations where it merely renormalizes the sound velocity $\rightarrow v_F/g$.

The tunnel barrier can be described [1] by

$$H_b = U_b [1 - \cos(2\sqrt{\pi}\theta(x=0))]. \quad (6)$$

Furthermore, we assume an electrostatic potential $(V/2) \text{sgn}(x)$ dropping discontinuously at the location $x=0$ of the tunnel barrier,

$$H_V = \frac{eV}{\sqrt{\pi}} \theta(x=0),$$

as in [1–3, 7, 8, 10]. In the limit of weak tunnelling it has been demonstrated [9] that the selfconsistently adjusted chemical potential indeed varies most pronouncedly close to $x=0$.

The DC-current

$$I = -\frac{e}{\sqrt{\pi}} \langle \partial_t q(t) \rangle \quad (7)$$

can be expressed in terms of the field θ at $x=0$,

$$q(t) \equiv \theta(t, x=0)$$

where both, the quantum average and the dynamics refer to the full Hamiltonian $H_w + H_b + H_V$.

Since H_w is purely quadratic in $\theta(x)$ all of the contributions away from the impurity $x \neq 0$ can be integrated out to obtain the reduced dynamics for $q(t)$. We are interested in the probability for transitions of q from a value θ_i to θ_f during the time t which can be expressed as a double integral

$$\int_{\theta_i}^{\theta_f} \mathcal{D}q \int_{\theta_f}^{\theta_i} \mathcal{D}q' \exp(iS[q]) \exp(-iS[q']) \mathcal{F}[q, q'] \quad (8)$$

over paths q and q' with endpoints $q(0) = q'(0) = \theta_i$ and $q(t) = q'(t) = \theta_f$. The action $S[q]$ contains all contributions to the Hamiltonian at $x=0$ while the influence of the bulk modes, $x \neq 0$, is exactly accounted for in the functional [21]

$$\mathcal{F}[q, q'] = \exp - \int_0^t dt' \int_0^t dt'' (\dot{q}(t') - \dot{q}'(t')) \times \quad (9)$$

$$(w(t' - t'') \dot{q}(t'') - w^*(t' - t'') \dot{q}'(t''))$$

where

$$w(t) = \int_0^\infty d\omega \frac{J(\omega)}{\omega^2} \left[(1 - \cos \omega t) \coth \frac{\beta\omega}{2} + i \sin \omega t \right], \quad (10)$$

and $1/\beta$ is the temperature. It will be important for the following that the non-linear dispersion (4) of the bulk modes, shown in the inset of Fig. 1, cause a non-Ohmic dissipative influence. The function $J(\omega)$ includes all of the details of the environmental modes in their efficiency to damp the frequency ω of the motion of $\theta(x=0, t)$.

Within the Feynman–Vernon technique [21] the quantum state for q is assumed to be initially known

(e.g. $\theta_i = q(0) = 0$) before the exact time evolution is switched on. With time t , q acquires probability $P_m(t)$ to assume the value $\theta_f = m\sqrt{\pi}$ (cf. (8)) where m elementary charges have been transferred through the barrier. The long time behaviour of the probability density distribution defines the stationary DC-current

$$I = -e \sum_m m \lim_{t \rightarrow \infty} \partial_t P_m(t),$$

according to (7), assuming ergodicity for the whole system.

For large U_b the potential (6) has deep minima at $\theta = m\sqrt{\pi}$ so that integer m contribute mainly to the saddle points of the action $S[q]$ in (8). In this limit the charge is transferred in integer units. Step like instantons dominate the path integral (8) [22, 23] for the low current properties, each instanton contributing with a factor $\pm i\Delta/2$ proportional to the tunnelling amplitude*.

The influence functional $\mathcal{F}[q, q']$ (9) introduces a temperature dependent coupling $w(t_i - t_j)$ between instantons centred at times t_i and t_j so that the sum over all possible instanton configurations in general cannot be performed analytically. For a barrier of low transmittivity the most important configurations are instanton - anti-instanton pairs that contribute in order Δ^2 . This leads to an expression for the current [3]

$$I(V) = e \frac{\Delta^2}{4} (1 - e^{-\beta eV}) \int dt e^{ieVt} e^{-w(t)}, \quad (11)$$

when the detailed balance property $\partial_t P_{-1} = e^{\beta eV} \partial_t P_{+1}$ is used [24].

For the case of Ohmic dissipation, $J(\omega) \propto \omega$, corresponding to a contact interaction, considerable progress has been made. To order Δ^2 the current has been calculated in [3] and to any order for weak interaction $1 - g \ll 1$ in [25]. Recently, the extension to arbitrary interaction strength has been achieved using conformal field theory techniques [7] and by systematically exploiting the duality symmetry between low and high transmittivities [10].

How the electron-electron interaction influences the transport properties is determined by $J(\omega)$ (cf. (10,11)). Its relationship to the bulk mode dispersion $\omega(k)$ can most easily be deduced from the partition function of the wire (3)

$$\begin{aligned} Z &= \text{Tr} e^{-\beta H_w} = \int \mathcal{D}[\theta(x, \tau)] e^{-S_w[\theta]} \\ &= \int \mathcal{D}[\hat{\theta}(k, \tau)] e^{-S_w[\hat{\theta}]} \end{aligned} \quad (12)$$

* The value of Δ can be related to U_b , cf. [10]. Through the one instanton action, Δ depends in principle also on α .

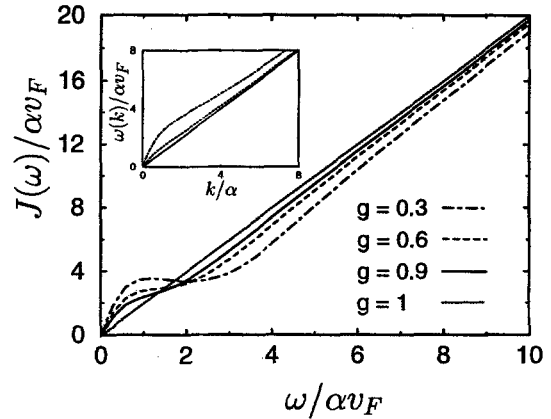


Fig. 1. Effective density $J(\omega)$ of charged modes in the wire that damp the motion of $\theta(x=0, t)$ at the frequency ω , according to equation (17) for different g . $J(\omega)$ is the crucial ingredient for the non-linear current. The inset shows the dispersion relation $\omega(k)$ according to equation (4). Natural units for wave vectors and frequencies are α and αv_F , respectively.

where

$$\begin{aligned} S_w[\hat{\theta}(k, \tau)] &= \frac{1}{2v_F} \int \frac{dk}{2\pi} \\ &\times \int_0^\beta d\tau \hat{\theta}(-k, \tau) (-\partial_\tau^2 + \omega^2(k)) \hat{\theta}(k, \tau) \end{aligned}$$

with $\hat{\theta}(k, \tau) = \int dx \theta(x, \tau) e^{ikx}$.

The modes $\theta(x \neq 0, \tau)$ act as a harmonic thermal environment on the mode of interest, $q(\tau) \equiv \theta(x=0, \tau)$,

$$\begin{aligned} Z &= \int \mathcal{D}[q(\tau)] \int \mathcal{D}[\theta(x \neq 0, \tau)] e^{-S_w[\theta]} \\ &\equiv \int \mathcal{D}[q] e[q]. \end{aligned} \quad (13)$$

The functional

$$e[q] \propto \exp -\frac{1}{2} \int_0^\beta d\tau \int_0^\beta d\tau' q(\tau) \hat{\mathcal{K}}(\tau - \tau') q(\tau') \quad (14)$$

for the reduced density contains the retarding effects, described by the Kernel [26]

$$\begin{aligned} \mathcal{K}(\omega_n) &= \int_0^\beta d\tau \hat{\mathcal{K}}(\tau) e^{-i\omega_n \tau} \\ &= \left[\int \frac{dk}{2\pi} \frac{v_F}{\omega_n^2 + \omega^2(k)} \right]^{-1} \end{aligned} \quad (15)$$

with $\omega_n = 2\pi n/\beta$. Analytic continuation,

$$J(\omega) = -\Im \lim_{\delta \rightarrow 0} \mathcal{K}(-i\omega + \delta) \quad (16)$$

relates $J(\omega)$ directly to $\mathcal{K}(\omega_n)$ [24].

The asymptotic behaviours $J(\omega \rightarrow 0) \approx 2|\omega|/g$ and $J(\omega \rightarrow \infty) \sim 2\omega$ can readily be deduced from (15) in view of $\omega^2(|k| \ll \alpha) \approx v_F^2 k^2/g^2$ and $\omega^2(|k| \gg \alpha) \approx v_F^2 k^2$. Here, $g \equiv (1 + v_0/\pi v_F)^{-1/2}$ has been defined in analogy to the Luttinger model with $v_0 = \hat{v}(k=0)$ (cf. (5)). For $\omega(k)$ as in (4) the integration (15) with (16) can be carried out analytically, yielding

$$\frac{J(\tilde{\omega})}{\alpha v_F} = 2\sqrt{2}\tilde{\omega} \frac{(N_+(\tilde{\omega}) + \tilde{\omega}N_-(\tilde{\omega}))M(\tilde{\omega})}{(1 + \tilde{\omega}^2)(|N_+(\tilde{\omega})|^2 + |N_-(\tilde{\omega})|^2)} \quad (17)$$

where

$$M(\tilde{\omega}) = \sqrt{4\tilde{\omega}^2 + (\tilde{\omega}^2 - 1/g^2)^2}$$

$$N_+(\tilde{\omega}) = \left| -\tilde{\omega}^2 + 1/g^2 + M(\tilde{\omega}) \right|^{1/2}$$

$$N_-(\tilde{\omega}) = \left| -\tilde{\omega}^2 + 1/g^2 - M(\tilde{\omega}) \right|^{1/2}$$

The time $(\alpha v_F)^{-1}$ for electrons of velocity v_F needed to traverse the interaction range establishes the natural frequency scale of the problem, $\tilde{\omega} = \omega/\alpha v_F$.

Figure 1 illustrates the result (17). At small $\omega \ll \alpha v_F$, $J(\omega) \sim 2\omega/g$ and the current-voltage relation (1) is recovered at long wave lengths and low energies.

With large $\omega \gg \alpha v_F$, $J(\omega)$ approaches the linear behaviour, $J(\omega) \rightarrow 2\omega$, that corresponds to the non-interacting case, $g = 1$, for the reasons motivated initially. Note, however that $J(\omega)$ does not simply interpolate between either of the Ohmic asymptotics but crosses the value 2ω so that the damping $J(\omega \gtrsim \alpha v_F) < 2\omega$ is *smaller* than it would be in the absence of interactions.

The current-voltage relation, obtained according to (11) for zero temperature, is depicted in Fig. 2. The crossover behaviour of $J(\omega)$ shows up in a transition from the power law at low voltages, $I(V) \sim V^{2/g-1}$ to the linear tunnel resistance behaviour, $I(V) \rightarrow V/R_T$. It takes place on the voltage scale $\alpha v_F(2/g - 1)/e$. The high voltage limit does *not* show any offset that would correspond to a Coulomb blockade since the charging energy

$$\begin{aligned} E_c/e &= \lim_{V \rightarrow \infty} (V/R_T - I(V)) \\ &\propto \int_0^\infty d\omega \left(\frac{J(\omega)}{\omega} - 2 \right) = 0 \end{aligned} \quad (18)$$

vanishes. The proportionality in (18) follows from the short time behaviour of $w(t)$ to the order $\sim t^2$ (cf. 10)

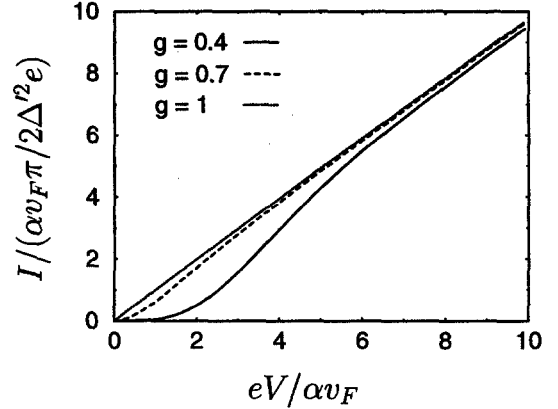


Fig. 2. Current through the tunnel barrier versus applied voltage for different g , $\Delta' \equiv \Delta/\omega_c$. The power-law at low voltages agrees with the Luttinger liquid behaviour. The crossover to the linear relation, $I(V) = V/R_T$, manifests the finite range of the $e-e$ -interaction.

and the right hand side expression vanishes since, for any dispersion (4), $\int d\omega J(\omega)/2\omega$ equals the number of modes in the wire, cf. (15,16). Two conditions are usually considered as being sufficient to establish a Coulomb blockade [27, 28]: the suppression of quantum fluctuations of the charges by low transmittivities and the presence of a nearby dissipative environment of high impedance for which the bulk modes serve [3]. Although both conditions are fulfilled in the present system no charging effects appear. The vanishing lateral extension of a 1D wire does not suffice to accumulate charging energy. Near a single barrier, a finite cross section is required for the capacitance C to be finite so that $I(V) \sim (V - E_c/e)/R_T$ at high voltages, with $E_c = e^2/2C$ [29]. This is consistent with the result for a selfconsistent determination of the charge distribution along the wire [9].

Equation (18) holds for any interaction potential of finite range. Another example is the screened Coulomb interaction $v(x - x') = e^2 e^{-\alpha|x-x'|} / \kappa \sqrt{(x-x')^2 + d^2}$ (κ : dielectric constant, d : width of the wire) which is again determined by two parameters, α and $\alpha v_0/2 \rightarrow e^2/\kappa d$. At low voltages $I(V)$ obeys a power law and at $V > 2\alpha v_F(\sqrt{1 + 2e^2 K_0(\alpha d)/\kappa \pi v_F} - 1)/e$ (K_0 : modified Bessel function) a crossover to the effectively non-interacting behaviour occurs. Only true long-range interaction $\alpha \rightarrow 0$ changes the power-law behaviour at low voltages [6, 19, 30] and the divergence of $\hat{v}(k \rightarrow 0)$ suppresses the crossover.

In principle, we can also infer the current-voltage characteristics for the case of a weak barrier and attractive, finite-range interactions by taking advantage of the exact duality relation [31] between the weak and

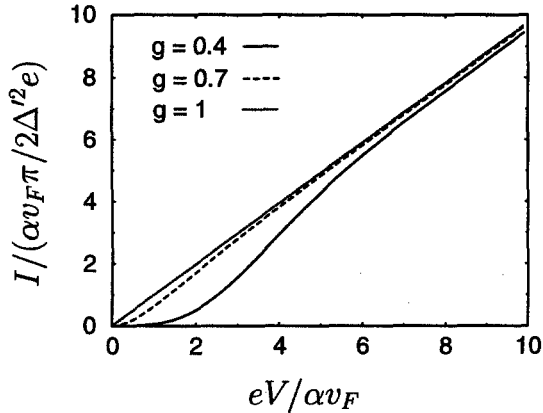


Fig. 3. Temperature dependence of the differential conductance for different voltages $eV/\alpha v_F = 0.1, \dots, 5$ as indicated. In a and b $g = 0.7$ and $g = 0.4$, respectively. At low temperatures and low voltages the power law behaviours [1] are recovered.

the strong barrier limit as it has been proven [32] for arbitrary $J(\omega)$. In our case, the dual $J(\omega)$ interpolates between $J(\omega \ll \alpha v_F) = 2g\omega$ and $J(\omega \gg \alpha v_F) = 2\omega$, since g maps to $1/g$, so that the interaction strength again vanishes at high energies. Correspondingly, to second order in the barrier height, the current exhibits a crossover from the Luttinger liquid behaviour $I(V) \sim (1 - c(U_b^2)V^{2g-2})V$ at small voltages to the Ohmic behaviour, $I(V) \sim (1 - c(U_b^2))V$, at large voltages.

Also the temperature dependence of the differential conductance $\partial I/\partial V$, Fig. 3, reveals a crossover around $T \approx \alpha v_F(2/g - 2)$ as can be deduced from a high temperature expansion up to the cubic term in (10) and (11). At small T the linear conductance varies $\sim T^{2/g-2}$, in agreement with [1], while the non-linear conductance can even decrease with temperature. At high temperatures $\partial I/\partial V$ approaches the constant value $1/R_T$, irrespective of the voltage, as has been demonstrated already for weak interaction [5].

To summarise, we have shown that finite ranges of the $e-e$ -interaction change the non-linear current through a tunnelling barrier in a 1D wire qualitatively, compared to the simple power-law behaviour. The latter is usually considered to be the main characteristic for one-dimensionality, but is only valid for short range interactions. At high voltages the influence of the interaction disappears and no Coulomb blockade remains. Similarly, the differential conductance becomes independent of high temperatures and assumes the value $1/R_T$ for all voltages.

Careful determination of the current voltage characteristics and also of its temperature dependence would allow to measure directly the range of the electron-electron interaction. This quantity is difficult

to access by other experimental means.

Acknowledgements—We would like to thank H. Grabert, B. Kramer, M. Sassetti, U. Weiss, and particularly R. Egger for many valuable discussions and the I.S.I. foundation for the fruitful atmosphere that stimulated the present work under contract ERBCHRX-CT920020. MS acknowledges support by the Gottlieb-Daimler- and Karl-Benz-Stiftung.

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